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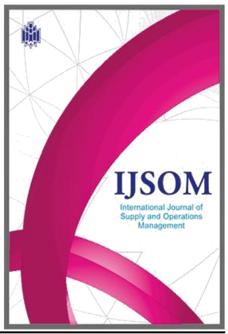

A Novel Algorithm for Estimating Reliability of Ready-to-use Systems in Designing Phase for Designed Lifetime Based on Markov Method and Fuzzy Approach

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Abstract
Reliability is one of the most important factors of complex systems which play a crucial role in the performance of modern systems. In this study, a novel algorithm for estimating reliability of ready-to-use systems in designing phase for designed lifetime is proposed. At the first stage, the related studies are checked, and then fundamental theories of each section are presented. With reference to the particular structure of ready-to-use systems and Markov Chain conditions, a new model based on Markov method and Fuzzy approach is suggested. The performance of proposed model is validated by testing on a real system. Therefore, the reliability and mean time to failure of the industrial system is estimated by the algorithm. Finally, practical suggestions are recommended for optimizing the system reliability.

Keywords: Reliability; Ready-to-use systems; Markov chain; Fuzzy theory; Designed lifetime; Designing phase.

1. Introduction

Reliability is the success probability measurement of a system in supporting its performance properly during its designed mission time. Complex systems are classified into some categories including ready-to-use systems, in which they are formed by diverse components with unknown reliabilities. Designed life cycle of ready-to-use systems is not restricted to one mission. In this type, the system is stored after the first operation and will begin the second mission after the depository process. By finishing the mission, the system is inspected and expected corrective maintenance is applied, followed by moving the system to storehouse, until the next mission turns. In case of occurring failures in ready-to-use systems, there will be no opportunity to compensate for the error. Hence, improving the reliability of ready-to-use systems has a far-reaching effect on the performance of the whole system. By the first failure during operation time, system will completely fail in mission performance (Billinton, R. & Allan, R. N., 1998). Therefore, failure probability of ready-to-use systems is accepted in a certain level. Consequently, an approach is required to estimate the reliability of a ready-to-use system in its designing phase. By means of that, reconnaissance of potential failures is performed in its infant life cycle and suggestions for improving the reliability may be made.

Bucci et al. (2008) proposed a dynamic model of fault tree service by mixing Markov theory and cell-to-cell mapping technique (CCMT). They demonstrated that service fault tree is applicable for all probability risk assessment (PRA) systems. Shalev & Tiran (2007) analyzed service fault tree in designing phase. They applied updated failure rate values to fault tree system by using Condition-based maintenance methods. In this method, by periodically calculating the highest failure rate and its effects, system failure probability is assessed. Jianzhong & Julian (2011) evaluated the safety of flight control system by applying Markov model. They compared the obtained results of Markov model by fault tree service. . The result showed that the Markov model was more accurate than the fault tree service approach. Dominguez-Garcia et al. (2008) proposed a model to assess the reliability of dynamic fault tolerant systems. They used Markov chainmodel to evaluate the performance of flight control system in a fighter plane. Lu & Wu (2014) considered Markov chain model for predicting the failure and repair time of each component, in order to evaluate the reliability of -

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Comprehensive fuzzy system. Bobbio et al. (2001) presented a generic model of system reliability by Bayesian network concept. Also they converted a fault tree into Bayesian network. Levitin et al. (2017) proposed an approach for evaluating reliability of standby systems composed of multi-state elements with constant state transition rates. They suggested an iterative algorithm for reliability evaluation based on element state probabilities. Li et al. (2017) concentrated on the development of reliability measures for a repairable multi-state system which operates under dynamic regimes under the discrete-time markov. Manesh et al. (2018) indicated that the state space diagram of the system must be simplified in order to use markov method for complex systems. For this purpose, they removed many states of the state space diagram and proposed a method that can calculate the probabilities by taking into account minimum states and also they indicated that the new procedure can predict state probabilities with high accuracy. Zhou, Q. et al. (2018) proposed a quantitative reliability analysis model based on fuzzy logic theory, Bayesian network, and cognitive reliability & error analysis method for the tanker shipping industry. VanDerHorn, E. et al. (2018) presented a methodology for updating the model parameters using abstracted data forms through a bayesian network. They stated in the context of reliability analysis, a common form of available information is summarized reliability data for various mechanical components (e.g., failure rates or failure probabilities) instead of detailed actual test data. Sihombing, F. et al. (2018) presented new algorithms that handle and solve a fault tree by taking advantage of the new state of the art in parallel computing. Kharazmi and Saadatinik (2016) introduced a new family of lifetime distributions called hyperbolic cosine – F (HCF) distribution. Kharazmi, O. (2017) focused on a special case of HCF family with exponentiated exponential distribution as a baseline distribution (HCEE). In the present study, we propose a novel algorithm for estimating reliability of ready-to-use systems. Fuzzy theory is benefitted, in order to improve the accuracy of results.

2. Formulation of the model

2.1. Markov Chain

Markov chain is a memoryless process. It means, the conditional probability of next event, only depends on the present state, not on previous ones. Markov chain is a discrete random process in time with a particular state, which changes randomly in each step. The changes in Markov chain model can be formulated by physical distance or any other discrete variables (Billinton, R. & Allan, R. N., 1998).

2.2. Fault Tree Analysis (FTA)

FTA is a logical and graphical diagram that describes the failures and their reasons. In graphical aspect, the FTA uses some special signs to mention all failures of system and subsystems (Clemens, P.L., 1992).

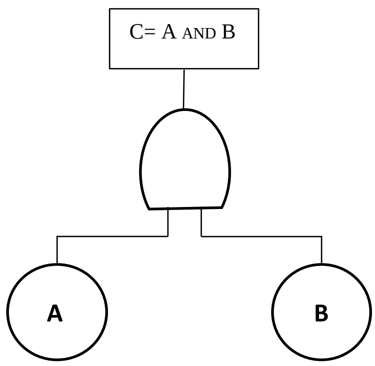
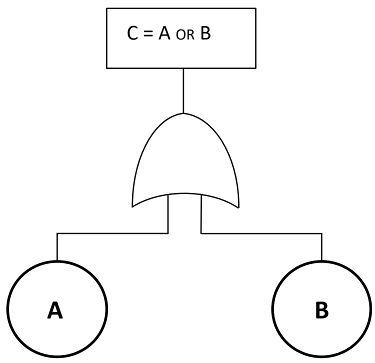


Figure 1. Parallel Fault Tree: AND Gate



Figiure 2. Series Fault Tree: OR Gate

2.3. Triangular fuzzy numbers

Triangular fuzzy numbers are described by three real numbers as (U, M, L). M is the most probable fuzzy number. These numbers can be described by a membership function which includes two linear sections connected at (M, 1) point. In previous studies, it was demonstrated that probable values of FTA can be considered as triangular fuzzy numbers (Verma, A. K et al., 2004).

2.4. Bayesian network

Bayesian network is a graphical diagram which includes knots for expressing variables, and arrows for representing the relationships between knots. Bayesian network is not a circular graph. This network is applicable to calculate the probability of any variable (knot) which is affected by some other variables (Bobbio, A. et al., 2001).

3. Assumptions

We consider four assumptions for the present study as follows:

- Ready-to-use systems are memoryless. It means the next state of the system is only affected by the present state.
- In idle time of a system, no failures are occurred. Moreover, depreciation rate is negligible.

- All essential repairs for ready-to-use systems are performed in idle state of system. So, repair times are also ignored.
- Failures are independent from each other.

4. Methodology of research

Discrete Markov method is the base of the proposed algorithm, due to considering the system as a repairable one. Also, we can consider a constant failure rate in designed lifetime of system. Moreover, the idle time of system can be ignored by the discrete behavior of Markov chain.

5. Steps of proposed algorithm

5.1. step 1: system inspection

It is mandatory to recognize all system features and its functions. It is impossible to define reliability indicators without understanding the properties of the system.

5.2. step 2: defining the number of subsystems and their connections

To facilitate reliability testing, the system should be split into several subsystems. The connection network of subsystems can be in series, parallel, series-parallel or parallel-series mode.

5.3. step 3: defining state space diagram and possible changes

The order of state space is all possible system conditions due to the performance or failure of subsystems. All possible states for a system are recognized and also possible state changes are defined (Billinton, R. & Allan, R. N., 1998).

5.4. step 4: failure rate calculation in each mission for all subsystems

Due to being in designing phase of a system and the lack of functional information about performance of the system, a FTA is needed to recognize the potential failures of system and their failure rates. First, all probable failure of each subsystem is recognized and fault tree of each subsystem is depicted in several levels. Then for calculating the marginal probability values of FTA, similar failure data of the system is used by referring to reference documents in intended industry

Then, the exact obtained results are converted into triangular fuzzy numbers as follows (Verma, A. K et al., 2004):

$$E^i = \left(m^i - d^i, m^i, m^i + d^i\right) \tag{1}$$

To obtain the unity value of each considered marginal probability, (2) can be expressed.

$$\begin{cases} g = \frac{1}{n}\sum_{i=1}^{n} d^i \\ f = \frac{\min m^i + \max m^i}{2} \end{cases} \quad F = (f - g\ , f\ , f + g) \tag{2}$$

FTA is converted into Bayesian network. In this step, the variables in FTA are changed into knots of Bayesian network, and the “OR” and “AND” gates are transformed into lines. Figure 3 and Figure 4 depict the related Bayesian network of FTA represented in Figure 1 and Figure 2 (Bobbio, A. et al., 2001).

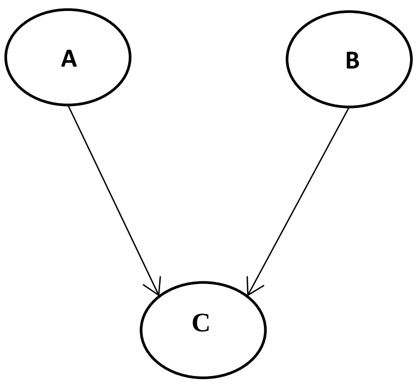


Figure 3. Parallel Bayesian Network: AND node

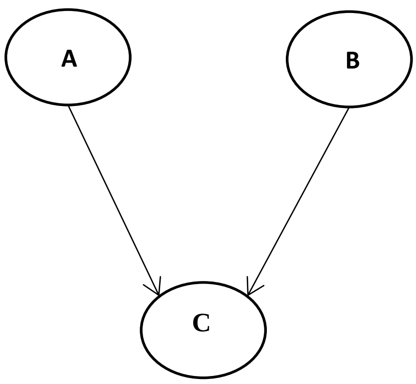


Figure 4. Series Bayesian Network: OR node

In the following, since failures are considered independent of one another (assumption number 4), a joint distribution function can be obtained to calculate the probability of failure. In cases where the above assumption is not true, we use the Markov-Monte Carlo method for deduction. (3) is used f for series bayesian network illustrated in Figure 4. In the following relationships, zero represents that event does not occur and one indicates occurrence (Bobbio, A. et al., 2001).

$$p\ (\ c = 1\ \mid\ A = 0, B = 0\ ) = 0$$
$$p\ (\ c = 1\ \mid\ A = 1, B = 0\ ) = 1$$
$$p\ (\ c = 1\ \mid\ A = 0, B = 1\ ) = 1 \quad (3)$$
$$p\ (\ c = 1\ \mid\ A = 1, B = 1\ ) = 1$$

Finally, the probability of event C can be obtained as follows:

$$p\ (c = 1) = \ p\ (c = 1 \mid\ A = 0, B = 0\ )p\ (\ A = 0, B = 0) +$$
$$p\ (\ c = 1\ \mid A = 0, B = 1\ ).\ p\ (\ A = 0, B = 1) +$$
$$p\ (\ c = 1\ \mid A = 1, B = 0\ ).\ p\ (\ A = 1, B = 0) + \quad (4)$$
$$p\ (\ c = 1\ \mid\ A = 1, B = 1\ ).\ p\ (\ A = 1, B = 1)$$

Assuming the independence of the occurrence of failures A and B with the above conditional relations, the following formula may be obtained.

$$p\ (c = 1) = p\ (\ A = 0, B = 1) + P\ (\ A = 1, B = 0) + P(A = 1, B = 1) \quad (5)$$

Complementary event probability can be used to simplify the above mentioned equations.

$$p\ (c = 1) = 1 - \ p\ (c = 0) = 1 - P[\ A = 0, B = 0] = 1 - P(\ A = 0) * P(B = 0) \quad (6)$$

It is possible to extend the (6) for Series Bayesian network. Therefore, by making failure C happen out-of n available failures, we have:

$$p\ (c = 1) = 1 - \ p\ (c = 0) = 1 - P[\ A_1 = 0, A_2 = 0, \dots, A_n = 0] \quad (7)$$

Regarding the independence of failure occurrences, it can be obtained:

$$p\ (c = 1) = 1 - [\ P(A_1\ = 0) * P(A_2 = 0) * \dots * \ P(A_n = 0)\ ] \quad (8)$$

Also for parallel bayesian network depicted in Figure 3, the following conditional relations are derived (Bobbio, A. et al., 2001):

$$p\ (\ c = 1\ \mid\ A = 0, B = 0\ ) = 0$$
$$p\ (\ c = 1\ \mid\ A = 0, B = 1\ ) = 0$$
$$p\ (\ c = 1\ \mid\ A = 1, B = 0\ ) = 0 \quad (9)$$
$$p\ (\ c = 1\ \mid\ A = 1, B = 1\ ) = 1$$

The calculation of the probability of occurrence in this case is similar to that of the previous relation to the probability of occurrence of C, and by placing the above conditional relations, we get the following relationship:

$$p\ (c = 1) = P(A = 1, B = 1) = P\ (\ A = 1) * P\ (\ B = 1) \quad (10)$$

We can extent the (10) for Parallel Bayesian network. Therefore, by making failure C happen out-of n available failures, we have:

$$p\ (c = 1) = \ P\ [\ A_1 = 1, A_2 = 1, \dots, A_n = 1] \quad (11)$$

According to independence of failure occurrence, (11) can be expressed as (12).

$$p(c = 1) = P(A_1\ = 1) * P(A_2 = 1) \dots P(A_n = 1) \quad (12)$$

5.5. step 5: calculation of the repair probability for subsystems

The total and conditional probability rules and Bayes' theorem are applied to calculate the repair probability of each component. The potential failures are categorized as repairable and non-repairable. Hence, the repair probability of potential repairable failures is considered as one, and for non-repairable failures as zero. For series bayesian network depicted in Figure 4, p (a), p(b) and p(c) indicate the failure probability of a, b and c respectively, and P(R_C) resembles the repair probability of c (Ross, S. M., 2014).

$$\mathrm{P}(\ R_C = 1) = \mathrm{P}(\ R_C = 1 \mid C = 1) = P(R_C = 1\ \mid \text{at least A = 1 or B = 1})$$
$$= (P[(R_C = 1) \cap (\ \text{at least A = 1 or B = 1})])/(P\ (\ \text{at least A = 1 or B = 1}\ )\ )$$
$$= (P\ [(R_C = 1) \cap\ (A \cup B)])/(P\ (A \cup B\ )\ ) \quad (13)$$
$$= (P\ [[(R_C = 1) \cap A] \cup [(R_C = 1) \cap B]])/(P(A \cup B\ )\ )$$
$$= (P[(R_C = 1) \cap\ A] + P[(R_C = 1) \cap B] - P[[(R_C = 1) \cap A] \cap [(R_C = 1)\ \cap B]])/(P(A) + P(B) - P(A \cap B))$$

In order to summarize the equations in this article $P(A = 1)$ is written as $P(A)$.
According to independence of events, the following equation can be made.

$$\mathrm{P}(R_C) = (P[(R_C = 1) \cap A] + P\,[(R_C = 1) \cap B] - P\ \ [[(R_C = 1) \cap (A \cap B)]])/(P(A) + P(B) - P(A) * P(B)) \tag{14}$$

The Bayes' theorem extents the Eq. (14) as follows:

$$P(R_C = 1) = (P[(R_C = 1)\ \mid A] * P(A)\ + P\,[(R_C = 1)\ \mid B] * P(B) - P\,[[(R_C = 1) \mid (A \cap B)]] * P(A) * P(B))/(P(A) + P\,(B) - P(A) * P\,(B)) \tag{15}$$

If in Series Bayesian network, one parent knot is made from more than 2 knots, the probability of repair on the condition of breakdown C is converted as follows, which is the main aspect of the presented method:

$$P(R_C = 1) = \sum_{i=1}^{n} P[(R_C = 1) \mid E_i\,] * P(E_i) - \sum_{i_1 < i_2}^{n} P\,\left[\left[(R_C = 1)\ \mid \left(E_{i_1} \cap E_{i_2}\right)\right]\right] * P\left(E_{i_1}\right) * P\left(E_{i_2}\right) + \sum_{i_1 < i_2 < i_3}^{n} P\,\left[\left[(R_C = 1)\ \mid \left(E_{i_1} \cap E_{i_2} \cap E_{i_3}\right)\right]\right] * P\left(E_{i_1}\right) * P\left(E_{i_2}\right) * P\left(E_{i_3}\right) - \cdots + (-1)^{n+1} P\,\left[\left[(R_C = 1)\ \mid \left(E_{i_1} \cap E_{i_2} \cap \ldots \cap E_n\right)\right]\right]\ /\ \sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2}^{n} P\left(E_{i_1}\right) * P\left(E_{i_2}\right) + \sum_{i_1 < i_2 < i_3}^{n} P\left(E_{i_1}\right) * P\left(E_{i_2}\right) * P\left(E_{i_3}\right) - \cdots + (-1)^{n+1}\ P\left(E_{i_1}\right) * P\left(E_{i_2}\right) * \ldots * P\left(E_n\right) \tag{16}$$

It should be noted that the following relation has been used to prove the Eq. (16).

$$p(E_1\ \cup \ldots \cup\ E_n) = \sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2}^{n} P\left(E_{i_1} \cap E_{i_2}\right) + \sum_{i_1 < i_2 < i_3}^{n} P\left(E_{i_1} \cap E_{i_2} \cap E_{i_3}\right) - \cdots + (-1)^{n+1}\ P\left(E_{i_1} \cap E_{i_2} \cap \ldots \cap E_n\right) \tag{17}$$

In parallel bayesian network, depicted in Figure 3, to calculate the repair probability on the condition of breakdown C, we have:

$$\begin{aligned} \mathrm{P}(R_C = 1) &= \mathrm{P}(R_C = 1 \mid C = 1) = P(R_C = 1\ \mid \mathrm{A} = 1\,, \mathrm{B} = 1) \\ &= (P[(R_C = 1) \cap (\mathrm{A} = 1\,, \mathrm{B} = 1)])/(P(\mathrm{A} = 1\,, \mathrm{B} = 1\,)\,) \\ &= (P\,[(R_C = 1) \cap (A \cap B)])/(P(A \cap B\,)\,) \end{aligned} \tag{18}$$

According to Bayes' theorem:

$$P(R_C = 1) = (P\,[[(R_C = 1) \mid (A \cap B)]] * P(A) * P(B))/(\,P(A) * P(B)\,) \tag{19}$$

As a result, if the value of $P\left[\left[(R_C = 1) \mid (A \cap B)\right]\right]$ is one, the repair probability of C would be one, and if the value of $P\left[\left[(R_C = 1)\ \mid (A \cap B)\right]\right]$ is zero, the repair probability of C would be zero, too.

5.6. step 6: calculation of failure rate and repair probability of main system

Draw the bayesian network for whole system. Similar to step 4 and 5, with failure rate and repair probability of each subsystem, calculate the failure rate and repair probability of whole system.

5.7. step 7: determining stochastic transitional probability matrix

According to probable spaces in step 3, probability matrix of variable states transition is defined. Elements of this matrix indicate the probability of possible states transition, which are failures probability and the whole system repair probability obtained from step 6. The sum of elements for each row is equal to one, but the sum of the elements of a column is not necessarily one (Billinton, R. & Allan, R. N., 1998).

5.8. step 8: evaluating time dependent probabilities

The probability that the system will function correctly after the m time stages: by considering the transition matrix, we can obtain distribution function of Xn|1 and by defining state *n+1*th distribution function of Xn|2 can be obtained. With this recursive method, Xn|m can be obtained. while the distribution function of system state in first state is obtained, to define a direct relation between Xn and Xn|m and calculating distribution function of system in state n, we can use (20) (Billinton, R. & Allan, R. N., 1998).

$$P(m)\ =\ p(0).\ p^{m} \tag{20}$$

Therefore, the element of transition matrix with m state is P_{ij}^{m} . This matrix should be evaluated by the first probable condition vector $p(0)$ which resembles the first state.

5.9. step 9: evaluating limiting states probabilities (reliability in designed lifetime)

Due to Ergodic system of Markov chain, by increasing m, the probability values of the states are closer to each other and eventually become the same value, and the probability values of the system state are obtained. Consequently, regardless of the system's start state, the probability of reaching the K state in the long run is constant. We show this value with p_k (Billinton, R. & Allan, R. N., 1998).

$$p_k = \lim_{m\to\infty} P_{ik}^{(m)} \tag{21}$$

Therefore, by increasing m, matrix p^m tends to a matrix in which all elements in each column, such as column I, would equal to Pi. Therefore by continuing the multiplication of the matrix in the preceding step, limit conditions are eventually achieved and the values of the matrix components does not change. We can define the probability vector (α) in the form of a row matrix with elements p_i and from the order of 1 × K (k is the number of possible states of the system) as follows:

$$\alpha * p = \alpha \tag{22}$$

After the formation of the above equations, for solving the model, we need an independent equation, since the two equations are obtained. The completed equation is:

$$\sum_{i=1}^{k} p_i = 1 \tag{23}$$

After the model is solved, the above equations determine the reliability of the system during the designed lifetime.

5.10. step 10: Determining the absorbing states and average stages before entering these states (MTTF)

The purpose of explaining the basics of absorbing states in this step of algorithm is to determine the moderate number that the system operates appropriately before entering the adverse conditions (MTTR). Absorbing mode refers to a condition in which it is not possible to exit the system. Of course, the adverse mode is not just an absorbing state, and in some cases it can be removed by performing repairs, but in practice it is considered to be a steady state, with the use of these methods, the number of steps before entering this states are defined. If P is the stochastic transitional probability matrix, then the Q matrix is formed by removing the rows and columns associated with the absorbing state. If such a matrix is formed, we will have the number of expected times as follows (Billinton, R. & Allan, R. N., 1998):

$$E\,(X) = \sum_{i=1}^{\infty} x_i p_i \tag{24}$$

The above equation is used not only for singular probabilistic members with probability of pi, but also for multi-probability matrices with Q matrices. Therefore, whenever n represents the number of times expected to occur from the repetition of the time intervals for transition states, we will have:

$$E\,(X) = 1.I + 1.Q + 1.Q^2 + \cdots + 1.Q^{n-1} \tag{25}$$

Where, I is a unit matrix representing the probability of all possible initial conditions, and the number one in each row indicates the share of the state with the same row number. Each of the numbers one in the above equation represents a stage of successive state and is equivalent to X(i) in the initial equation. The first state occurs with the probability matrix I. The second state with the Q matrix, the third state with Q ^ 2, and ultimately the system at the nth state of the time is in permanent mode. Hence, the following equation is obtained:

$$E\,(X) = N = I + Q + Q^2 + \cdots + Q^{n-1} \tag{26}$$

Because the above equation cannot be easily evaluated, instead of the (26), the following equation is used, which is actually a binomial expansion,:

$$[I - Q][I + Q^2 + \cdots + Q^{n-1}] = I - Q^n \tag{27}$$

Also when n → ∞

$$\lim_{n\to\infty} Q^n = o \tag{28}$$

Then:

$$I - Q^n \to I \tag{29}$$

As a result, (27) changes into (30) as follows:

$$[I - Q][I + Q^2 + \cdots + Q^{n-1}] = I \tag{30}$$

Or

$$[I + Q^2 + \cdots + Q^{n-1}] = [I - Q]^{-1} \tag{31}$$

Based on previous equations we have:

$$N = [I - Q]^{-1} \tag{32}$$

6. experimental results

Ultimately, the proposed algorithm has been applied to an intelligent industrial system sample to evaluate its reliability. According to the system inspection, it is clear that the system should be divided into two separate subsystems (A and B) for ease of reference. The possible mode space for the system consists of 2 modes. Mode 1 is the correct operating mode of the system and mode 2 is the state where the system is not functioning properly.

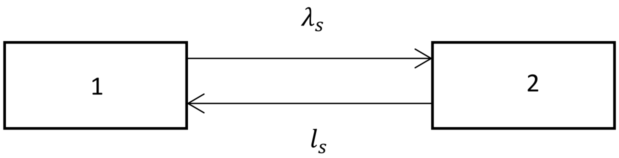


Figure 5. State space diagram of system

After plotting the Fault Tree for each subsystem and mapping it to the Bayesian network and collecting data from similar system's failure data and referring to reference documents in intended industry to determine the marginal probabilities and converting the data into fuzzy numbers, we calculate the failure rate for each mission in accordance to (8). Consequently, according to the experts' opinion, for potential failures that can be repaired, we will consider the possibility of repair on the condition of failure as one and for potential failures that are not repairable, the possibility of repair on condition of failures are zero. Then, with respect to (16), we obtain the probability of repair on the condition of failure of each subsystem. And in the next step, we calculate the total system failure rate (λ_s) and the probability of repairing the entire system (L_s). Then, we obtain the stochastic transitional probability matrix based on the state space diagram of system.

$$P = \begin{bmatrix} 1-\lambda_S & \lambda_S \\ L_S & 1-L_S \end{bmatrix} \tag{33}$$

In the next step, we calculate the probability that the system will work correctly for different missions, based on (20).

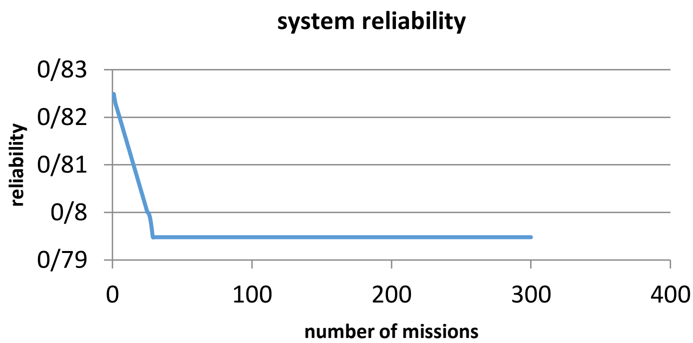


Figure 6. System reliability diagram

Then, we will determine the limiting state probabilities of the system lifetime (reliability). α is defined as follows.

$$\alpha = [\, p_1 \quad p_2 \,] \tag{34}$$

According to (22), we have:

$$[\, p_1 \quad p_2 \,] * \begin{bmatrix} 1-\lambda_S & \lambda_S \\ L_S & 1-L_S \end{bmatrix} = [\, p_1 \quad p_2 \,] \tag{35}$$

The following system of equations can be obtained:

$$\begin{cases} (1-\lambda_S)\, p_1 + L_S . p_2 = p_1 \\ \lambda_S . p_1 + (1-L_S)\, p_2 = p_2 \end{cases} \tag{36}$$

Due to recursive equations, (23) can be used to complete the system of equations. So we have:

$$\begin{cases}(1-\lambda_S)\,p_1 + L_S.p_2 = p_1 \\ \qquad p_1 + p_2 = 1\end{cases} \tag{37}$$

After solving the above system equations, system reliability (p_1) will be obtained based on given parameters.

$$p_1 = \frac{L_S}{L_S + \lambda_S} \tag{38}$$

By setting the values of the parameters (which are fuzzy numbers) in the above equation and performing fuzzy calculations, the reliability value for this system is obtained as follows.

$$(R_S) = p_1 = [0/728\,,\ 0/794\,,\ 0/889\,] \tag{39}$$

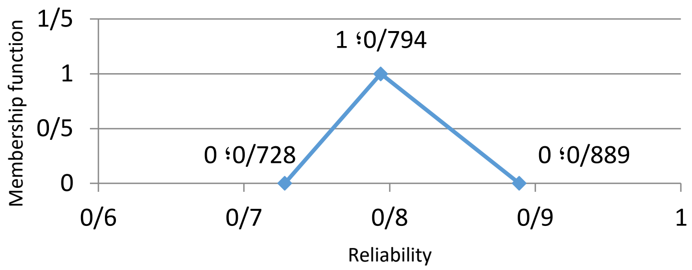


Figure 7. Representation of fuzzy system reliability

In the final step, according to (32), the value of MTTF of the system can be calculated as 5/70. With regard to the calculated system reliability, which is fuzzy and has a core of approximately 79.4%, as well as MTTF of 5/70 missions, it is obvious that the improvements are essential. It should be noted that the multiplicity of series systems in the structure of the system, as found in the associated Fault Tree, justifies this value.

7. Conclusion

These calculations help in two ways. First, the failure mode and effects analysis (FMEA) tool identifies errors in design phase and prioritizes them. Second, in terms of budgeting, it determines how much to invest in each piece and how much it will cost. Because the final result should always keep the system in terms of cost-effectiveness and efficiency at the optimal point, by increasing the complexity of a system, whether in terms of increasing the number of components or the combination of multiple mechanical and electronic systems, as well as evaluating the construction and design costs to meet the reliability limitation, the approaches of the meta-heuristic algorithms such as Genetic Algorithm or Hybrid Algorithms would be applicable. Creating a new method for calculating marginal probabilities and introducing a new method for calculating conditional probabilities that are used to calculate the probability of repair on the condition of failure of each subsystem are the most important suggestions that can be made to further this research.

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