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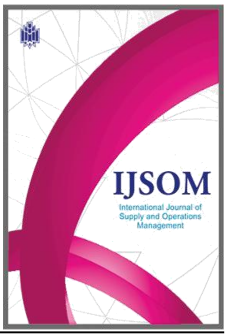

A Markov Chain Analysis of the Effectiveness of Kanban Card with Dynamic Information

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Abstract

A pull system produces products based on customer demands. Each station is isolated until a customer order is placed; then a signal or Kanban is sent from downstream station to upstream station and continues until the first station. Most studies have investigated pull systems in deterministic environments while many real production lines are subjected to different types of uncertainties. Thus, the aim of this paper is to apply a dynamic Kanban system that changes the information on the Kanban cards based on the remained inventory in the buffer. The proposed approach uses a Markov chain analysis to compare the effectiveness of the Kanban card with dynamic information to the Kanban card with static information. In the present study, the production line of two work stations and two inventory buffers is modeled. Throughput, shortage, work-in-process and cycle time are the model measurement parameters. The results show the advantages of the proposed approach.

Keywords: Pull system; Kanban; Buffer; Dynamic environment; Markov chain.

1. Introduction

The pull type production control manufactures products based on customer demands. Each station is isolated until a customer order is placed; then a signal or Kanban is sent from downstream station to upstream station and continues until the first station. This type of production control system reduces work-in-process. The production is controlled by cards known as Kanban (Gupta and Turki, 1997). Neither the flow of a product nor the processing of a part takes place without the signal of a Kanban. Kanban supports visual production control using the card of providing information to regulate the flow of inventory and materials (Jou Lin et al., 2013). It contains information such as the production rate and product name.

The number of Kanban control cards significantly influences the performance of the production system. Framinan et al. (2003) considered two aspects of this problem:

1. Card control: it changes or maintains the number of cards in pull systems based on the current conditions. Therefore, the number of cards is variable, and this can improve the performance of the production system.

2. Card setting: it determines the number of Kanban cards in order to obtain appropriate performance. In this case the number of cards is constant.

In a real-life environment, production systems are subjected to different types of uncertainties such as variable processing time and stochastic demand. Since determining the fix control system is only appropriate in a deterministic environment, the number of Kanban cards and also their information such as the production rate are seriously affected by variations in real-life environments. Most researchers did their studies on pull systems in deterministic environments while many real production lines are subjected to different types of uncertainties.

In this paper, a dynamic Kanban system is developed that changes the Kanban card information (production rate) based

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on the remained inventory in the buffer. The production line of two work stations and two inventory buffers is modeled based on Miltenburg (2010)'s model. Results of the model measurement parameters (throughput, shortage, work-in-process and cycle time) show the advantage of the proposed approach. Miltenburg modeled the simple production line and used a Markov chain analysis to examine the effect of different material flow management on the production line.
Markov Chain analysis is a well-founded analytical tool for analyzing dynamic system behavior (Kemeny and Snell, 1976). This paper begins with a description of pull systems and a literature review related to this study is presented in Section 2. Then the problem is described in Section 3. In Sections 4 and 5 a Markov chain analysis is applied to compare the effectiveness of the Kanban card with dynamic information to the Kanban card with static information. Finally, conclusions and suggestions for further research are given in Section 6.

2. Literature Review

A review of the literature on pull systems and Kanban control cards indicates that most studies investigated pull systems in deterministic environments (see Moeeni & Chang, 1990; Bitran & Chang, 1987). There are few studies on developing methods of dynamic card control. For instance, Renna (2010) considered a set of rules to change the number of cards in the system. He developed a dynamic simulation environment to study the effect of Kanban card in different dynamic states. Some performance measurements like throughput and work-in-process were evaluated and finally the proposed method was compared with a Kanban control.

Gupta and Turki (1997) developed a Kanban system which uses an algorithm to dynamically manipulate the number of Kanbans. The proposed method is a flexible Kanban system that controls the blocking and starvation under different conditions during a production cycle. Kumar and Panneerselvam (2007) examined Kanban systems and showed the importance of the dynamic number of Kanban control cards.

In another study, Shahabudeen et al. (2003) focused on the number of Kanbans at each workstation by applying a simulated annealing algorithm approach. They modeled a two-card dynamic Kanban system with different demand requirements and measured the performance of the proposed algorithm under different conditions. Schmidbauer and Riisch (1994) analyzed a dynamic model of a production pull system with *k* stations in series. They considered finite capacity buffers between the machines and end of the production line. In their model, demand for products and breakdown and repair time for the machines were stochastic. The decomposition algorithm was used to solve the model and finally simulation results showed the advantages of the model. Hopp and Roof (1998) presented an approach that dynamically changed the number of control Kanban cards based on the throughput of the system. They measured a throughput rate and then decided based on its level. If the throughput was over or under the predefined level, they added or deleted the cards to the system.

Shahabudeen and Sivakumar (2008) developed an adaptive Kanban system and applied Genetic and simulated annealing algorithms to set the design parameters of the system. They found out that simulated annealing algorithms reached better solutions with improved computational efficiency.

Conwip pull system is similar to Kanban pull system while easier to implement and apply a single card type to control the total amount of work-in-process in the production system. When a job is released to a system, a card is attached to it if cards are available in the system, otherwise, the job must wait in a backlog until another job is completed and its card is sent to the beginning of the system. In this control pull system determining the number of cards is important (Spearman et al., 1990).

Huang et al. (2013) explored the potential to combine CONWIP and Kanban methods into a hybrid pull system to have the advantages of both methods. Using the theory of token transaction systems, Sato and Khojasteh-Ghamari (2010) proposed a novel approach for card-based control of production processes, and clarified the relation of cycle time and throughput in the specific sub-network of a production process between CONWIP and Kanban methods. Gong et al. (2014) studied the performance of different production control systems (MRP, KANBAN, and CONWIP) to investigate the different amounts of position information. They concluded that the production control system with the lowest amount of information spends the least amount of time on decision-making.

Khojasteh-Ghamari (2009) showed that the Kanban is more flexible for the assembly system than the CONWIP. Specifically, sometimes if the number of Kanbans is optimally set, the Kanban system outperforms CONWIP with a lower average inventory and the same throughput. Tardif and Maaseidvaag (2001) developed a method in a make to stock environments controlled by a Conwip system. They aimed to determine the tradeoff between the backordered demand and inventory level of finished goods. They added a cart to the system if the inventory level was under a predefined control limit, and they deleted a card from the system or an extra card was added, if the inventory level was over a predefined control.

Framinan et al. (2006) presented a method for controlling cards in a Conwip system. They defined throughput rate for make to order environments, measured a throughput rate and decided based on its level. If the throughput is over a predefined upper control limit, the card is deleted from the system. In this method, a defined level is considered for the maximum number of extra cards.

3. The problem description

The proposed work is to use Markov chain in the pull system and compare effectiveness of the Kanban card with dynamic information to the Kanban card with static information. It is practically not feasible to calculate the steady-state distribution of the complex model because the state space is too large. So, the proposed production line consists of two work stations and two inventory buffers based on the model of Miltenburg (2010).

3.1. A production system model

Consider a production line of two work stations and two inventory buffers. Assume a product enters Station one and when its operation is completed it moves to the first inventory buffer and waits until it can enter Station two. At Station two another operation is completed and finally the product leaves Station two and enters the second inventory buffer (Figure. 1).

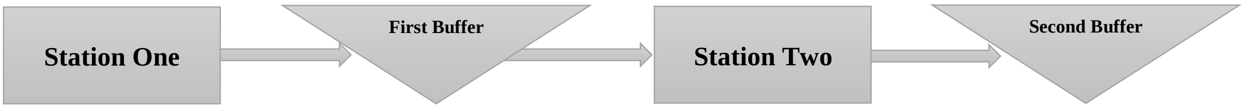

Figure 1. A simple flow shop production system

Stations 1 and 2 can break down and repair so they are up or down. Station one is blocked when the first inventory buffer is full. It can never be starved because it is that assumed there is an infinite supply of products that are released to Station one. Station two is blocked when the second inventory buffer is full and is starved when the first inventory buffer is empty.

In this work the following assumptions were considered:

1. Throughput, shortage, work-in-process and cycle time are the model measurement parameters. Throughput was defined as the rate the system generates money through sales, or the selling price minus total variable costs (Gupta, 2003). Shortage is a situation where demand for a product exceeds the available supply. Work-in-process is a company's partially finished goods waiting for completion and eventual sale or the value of these items. The items are waiting for further processing in a queue or a buffer storage. Cycle time is the period required to complete a job or task from start to finish.
2. Station one completes the operation of product with the probability of $\gamma 1$.
3. Station two completes the operation of product with the probability of $\gamma 2$.
4. Station one breaks down with the probability of $\alpha 1$.
5. Station two breaks down with the probability of $\alpha 2$.
6. Station one repairs with the probability of $\beta 1$.
7. Station two repairs with the probability of $\beta 2$.
8. Second inventory buffer reduces with one product demand with the probability of λ.

A Markov chain model is developed to analyze the production line. Each state of the Markov chain is showed by the (S1, I1, S2, I2) where S1 = {U, D}, I1= {0, 1, 2}, S1 = {U, D} and I1= {0, 1, 2}. Stations one and two are either up (U) or down (D). The inventory in the first and second inventory buffer can be {I= 0, 1, 2}. So, 2*3*2*3=36 states that can be produced in this Markov chain are listed in Table 1:

Table 1. States of the Markov chain

States number	States	States number	States	States number	States
1	D0D0	13	U0D1	25	D0U2
2	D1D0	14	U1D1	26	D1U2
3	D2D0	15	U2D1	27	D2U2
4	D0D1	16	U0D2	28	U0U0
5	D1D1	17	U1D2	29	U1U0
6	D2D1	19	D0U0	30	U2U0
7	D0D2	20	D1U0	31	U0U1
8	D1D2	21	D2U0	32	U1U1

Table 1. Continued

States number	States	States number	States	States number	States
9	D2D2	22	D0U1	33	U2U1
10	U0D0	23	D1U1	34	U0U2
11	U1D0	24	D2U1	35	U1U2
12	U2D0	19	D0U0	36	U2U2

The transition diagram for the model of considered production system is shown in Fig. 2 (parts a, b and c). To improve the readability of the transition diagram, it has been divided into three parts with each part showing some transition arc of the system.

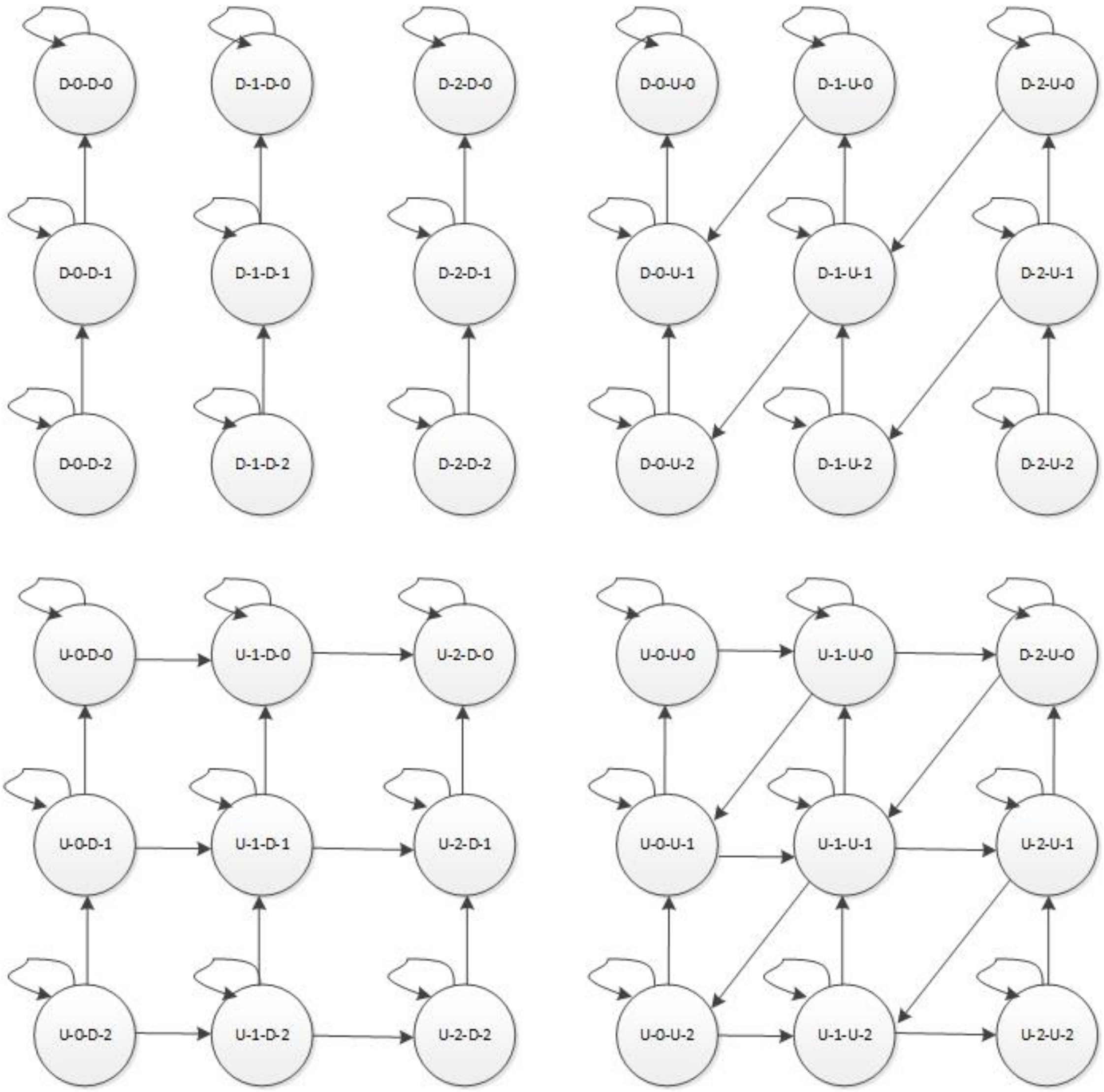

Figure 2(a). Transition diagram

The transition diagram is divided into three parts and different colors to make reading of it as simple as possible. To improve the readability of transition probability matrixes, it has been divided into four matrixes and each matrix is presented separately.

Transition Probability Matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

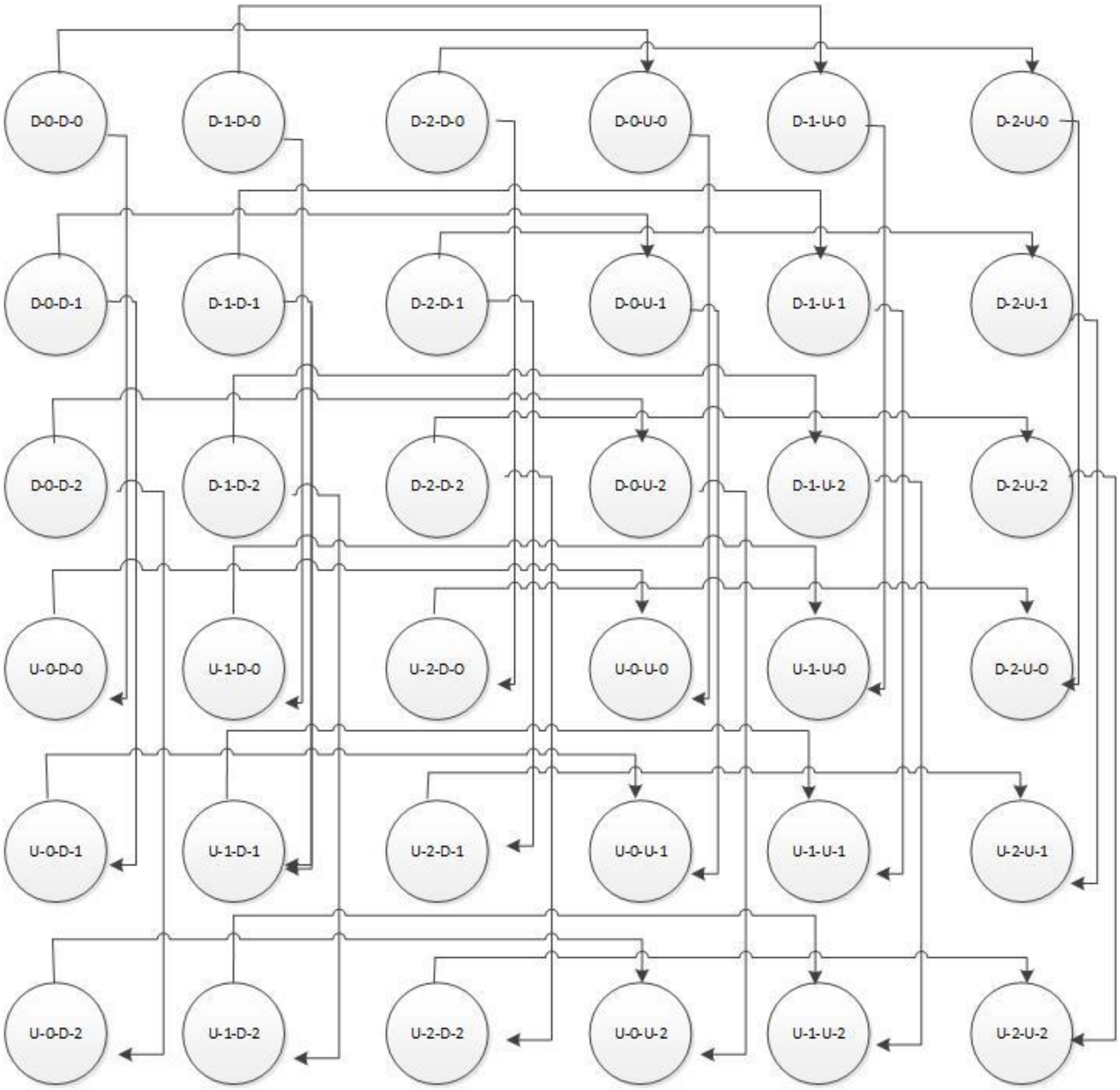

Figure 2(b). Transition diagram

Transition Probability Matrix. (A)

States	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	*	0	0	0	0	0	0	0	0	β1	0	0	0	0	0	0	0	0
2	0	*	0	0	0	0	0	0	0	0	β1	0	0	0	0	0	0	0
3	0	0	*	0	0	0	0	0	0	0	0	β1	0	0	0	0	0	0
4	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0	0	0	0	0
5	0	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0	0	0	0
6	0	0	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0	0	0
7	0	0	0	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0	0
8	0	0	0	0	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0
9	0	0	0	0	0	λ	0	0	*	0	0	0	0	0	0	0	0	β1
10	α1	0	0	0	0	0	0	0	0	*	γ1	0	0	0	0	0	0	0
11	0	α1	0	0	0	0	0	0	0	0	*	γ1	0	0	0	0	0	0
12	0	0	α1	0	0	0	0	0	0	0	0	*	0	0	0	0	0	0
13	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1	0	0	0	0
14	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1	0	0	0
15	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	0	0	0
16	0	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1	0
17	0	0	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1
18	0	0	0	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*

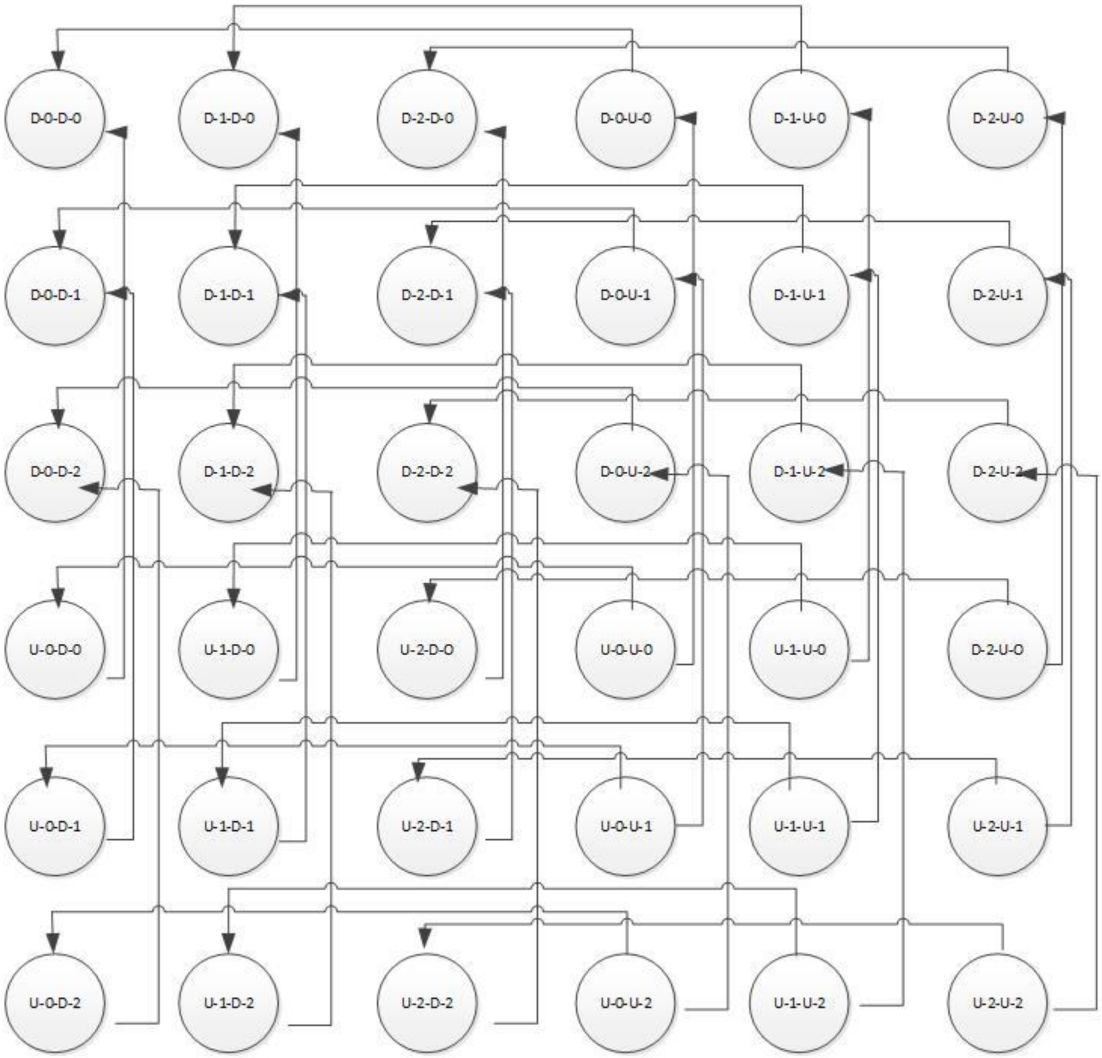

Figure 2(c). Transition diagram

Transition Probability Matrix. (B)

States	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
1	β2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	β2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	β2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	β2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	β2	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	β2	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	β2	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	β2	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	β2	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	β2	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	β2	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	β2	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	β2	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	β2	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	β2	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	β2	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	β2	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	β2

Transition Probability Matrix. (C)

States	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
19	α2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	α2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	α2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	α2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	α2	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	α2	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	α2	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	α2	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	α2	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	α2	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	α2	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	α2	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	α2	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	α2	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	α2	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	α2	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	α2	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	α2

Transition Probability Matrix. (D)

States	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
19	*	0	0	0	0	0	0	0	0	β1	0	0	0	0	0	0	0	0
20	0	*	0	γ2	0	0	0	0	0	0	β1	0	0	0	0	0	0	0
21	0	0	*	0	γ2	0	0	0	0	0	0	β1	0	0	0	0	0	0
22	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0	0	0	0	0
23	0	λ	0	0	*	0	γ2	0	0	0	0	0	0	β1	0	0	0	0
24	0	0	λ	0	0	*	0	γ2	0	0	0	0	0	0	β1	0	0	0
25	0	0	0	λ	0	0	*	0	0	0	0	0	0	0	0	β1	0	0
26	0	0	0	0	λ	0	0	*	0	γ2	0	0	0	0	0	0	β1	0
27	0	0	0	0	0	λ	0	0	*	0	γ2	0	0	0	0	0	0	β1
28	α1	0	0	0	0	0	0	0	0	*	γ1	0	0	0	0	0	0	0
29	0	α1	0	0	0	0	0	0	0	0	*	γ1	γ2	0	0	0	0	0
30	0	0	α1	0	0	0	0	0	0	0	0	*	0	γ2	0	0	0	0
31	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1	0	0	0	0
32	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1	γ2	0	0
33	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	0	γ2	0
34	0	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1	0
35	0	0	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*	γ1
36	0	0	0	0	0	0	0	0	α1	0	0	0	0	0	λ	0	0	*

*=1-(the summation of each row)

The matrixes were filled based on the states and their probabilities. For example, we explained one state as follows:

State 20: this state is (D1U0). It means statin one is up, first inventory buffer is 1(zero in the inventory buffer and one in Station two), Station two is up, and there is no product in the second inventory buffer. In the next transition, the states of Markov model will change if the following condition is met:

It goes to state two (D1D0) if machine in station two breaks down with the probability of α2. It remains in its state (D1U0) if none of other states take place. It goes to state twenty two (D0U1) if Station two completes the operation on product with the probability of γ2. In this state the first inventory buffer decreases because the inventory in Station two leaves the station and the second inventory buffer increases. It goes to state twenty nine (U1U0) if the machine in station one repairs with the probability of β1.

In the next section, we calculate the transition probability with the assumed input of the pull system and apply a Markov chain analysis to compare effectiveness of the Kanban card with dynamic information to the Kanban card with static information.

4. Kanban card with static information as a signal to handle the pull system

As mentioned before, the pull system makes products based on customer demands. Each station is isolated until a customer order is placed, then a signal or Kanban is sent from downstream station to upstream station and continues until the first station (Figure 3).

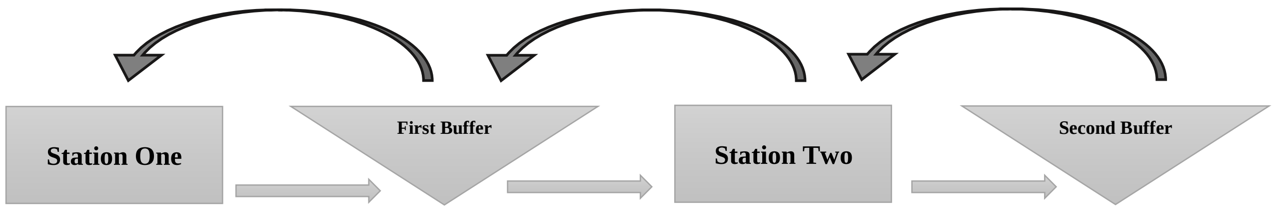

Figure 3. Pull control system

When the first and second inventory buffers decease to zero or one, Kanban cards send the signal to stations one and two to produce based on the predefined production rate. When the first and second inventory buffers are full, the Kanban cards send the signal to stations one and two to stop production temporarily. This control of the system is the control Kanban card with static information.

The predefined probability levels (α1, α1, β1, β2, γ1, λ) are calculated in (Table 2). See Miltenburg (2010). Some assumptions are considered for calculating the probabilities:

1. The planning period is 1000 hours.
2. In the production plan 240 units of products are made.
3. The number of product demands during a period of 100 hours is 40.
4. Production time at Station one is 3 hours for each unit.
5. Production time at Station two is 5 hours for each unit.
6. Time between breakdowns at station one is 9 hours.
7. Time between breakdowns at station one is 9 hours.
8. Expected repair time at station one is 1 hours.
9. Expected repair time at station two is 1 hours.

The following occurrences are expected to happen during an arbitrary length period (for example a period of 100 hours).

Table 2. Transition probability

Event	Frequency	Probability
product demand during a period	40	40/288=0.139 (λ)
Station one is up	100*(9/10)=90	90/288=0.312(β1)
Station two is up	100*(9/10)=90	90/288=0.312 (β2)
Station one is under repair	100*(1/10)=10	10/288=0.035 (α1)
Station two is under repair	100*(1/10)=10	10/288=0.035 (α2)
Station one completes the operation	90/3=30	30/288=0.104 (γ1)
Station two completes the operation	90/5=18	18/288=0.063 (γ2)
	Total=288	Total=1

Now, we can calculate the numerical transition probability matrix base on the attained probability. This matrix is called P and each cell of it is p_{ij} (transition probability from state i to state j). We define some notations and then calculate them for our Markov chain as follows. See Miltenburg (2010).

A= limiting behavior of the transition probability matrix. Each row is Π.

$$\bar{\Pi} = \{\pi_j\}$$
$$\pi_j = \sum_{i=1}^{n} \pi_i * p_{ij} \tag{1}$$
$$\sum_{j=1}^{n} \pi_j = 1$$

Π = [0.0011 0.0019 0.0047 0.0005 0.0009 0.0005 0.0002 0.0002 0.0001 0.0072 0.0151 0.0466 0.0034 0.0071 0.0064 0.0015 0.0017 0.0016 0.0107 0.0162 0.0349 0.0067 0.0111 0.0041 0.0033 0.0020 0.0011 0.0700 0.1280 0.3504 0.0465 0.0952 0.0564 0.0240 0.0231 0.0154]

Z= {zij}, Fundamental matrix. We only need the diagonal entries, so we write it here.

$$Z = \{z_{ij}\} = (I - P + A)^{-1} \quad (2)$$

Zjj= [1.8272 1.7560 1.7678 1.3921 1.3769 1.3994 1.3928 1.3909 1.4100 2.7953 2.4382 3.2140 1.8498 1.7769 2.3173 1.8479 1.8782 2.4616 3.8650 2.7618 2.6827 2.2702 1.8434 2.0418 2.2569 2.0293 2.1333 8.2289 4.2177 5.1433 3.6810 2.2443 4.4303 3.4638 3.9890 6.4405]

I=Unit matrix.

B= {bj} = the limiting variance of the number that the Markov chain is in each state;

$$b_j = \pi_j (2z_{jj} - 1 - \pi_j) \quad (3)$$

B = [0.0030 0.0049 0.0119 0.0009 0.0015 0.0009 0.0004 0.0003 0.0002 0.0332 0.0585 0.2510 0.0091 0.0181 0.0231 0.0042 0.0048 0.0061 0.0722 0.0729 0.1511 0.0237 0.0297 0.0128 0.0116 0.0063 0.0034 1.0772 0.9352 3.1310 0.2935 0.3231 0.4399 0.1417 0.1607 0.1830]

Before calculating the distribution of the number of units produced for each job we determine two sets from the Markov chain as follows:

Set 1=the states of Markov chain that product is at Station one (number of states: 10, 11, 13, 14, 16, 17, 28, 29, 31, 32, 34 and 35).

Set 2=the states of Markov chain that product is at Station two (number of states: 20, 21, 23, 24, 26, 27, 29, 30, 32, 33, 35 and 36).

The number of units that are produced in a period of transition has a normal distribution (the mean is $\sum \pi j \in set / RT$ and the variance is $\sum bj \in set / RT^2$; here RT is the Production time at each Station) and is calculated in (Table 3). See Kemeny and Snell (1960) and Miltenburg (2010).

Table 3. Production time distribution

Station	Production time	Normal Distribution
One (X1)	3 hours per unit	$mean = \sum \pi j \in set_1 / RT_1 =$ 0.1410 $variance = \sum bj \in set_1 / RT_1^2$ = 0.3399
Two (X2)	5 hours per unit	$mean = \sum \pi j \in set_2 / RT_2 =$ 0.1476 $variance = \sum bj \in set_2 / RT_2^2$ = 0.2180

4.1. Throughput

The number of units produced over the production planning period has the following mean and variance:

Mean=1000*mean of the number of units that are produced =1000*0.1476=147.6 unit

Variance= 1000* variance of the number of units that are produced = 1000* 0.218=218

4.2. Shortage

Shortage is a situation where demand for a product exceeds the available output.

$$E(shortage) = \int_{-\infty}^{pp} (pp - x_{21}) * f(x_{21}) dx_{21} \quad pp = production\_plan \quad (4)$$

After some steps we have:

$$E(shortage) = \sigma(Throughput) * (f_z(s) + sF_z \leq (s)) \quad (5)$$

While

$$s = (pp - E(Throughput)) / \sigma(Throughput) \quad (6)$$

f_z and F_z = unit normal distribution

See Miltenburg (1997).

$$s = \frac{240 - 147.6}{14.76} = 6.26$$

$$E(shortage) = 14.76 * (f_z(6.26) + 6.26 F_z \leq (6.26)) = 110.655$$

4.3. Work-in-process

To calculate the mean and variance of the work-in-process, first we have to consider the inventory in the production line in each state as shown in (Table 4).

Table 4. Transition probability

State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
I_j	0	1	2	1	2	3	2	3	4	0	1	2	1	2	3	2	3	4
State	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
I_j	0	1	2	1	2	3	2	3	4	0	1	2	1	2	3	2	3	4

$$Mean = \sum_{j=1}^{36} I_j * \pi_j = 1.7342 \tag{7}$$

4.4. Cycle time

Cycle time is the period required to complete a job from start to finish. In the defined production system cycle time is the time at Station 1 added to the time at Station two.

$$\frac{1}{E(X_1)} + \frac{1}{E(X_2)} = 13.867 \text{ } hours \tag{8}$$

5. Kanban card with dynamic information as a signal to handle the pull system

In this Section, a dynamic Kanban system is developed that changes the Kanban card information (production rate) based on the remained inventory in the buffer. When the complete product in the second inventory buffer deceases to zero and system is in the shortage danger, the information on the dynamic Kanban card changes and sends the signal to station two to reduce the production time to 4 hours. When the completed product in the second inventory buffer deceases to one, again the information on the dynamic Kanban card changes and sends the signal to station two to change the production time to 4.5 hours. When capacity of the second inventory buffer is full and system is not in the shortage danger the information on the dynamic Kanban card is similar to the information on the static Kanban card and sends the signal to station two to produce like before (5 hours for each unit).

When the inventory in the first inventory buffer deceases to zero and Station two is in the starvation danger the information on the dynamic Kanban card changes and sends the signal to station one to reduce the production time to 2 hours. When the inventory in the first inventory buffer deceases to one, again the information on the dynamic Kanban card changes and sends the signal to station one to change the production time to 2.5 hours. When capacity of the inventory in the first inventory buffer is full and Station one is in the danger to be blocked the information on the dynamic Kanban card is similar to information on the static Kanban card and sends the signal to station one to produce like before (3 hours for each unit). So, again the Markov chain parameters and its calculation are shown in (Table 5).

Table 5. Transition probability

	First buffer: 0 Second buffer: 0	**First buffer: 0 Second buffer: 1**	**First buffer: 0 Second buffer: 2**	**First buffer: 1 Second buffer: 0**
Event	Frequency probability	Frequency probability	Frequency probability	Frequency probability
Λ	40 40/307=0.13	40 40/305=0.132	40 40/303=0.132	40 40/298=0.134
β1	90 90/307=.294	90 90/305=.295	90 90/303=.297	90 90/298=.302
β2	90 90/307=.294	90 90/305=.295	90 90/303=.297	90 90/298=.302
α1	10 10/307=.032	10 10/305=.033	10 10/303=.033	10 10/298=.034
α2	10 10/307=.032	10 10/305=.033	10 10/303=.033	10 10/298=.034
γ1	90/2=45 45/307=0.146	90/2=45 45/305=0.147	90/2=45 45/303=0.149	90/2.5=36 36/298=0.12
γ2	90/4≈22 22/307=0.072	90/4.5=20 20/305=0.065	90/5=18 18/303=0.059	90/4≈22 22/298=0.074
Total	307 1	305 1	303 1	298 1

Table 5. Continued

	First buffer: 0 **Second buffer: 0**	**First buffer: 0** **Second buffer: 1**	**First buffer: 0** **Second buffer: 2**	**First buffer: 1** **Second buffer: 0**
	First buffer: 1 Second buffer: 1	First buffer: 1 Second buffer: 2	First buffer: 2 Second buffer: 0	First buffer: 2 Second buffer: 1
Λ	40 40/296=0.135	40 40/294=0.137	40 40/292=0.137	40 40/290=0.138
β1	90 90/296=.304	90 90/294=.306	90 90/292=.308	90 90/290=.31
β2	90 90/296=.304	90 90/294=.306	90 90/292=.308	90 90/290=.31
α1	10 10/296=.034	10 10/294=.034	10 10/292=.035	10 10/290=.035
α2	10 10/296=.034	10 10/294=.034	10 10/292=.035	10 10/290=.035
γ1	90/2.5=36 36/296=0.122	90/2.5=36 36/294=0.123	90/3=30 30/292=0.102	90/3=30 30/290=0.103
γ2	90/4.5=20 20/296=0.067	90/5=18 18/304=0.06	90/4≈22 22/292=0.075	90/4.5=20 20/290=0.069
Total	296 1	294 1	292 1	290 1

Now, we can calculate the numerical transition probability matrix base on the attained probability and the first and second inventory buffers in their states. We calculate all predefined matrixes to compare them with each other.
A= limiting behavior of the transition probability matrix. Each row is Π .

$$\bar{\Pi} = \{\pi_j\}$$

$$\pi_j = \sum_{i=1}^{n} \pi_i * p_{ij}$$

$$\sum_{j=1}^{n} \pi_j = 1$$

Π = [0.0008 0.0018 0.0047 0.0005 0.0010 0.0007 0.0002 0.0003 0.0002 0.0043 0.0135 0.0470 0.0028 0.0081 0.0086 0.0015 0.0026 0.0023 0.0078 0.0151 0.0337 0.0068 0.0129 0.0056 0.0039 0.0031 0.0016 0.0448 0.1144 0.3381 0.0428 0.1093 0.0757 0.0255 0.0350 0.0230]

Z= {zij} = $\left(I - P + A\right)^{-1}$, Fundamental matrix. We only need the diagonal entries, so we write it here.

Zjj= [1.8995 1.8058 1.7806 1.4637 1.4161 1.4043 1.4575 1.4079 1.4084 2.4916 2.3921 3.1670 1.7722 1.7409 2.2831 1.7532 1.8928 2.4419 3.8479 2.7278 2.5608 2.3781 1.8542 1.9957 2.3420 1.9889 2.1199 6.3137 3.9275 4.4298 3.3692 2.0569 4.0128 3.0536 3.8918 6.2594]
I=Unit matrix.
B= {bj} = the limiting variance of the number that the Markov chain is in each state; bj= πj(2zjj-1- πj)
B = [0.0021 0.0048 0.0121 0.0009 0.0019 0.0012 0.0005 0.0005 0.0003 0.0171 0.0509 0.2483 0.0072 0.0200 0.0308 0.0038 0.0048 0.0061 0.0722 0.0729 0.1511 0.0237 0.0297 0.0128 0.0116 0.0063 0.0034 1.0772 0.9352 3.1310 0.2935 0.3231 0.4399 0.1417 0.1607 0.1830]

The number of units that are produced in a period of transition (for Jobs one and two) has a normal distribution (the mean is $\sum \pi j \in set / RT$ and variance is $\sum bj \in set / RT^2$; here RT is the weighted mean of production time based on the probability of amount of inventory) and is calculated in (Table 6).
RT_1=2*(probability that the first buffer is 0) +2.5*(probability that the first buffer is 1) +3*(probability that the first buffer is 2) = 2.6997
RT_2=4*(probability that the second buffer is 0) +4.5*(probability that the second buffer is 1) +5*(probability that the second buffer is 2) = 4.2366

5.1. Throughput

Mean=1000*mean of the number of units that are produced =1000*0.1811=181.1 unit
Variance= 1000* variance of the number of units that are produced = 1000* 0.303=303

Table 6. Production time distribution

Station	Production time	Normal Distribution
One (X_1)	2.6997 hours per unit	$mean = \sum \pi j \in set_1 / RT_1 =$ 0.1499 $\text{var}iance = \sum bj \in set_1 / RT_1^2 =$ 0.416
Two (X_2)	4.2366 hours per unit	$mean = \sum \pi j \in set_2 / RT_2 =$ 0.1811 $\text{var}iance = \sum bj \in set_2 / RT_2^2 =$ 0.303

5.2. Shortage

$$E(shortage) = \int_{-\infty}^{pp} (pp - x_2) * f(x_2) dx_2$$ pp=production plan

After some calculations: $E(shortage) = \sigma(Throughput) * (f_z(s) + sF_z \leq (s))$, while

$s = \dfrac{pp - E(Throughput)}{\sigma(Throughput)}$, f_z and F_z =unit normal distribution

See Miltenburg (2010).

$s = \dfrac{240 - 181.1}{17.4}$ =3.38

$E(shortage) = 17.4 * (f_z(3.38) + 3.38F_z \leq (3.38))$ =58.83

5.3. Work-in-process

To calculate the mean and variance of the work-in-process, first we have to consider the inventory in the production line in each state as shown in (Table 7).

Table 7. Production time distribution

State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
I_j	0	1	2	1	2	3	2	3	4	0	1	2	1	2	3	2	3	4
State	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
I_j	0	1	2	1	2	3	2	3	4	0	1	2	1	2	3	2	3	4

Mean= $\sum_{j=1}^{36} I_j * \pi_j$ = 1.872

5.4. Cycle time

In the defined production system, cycle time is the time at Station 1 added to the time at Station two.

$$\frac{1}{E(X_1)} + \frac{1}{E(X_2)} = 12.192 \ hours$$

5.5. Comparison

The results are summarized in (Table 8). The table compares the Kanban card with static information to the Kanban card with dynamic information as a signal to handle the pull system. Some measurement parameters were considered.

Table 8. Comparison of the two approaches

Approach	Throughput (unit)	WIP(unit)	Shortage(unit)	Cycle Time(hours)
static information	147.6	1.734	110.655	13.867
dynamic information	181.1	1.872	58.83	12.192

As the table reveals, improvement by the Kanban card with dynamic information is on all measurement parameters except for work-in-process. This approach could not improve work-in-process because when the information on the dynamic Kanban card changes and sends the signal to station one or two to reduce the production time, work-in-process produces more. While applying the Kanban card with static information is an easy approach to manage the pull production system, it is not useful in competitive conditions. In competitive environments, the Kanban card with dynamic information gives the best competitive advantage.

6. Conclusions and further research

The pull type production control produces products based on customer demands. Each station is isolated until a customer order is placed, then a signal or Kanban is sent from downstream station to upstream station and continues until the first station. Most researchers did their studies on pull systems in deterministic environments while many real production lines are subjected to different types of uncertainties. In this paper, a dynamic Kanban system is developed that changes the Kanban card information (production rate) base on the remained inventory in the buffer.

The proposed work here used Markov chain in the pull system and compared the effectiveness of the Kanban card with dynamic information to the Kanban card with static information. It showed the Markov chain analysis in handling pull system by dynamic information on the Kanban card. So, there is scope for further research to extend Markov chain analyses for controlling different control systems in stochastic environments.

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References

Bitran, G., and Chang, L. (1987). A Mathematical Programming Approach to a Deterministic Kanban System. *Management Science*, Vol. 33(4), pp. 427-441.

Framinan, J., González, P., and Ruiz-Usano, R. (2003). The CONWIP production control system: Review and research issues. *Production Planning & Control*, Vol. 14(3), pp. 255-265.

Framinan, J., González, P., and Ruiz-Usano, R. (2006). Dynamic card controlling in a Conwip system. *International Journal of Production Economics*, Vol. 99(1-2), pp. 102-116.

Gong, Q., Yang, Y., and Wang, S. (2014). Information and decision-making delays in MRP, KANBAN, and CONWIP. *International Journal of Production Economics*, Vol. 156, pp. 208-213.

González-R, P., Framinan, J., and Pierreval, H. (2011). Token-based pull production control systems: an introductory overview. *Journal of Intelligent Manufacturing*, Vol. 23(1), pp. 5-22.

Gupta, S., and Al-Turki, Y. (1997). An algorithm to dynamically adjust the number of Kanbans in stochastic processing times and variable demand environment. *Production Planning & Control*, Vol. 8(2), pp. 133-141.

Gupta, M. (2003). Constraints management--recent advances and practices. *International Journal of Production Research*, Vol. 41(4), pp. 647-659.

Hopp, W., and Roof, M. (1998). Setting WIP levels with statistical throughput control (STC) in CONWIP production lines. *International Journal of Production Research*, Vol. 36(4), pp. 867-882.

Huang, Y., Wan, H., Kuriger, G., and Chen, F. (2013). Simulation Studies of Hybrid Pull Systems of Kanban and CONWIP in an Assembly Line. *Advances in Sustainable and Competitive Manufacturing Systems*, pp. 1553-1563.

Jou Lin, C., Frank Chen, F., and Min Chen, Y. (2013). Knowledge kanban system for virtual research and development. *Robotics and Computer-Integrated Manufacturing*, Vol. 29(3), pp. 119-134.

Kemeny, J., & Snell, J. (1960). *Finite markov chains*. Princeton, N.J.: Van Nostrand.

Khojasteh-Ghamari, Y. (2008). A performance comparison between Kanban and CONWIP controlled assembly systems. *Journal of Intelligent Manufacturing*, Vol. 20(6), pp. 751-760.

Miltenburg, J. (1997). Comparing JIT, MRP and TOC, and embedding TOC into MRP. *International Journal of Production Research*, Vol. 35(4), pp. 1147-1169.

Moeeni, F., and Chang, Y. (1990). An Approximate Solution to Deterministic Kanban Systems. *Decision Sciences*, Vol. 21(3), pp. 596-607.

Renna, P. (2010). Dynamic Control Card in a Production System Controlled by Conwip Approach. *Jordan Journal of Mechanical and Industrial Engineering*, Vol. 4(4), pp.425-432.

Sato, R., and Khojasteh-Ghamari, Y. (2010). An integrated framework for card-based production control systems. *Journal of Intelligent Manufacturing*, Vol. 23(3), pp. 717-731.

Schmidbauer, H., and Rösch, A. (1994). A stochastic model of an unreliable Kanban production system. *OR Spektrum*, Vol. 16(2), pp. 155-160.

Sendil Kumar, C., and Panneerselvam, R. (2006). Literature review of JIT-KANBAN system. *Int J Adv Manuf Technol*, Vol. *32*(3-4), pp. 393-408.

Shahabudeen, P., Krishnaiah, K., and Thulasi Narayanan, M. (2003). Design of a Two-Card Dynamic Kanban System Using a Simulated Annealing Algorithm. *The International Journal of Advanced Manufacturing Technology*, Vol. 21(10-11), pp. 754-759.

Shahabudeen, P., and Sivakumar, G. (2008). Algorithm for the design of single-stage adaptive kanban system. *Computers & Industrial Engineering*, Vol. 54(4), pp. 800-820.

Spearman, M., Woodruff, D., and Hopp, W. (1990). CONWIP: a pull alternative to Kanban. *Int. J. Of Prodn. Res.*, Vol. 28(5), pp. 879-894.

Tardif, V., and Maaseidvaag, L. (2001). An adaptive approach to controlling Kanban systems. *European Journal of Operational Research*, Vol. 132(2), pp. 411-424.