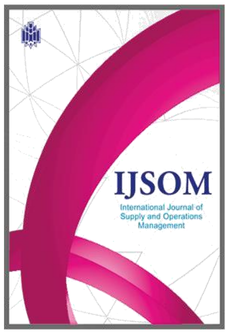
International Journal of Supply and Operations Management

IJSOM

February 2019, Volume 6, Issue 1, pp. 30-50
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com



Integrated production and distribution in a milk supply chain under uncertainty with the Hurwicz criterion

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Abstract
In this paper, we propose a credibility-based fuzzy mathematical programming model for integrating the production and distribution in a milk supply chain under uncertainty. The proposed model is a mixed integer linear programming, which takes into account technological constraints and aims to maximize the total profit including the total costs such as production, storage, and distribution. To bring the model closer to real-world planning problems, the objective function coefficients (e.g. production cost, inventory holding and transport costs) and other parameters (e.g. demand, production capacity, and safety stock level) are all considered fuzzy numbers. In the uncertain environment, the most known criteria widely employed are optimistic and pessimistic value criterions. Both criteria present some deficiency. For the optimistic criterion, it suggests an audacious decision maker who is attracted by high payoffs (low cost), while for the pessimistic criterion, it suggests a conservative decision-maker who tries to make sure that in the case of an unfavorable outcome (loss), there is at least (in most) a known minimum payoff (maximum loss). The Hurwicz criterion is used to overcome these problems. By varying the value of θ, it can balance the optimistic and pessimistic levels of the decision makers. Moreover, the different property of the credibility measure is used to build the crisp equivalent model, which is a MILP model that can solve problems by using a commercial solver such as GAMS. Finally, numerical results are reported for a real case study to demonstrate the efficiency and applicability of the proposed model.

Keywords: Milk Supply Chain, Production-Distribution, Credibility Theory, Hurwicz Criterion.

1. Introduction

The dairy industry is an important part of the food sector in Morocco. It contributes to the development of the economy by an agricultural GDP of 30% and a coverage rate of approximately 90% of the national demand for milk and dairy products. However, this sector is characterized by unique characteristics that differentiate it from other industries, for example, the high number of product variants, the divergent structure of products, shared resources (often identical machines), traceability and hygiene requirements, and respect for the shelf life of raw materials, finished products. These factors directly influence the rate of return and stock levels (Amorim et al., 2011, Entrup, 2006, Nakhla, 1995). As a result, dairy production planning must meet a number of constraints like the quantity of white mass to be produced in each tank and the synchronization of the production steps. In addition, other technical constraints affect sequencing and assignment decisions such as cleaning requirements and traceability. It should also be noted that the length of the manufacturing period and the short shelf life of the finished products and finally the transport conditions accentuate the challenges associated with the variability of demand. All these factors make planning and scheduling of the dairy supply chain a difficult task (Kılıç 2011). Faced with this situation, the enterprises investigate the possibility to turn these challenges to a competitive advantage while continuously improving their operation. Over the past two decades, supply chain management has proven to be an unavoidable solution to meet these contradictory constraints. As a result, several researchers and practitioners focus on integrating the various supply chain functions to increase flexibility, improve cycle times, and reduce costs.

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In supply chain management, both functions, production and distribution operations, can be decoupled if there is a sufficient buffer between them. Related to the dairy supply chain, a poor integration within a network may be excessive inventories at the manufacturers' warehouses. Regarding the perishability characteristic, the finished products should be kept refrigerated at a controlled temperature that results in a high cost of storage that may influence the efficiencies.

A considerable number of academic studies have been done to examine the importance of production-distribution problem in the dairy supply chain. In the mentioned models all parameters were considered deterministic. However, according to Klibi et al., (2010), any supply chain planning that relies on deterministic conditions risks losing its durability. They also mention that in some cases, it is not enough for the company to consider usual parameters such as demand, prices or other parameters such as random variables, but undesirable events such as terrorist attacks and natural disasters. Therefore, it is necessary to have a specific strategy that integrates uncertainty into supply chain planning.

Our research is in line with this trend, with the aim of studying the problems of integrating production and distribution into the supply chain planning of the dairy industry in Morocco. Despite the importance of milk production in the supply chain of dairy products, little research has been conducted into the subject. Inspired by some research on other dairy products such as yogurt and ice cream, this work develops a decision-making framework to deal with the inherent characteristics of milk supply chain under uncertainty. Our main contributions can be summarized as follows:

(i) Proposing a MILP formulation for integrating the production and distribution problem with the milk supply chain;
(ii) Taking into account different sources of uncertainty such as costs, demand, production capacity, and safety stock level;
(iii) Utilizing the credibility theory as an appropriate framework to deal with the uncertainty;
(iv) Overcoming the extreme cases of the optimistic and pessimistic criteria by using the Hurwicz criterion which attempts to strike a balance between those extreme criteria;
(v) Testing the proposed model against a real-world production scheduling and distribution-planning problem in a leading diary company in Morocco.

The rest of the paper is organized as follows. In Section 2 a short review of the relevant literature is given. Section 3 describes a brief presentation of the credibility theory, the Hurwicz criterion. Section 4 gives the structure of supply chain network, a detailed description of the problem, and the mathematical formulation. Section 5 describes the crisp equivalent model. Section 6 presents the case study in order to illustrate the highlight characteristics of the proposed model and methodology. Finally in Section 7 some conclusions and future research directions are provided.

2. Review of the literature

A considerable number of studies have examined the importance of the production-distribution problem in supply chain management. A comprehensive review of integrating production-distribution problems has been presented by Erengüç et al. (2015).

Elbounjimi et al. (2015) propose a model to design a new integrated multi-period and multi-product closed-loop supply chain network considering reused costs and capacity constraints for all stages. The considered objective function is total cost factors that consist of seven components: costs of locating the plants and retailers, purchasing, transportation, collection of used products from customers, disposal for subassemblies, refurbishing, and finally refund to customers. First, parameters and decision variables of this problem are defined, and then a mixed integer linear programming mathematical model is presented. The proposed mathematical model is run on the GAMS software. Two real examples (shed light and power-outlet) are considered to be solved using the proposed mathematical model.

Derakhshan et al. (2017**)** address the problem of designing a multi-echelon, multi-product and capacitated closed-loop supply chain network. First, the problem is modeled with a mixed-integer, non-linear programming model that aims to maximize the total closed-loop supply chain profit. To reduce the complexity of the model, it was linearized and solved by LINGO.

Najafi et al. (2017) address the single-objective optimization of multi-product for a three-echelon supply chain. A mathematical model is proposed by considering the associated constraints, production, capacity and shipment costs. The objective is to minimize the total costs along the supply chain network. To solve the proposed model, a genetic algorithm (GA) and a Simulated Annealing (SA) are proposed.

In the dairy supply chain, market demand is no longer limited to local or regional supply. The highly competitive market, product variety, and short shelf life require companies to coordinate more closely between the function of production and the function of distribution for more flexible use of resources and faster response to demands while reducing costs and increasing productivity. Above all, there are strict shelf-life restrictions for dairy products, and the customer prefers to have a product with a maximum shelf life. To avoid excessive inventory and to allow a quick response to customer requests are important aspects that require more attention. Therefore, an effective integration of production and distribution plans into a unified framework is essential to obtaining a competitive advantage. In the remainder of this section, the literature review of the integration of the production function and the distribution function into the dairy supply chain is presented according to the methodologies of solutions, especially the mixed integer linear programming.

The literature deals with the planning of the production function and the distribution function from a strategic, tactical and operational point of view. Most studies use the Mixed-Integer Linear Programming - MILP to formulate problems of planning production and distribution in the dairy supply chain.

Wouda et al., (2002) propose an MILP to optimize a dairy supply chain network. The aim of the work is to find the optimal number of production sites, their locations, and production capacities by product family in order to minimize the cost of production and distribution of the whole supply chain network.

Studying a dairy supply chain model, Subbaiah et al. (2009**)** primarily focus on production and distribution activities. The proposed model consists of four echelons, suppliers of raw milk, production sites, distribution centers and customers. It integrates the raw milk purchase plan, the production plan, and the transport plan of the finished products. They propose a linear programming model (Linear programming - LP) with a single objective function integrating the various costs of the supply chain, which includes plant, production and transport costs.

Kopanos et al (2012) develop a novel MILP framework for the dairy industry based on a representation combining discrete time/continuous time for the simultaneous detailed production and distribution-planning problem of a multi-site company that produces multiple products. The proposed mathematical formulation by the authors integrates the different modeling approaches as well as the detailed examination of the operations of production and distribution.

Amorim et al (2012) propose multi-objective mixed integer programming (MIP) models using the block planning approach for solving the integration problem of production and distribution by considering the total costs and freshness at an operational level. The proposed models considered two separate case studies with a fixed and a loose shelf life of rapidly deteriorating goods. The first objective function aims to minimize the total costs of the supply chain covering transport production set-up and deterioration costs, the second maximizes the average shelf life of products in distribution centers over the planning horizon. To solve the models, the researchers propose a genetic algorithm where the shelf life is loose.

Van Elzakker et al (2013, 2014) discuss modeling of the tactical level of production planning and distribution of fast moving consumer goods that present challenges to the optimization of the quantity produced storage times and waste. Van Elzakker et al (2013) propose a decomposition algorithm, which decomposes the problems related to complex cases into sub-models. This decomposition allows the optimization of stocks in a sequential manner and integrates a penalty cost for violating capacity. After each optimization, the penalty cost is increased until it is high enough to get a feasible solution. Van Elzakker et al (2014) present computationally efficient methods namely, direct, indirect and hybrid to accurately track shelf life. The direct method allows for following the shelf life of each product. This method guarantees to have an optimum but shows deficiencies in computational complexity. The indirect method assumes that products must leave the inventory at the end of their shelf life. This method does not guarantee the optimal value but it obtains results close to the optimum. The hybrid method uses the advantages of both approaches, that is, the shelf life of the product is treated directly in the first step while considering the shelf life indirectly in the second step, it provides near-optimal solutions with reasonable computational times.

Bilgen and Çelebi (2013) propose a hybrid framework combining a mixed integer-programming model and simulation to solve an integrated production scheduling and distribution-planning problem in yoghurt production. The model gives an optimal production plan and distribution plan. A hybrid approach is used to explore the dynamic behavior of the system in a real way. Operation times are considered as dynamic variables and adjusted by optimization and simulation in an iterative manner.

Touil et al., (2016) provide a bi-objective possibilistic mixed integer-programming model to deal with the production-distribution problem in a dairy company in Morocco. The model seeks to integrate two conflicting simultaneous objectives: maximizing benefit as well as maximizing the service level subject to several technological constraints that typically arise in the dairy industry. Because of the highly dynamic and uncertain characteristics related to the aspiration level of objectives, coefficient parameters of objectives and some data, such as process production, transport parameters, and customer demand, an interactive approach is proposed to find an efficient compromise solution.

Table 1. Classifies the studies and shows the relation of the proposed model to the related literature.

Reference	Decision level	Type de Products	SC Process				Objective	Perishability	Transportation	Uncertainty	Solution Approach
Wouda et al., (2002)	T	D	PRC	PRO	ST	DS	S	×	×	×	MILP
Subbaiah et al., (2009)	T	D	PRC	PRO	ST	DS	S	×	×	×	LP
Kopanos et al (2012)	O/T	Y	×	PRO	ST	DS	S	×	×	×	MILP
Amorim et al (2012)	T	D	PRC	PRO	ST	DS	S	×	×	×	MILP
Van Elzakker et al (2013)	T	OT	PRC	PRO	×	DS	S	×	×	×	MILP
Van Elzakker et al (2014)	T	OT	PRC	PRO	×	DS	S	P	×	×	MILP
Bilgen and Çelebi (2013)	O	Y	×	PRO	×	DS	S	P	×	×	MILP-SM
Touil et al (2016)	O	M	×	PRO	×	DS	S	P	×	Pos	MILP
This Research	O/T	M	×	PRO	ST	DS	M	×	MV	Cr	MILP

- Decision level: O: Operational ; T: Tactical
- Type of products: D: Other dairy products; Y: Yogurt; M: Milk
- Supply Process: PRC: Procurement; PRO: Production; ST: Storage; DS: Distribution
- Number of Objective: S: Single; M: Multiple;
- P: Perishability
- Transportation : MV : multiple-vehicles
- Uncertainty: Cr: Credibility theory, Pos: Possibility theory
- Solution Approach: MILP: Mixed-Integer-Linear-Programming; SM: Simulation; LP: Linear Programming

By analyzing the table, we see that optimization problems in the dairy industry have received considerable attention in recent years. But this research differs from the previous studies in several ways. First, those studies did not consider milk as products. Second, none of them took into consideration the transportation vehicles. Third, no strategy has been specified to deal with the sources of uncertainties, which may relate to target values or coefficients of objectives; or other data such as customer demand, transport costs and transport lead-time. These gaps motivate and guide our study.

3. Credibility theory

Fuzzy Set theory (FST) was introduced by Zadeh in 1965 and has been well employed to solve a variety of practical problems since then. Fuzzy variable is a type of mathematical tool to describe fuzzy uncertainty. In the FST, there are three types of measures for a fuzzy event namely the possibility, the necessity, and the credibility measure. Possibility theory was considered as a mathematical counterpart of probability theory by Dubois (2004) and widely used to treat the fuzzy variables. Further, it is inconsistent with the law of excluded middle and the law of contradiction. For example, a fuzzy event may fail even though its possibility value is 1 and hold even though its necessity value is 0. This is mainly because the possibility measure does not satisfy self-duality property, which is absolutely needed in both theory and practice. In order to overcome this difficulty, Liu and Liu (2002) proposed a self-dual measure, namely, credibility measure. The credibility measure is a more reasonable fuzzy inequality indicator than possibility and necessity because it compensates for their disadvantages. For example, a fuzzy event with maximum possibility 1 sometimes carries no information while a fuzzy event with maximum credibility 1 means that the event will happen at the greatest chance (Huang 2006). Furthermore, credibility theory, developed by Liu (2004), is a branch of mathematics for studying the behavior of fuzzy phenomena.

In this subsection, we introduce some basic concepts, which will be helpful in establishing the integration of production-distribution problem in the milk supply chain under the Hurwicz criterion.

Definition 1 (Liu 2004). Let Θ be a nonempty set, $\mathcal{P}$ be the power set of Θ, and Cr a credibility measure. Then, the triplet $(\Theta, \mathcal{P}, \mathrm{Cr})$ is called a credibility space.

Definition 2 (Liu 2004). A fuzzy variable is a function from a credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$ to the set of real numbers.

With the concept of fuzzy variable, we can define the membership function of a fuzzy variable.

Definition 3 (Liu 2004). Let ξ be a fuzzy variable defined on the credibility space $(\Theta, \mathcal{P}, \mathrm{Cr})$. Then its membership function is derived from the credibility measure by:

$$\mu(\xi) = (2\mathrm{Cr}\{\xi = \mathrm{r}\}) \wedge 1; \quad \mathrm{r} \in \mathbb{R} \tag{1}$$

Where $\wedge$ is the minimum operator, i.e., for $\mathrm{p}, \mathrm{k}\ \mathbb{R}$, $\mathrm{p} \wedge \mathrm{k}$ equals the smaller one of p and k.

Actually, the credibility measure can also be derived from the membership function of a fuzzy variable, which is called the credibility inversion theorem.

Theorem 1 (Liu 2006a). Let ξ be a fuzzy variable with membership function μ. Then for any set $\mathcal{B}$ of real numbers, we have

$$\mathrm{Cr}\{\xi \le \mathrm{t}\} = \frac{1}{2}\left(\operatorname*{Sup}_{\mathrm{x} \le \mathrm{t}} \mu(\mathrm{x}) + 1 - \operatorname*{Sup}_{\mathrm{x} > \mathrm{t}} \mu(\mathrm{x})\right), \tag{2}$$

It is noteworthy that since $\mathrm{Pos}\{\xi \le \mathrm{t}\} = \operatorname*{Sup}_{\mathrm{x} \le \mathrm{t}} \mu(\mathrm{x})$ and $\mathrm{Nec}\{\xi \le \mathrm{t}\} = 1 - \operatorname*{Sup}_{\mathrm{x} > \mathrm{t}} \mu(\mathrm{x})$, the credibility measure can be defined as follows:

$$\mathrm{Cr}\{\xi \le \mathrm{t}\} = \frac{1}{2}(\mathrm{Pos}\{\xi \le \mathrm{t}\} + \mathrm{Nec}\{\xi \le \mathrm{t}\}), \tag{3}$$

Accordingly, the credibility measure could be defined as an average of the possibility (Pos) and necessity (Nec) measures.

Definition 4 (Liu and Liu 2002). Let ξ be a fuzzy variable. The expected value of ξ can be determined based on the credibility measure as follows:

$$\mathrm{E}[\xi] = \int_0^\infty \mathrm{Cr}\{\xi \ge \mathrm{t}\}\mathrm{dt} - \int_{-\infty}^0 \mathrm{Cr}\{\xi \le \mathrm{t}\}\,\mathrm{dt}, \tag{4}$$

provided that at least one of the above two integrals is finite.

Definition 5 (Liu and Liu 2002). Let ξ be a fuzzy variable, and $\alpha \in (0,1]$. Then

$$\mathrm{f_{opt}}(\alpha) = \left\{\mathrm{f}|\mathrm{Cr}\left\{\mathrm{f} \le \mathrm{f_{opt}}\right\} \ge \alpha\right\}, \tag{5}$$

is called the α-pessimistic value of ξ, and

$$\mathrm{f_{pess}}(\alpha) = \left\{\mathrm{f}|\mathrm{Cr}\left\{\mathrm{f} \ge \mathrm{f_{pess}}\right\} \ge \alpha\right\}, \tag{6}$$

is called the α-optimistic value of ξ.

Now, assume that ξ is a triangular fuzzy number denoted by three prominent points as $\xi = (\mathrm{a}, \mathrm{b}, \mathrm{c})$. According to Eq. (4), the expected value is as follows:

$$\mathrm{E}[\xi] = \frac{a + 2 * b + c}{4} \tag{7}$$

The corresponding credibility measures are

$$\mathrm{Cr}\{\xi \le t\} = \begin{cases} 0, & \text{if } t \le a \\ (t-a)/2(b-a), & \text{if } a < t \le b \\ (c-2b+t)/2(c-b), & \text{if } b < t \le c' \\ 1, & \text{if } t > c \end{cases} \tag{8}$$

$$\mathrm{Cr}\{\xi \ge t\} = \begin{cases} 1, & \text{if } t \le a \\ (2b-a-t)/2(b-a), & \text{if } a < t \le b \\ (c-t)/2(c-b), & \text{if } b < t \le c' \\ 0, & \text{if } t > c \end{cases} \tag{9}$$

Based on (8) and (9), it can be proven (Zhu and Zhang, 2009) that if ξ is a triangular fuzzy number and $\beta > 0.5$, then:

$$\mathrm{Cr}\{\xi \le t\} \ge \beta \Leftrightarrow (2\beta-1)c + (2-2\beta)b, \tag{10}$$

$$\mathrm{Cr}\{\xi \ge t\} \ge \beta \Leftrightarrow (2\beta-1)a + (2-2\beta)b, \tag{11}$$

Eqs. (10) and (11) can be applied directly and more conveniently when compared to a-critical values proposed by Liu (2004), to convert fuzzy chance constraints into their equivalent crisp ones.

In credibility programming, there are three types of credibility-based fuzzy mathematical programming approaches: the chance-constrained programming (Liu and Iwamura 1998), the expected value (Liu and Liu 2002), and the dependent chance-constrained programming (Liu 1999). The first model uses the expected value operator for each imprecise coefficient in the objective function and constraints. It can be applied easily without increasing the complexity of the original model compared to the other two methods, but at the same time, it has no control over the level of confidence of the fuzzy chance constraint. The second model is able to control the level of satisfaction of the fuzzy chance constraint by using the concept of $\alpha -$ level. But it also increases the complexity of the model since it adds a new constraint for each objective function of the main model. The third model is similar to the second in some way, but it provides a more conservative decision for the decision maker as it attaches more importance to maximizing levels of satisfaction.

In an uncertain environment, the most known criteria widely employed are optimistic value criterion and pessimistic value criterion. On the one hand, by using the optimistic criterion, the decision-maker handles the maximum payoffs of alternatives and chooses the suitable alternative whose outcome is the best (i.e. the cost is the lowest). This criterion suggests an audacious decision-maker who is attracted by high payoffs (low costs). On the other hand, using the pessimistic criterion, the decision-maker handles only the minimum payoffs of alternatives and chooses the suitable alternative whose outcome is the worst. This criterion suggests a conservative decision-maker who tries to make sure that in the case of an unfavorable outcome (loss), there is at least (in most) a known minimum payoff (maximum loss). To overcome the extreme cases of these two criteria, several other criteria are proposed. The Hurwicz criterion is one of the best-known criteria, proposed by Hurwicz (1951), which attempts to strike a balance between the extreme criteria. It ensures that both the optimistic and pessimistic criteria should be averaged using the weights λ and $(1-\lambda)$, then associates to each action x the following index: $\lambda \min(x) + (1-\lambda)\max(x)$. Using the Hurwicz criterion in this paper, we get it under the fuzzy environment as follows:

$$\lambda * \min_{f_{opt}} f_{opt}(\alpha) + (1-\lambda) * \max_{f_{pess}} f_{pess}(\alpha),$$

where $f_{opt}(\alpha)$and $f_{pess}(\alpha)$ are the α-optimistic and α-pessimistic values. The parameter $\alpha \in (0,1]$ reflects the level of satisfying the event $\mathrm{Cr}\{f \le f_{opt}\}$ or $\mathrm{Cr}\{f \ge f_{pess}\}$. Therefore, by changing the value of θ, the Hurwicz criterion degenerates various criteria (e.g. $\lambda = 1$ leads to an optimistic criterion; $\lambda = 0$ degenerates a pessimistic criterion).

4. Milk Supply Chain

4.1. Description of the Supply Chain network

This work is motivated by the problem of integrating the production function and distribution function of a leading company that produces a wide range of dairy products (milk-yogurt-cheese-butter) in Morocco, and which has several geographically dispersed production sites. Each site has a limited production capacity, which depends essentially on the installed processing units and the speed of each unit. The utilization of a production unit generates production costs that must be integrated into the model. Due to hygiene and safety, setup time, sterilization and cleaning often depend on the sequence of batches on a specific unit. In general, the optimization procedure must take into account the setup times and costs. The finished products are perishable, are not simply stored in a warehouse, and are kept at a controlled temperature. This translates into a significant storage cost to be integrated into the model. Homogeneous vehicles ensure the delivery with a limited capacity towards the distribution centers DCs while respecting the shelf life of the products. For other ranges of products that are not manufactured in the corresponding site and are provided by other sites, each DC is able to supply the full range of products. Figure 1 represents an illustration of the dairy supply chain.

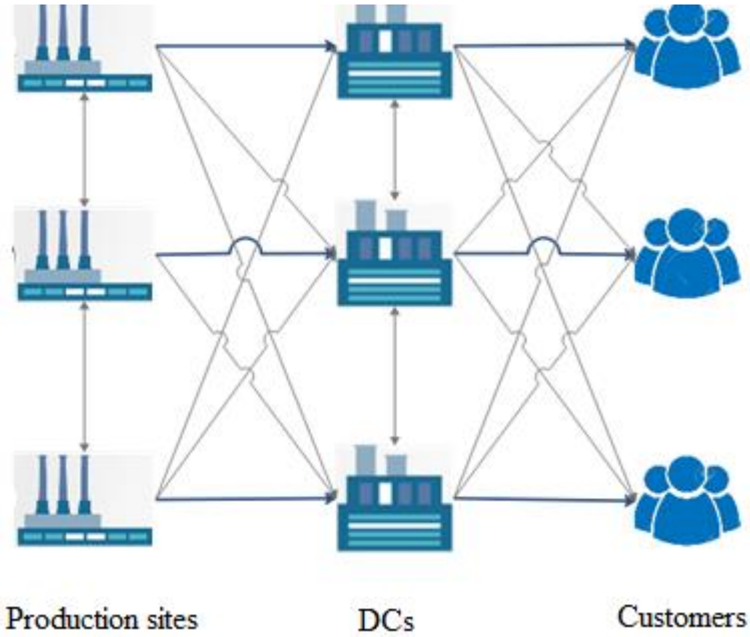

Figure 1. The milk supply chain network

The basic considerations of the problem at hand are summarized as follows:

- The planning horizon is known and divided into a set of periods $t \in T$
- A set of production sites $s \in S$, a set of DCs $d \in D$, and a set of customers $c \in C$
- A set of vehicles type $v, v' \in V$. The vehicles type v are owned by the production site and can transfer final products between production sites to DCs. The vehicles type v' are owned by the DC d and can transfer final products between DC d and customers. Each vehicle type is characterized by a maximum capacity CAP_v, $CAP_{v'}$ respectively, with a fixed utilization cost $\boldsymbol{\tau_{s,v}}$, $\boldsymbol{\tau_{d,v'}}$
- Each vehicle type v owned by the production site can make more than one trip during each period but to different distribution centers. The total quantity delivered by a vehicle in each period should not exceed its capacity. The same assumption holds for the vehicle type v' owned by DCs.
- A set of processing units $j \in J_s$ which are installed on a production site s, with a maximum capacity in period n equal to $CAPJ_{s,j,t}$.
- A set of products $p \in P$ with specific demand in period n $DPC_{c,p,t}$, $LSC_{c,p,t}$ lost cost, $\varpi_{p,k,s,j}$ processing costs, $SSPS_{s,p,t}$, $SSPD_{d,p,t}$ safety stock and $IHCPS_{s,p,t}$, $IHCPD_{d,p,t}$ inventory costs. P_j Is the subset of products that can be assigned to the unit j, and J_p is the subset of units that can produce product p.
- Due to hygiene reasons, a sequence-dependent changeover time is needed between two different products on each processing unit with a changeover cost.
- Product processing rates $rate_{p,s,j}$ are considered fixed as potential fluctuations may provoke quality problems (Soman et al., 2004).

The key decision variables are:

- The assignment of products to units in each period t.
- The sequencing between products p and k on each processing unit in each period t.
- The utilization of each vehicle types v and v' in each period t.
- The assignment of each vehicle to Sites-DCs and DCs-Customers.
- The quantity produced of each product on each processing unit at each period t.
- The total quantity dispatched between production sites-DCs and between DCs-Customers respectively.
- The inventory level of each product at sites and DCs at the end of each period.
- The lost demand of each product p provided by the customer at each period.

4.2. Mathematical formulation

4.2.1. Indices and Sets

s: Index used for a production site $s = 1, \dots, S$

j: Index used for a processing unit $j = 1, \ldots, J$
p, k, l, m: Index used for finished products $p, k = 1, \ldots, P$
d: Index used for a distribution center (DC) $d = 1, \ldots, D$
v, v': Index used for a type of vehicle $v, v' = 1, \ldots, V$
t: Index used for a planning period $t = 1, \ldots, T$
S_d : Production sites s that can supply DC d.
S_v : Production sites s that are owners of vehicle v.
J_p : Processing units j that can process product p.
J_s : Processing units j that are installed on production site s.
$V_{s,d}$:Vehicles v that can transfer products from production site s to DC d.
D_v : DCs d that can be supplied by vehicle v.
D_s : DCs d that can be supplied by production site s.

4.2.2. Parameters

$DPC_{c,p,t}$: Demand of product p required by customer region in period t
Ben_p : Unit price of product p;
$LSC_{c,p,t}$: Lost sale cost per unit of product p at customer group g in period t
$IHCPS_{s,p,t}$Inventory holding cost for product p in production site s in period t
$IHCPD_{d,p,t}$Inventory holding cost for product p in DC d in period t
$\psi_{p,k,s,j,t}$: Changeover cost between product p and product k on processing unit j of production site s in period t
$\phi_{p,k,s,j}$: Changeover time between product p and product k on processing unit j of production site s;
π_p: Volume of product.
CAP_v: Maximum capacity of vehicle type v
$CAP_{v'}$: Maximum capacity of vehicle type v'
$\varpi_{p,k,s,j}$: Production cost of product p on processing unit j of production site s in period t
$TCSD_{s,d,v}$: Cost of transferring finished products from production site s to DC d by vehicle type v;
$TCDC_{d,c,v'}$: Cost of transferring finished products from DC d to customer c by vehicle type v'
$\tau_{s,v}$: Cost of using vehicle type v to carry out products from site s
$\tau\tau_{d,v'}$: Cost of using vehicle type v' to carry out finished products from DC d;
$SCAPS_{s,t}$: Storage capacity of production site s in period t.
$SCAPD_{d,t}$: Storage capacity of DC d in period t.
$\theta_{p,j}$: Required production capacity of processing unit $j \in J_p$ of production s to produce one unit of product p;
$CAPJ_{s,j,t}$: Available processing time capacity for processing unit j of production site s in period t;
$SSPS_{s,p,t}$: Safety stock of product p at production site s in period t;
$SSPD_{d,p,t}$: Safety stock of product p at DC d in period t.
$TLT1_{s,d}$: Transport lead-time between the site s and the DC d
$TLT2_{d,c}$: Transport lead-time between the DC d and customer region c
M: A big number

4.2.3. Decision Variables

- **Binary variables**

$X_{p,s,j,t}$: 1, if product p is assigned to processing unit j of production site s in period t. 0, otherwise
$Y_{p,k,s,j,t}$: 1 if product p is processed immediately after product k on processing unit j of production site s in period t. 0, otherwise.
$U_{s,v,t}$: 1, if vehicle of type v owned by production site s is used in period t. 0, otherwise.
$W_{s,d,v,t}$: 1, if vehicle of type v owned by site s transfers products to DC d in period t. 0, otherwise.
$U'_{d,v,t}$: 1, if vehicle of type v' owned by DC d is used in period t. 0, otherwise.
$W'_{d,c,v,t}$: 1, if vehicle of type v' owned by DC d transfers products to customer region c in period t. 0, otherwise.

- **Continuous variables**

$CT_{p,s,j,t}$: Completion time of product p in unit j of production site s in period t
$PT_{p,s,j,t}$: Processing time of product p in unit j of production site s in period t
$QTPPSJ_{p,s,j,t}$: Quantity of product p produced in unit j of production site s in period t
$IPS_{s,p,t}$: Inventory level of product p in production site s at the end of period t.
$QTPSD_{s,d,v,p,t}$: Quantity of product p dispatched from production site s to DC d by vehicle v in period t
$\overline{QTSD}_{s,d,v,t}$: Total quantity dispatched from production site s to DC d by vehicle v in period t
$IPD_{d,p,t}$: Inventory level of product p in DC d at the end of period t.
$QTPDC_{d,c,v,p,t}$: Quantity of product p dispatched from DC d to customer region c by vehicle v in period t

$\overline{\mathrm{QTDC}}_{d,c,v',t}$: Total quantity dispatched from DC d to customer region c by vehicle v' in period t
$LS_{c,p,t}$: Quantity of lost sale for product p incurred by customer region c in period t.

4.2.4. Objective function

Objective function (I) maximizes the total profit including the total costs such as (i) operating costs, (ii) inventory costs, (iii) vehicle type utilization costs, (iii) products changeover costs, (v) transportation costs, and (vi) lost costs.

$$\begin{aligned}\text{Max OF} = & \sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathrm{Ben}_p * \mathrm{QTPDC}_{p,d,c,v',t} - \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T} \varpi_{p,k,s,j} * \mathrm{QTPPS}_{p,s,j,t} \\ & - \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T} \psi_{p,k,s,j,t} * Y_{p,k,s,j,t} - \sum_{s\in S}\sum_{p\in P}\sum_{t\in T} \mathrm{IHCPS}_{s,p,t} * \mathrm{IPS}_{s,p,t} \\ & - \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{t\in T} \mathrm{TCSD}_{s,d,v} * \overline{\mathrm{QTPSD}}_{s,d,v,t} - \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T} \tau_{s,v} * U_{s,v,t} \\ & - \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} \mathrm{IHCPD}_{d,p,t} * \mathrm{IPD}_{d,p,t} - \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T} \tau_{d,v'} * U'_{d,v',t} \\ & - \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{t\in T} \mathrm{TCDC}_{d,c,v'} * \overline{\mathrm{QTPDC}}_{d,c,v',t} - \sum_{c\in C}\sum_{p\in P}\sum_{t\in T} \mathrm{LSC}_{c,p,t} * \mathrm{LS}_{c,p,t} \qquad \text{(I)}\end{aligned}$$

4.2.5. Constraints

$$Q^{Min}_{p,s,j,t} * X_{p,s,j,t} \leq \mathrm{QTPPS}_{p,s,j,t} \leq Q^{Max}_{p,s,j,t} * X_{p,s,j,t},\ \forall p,s,j\in J_s\cap J_p,t \qquad (12)$$

$$PT_{p,s,j,t} = {\mathrm{QTPPS}_{p,s,j,t}}/{\mathrm{rate}_{p,s,j}}\ \ \forall p,s,j\in J_s\cap J_p,t \qquad (13)$$

$$CT_{p,s,j,t} + Y_{p,k,s,j,t} * \phi_{p,k,s,j} \geq CT_{k,s,j,t} - PT_{k,s,j,t} + M * (1 - Y_{p,k,s,j,t}), \qquad (14)$$

$$\forall p, p\neq k, s, j\in J_s\cap J_p\cap J_k, t$$

$$CT_{k,s,j,t} - PT_{k,s,j,t} \geq (pt_{past} + \delta_c) * X_{p,s,j,t} + \textstyle\sum_{k\in P_j,k\neq p} Y_{p,k,s,j,t} * \phi_{p,k,s,j}, \qquad (15)$$

$$\forall p,s,j\in J_s\cap J_p,t$$

$$CT_{p,s,j,t} \leq \mathrm{CAPJ}_{s,j,t} * X_{p,s,j,t},\ \forall p,s,j\in J_s\cap J_p,t \qquad (16)$$

$$\textstyle\sum_{k\in P_j\neq p}\sum_{p\in P_j} Y_{p,k,s,j,t} + 1 \geq \sum_{p\in P_j} X_{p,s,j,t},\ \forall s,j\in J_s,t \qquad (17)$$

$$\textstyle\sum_{k\in P_j,k\neq p} Y_{k,p,s,j,t} \leq X_{p,s,j,t},\ \forall p,s,j\in J_s\cap J_p,t \qquad (18)$$

$$\textstyle\sum_{k\in P_j,k\neq p} Y_{p,k,s,j,t} \leq X_{p,s,j,t},\ \forall p,s,j\in J_s\cap J_p,t \qquad (19)$$

$$\mathrm{IPS}_{s,p,t} = \mathrm{IPS}_{s,p,t} + \textstyle\sum_{j\in J_s\cap J_p} \mathrm{QTPPS}_{p,s,j,t} - \sum_{d\in D_s}\sum_{v\in V_{s,d}} \mathrm{QTPSD}_{s,d,v,p,t},\ \ \forall p,s,t \qquad (20)$$

$$\mathrm{IPS}_{s,p,t} \geq \mathrm{SSPS}_{s,p,t},\ \forall s,p,t \qquad (21)$$

$$\textstyle\sum_p\sum_j \pi_p * \mathrm{QTPPS}_{p,s,j,t} \leq \mathrm{SCAPS}_{s,t},\ \forall s,t \qquad (21)$$

$$\textstyle\sum_p \pi_p * \mathrm{IPS}_{s,p,t} \leq \mathrm{SCAPS}_{s,t},\ \forall s,t \qquad (23)$$

$$\mathrm{IPD}_{d,p,t} = \mathrm{IPD}_{d,p,t-1} + \textstyle\sum_{s\in S_d}\sum_{v\in V_{s,d}} \mathrm{QTPSD}_{s,d,v,p,t-TLT1} - \sum_{c\in C_d}\sum_{v'\in V'_{d,c}} \mathrm{QTPDC}_{d,c,v',p,t}, \forall p,d,t \qquad (24)$$

$$\mathrm{IPD}_{d,p,t} \geq \mathrm{SSPD}_{d,p,t},\ \forall d,p,t \qquad (25)$$

$$\textstyle\sum_p\sum_{s\in S_d}\sum_{v\in V_{s,d}} \pi_p * \mathrm{QTPSD}_{s,d,v,p,t} \leq \mathrm{SCAPD}_{d,t},\ \forall d,t \qquad (26)$$

$$\textstyle\sum_p \pi_p * \mathrm{IPD}_{d,p,t} \leq \mathrm{SCAPD}_{d,t},\quad \forall d,t \qquad (27)$$

$$\overline{\mathrm{QTSD}}_{s,d,v,t} = \sum_p \pi_p * \mathrm{QTPSD}_{s,d,v,p,t}, \forall s, d, v \in V_{s,d}, t \quad (28)$$

$$\sum_{d \in D_s} \overline{\mathrm{QTSD}}_{s,d,v,t} \leq \mathrm{CAP}_v * U_{s,v,t}, \quad \forall s, v \in V_{s,d}, t \quad (29)$$

$$\overline{\mathrm{QTSD}}_{s,d,v,t} \leq \mathrm{CAP}_v * W_{s,d,v,t}, \quad \forall s, d, v, t \quad (30)$$

$$\sum_{d \in D_s} W_{s,d,v,t} \leq \mathrm{CAP}_v * U_{s,v,t}, \quad \forall s, v \in V_{s,d}, t \quad (31)$$

$$\overline{\mathrm{QTDC}}_{d,c,v',t} = \sum_p \pi_p * \mathrm{QTPDC}_{p,d,c,v',t}, \forall d, c, v' \in V'_{d,c}, t \quad (32)$$

$$\sum_{c \in C_d} \overline{\mathrm{QTDC}}_{d,c,v',t} \leq \mathrm{CAP}_{v'} * U'_{d,v',t}, \ \forall d, v' \in V'_{d,c}, t \quad (33)$$

$$\overline{\mathrm{QTDC}}_{d,c,v',t} \leq \mathrm{CAP}_{v'} * W'_{d,c,v',t}, \ \forall d, c, v' \in V'_{d,c}, t \quad (34)$$

$$\sum_{c \in C_d} W'_{d,c,v',t} \leq \mathrm{CAP}_{v'} * U'_{d,v',t}, \ \forall d, v' \in V'_{d,c}, t \quad (35)$$

$$\sum_{d \in D_c} \sum_{v' \in V'_{d,c}} \mathrm{QTPDC}_{d,c,v',p,t-\mathrm{TLT3}} \leq \mathrm{DPC}_{c,p,t}, \ \forall c, p, t \quad (36)$$

$$\mathrm{DPC}_{c,p,t} - \sum_d \sum_{v'} \mathrm{QTPDC}_{d,c,v',p,t} \leq \mathrm{LS}_{c,p,t}, \ \forall c, p, t \quad (37)$$

$$\mathrm{CT}, \mathrm{ST}, \mathrm{TP}, \mathrm{QTPPS}, \mathrm{IPS}, \mathrm{QTPSD}, \mathrm{IPD}, \mathrm{QTPDC}, \mathrm{LS} \geq 0 \quad (38)$$

$$X, Y, U, W, U', W' \in \{0,1\} \quad (39)$$

Constraint set (12) assures that the produced quantity $\mathrm{QTPPS}_{p,s,j,t}$ of product p is greater than its corresponding minimum lot $Q^{Min}_{p,s,j,t}$ and lower than its maximum $Q^{Max}_{p,s,j,t}$. Constraint set (13) indicates that the processing time is equal to the packaged amount $\mathrm{QTPPS}_{p,s,j,t}$ of product p at the processing unit j of site s, divided by the processing rate of the same unit. Eq. (14) ensures that the starting time of product k, which followed directly another product p on a processing unit j in site s at period t ($Y_{p,k,s,j,t} = 1$) should be greater than the completion time of the product p ($\mathrm{CT}_{p,s,j,t}$), plus the required setup time $\phi_{p,k,s,j}$ between those products. Eq. (15) guarantees that the starting time (i.e.,$\mathrm{CT}_{p,s,j,t} - \mathrm{PT}_{p,s,j,t}$) should be greater than the required pasteurization time pt_{past}, plus the necessary time quality control time δ_c, plus the setup time $\phi_{p,k,s,j}$ for changing the production to product p. Constraint set (16) ensures that the completion time of each product should be less than the maximum capacity of the unit. Constraints (17) and (18) ensure that if a product p is assigned to unit j of site s at period t ($X_{p,s,j,t} = 1$) then at most one product k is processed before and after it respectively. Constraint set (19) enforces that the total number of active sequencing binary variable $Y_{p,k,s,j,t}$should be greater than the total number of active assignment binary variables $X_{p,s,j,t}$ in the unit j at site s in period n minus one. Constraint set (20) guarantees that the material balance equation for production sites, the quantity of product p produced in site s must be equal to the quantity stored at this and transported to DC d by vehicle type v. Constraint set (21) ensures that the inventory level of product p in site s at the end of period should be greater than or equal to a given safety stock. Eq. (22) denotes that the total produced quantity in production site s at period must be less than the maximum storage capacity of this site. Eq. (23) guarantees that the inventory level at the end of period t must be less than the maximum storage capacity of site s. Constraints (24) and (25) denote the material balance and safety stock of products at DCs. Constraint set (26) shows that the total quantity received from sites by vehicle type v must be less than the maximum storage capacity of the DC. Eq. (27) guarantees that the inventory level at the end of period t must be less than the maximum storage capacity of the DC d. Constraint set (28) denotes the total amount transported from site s to DC d by vehicle type v in period t. Constraint set (29) ensures that the quantity transported from site s to DCs d by a vehicle of type v (owned by site s) does not exceed its maximum loading capacity. Constraint set (30) links the binary variable $W_{s,d,v,t}$ to $\mathrm{QTPSD}_{s,d,v,t}$. In other words, there are no flow material between sites and DCs if when $W_{s,d,v,t} = 1$. Constraint (31) ensures that $W_{s,d,v,t}$ is positive just when vehicle v owned by production site s is used in period t. Constraint set (32) denotes the total amount transported from DC d to site s d by vehicle type v' in period t. Constraints (33), (34) and (35) describe the similar logic as constraints (29), (30) and (31) for the utilization of vehicles types between DCs and customers. Constraint set (36) enforces that the quantity of a product transported from DC d to each customer by vehicle type v' in each period t cannot exceed the demand. Constraint set (37) guarantees that the unmet demand is lost. Finally, constraints (38) and (39) describe the nature of decision variables. In addition, Tables 2 and 3 represent the total number of variables and constraints of the proposed model.

Table 2. Number of variables in the model

Variable	Dim	Variable	Dim
$X_{p,s,j,t}$	$P \times S \times J \times T$	$PT_{p,s,j,t}$	$P \times S \times J \times T$
$Y_{p,k,s,j,t}$	$P \times K \times S \times J \times T$	$QTPPSJ_{p,s,j,t}$	$P \times S \times J \times T$
$U_{s,v,t}$	$S \times V \times T$	$IPS_{s,p,t}$	$S \times P \times T$
$W_{s,d,v,t}$	$S \times D \times V \times T$	$QTPSD_{s,d,v,p,t}$	$S \times D \times V \times P \times T$
$\overline{QTSD}_{s,d,v,t}$	$S \times D \times V \times T$	$\overline{QTDC}_{d,c,v',t}$	$D \times C \times V' \times T$
$U'_{d,v',t}$	$D \times V' \times T$	$IPD_{d,p,t}$	$D \times P \times T$
$W'_{d,c,v',t}$	$D \times C \times V' \times T$	$QTPDC_{d,c,v',p,t}$	$D \times C \times V' \times P \times T$
$CT_{p,s,j,t}$	$P \times S \times J \times T$	$LS_{c,p,t}$	$C \times P \times T$
$\text{Sum} = 4 * (P \times S \times J \times T) + (P \times K \times S \times J \times T) + (S \times V \times T) + (S \times D \times V \times T) + (D \times V' \times T) + (D \times C \times V' \times T) + (S \times P \times T) + (S \times D \times V \times P \times T) + (D \times P \times T) + (D \times C \times V' \times P \times T) + (C \times P \times T)$			

Table 3. Number of constraints in the model

Constraint	Dim	Constraint	Dim
(12)	$P \times S \times J \times T$	(25)	$D \times P \times T$
(13)	$P \times S \times J \times T$	(26)	$D \times T$
(14)	$P \times K \times S \times J \times T$	(27)	$D \times T$
(15)	$P \times S \times J \times T$	(28)	$S \times D \times V \times T$
(16)	$P \times S \times J \times T$	(29)	$S \times V \times T$
(17)	$S \times J \times T$	(30)	$S \times D \times V \times T$
(18)	$P \times S \times J \times T$	(31)	$S \times V \times T$
(19)	$P \times S \times J \times T$	(32)	$D \times C \times V \times T$
(20)	$S \times P \times T$	(33)	$D \times V' \times T$
(21)	$S \times P \times T$	(34)	$D \times C \times V' \times T$
(22)	$S \times T$	(35)	$D \times V' \times T$
(23)	$S \times T$	(36)	$C \times P \times T$
(24)	$D \times P \times T$	(37)	$C \times P \times T$
$\text{Sum} = 6 * (P \times S \times J \times T) + (P \times K \times S \times J \times T) + (S \times J \times T) + 2 * (S \times P \times T) + 2 * (S \times T) + 2 * (D \times P \times T) + 2 * (D \times T) + 2 * (S \times D \times V \times T) + 2 * (S \times V \times T) + 2 * (D \times C \times V' \times T) + 2 * (D \times V' \times T) + 2 * (C \times P \times T)$			

5. Milk Supply Chain management under uncertainty

In supply chain management, decision-making problems consist of strategic, tactical and operational decisions. The objective of strategic decisions is to define the design and configuration of the network, while tactical decisions are related to the efficient use of different resources. Operational decisions, on the other hand, are linked to scheduling, sequencing, allocation of costs, etc., within a short-term planning horizon. All the problems mentioned above are often dealt with under specific conditions. According to Klibi et al., (2010), any supply chain planning that relies on deterministic conditions risks losing its durability. They also mention that in some cases, it is not enough for the company to consider usual parameters such as demand, prices or other parameters such as random variables, but undesirable events such as terrorist attacks and natural disasters. Therefore, it is necessary to have a specific strategy that integrates uncertainty into supply chain planning.
The theory of probability proved to be the old theory to deal with uncertain parameters. However, in practice, several disadvantages are related to the use of the probability theory including the need for a sufficient and reliable history and the problem of modeling subjective parameters (Pishvaee and Torabi 2010, Liu and Kao 2004). The fuzzy set theory (FST) is an alternative for treating uncertainty by taking local preferences into account in optimization problems. In this paper, the credibility theory is used to deal with the sources of uncertainties, which may relate to target values or coefficients of objectives, or other data such as customer demand, transport costs and transport lead-time.

5.1. The proposed credibility-based fuzzy model with the Hurwicz criterion

According to above-mentioned descriptions and justifications, the proposed credibility-based fuzzy mathematical programming with the Hurwicz criterion for integrated production-distribution in the dairy supply chain can be formulated as follows:

$$\max\left\{\lambda * \max_{OF_{opt}} OF_{opt} + (1-\lambda) * \min_{OF_{pess}} OF_{pess}\right\} \qquad (I')$$

S.t

$$\mathrm{Cr}\left\{\sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T}\mathrm{Ben}_p * \mathrm{QTPDC}_{p,d,c,v',t} - \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T}\varpi_{p,k,s,j} * \mathrm{QTPPS}_{p,s,j,t} \right.$$
$$- \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T}\psi_{p,k,s,j,t} * Y_{p,k,s,j,t} - \sum_{s\in S}\sum_{p\in P}\sum_{t\in T}\mathrm{IHCPS}_{p,s,t} * \mathrm{IPS}_{p,s,t}$$
$$- \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{t\in T}\mathrm{TCSD}_{s,d,v} * \overline{\mathrm{QTPSD}}_{s,d,v,t} - \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T}\tau_{s,v} * U_{s,v,t}$$
$$- \sum_{d\in D}\sum_{p\in P}\sum_{t\in T}\mathrm{IHCPD}_{p,d,t} * \mathrm{IPD}_{p,d,t} - \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T}\tau_{d,v'} * U'_{d,v',t}$$
$$\left. - \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{t\in T}\mathrm{TCDC}_{d,c,v'} * \overline{\mathrm{QTPDC}}_{d,c,v',t} - \sum_{c\in C}\sum_{p\in P}\sum_{t\in T}\mathrm{LSC}_{p,c,t} * \mathrm{LS}_{p,c,t} \geq \mathrm{OF}_{opt}\right\}$$
$$\geq \alpha \qquad (\mathrm{II}')$$

$$\mathrm{Cr}\left\{\sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T}\mathrm{Ben}_p * \mathrm{QTPDC}_{p,d,c,v',t} - \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T}\varpi_{p,k,s,j} * \mathrm{QTPPS}_{p,s,j,t} \right.$$
$$- \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T}\psi_{p,k,s,j,t} * Y_{p,k,s,j,t} - \sum_{s\in S}\sum_{p\in P}\sum_{t\in T}\mathrm{IHCPS}_{p,s,t} * \mathrm{IPS}_{p,s,t}$$
$$- \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{t\in T}\mathrm{TCSD}_{s,d,v} * \overline{\mathrm{QTPSD}}_{s,d,v,t} - \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T}\tau_{s,v} * U_{s,v,t} - \sum_{d\in D}\sum_{p\in P}\sum_{t\in T}\mathrm{IHCPD}_{p,d,t} * \mathrm{IPD}_{p,d,t}$$
$$\left. - \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T}\tau_{d,v'} * U'_{d,v',t} - \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{t\in T}\mathrm{TCDC}_{d,c,v'} * \overline{\mathrm{QTPDC}}_{d,c,v',t} - \sum_{c\in C}\sum_{p\in P}\sum_{t\in T}\mathrm{LSC}_{p,c,t} * \mathrm{LS}_{p,c,t} \leq \mathrm{OF}_{pess}\right\}$$
$$\geq \alpha \qquad (\mathrm{III}')$$

$$\mathrm{Cr}\{\mathrm{CT}_{p,s,j,t} \leq \mathrm{CAPJ}_{s,j,t} * X_{p,s,j,t}\} \geq \beta_{p,s,j,t}, \quad \forall p,s,j \in J_s \cap J_p, t, \qquad (40)$$
$$\mathrm{Cr}\{\mathrm{IPS}_{s,p,t} \geq \mathrm{SSPS}_{s,p,t}\} \geq \beta_{s,p,t}, \quad \forall s,p,t, \qquad (41)$$
$$\mathrm{Cr}\{\mathrm{IPD}_{d,p,t} \geq \mathrm{SSPD}_{d,p,t}\} \geq \beta_{d,p,t}, \quad \forall d,p,t, \qquad (42)$$
$$\mathrm{Cr}\left\{\sum_{d\in D_c}\sum_{v'\in V'_{d,c}}\mathrm{QTPDC}_{d,c,v',p,t-\mathrm{TLT3}} \leq \mathrm{DPC}_{c,p,t}\right\} \geq \beta_{c,p,t}, \quad \forall c,p,t, \qquad (43)$$
$$\mathrm{Cr}\left\{\mathrm{DPC}_{c,p,t} - \sum_{d}\sum_{v'}\mathrm{QTPDC}_{d,c,v',p,t} \leq \mathrm{LS}_{c,p,t}\right\} \geq \beta_{c,p,t}, \quad \forall c,p,t, \qquad (44)$$

$(12-15), (16-19), (21-33), (24-34), (37), (38)$

Where the parameter $\alpha \in (0,1]$ reflects the level of satisfying the decision maker and $\beta_{p,s,j,t}, \beta_{s,p,t}, \beta_{d,p,t}, \beta_{c,p,t}$ are predetermined confidence levels, $\mathrm{Cr}\{\cdot\}$ denotes the credibility of the event $\{\cdot\}$.

5.2. The equivalents crisp model

To solve the FCCP, it has become evident to transform the fuzzy chance constraints into their crisp equivalents with respect to the predetermined confidence level, and then solve the equivalent crisp model (Liu 2009; Lau et al., 2010). In the present study, the uncertain parameters are represented by triangular fuzzy numbers. So far, the crisp equivalents of the fuzzy constraints are given based on the property of the triangular fuzzy numbers, then constraints (II'), (III'), and (40-44) can be deduced below. The crisp equivalent constraint of the fuzzy chance constraint (II') is:

$$
\begin{aligned}
\mathbf{OF_{opt}} \leq (\mathbf{2-2\alpha}) \\
&* \Bigg[\sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{Ben_p^m} * \mathbf{QTPDC_{p,d,c,v',t}} \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T} \boldsymbol{\varpi}^{\mathbf{m}}_{\mathbf{p,k,s,j}} * \mathbf{QTPPS_{p,s,j,t}} \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T} \boldsymbol{\psi}^{\mathbf{m}}_{\mathbf{p,k,s,j,t}} * \mathbf{Y_{p,k,s,j,t}} - \sum_{s\in S}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPS^m_{s,p,t}} * \mathbf{IPS_{s,p,t}} \\
&- \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{t\in T} \mathbf{TCSD^m_{s,d,v}} * \overline{\mathbf{QTPSD}}_{\mathbf{s,d,v,t}} - \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{s,v}} * \mathbf{U_{s,v,t}} \\
&- \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPD^m_{d,p,t}} * \mathbf{IPD_{d,p,t}} - \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{d,v'}} * \mathbf{U'_{d,v',t}} \\
&- \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCDC^m_{d,c,v'}} * \overline{\mathbf{QTPDC}}_{\mathbf{d,c,v',t}} - \sum_{c\in C}\sum_{p\in P}\sum_{t\in T} \mathbf{LSC^m_{c,p,t}} * \mathbf{LS_{c,p,t}}\Bigg] \\
&+ (\mathbf{2\alpha-1}) \\
&* \Bigg[\sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{Ben_p^p} * \mathbf{QTPDC_{p,d,c,v',t}} \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T} \boldsymbol{\varpi}^{\mathbf{o}}_{\mathbf{p,k,s,j}} * \mathbf{QTPPS_{p,s,j,t}} \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T} \boldsymbol{\psi}^{\mathbf{o}}_{\mathbf{p,k,s,j,t}} * \mathbf{Y_{p,k,s,j,t}} - \sum_{s\in S}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPS^o_{s,p,t}} * \mathbf{IPS_{s,p,t}} \\
&- \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCSD^o_{s,d,v}} * \overline{\mathbf{QTPSD}}_{\mathbf{s,d,v,t}} - \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{s,v}} * \mathbf{U_{s,v,t}} \\
&- \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPD^o_{d,p,t}} * \mathbf{IPD_{d,p,t}} - \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{d,v'}} * \mathbf{U'_{d,v',t}} \\
&- \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCDC^o_{d,c,v'}} * \overline{\mathbf{QTPDC}}_{\mathbf{d,c,v',t}} - \sum_{c\in C}\sum_{p\in P}\sum_{t\in T} \mathbf{LSC^o_{c,p,t}} * \mathbf{LS_{c,p,t}}\Bigg]
\end{aligned}
\tag{II''}
$$

By the same manner as the equation (II'), the crisp equivalent constraint of the fuzzy chance constraint (III') is III".

For the fuzzy chance constraint (40), the crisp equivalent constraint is:

$$\mathrm{CT_{p,s,j,t}} \leq \left[\left(2\beta_{c,p,t}-1\right) * \mathrm{CAPJ^p_{s,j,t}} + \left(2-2\beta_{c,p,t}\right) * \mathrm{CAPJ^m_{s,j,t}}\right] * \mathrm{X_{p,s,j,t}}, \quad \forall p, s, j \in J_s \cap J_p, t \tag{40'}$$

For the fuzzy chance constraint (41), the crisp equivalent constraint is:

$$\mathrm{IPS_{s,p,t}} \geq \left(2\beta_{s,p,t}-1\right) * \mathrm{SSPS^o_{s,p,t}} + \left(2-2\beta_{s,p,t}\right) * \mathrm{SSPS^m_{s,p,t}}, \ \forall s, p, t \tag{41'}$$

For the fuzzy chance constraint (42'), the crisp equivalent constraint is

$$\mathrm{IPD_{d,p,t}} \geq \left(2\beta_{d,p,t}-1\right) * \mathrm{SSPD^o_{d,p,t}} + \left(2-2\beta_{d,p,t}\right) * \mathrm{SSPD^m_{d,p,t}}, \ \forall d, p, t \tag{42'}$$

For the fuzzy chance constraint (43), the crisp equivalent constraint is

$$\textstyle\sum_{d\in D_c}\sum_{v'\in V'_{d,c}} \mathrm{QTPDC_{d,c,v',p,t-TLT3}} \leq \left[\left(2\beta_{c,p,t}-1\right) * \mathrm{DPC^p_{c,p,t}} + \left(2-2\beta_{c,p,t}\right) * \mathrm{DPC^m_{c,p,t}}\right], \ \forall c, p, t' \tag{43'}$$

For the fuzzy chance constraint (44), the crisp equivalent constraint is

$$\left[\left(2\beta_{c,p,t}-1\right) * \mathrm{DPC^o_{c,p,t}} + \left(2-2\beta_{c,p,t}\right) * \mathrm{DPC^m_{c,p,t}}\right] - \textstyle\sum_d\sum_{v'} \mathrm{QTPDC_{d,c,v',p,t}} \leq \mathrm{LS_{c,p,t}}, \ \forall c, p, t \tag{44'}$$

Now, equations (II') and (III') are replaced with the right hand sides of equations (II'') and (III''). Also, the fuzzy chance constraints (40 - 44) are replaced with constraints (40'- 44'), then we get the crisp equivalent model of IPDP-DSC. The equivalent model is a mixed integer linear program (MILP) and we can use the related optimization software like GMAS to solve it.

$$\begin{aligned}
\mathbf{OF_{pess}} \geq (\mathbf{2}-\mathbf{2\alpha}) \\
&* \left[\sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{Ben_p^m} * \mathbf{QTPDC_{p,d,c,v',t}}\right. \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T} \boldsymbol{\varpi}^{\mathbf{m}}_{\mathbf{p,k,s,j}} * \mathbf{QTPPS_{p,s,j,t}} - \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T} \boldsymbol{\psi}^{\mathbf{m}}_{\mathbf{p,k,s,j,t}} * \mathbf{Y_{p,k,s,j,t}} \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPS^m_{s,p,t}} * \mathbf{IPS_{s,p,t}} - \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCSD^m_{s,d,v}} * \overline{\mathbf{QTPSD}}_{\mathbf{s,d,v,t}} \\
&- \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{s,v}} * \mathbf{U_{s,v,t}} - \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPD^m_{d,p,t}} * \mathbf{IPD_{d,p,t}} \\
&- \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{d,v'}} * \mathbf{U'_{d,v',t}} - \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCDC^m_{d,c,v'}} * \overline{\mathbf{QTPDC}}_{\mathbf{d,c,v',t}} \\
&\left.- \sum_{c\in C}\sum_{p\in P}\sum_{t\in T} \mathbf{LSC^m_{c,p,t}} * \mathbf{LS_{c,p,t}}\right] + (\mathbf{2\alpha}-\mathbf{1}) \\
&* \left[\sum_{d\in D}\sum_{c\in C_d}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{Ben_p^o} * \mathbf{QTPDC_{p,d,c,v',t}}\right. \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{j\in J_p\cap J_s}\sum_{t\in T} \boldsymbol{\varpi}^{\mathbf{p}}_{\mathbf{p,k,s,j}} * \mathbf{QTPPS_{p,s,j,t}} - \sum_{s\in S}\sum_{p\in P}\sum_{k\in P,k\neq p}\sum_{j\in J_p\cap J_k\cap J_s}\sum_{t\in T} \boldsymbol{\psi}^{\mathbf{p}}_{\mathbf{p,k,s,j,t}} * \mathbf{Y_{p,k,s,j,t}} \\
&- \sum_{s\in S}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPS^p_{s,p,t}} * \mathbf{IPS_{s,p,t}} - \sum_{s\in S}\sum_{d\in D}\sum_{v\in V_{s,d}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCSD^p_{s,d,v}} * \overline{\mathbf{QTPSD}}_{\mathbf{s,d,v,t}} \\
&- \sum_{s\in S}\sum_{v\in V_s}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{s,v}} * \mathbf{U_{s,v,t}} - \sum_{d\in D}\sum_{p\in P}\sum_{t\in T} \mathbf{IHCPD^p_{d,p,t}} * \mathbf{IPD_{d,p,t}} \\
&- \sum_{d\in D}\sum_{v'\in V_d}\sum_{t\in T} \boldsymbol{\tau}_{\mathbf{d,v'}} * \mathbf{U'_{d,v',t}} - \sum_{d\in D}\sum_{c\in C}\sum_{v'\in V_{d,c}}\sum_{p\in P}\sum_{t\in T} \mathbf{TCDC^p_{d,c,v'}} * \overline{\mathbf{QTPDC}}_{\mathbf{d,c,v',t}} \\
&\left.- \sum_{c\in C}\sum_{p\in P}\sum_{t\in T} \mathbf{LSC^p_{c,p,t}} * \mathbf{LS_{c,p,t}}\right]
\end{aligned} \tag{III''}$$

6. Computational results

In this paper, the problem of integrating the production and distribution functions of a three-echelon milk supply chain in Morocco is presented. The manufacturer is a leading company with four dispersed geographic production sites. Seven products types are considered in this work (two products belong to pasteurized milk and five products to UHT). The site N°1 can produce the whole range of products while the other sites, depending mainly on the installed processing units, produce just a range of products. Table 5 shows the sites-units matrix allocation, while Table 6 displays the units-products matrix allocation. Tables 7 and 8 indicate the sites-DCs and DCs-customers matrix allocation. Tables 9 and 10 report the vehicle type v owned by each site as well as the vehicle type v' owned by each DC. Table 11 lists the processing speeds of each processing unit for each product. Finally, Table 12 shows the related random distribution of each parameter.

Table 4. Sites - Units matrix; (*) the unit u is installed in the site s; Otherwise

Sites | Unites	U1	U2	U3	U4
S1	*	*	*	*
S2		*		
S3	*			
S4		*	*	

Table 5. Units - Products matrix; (*) unit u that can process the product p; Otherwise

Units |Products	P1	P2	P3	P4	P5	P6	P7
U1	*		*				
U2						*	*
U3		*					
U4				*	*		

Table 6. Sites-DCs matrix; (*) the site s can supply DC; Otherwise

Sites | DCs	D1	D2	D3	D4
S1	*	*	*	*
S2	*			*
S3		*	*	*
S4		*		*

Table 7. DCs - Customers matrix; (*) DC d that can supply customer c; Otherwise

DCs | Customers	C1	C2	C3	C4	C5	C6
D1	*	*	*	*	*	*
D2	*			*	*	*
D3		*	*	*	*	*
D4		*		*	*	*

Table 8. Sites - Vehicles matrix; (*) Site s owned by vehicle v; Otherwise

	V1	V2	V3	V4	V5	V6	V7	V8	V9
S1	*	*			*				
S2			*	*					
S3						*	*		
S4								*	*

Table 9. DCs - Vehicles matrix; (*) DC d owned by vehicle v'; Otherwise

	V'1	V'2	V'3	V'4	V'5	V'6	V'7	V'8	V'9
D1	*	*			*				
D2			*	*					
D3						*	*		
D4								*	*

Table 10. Product Sequences

	P1	P2	P3	P4	P5	P6	P7
P1	--		*				
P2		--					
P3	*		--				
P4				--	*		
P5				*	--		
P6						--	*
P7						*	--

Table 11. Machine speeds for Product $\mathbf{rate_{p,s,j}}$

Units |Products	P1	P2	P3	P4	P5	P6	P7
U1	2500	-	2500	-	-	-	-
U2	-	-	-	-	-	1700	2000
U3	-	1700	-	-	-	-	-
U4	-	-	-	1450	1700	-	-

In this work, the fuzzy parameters are considered as fuzzy triangular numbers $\xi = (\xi^p, \xi^m, \xi^o)$ where, ξ^p is the most pessimistic, ξ^m is the most likely and ξ^o is most optimistic value. These values must be estimated for each fuzzy parameter. To do so, the method proposed by Lai and Hwang (1992) is used. First, the most likely ξ^m value for each imprecise parameter is specified randomly according to the uniform distribution given in Table 12. Then, the most pessimistic ξ^p and the most optimistic ξ^o values of a fuzzy number ξ are obtained as $\xi^p = (1 - r_1)\xi^m$, $\xi^o = (1 + r_2)\xi^m$ where (r_1, r_2) are two numbers randomly generated according to the uniform distribution [0.1, 0.3]. The equivalent auxiliary crisp model is coded in GAMS 22.5/CPLEX 12.2 optimization software and all numerical experiments are solved using a Core i5 2.10 GHz computer with 4 GB RAM. The proposed model is dependent on the values of confidence levels α, β, and Hurwicz criterion γ, thus it is an importing task to design a sensitivity analysis in order to study the influence of these parameters on the optimal value of the objective function. Designing a good experiment requires minimizing the number of experiments to acquire as much information as possible. The uniform design method was first proposed by Fang in 1980. Compared to the traditional experimental design methods such as the orthogonal design, it has the advantages of shorter test times, higher work efficiency, and being more economical as the number of experimental tests equals the level figure of independent variables. UD tables can be described as $U_n(q^s)$, where U stand for the uniform design, n the number of experimental trials, q the number of levels, and s the number of factors, respectively. For a given measure of uniformity M (Centered L_2, Wrap around L_2), a uniform design has the smallest M-value overall fractional factorial design with n runs and s q-level factors. In this paper, α, β and Hurwicz criterion γ are considered three independent variables. The value of β is fixed to 0.9. Five levels for factors $\alpha, \gamma \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ were selected to investigate the influence and interaction of the factors. In order to improve the accuracy, the experiments were carried out using the asymmetrical table $U_5(5^2)$. The range and levels of each factor are showed in Table 13.

Table 12. Model parameters

Deterministic Parameters			
Parameter	Related random distribution	Parameter	Related random distribution
$\mathrm{SCAPD}_{d,t}$	$\sim U(2500{,}3000)$	$\phi_{p,k,s,j}$	$\sim U(1.2{,}1.5)$
$\mathrm{SCAPS}_{s,t}$	$\sim U(3500{,}4500)$	$\mathrm{CAP}_{v'}$	$\sim U(600{,}900)$
CAP_{v}	$\sim U(600{,}900)$	π_{p}	$\sim U(0.05{,}0.1)$
$\tau_{s,v}$	$\sim U(10{,}15)$	$\mathrm{TLT1}_{s,d}$	$\sim U(1{,}2)$
$\tau\tau_{d,v'}$	$\sim U(11{,}16)$	$\mathrm{TLT2}_{d,c}$	$\sim U(1{,}2)$
Imprecise Parameters			
Parameter	Related random distribution	Parameter	Related random distribution
$\mathrm{DPC}^{m}_{c,p1,t}$	$\sim \mathcal{N}(900{,}20)$	$\mathrm{IHCPS}^{m}_{s,p,t}$	$\sim \mathcal{N}(6{,}1)$
$\mathrm{DPC}^{m}_{c,p2,t}$	$\sim \mathcal{N}(800{,}20)$		
$\mathrm{DPC}^{m}_{c,p3,t}$	$\sim \mathcal{N}(650{,}20)$		
$\mathrm{DPC}^{m}_{c,p4,t}$	$\sim \mathcal{N}(650{,}20)$		
$\mathrm{DPC}^{m}_{c,p5,t}$	$\sim \mathcal{N}(450{,}10)$		
$\mathrm{DPC}^{m}_{c,p6,t}$	$\sim \mathcal{N}(400{,}10)$		
$\mathrm{DPC}^{m}_{c,p7,t}$	$\sim \mathcal{N}(300{,}10)$		
$\mathrm{LSC}^{m}_{c,p,t}$	$\sim U(7{,}9)$	$\psi^{m}_{p,k,s,j,t}$	$\sim \mathcal{N}(15{,}2)$
Ben^{m}_{p}	$\sim U(50{,}60)$	$\mathrm{TCSD}^{m}_{s,d,v}$	$\sim \mathcal{N}(15{,}2)$
$\varpi^{m}_{p,k,s,j}$	$\sim \mathcal{N}(8{,}1.5)$	$\mathrm{TCDC}^{m}_{d,c,v'}$	$\sim \mathcal{N}(14{,}1.5)$
$\mathrm{CAPJ}^{m}_{s,j,t}$	$\sim \mathcal{N}(21{,}2)$	$\mathrm{SSPS}^{m}_{s,p,t}$	$\sim U(100{,}120)$
$\mathrm{IHCPD}^{m}_{d,p,t}$	$\sim \mathcal{N}(5{,}1)$	$\mathrm{SSPD}^{m}_{d,p,t}$	$\sim U(100{,}120)$

Table 13. Numerical results

No	α	β	γ	$1-\gamma$	$\mathrm{OF}_{\mathrm{pess}}$	$\mathrm{OF}_{\mathrm{opt}}$	$\mathrm{OF}_{\mathrm{HC}}$	CPU
1	0.1	0.9	0.1	0.9	5806307.718	7399502.389	5965627.185	5.163
2	0.3	0.9	0.5	0.5	5090273.767	8256133.727	6673203.747	5.304
3	0.5	0.9	0.7	0.3	6632577.814	6632577.814	6632577.814	5.919
4	0.7	0.9	0.9	0.1	8227141.566	5062803.600	5379237.397	5.895
5	0.9	0.9	0.3	0.7	7445912.271	5855986.948	6968934.674	6.114

It can be seen from Table 12 that when $\alpha \leq 0.5$, the optimal objective value increases with the increase of α; when $\alpha > 0.5$, the optimal objective value increases with the increase of α. Furthermore, the optimal objective values are symmetrical with respect to $\alpha = 0.5$. When $\alpha \leq 0.5$ and γ increases (between N° 1 and N°2), the optimal objective value increase; when $\alpha > 0.5$ and γ increases (between N° 4 and N°5), the optimal objective value increases. Thus, the sensitivity analysis can help the company to make decisions on different optimal trade matching pairs by varying parameters. The optimal solutions with respect to the case when run N°3 ($\alpha = 0.5, \beta = 0.9, \gamma = 0.7$) are presented in Tables 14–19.

Table 14. Quantity of product p produced at each unit j of each site s in each period t

			T1	T2	T3	T4	T5	T6
P1	S1	J1	2703,47	2703,47	2703,47	2703,47	2703,47	2703,47
	S3	J1	2494,109	2648,288	2648,288	109,6456	2648,288	
P2	S1	J3	2586,503	2586,503	2178,817	2586,503	2586,503	2586,503
	S4	J3	2863,565	115,7333	1900,267	1331,4	2247,555	
P3	S1	J1	1998,6	1653,96	2325,213	2905,361	1636,32	2589,48
	S3	J1	2009,255	1375,185	1395,346		1755	
P4	S1	J4	2635,886	2635,886	2635,886	2635,886	2635,886	2635,886
P5	S1	J4	2787,47	2864,546	2449,666	2154,198	1124,76	2304,12
P6	S1	J2	2621,754	2919,354	1088,984	1721,331	1863,266	2121,808
	S2	J2					2048,76	
	S4	J2			1035,443	1422,45	916,243	
P7	S1	J2	1115,76	2078,815	803,0201	1857,309		1347,348
	S2	J2					923,1719	
	S4	J2	1006,605			141,8908		

Table 15. Inventory level of product p in production site s at the end of period t.

		T2	T3	T4	T5
S1	P1				238,2841
	P2		134,2574		
S3	P1	482,8371		109,6456	
S4	P2	115,7333			

Table 16. Total quantity dispatched from production site s to DC d by vehicle v in period t

			T1	T2	T3	T4	T5	T6
S1	D1	V1		297,597	366,8156	430,7122		547,0888
		V2					539,9417	
		V5	294,1428					
	D2	V2	284,2122	326,8436	284,2122			284,2122
	D3	V1		297,597		390,6227		
		V2	285,0837					
		V5	866,9433	866,9433	866,9433	866,9433	1161,086	866,9433
	D4	V1	1191,085	595,8908	824,2693	369,7499	1191,085	643,996
		V2	284,2122				284,2122	
		V5		294,1428	294,1428	294,1428		294,1428
S2	D1	V4					253,663	
S3	D1	V6	798,4044	1095,436	798,4044	1095,436	798,4044	1095,436
		V7	1067,804	834,7718			1067,804	
	D2	V6	297,0312					
		V7		233,0326				
	D4	V6			297,0312		297,0312	
		V7			1067,804	1067,804		1067,804
S4	D1	V8	1027,846	1027,846	1027,846	813,8158	1027,846	1027,846
		V9	803,1557	1005,565	803,1557	1005,565	803,1557	1005,565
	D2	V9	202,4089					
	D4	V8				214,0306		
		V9			202,4089		202,4089	

7. Conclusion

To cope with the problem of integrating production and distribution in a milk supply chain under uncertainty, a credibility-based fuzzy mathematical programming model with the Hurwicz criterion is proposed in this work. The proposed model is a mixed integer linear programming which takes into account technological constraints arising in dairy industry and aims to maximize the total profit including the total costs such as production, storage, and distribution. To bring the model closer to the real-world planning problems, the objective function coefficients (e.g. production cost, inventory holding and transport costs) and other parameters (e.g., demand, production capacity, and safety stock level) are all considered fuzzy numbers. To overcome the extreme cases of optimistic and pessimistic criterions, the Hurwicz criterion is used for the problem. By varying the value of θ, it can balance the optimistic and pessimistic levels of the decision makers. Moreover, we use the different property of the credibility measure to build the crisp equivalent model which is a MILP model that can solve problems by using a commercial solver such as GAMS software. Finally, a real case study is also reported to show the practicality of the proposed model. The proposed model considers the fuzziness as the source of uncertainty. However, future research could be aimed at addressing hybrid uncertainties, such as an encounter with fuzziness and roughness simultaneously. For example, it is widely accepted that the demand of customer is presented as a triangular fuzzy number (a, b, c) variable from the viewpoint of the fuzzy theory, but the values of a, b and c may emerge with incomplete or uncertain information. In a sense, the values of a, b and c are of rough characteristics. Thus decision-makers have to face the fuzzy number with rough parameters. In this case, the client's demand should be more appropriately represented as the so-called fuzzy rough variable (Liu and Liu 2002), and also, considered multi-objective such as the service level and environmental aspects.

Table 17. Inventory level of product p in DC d at the end of period t.

		T1	T2	T3	T4	T5	T6
D1	P1	188,3501	116	116	116	116	116
	P2	116	116	116	116	278,8174	116
	P3	116	116	116	116	116	116
	P4	116	116	116	116	116	116
	P5	116	116	116	116	116	116
	P6	198,4846	677,3058	116	116	1143,32	116
	P7	116	629,24	116	116	309,2119	116
D2	P1	221,9086	116	206,6458	206,6458	206,6458	116
	P2	1081,827	116	116	116	116	116
	P3	356,9749	116	116	116	116	116
	P4	116	116	116	116	116	116
	P5	116	116	116	116	116	116
	P6	173,5895	116	768,37	768,37	768,37	1335,547
	P7	232,965	116	116	116	116	116
D3	P1	116	619,6201	619,6201	116	116	116
	P2	116	116	116	116	116	116
	P3	116	116	116	116	116	116
	P4	116	116	116	116	116	116
	P5	116	116	116	116	116	116
	P6	116	479,8414	479,8414	116	116	116
	P7	116	116	116	116	116	116
D4	P1	116	116	1344,11	116	1611,2	116
	P2	116	116	116	116	1428,08	116
	P3	116	116	533,6786	116	756,3197	116
	P4	116	116	116	116	252,6259	116
	P5	179,03	753,7361	963,9622	869,48	116	116
	P6	116	246,0412	284,4045	1723,947	3832,296	4381,447
	P7	116	305,8599	116	609,08	116	116

Table 18. Total quantity dispatched from DC d to customer region c by vehicle v' in period t

			T1	T2	T3	T4	T5	T6
D1	C1	W5				215,3561	215,3561	
	C2	W5				215,3561		
	C3	W2		225,7196	225,7196			225,7196
		W5	1049,883		1049,883	619,1713	834,5274	834,5274
	C4	W5						215,3561
	C5	W2	225,7196		225,7196		229,0336	225,7196
		W5		1049,883				
	C6	W2					229,5086	
D2	C1	W3	214,6419	214,6419	214,6419			234,7192
	C4	W3	214,6419	214,6419				
	C6	W3	214,6419	214,6419				
D3	C1	W7	910,4892	910,4892	1139,896	681,0826	1139,896	1139,896
	C3	W7	229,4066			229,4066		
	C5	W7		229,4066		229,4066		

Table 18. Continued

			T1	T2	T3	T4	T5	T6
D4	C1	W9	794,0353	796,7066	342,2958	562,097	558,5211	563,042
	C2	W9	223,0472	220,376	217,2707		232,7639	227,8817
	C4	W9			231,3531	228,589	225,7976	
	C6	W9			226,163	226,3965		226,1589

Table 19. Quantity of lost sale for product p incurred by customer region c in period t.

		T1	T2	T3	T4	T5	T6
C1	P1	282,56	288	287,36	280,96	280,96	298,24
	P2	244,48	604,8853	274,88	259,84	248	268,16
	P3	208,32	210,56	215,68	213,44	208,96	203,52
	P4	454,0245	218,24	469,8192	466,652	418,009	204,48
	P5	140,8	146,56	136	142,08	142,72	152,64
	P6	135,68	123,84	125,44	135,04	130,88	131,84
	P7	98,24	100,16	88,96	93,76	87,36	95,36
C2	P1	293,76	286,72	285,76	277,12	290,24	281,92
	P2	246,08	250,56	261,76	261,12	256,32	246,08
	P3	210,56	213,44	212,16	217,92	212,48	200,96
	P4	300,4671	383,8912	464,7357	502,1585	213,44	214,08
	P5	141,76	143,36	144	148,8	144,32	145,28
	P6	125,76	129,92	127,68	123,2	131,84	131,84
	P7	98,24	95,68	93,76	96	95,36	92,8
C3	P1	291,84	282,56	276,8	283,52	1045,16	289,6
	P2	253,44	254,72	251,2	258,88	918,72	258,56
	P3	213,12	207,04	215,36	209,92	741,24	207,36
	P4	235,1953	265,4873	268,1948	225,8028	697,16	358,913
	P5	145,28	144,96	142,08	144	498,8	145,92
	P6	132,8	126,08	125,76	130,56	436,16	129,92
	P7	98,88	94,08	104,32	94,08	342,2	104
C4	P1	285,12	289,92	290,24	276,16	280	284,8
	P2	269,44	333,6142	257,28	254,72	253,44	254,08
	P3	207,36	195,2	200,32	206,72	212,16	201,92
	P4	509,3988	327,7586	210,24	210,56	200	438,2456
	P5	143,36	146,24	152,32	140,8	142,72	143,68
	P6	120,32	117,44	127,36	133,12	123,84	133,12
	P7	97,6	99,52	97,92	97,6	92,48	93,44
C5	P1	284,16	296,32	291,52	289,92	287,68	282,56
	P2	259,84	255,8105	252,8	256,64	258,88	266,56
	P3	205,12	209,6	207,04	209,28	213,44	214,72
	P4	267,0339	203,2	247,9247	215,61	212,48	293,0498
	P5	148,48	145,92	138,88	137,92	136,32	149,12
	P6	126,08	130,56	128,96	136,96	125,44	128,32
	P7	95,36	84,48	91,2	94,72	94,08	94,4
C6	P1	297,92	287,36	288,64	281,92	281,28	287,68
	P2	258,24	253,44	248,96	252,48	263,04	253,76
	P3	213,76	209,92	207,68	208,64	200,96	201,92
	P4	543,0747	523,1772	199,36	263,851	218,6912	200,96

Table 19. Continued

C6	P5	141,44	145,28	139,84	143,04	149,44	141,12
	P6	128	135,68	124,8	128,96	132,8	119,36
	P7	98,88	94,72	97,6	97,6	96,64	106,88

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