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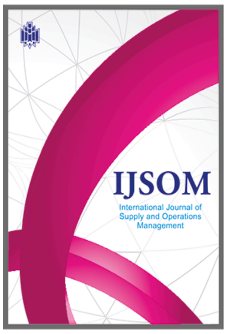

Inventory model for deteriorating items involving fuzzy with shortages and exponential demand

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Abstract
This paper considers the fuzzy inventory model for deteriorating items for power demand under fully backlogged conditions. We define various factors which are affecting the inventory cost by using the shortage costs. An intention of this paper is to study the inventory modelling through fuzzy environment. Inventory parameters, such as holding cost, shortage cost, purchasing cost and deterioration cost are assumed to be the trapezoidal fuzzy numbers. In addition, an efficient algorithm is developed to determine the optimal policy, and the computational effort and time are small for the proposed algorithm. It is simple to implement, and our approach is illustrated through some numerical examples to demonstrate the application and the performance of the proposed methodology.

Keywords: Exponential Demand; Deterioration; Shortages; Trapezoidal Fuzzy Numbers; Fuzzy Demand; Fuzzy Deterioration.

1. Introduction

Lofti Zadeh(1965) introduced "Fuzzy Sets". They develop fuzzy logic at that time. The idea of fuzzy sets and fuzzy logic were not accepted well within academic circles, because some of the underlying mathematics had not yet been explored. So that, the applications of fuzzy logic were slow to develop, except in the east. In Japan specifically fuzzy logic was fully accepted and implemented in products simply because fuzzy logic worked. The success of many fuzzy logic products in Japan in led to a revival in fuzzy logic in the US in the late 80s. Since that time America has been playing catch up with the east in the area of fuzzy logic.

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The effects of deteriorating are important in many inventory systems. Deterioration means the falling from a higher to a lower level in quality, character, or vitality. Deterioration stresses physical, intellectual, or especially moral retrogression. Decadence presupposes a reaching and passing the peak of development and implies a turn downward with loss in vitality or energy.

Exponential demand is an average method. This is useful if the recent changes in the data result from a change such as a seasonal pattern instead of just random fluctuations

This paper consists of shortage cost. Usually, an approximate figure is arrived at after our assumption of the several values such as lost of customer, lost sale, stock-out penalties and disputes in contract. In that way the inventory shortage does not cost an immediate loss in sales or profit. The vendor may commit to deliver the product within a particular lead time. The cost incurred in that case is the ‘back-order cost’.

We convert the inventory model into fuzzy inventory model. The holding costs, shortage cost, purchasing cost are assumed to be the trapezoidal fuzzy numbers. Trapezoidal fuzzy number is the fuzzy number represented with three points as follows $\left(a_1, a_2, a_3\right)$. This paper has developed an effective procedure for determining the optimal solutions of inventory order quantity, time, and total cost. Besides, an efficient algorithm is developed to determine the optimal solution, and our approach is illustrated through a numerical example. Sensitivity analysis has been carried out to illustrate the behaviors of the proposed model and some managerial implications are also included.

2. Literature review

Inventory systems with deteriorating items have received considerable attention in recent years. These systems are held in stock experience continuous deterioration over time. Examples of products that experience deterioration while in stock include food stuff, medicines, volatile liquids, blood banks, etc. Details regarding inventory models with deteriorating items were found in the recent review by Raafat (Raafat, F. 1991).

Aggarwal, and Jaggi, C. K. (1995) extended Goyal’s model to consider the deteriorating items. Chandrasekhara Reddy and Ranganatham (2012) discussed about the demand changes from time to time, the inventory problem becomes dynamic. Chang and Dye (2001) developed a partial backlogging inventory model for deteriorating items with Weibull distribution and permissible delay in payments. Concurrently, Chang *et al*. (2001) presented an inventory model for deteriorating items with linear trend under the condition of permissible delay in payments. Chang *et al*. (2008) made a review on previous related literatures under trade credit.

Chang *et al*. (2009) proposed an optimal payment time for deteriorating items under inflation and permissible delay in payments during a finite planning horizon. Dutta and Pavan Kumar (2013) were described the trapezoidal fuzzy number in normal inventory models on purpose of reducing the total costs. Goyal (1985) was the first to establish an economic order quantity model with a constant demand rate under the condition of a permissible delay in payments.

Halkos et al. (2012) described the estimator of the second estimation policy to ensure that the requested critical factors are attained. And also they told that the third estimation policy, the corresponding estimator is obtained maximizing profit with respect to a constant which included the form of the estimator. Halkos et al. (2013) established the values for the two criteria. And the relative-expected half-length, values are computed also analytically.

Horng-Jinh Chang and Chung-Yuan Dye (1999) developed the backlogging rate. The backlogging rate function is considered an exponential decreasing function of the waiting time for the next replenishment. Hwang and Shinn (1997) added the pricing strategy to the model, and developed the optimal price and lot-size for a retailer under the condition of a permissible delay in payments. Jaggi et al. (2012) presented a fuzzy inventory model for deteriorating items with time-varying demand and shortages.

Jamal et al. (1997) proposed an inventory model with deteriorating items under inflation when a delay in payment is permissible. Kapil Kumar Bansal and Navin (2012) described exponentially increasing demand has been considered in place of constant demand. Since the exponentially increasing demand, whose demand changes steadily along with a steady increase in population density. Liang and Zhou (2011) provided a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment.

Maragatham and Lakshmidevi (2014) developed a proper EOQ deterioration inventory model, there exists the unique optimal solution to minimize total cost and the analytic solution of the optimal order cycle was derived. Instead of having on hand inventory, allowing shortages was the best method to minimize the total cost. Mary Latha and Uthayakumar (2014) described the deterioration was probabilistic to find the associate total cost. Nithya and Ritha [17] inventory models discussed with fuzzy parameters for crisp order quantity, or for fuzzy order quantity. And function principle was proposed as an arithmetic operation of fuzzy trapezoidal number to obtain fuzzy economic order quantity and fuzzy annual profit.

Nirmal Kumar Duari and Tripti Chakraborty (2012) assumed that the demand is as exponential distribution, they expected to induce increases in demand and sales in marketing. Ritha and Rexlin Jeyakumari (2013) described the optimum order quantity is in fuzzy sense with the help of signed distance method. Sarah Ryan (2003) argued the capacity when significant excess capacity remains, or to install large capacity increments. And also discussed about the lead times, the cost parameters determined expansion size, demand characteristics are affecting both policy dimensions but in different ways.

A high expectation of demand growth motivates large expansions that occur somewhat earlier. Sanhita Banerjee and Tapan Kumar Roy (2012) were consulted about on extension principle, interval method and vertex method and compare three methods. And also solved some numerical problems with various values. Savitha Pathak and Seema Sarkar (Mondal) (2012) considered that the objectives were maximized and also the costs were taken in fuzzy environment as triangular fuzzy and trapezoidal fuzzy. Shah (1993) considered a stochastic inventory model when delays in payments are permissible. Shah (2006) considered an inventory model for deteriorating items and time value of money under permissible delay in payments during a finite planning horizon.

Soni et al. (2006) discussed an EOQ model for progressive payment scheme under discounted cash flow (DCF) approach. Sushil Kumar and Rajput (2015) discussed about a fuzzy inventory model for deteriorating items with time dependent demand and also shortages were permitted. In this discussion they considered the demand rate, deterioration rate and backlogging rate were assumed as a triangular fuzzy numbers. Syed and Aziz [28] were calculated the optimal order quantity by using signed distance method for defuzzification.

In this paper, we consider a fuzzy inventory model for deteriorating items with shortages under fully-backlogged condition and exponential demand. The inventory costs are assumed to be the trapezoidal fuzzy numbers. Numerical examples and sensitivity are analyzed and calculated. We can also afford the notations and assumptions for the assumed model in section 3. A Mathematical model is recognized in section 4. In section 4.1 contains fuzzy model and solution procedure. In section 5 an efficient algorithm is developed to obtain the optimal solution. Numerical analysis for inventory control and fuzzy model are presented in section 6. In section 6.3 the sensitivity analysis of the optimal solution with respect values of the system is obtained in the same section. In section 6.4 there were the managerial implications of the inventory control and the fuzzy model. Finally, we provide the conclusions and future research in section 7.

3. Notations and Assumptions

For developing the proposed models, the following assumptions and notations are used throughout this chapter.

3.1. Notations

The following notations and assumptions are used here:

C_0	Ordering cost per order
C_h	Holding cost per unit per unit time
C_s	Shortage cost per unit time
C_p	Purchasing cost per unit per unit time
D	Demand rate at any time t per unit time $(D(t) = ae^{bt} \ a > 0; b > 0)$
a_1	Deterioration function $(0 < a_1 < 1)$
T	Length of ordering cycle
Q	Order quantity per unit
C_{Ts}	Total shortage cost per unit time
ς_{Ts}	Fuzzy total shortage cost per unit time
$(C_{Ts})ds$	Defuzzified value of fuzzy number ς_{Ts} by using signed distance method

$Tc(t_1,T)$ Total inventory cost per unit time

$T\varsigma(t_1,T)$ Fuzzy total cost per unit time

$(C_{Ts})ds\ (t_1,T)$ Defuzzified value of fuzzy number $T\varsigma(t_1,T)$ by using signed distance method

3.2 Assumptions

To develop the proposed model, we adopt the following assumptions

1. Demand rate is exponential function of time t $(D(t)=ae^{bt}\ a>0; b>0)$.
2. Lead time is zero.
3. Shortages are allowed and fully backlogged.
4. During the cycle deterioration is not repaired or replaced.
5. Replenishment rate is infinite.
6. Holding cost is as time dependent.

4. Mathematical Modeling

The beginning of the product or purchased the product based on Q and after fulfilling backorders. During the period $[0, t_1]$ the inventory level gradually diminishes and ultimately falls to zero. From these time interval shortages may occur and fully backlogged. Let $I_1(t)$ be the on – hand inventory level at time t, which is developed from the following equations:

$$\frac{dI_1(t)}{dt}+a_1I_1(t)=-ae^{bt} \quad \text{For} \quad 0\le t\le t_1 \tag{1}$$

and
$$\frac{dI_2(t)}{dt}=-ae^{bt} \quad \text{for} \quad t_1\le t\le T \tag{2}$$

with
$$I_1(0) = Q \quad \text{and} \quad I_1(t_1)=0 \tag{3}$$

Now solve in (1) and (2) using (3) we get the final solutions, which is given by

$$I_1(t) = -\left[\frac{ae^{bt}}{(b+a_1)}\right]+\left[\frac{ae^{bt_1}}{(b+a_1)}\right] \quad \text{for} \quad 0\le t\le t_1 \tag{4}$$

And

$$I_2(t) = \left[\frac{ae^{bt}}{b}\right] \quad \text{for} \quad t_1\le t\le T \tag{5}$$

Using the condition $I_1(0) = Q$ we get the value of $Q = -\left[\frac{a}{(b+a_1)}\right]+\left[\frac{ae^{bt_1}}{(b+a_1)}\right]$

(6) Total average number of holding costs is I_h , during the period [0, T] is given by,

$$I_h = \int_0^{t_1} I_1(t)dt = \left[\frac{t_1 a e^{bt_1}}{(b+a_1)}\right] - \left[\frac{a e^{bt_1}}{b(b+a_1)}\right] \tag{7}$$

Total number of deteriorated units I_d during the period [0, T] is given by,

$$I_D = \frac{C_0}{T}\int_0^{T} a e^{bt} I_1(t)dt = \frac{C_0}{T}\left[\frac{a^2 e^{b(t_1+T)}}{b(b+a_1)}\right] - \left[\frac{a^2 e^{2bT}}{2b(b+a_1)}\right] \tag{8}$$

Total number of shortage units I_s during the period [0, T] is given by,

$$I_s = \int_{t_1}^{T} I_2(t)dt = \left[\frac{a e^{b(T-t_1)}}{b^2}\right] \tag{9}$$

Total costs per unit time

$$C_{Ts} = \frac{1}{T}\left[C_s I_s\right] \tag{10}$$

Total cost of the system per unit time

$$Tc(t_1,T) = \frac{1}{T}\left[C_0 + C_h I_h + C_p I_D + C_s I_s\right] \tag{11}$$

$$Tc(t_1,T) =$$

$$\frac{1}{T}\left[C_0 + C_h\left[\frac{t_1 a e^{bt_1}}{(b+a_1)} - \frac{a e^{bt_1}}{b(b+a_1)}\right] + C_p\left(C_0\left[\frac{a^2 e^{b(t_1+T)}}{b(b+a_1)} - \frac{a^2 e^{2bT}}{2b(b+a_1)}\right]\right) + C_s\left[\frac{a e^{b(T-t_1)}}{b^2}\right]\right] \tag{12}$$

To minimize the total cost per unit time $Tc(t_1,T)$, the optimal value of T and t_1 can be obtained by solving the following equations:

$$\frac{\partial Tc(t_1,T)}{\partial t_1} = 0 \quad \text{and} \quad \frac{\partial Tc(t_1,T)}{\partial T} = 0 \tag{13}$$

Now, $\dfrac{\partial Tc(t_1,T)}{\partial t_1}$

$$= \frac{1}{T}\left[C_h\left(\frac{t_1 a e^{bt_1}}{b(b+a_1)} + \frac{a e^{bt_1}}{(b+a_1)} - \frac{a e^{bt_1}}{b(b+a_1)}\right) + C_p C_0\left(\frac{a^2 e^{b(t_1+T)}}{b^2(b+a_1)}\right) - C_s\left(\frac{a e^{b(T-t_1)}}{b^3}\right)\right] = 0 \tag{14}$$

And

$$\frac{\partial Tc(t_1,T)}{\partial T} = -\frac{1}{T^2}\left[C_0 + C_h\left(\frac{t_1 ae^{bt_1}}{(b+a_1)} - \frac{ae^{bt_1}}{b(b+a_1)}\right) + C_p C_0\left(\frac{a^2 e^{b(t_1+T)}}{b(b+a_1)} - \frac{a^2 e^{2bT}}{2b(b+a_1)}\right) + C_s \frac{ae^{b(T-t_1)}}{b^2}\right]$$
$$+\frac{1}{T}\left[C_p C_0\left(\frac{a^2 e^{b(t_1+T)}}{b(b+a_1)} - \frac{a^2 e^{2bT}}{4b^2(b+a_1)}\right) + C_s\left(\frac{ae^{b(T-t_1)}}{b^3}\right)\right] \quad (15)$$

We solve the non-linear equations (14) and (15) by using the computer software Matlab, We can easily prove the total cost $Tc(t_1,T)$.

4.1 Fuzzy Model and Solution Procedure

We consider the model in fuzzy environment. Due to uncertainty, it is not easy to define all parameters exactly.

Let $C_h = (\varsigma_{h1},\varsigma_{h2},\varsigma_{h3},\varsigma_{h4})$, $C_p = (\varsigma_{p1},\varsigma_{p2},\varsigma_{p3},\varsigma_{p4},)$, $C_s = (\varsigma_{s1},\varsigma_{s2},\varsigma_{s3},\varsigma_{s4})$, $a_1 = (a_{11},a_{12},a_{13},a_{14})$ be trapezoidal fuzzy numbers. Then the total cost of the system per unit time in fuzzy sense is given by,

$$T\varsigma(t_1,T) = \left[\frac{1}{T}\left(C_0 + \varsigma_{h1}\left(\frac{-ae^{bt_1}}{b(b+a_{11})}\right) + \varsigma_{p1}\left(\frac{-a}{(b+a_{11})} - \frac{ae^{bt_1}}{b}\right) + \varsigma_{s1}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right],$$
$$\left[\frac{1}{T}\left(C_0 + \varsigma_{h2}\left(\frac{-ae^{bt_1}}{b(b+a_{12})}\right) + \varsigma_{p2}\left(\frac{-a}{(b+a_{12})} - \frac{ae^{bt_1}}{b}\right) + \varsigma_{s2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right],$$
$$\left[\frac{1}{T}\left(C_0 + \varsigma_{h3}\left(\frac{-ae^{bt_1}}{b(b+a_{13})}\right) + \varsigma_{p3}\left(\frac{-a}{(b+a_{13})} - \frac{ae^{bt_1}}{b}\right) + \varsigma_{s3}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right],$$
$$\left[\frac{1}{T}\left(C_0 + \varsigma_{h4}\left(\frac{-ae^{bt_1}}{b(b+a_{14})}\right) + \varsigma_{p1}\left(\frac{-a}{(b+a_{14})} - \frac{ae^{bt_1}}{b}\right) + \varsigma_{s4}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]. \quad (16)$$

$$T\varsigma(t_1,T) = \left[\frac{1}{T}\left(C_0 + (\varsigma_{h1},\varsigma_{h2},\varsigma_{h3},\varsigma_{h4}) + (\varsigma_{p1},\varsigma_{p2},\varsigma_{p3},\varsigma_{p4}) + (\varsigma_{s1},\varsigma_{s2},\varsigma_{s3},\varsigma_{s4})\right)\right] \quad (17)$$

$$T\varsigma(t_1,T) = W, X, Y, Z \quad (18)$$

Where $W = \left[\frac{1}{T}\left(C_0 + \varsigma_{h1}\left(\frac{-ae^{bt_1}}{b(b+a_{11})}\right) + \varsigma_{p1}\left(\frac{-a}{(b+a_{11})} - \frac{ae^{bt_1}}{b}\right) + \varsigma_{s1}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right];$

$$X = \left[\frac{1}{T}\left(C_0 + \varsigma_{h2}\left(\frac{-ae^{bt_1}}{b(b+a_{12})}\right) + \varsigma_{p2}\left(\frac{-a}{(b+a_{12})} - \frac{ae^{bt_1}}{b}\right) + \varsigma_{s2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right];$$

$$Y=\left[\frac{1}{T}\left(C_0+\varsigma_{h3}\left(\frac{-ae^{bt_1}}{b(b+a_{13})}\right)+\varsigma_{p3}\left(\frac{-a}{(b+a_{13})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s3}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]$$

And

$$Z=\left[\frac{1}{T}\left(C_0+\varsigma_{h4}\left(\frac{-ae^{bt_1}}{b(b+a_{14})}\right)+\varsigma_{p1}\left(\frac{-a}{(b+a_{14})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s4}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right].$$

The $\alpha-$cuts, $C_L(u)$ and $C_R(u)$ of trapezoidal fuzzy number $T\varsigma(t_1,T)$ all given,

$$C_L(\alpha)=W+(X-W)\alpha$$

$$=\left[\frac{1}{T}\left(C_0+\varsigma_{h1}\left(\frac{-ae^{bt_1}}{b(b+a_{11})}\right)+\varsigma_{p1}\left(\frac{-a}{(b+a_{11})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s1}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]+$$

$$\left(\alpha\left\{\left[\frac{1}{T}\left(C_0+\varsigma_{h2}\left(\frac{-ae^{bt_1}}{b(b+a_{12})}\right)+\varsigma_{p2}\left(\frac{-a}{(b+a_{12})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]\right.\right.$$

$$\left.\left.-\left[\frac{1}{T}\left(C_0+\varsigma_{h1}\left(\frac{-ae^{bt_1}}{b(b+a_{11})}\right)+\varsigma_{p1}\left(\frac{-a}{(b+a_{11})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s1}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]\right\}\right) \tag{19}$$

And $C_R(\alpha)=Z+(Z-Y)\alpha$

$$=\left[\frac{1}{T}\left(C_0+\varsigma_{h4}\left(\frac{-ae^{bt_1}}{b(b+a_{14})}\right)+\varsigma_{p1}\left(\frac{-a}{(b+a_{14})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s4}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]-$$

$$\left(\alpha\left\{\left[\frac{1}{T}\left(C_0+\varsigma_{h4}\left(\frac{-ae^{bt_1}}{b(b+a_{14})}\right)+\varsigma_{p1}\left(\frac{-a}{(b+a_{14})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s4}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]\right.\right.$$

$$\left.\left.-\left[\frac{1}{T}\left(C_0+\varsigma_{h3}\left(\frac{-ae^{bt_1}}{b(b+a_{13})}\right)+\varsigma_{p3}\left(\frac{-a}{(b+a_{13})}-\frac{ae^{bt_1}}{b}\right)+\varsigma_{s3}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right)\right]\right\}\right) \tag{20}$$

By using signed distance method, the defuzzified value of fuzzy number $\varsigma(t_1,T)$ is given by

$$T\varsigma_{ds}(t_1,T)=\frac{1}{2}\int_0^1[C_L(\alpha)+C_R(\alpha)]d\alpha \tag{21}$$

$$T\varsigma_{ds}(t_1,T)=\frac{1}{2T}\left[C_0T-C_{h1}T\left(\frac{ae^{bt_1}}{b(b+a_{11})}\right)-C_{p1}T\left(\frac{a}{b+a_{11}}+\frac{ae^{bt_1}}{b}\right)+C_{s1}T\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\right.$$

$$-C_{h2}\left(\frac{ae^{bt_1}}{b(b+a_{12})}\right)\frac{T^2}{2}-C_{p2}\left(\frac{a}{b+a_{12}}+\frac{ae^{bt_1}}{b}\right)\frac{T^2}{2}+C_{s2}\,\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)$$

$$+C_{h1}\left(\frac{ae^{bt_1}}{b(b+a_{11})}\right)\frac{T^2}{2}+C_{p1}\left(\frac{a}{b+a_{11}}+\frac{ae^{bt_1}}{b}\right)\frac{T^2}{2}-C_{s1}\,\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)$$

$$C_0T-C_{h4}\left(\frac{ae^{bt_1}}{b(b+a_{14})}\right)T-C_{p4}T\left(\frac{a}{b+a_{14}}+\frac{ae^{bt_1}}{b}\right)+C_{s2}T\left(\frac{ae^{b(T-t_1)}}{b^2}\right)$$

$$+C_{h4}\left(\frac{ae^{bt_1}}{b(b+a_{14})}\right)\frac{T^2}{2}+C_{p4}\,\frac{T^2}{2}\left(\frac{a}{b+a_{14}}+\frac{ae^{bt_1}}{b}\right)-C_{s2}\,\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)$$

$$-C_{h3}\left(\frac{ae^{bt_1}}{b(b+a_{13})}\right)\frac{T^2}{2}-C_{p4}\,\frac{T^2}{2}\left(\frac{a}{b+a_{13}}+\frac{ae^{bt_1}}{b}\right)+C_{s3}\,\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^2}\right) \qquad (22)$$

To minimize the total costs function per time $T\varsigma_{ds}(t_1,T)$ the optimal value of t_1 and T can be obtained by solving the following equations

$$\frac{\partial T\varsigma_{ds}}{\partial t_1}=0 \quad \text{and} \quad \frac{\partial T\varsigma_{ds}}{\partial T}=0 \qquad (23)$$

$$\frac{1}{2T}\Bigg[\varsigma_{h1}\left(\frac{ae^{bt_1}}{b^2(b+a_{11}}\left(-T+\frac{T^2}{2}\right)\right)+\varsigma_{p1}\left(\frac{ae^{bt_1}}{b^3}\left(\frac{T^2}{2}-T\right)\right)-\varsigma_{s1}\left(\frac{ae^{b(T-t_1)}}{b^3}\left(\frac{T^2}{2}-T\right)\right)$$

$$+\varsigma_{h2}\left(\frac{ae^{bt_1}}{b^2(b+a_{12}}\frac{T^2}{2}\right)\varsigma_{p2}\left(\frac{ae^{bt_1}}{b^3}\left(\frac{T^2}{2}\right)-\varsigma_{s2}\left(\frac{ae^{b(T-t_1)}}{b^3}\frac{T^2}{2}\right)+\varsigma_{h4}\left(\frac{ae^{bt_1}}{b^2(b+a_{14})}\left(\frac{T^2}{2}-T\right)+\right.\right.$$

$$\varsigma_{p4}\left(\frac{ae^{bt_1}}{b^3}\left(\frac{T^2}{2}-T\right)+\varsigma_{s4}\left(\frac{ae^{b(T-t_1)}}{b^3}\left(\frac{T^2}{2}-T\right)\right)-\varsigma_{h3}\left(\frac{ae^{bt_1}}{b^2(b+a_{13})}\left(\frac{T^2}{2}\right)\right)-\varsigma_{p3}\left(\frac{ae^{bt_1}}{b^3}\left(\frac{T^2}{2}\right)\right)\right.$$

$$\left.+\varsigma_{s3}\left(\frac{ae^{b(T-t_1)}}{b^3}\left(\frac{T^2}{2}\right)\right)\right)\Bigg]=0 \qquad (24)$$

$$\frac{1}{2T}\Bigg[C_0-\varsigma_{h1}\left(\frac{ae^{bt_1}}{b(b+a_{11})}\right)-\varsigma_{p1}\left(\frac{a}{b+a_{11}}+\frac{ae^{bt_1}}{b}\right)+\varsigma_{s1}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)+\varsigma_{s1}T\left(\frac{ae^{b(T-t_1)}}{b^3}\right)$$

$$-\varsigma_{h2}T\left(\frac{ae^{bt_1}}{b(b+a_{12})}\right)-\varsigma_{p2}T\left(\frac{a}{b+a_{12}}+\frac{ae^{bt_1}}{b}\right)+\varsigma_{s1}T\left(\frac{ae^{b(T-t_1)}}{b^2}\right)+\varsigma_{s1}\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^3}\right)$$

$$+\varsigma_{h1}T\left(\frac{ae^{bt_1}}{b(b+a_{11})}\right)+\varsigma_{p1}T\left(\frac{a}{b+a_{11}}+\frac{ae^{bt_1}}{b}\right)-\varsigma_{s1}T\left(\frac{ae^{b(T-t_1)}}{b^2}\right)-\varsigma_{s1}\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^3}\right)$$

$$C_0-\varsigma_{h4}\left(\frac{ae^{bt_1}}{b(b+a_{14})}\right)-\varsigma_{p4}\left(\frac{a}{b+a_{14}}+\frac{ae^{bt_1}}{b}\right)+\varsigma_{s4}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)+\varsigma_{s4}T\left(\frac{ae^{b(T-t_1)}}{b^3}\right)$$

$$+\varsigma_{h4}T\left(\frac{ae^{bt_1}}{b(b+a_{14})}\right)+\varsigma_{p4}T\left(\frac{a}{b+a_{14}}+\frac{ae^{bt_1}}{b}\right)-\varsigma_{s4}T\left(\frac{ae^{b(T-t_1)}}{b^2}\right)-\varsigma_{s4}\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^3}\right)$$

$$-\varsigma_{h3}T\left(\frac{ae^{bt_1}}{b(b+a_{13})}\right)-\varsigma_{p3}T\left(\frac{a}{b+a_{13}}+\frac{ae^{bt_1}}{b}\right)+\varsigma_{s3}T\left(\frac{ae^{b(T-t_1)}}{b^2}\right)+\varsigma_{s3}\frac{T^2}{2}\left(\frac{ae^{b(T-t_1)}}{b^3}\right)=0 \qquad (25)$$

We solve the equation (23) and (24) by using the computer software Matlab. But we get the second derivatives of the total cost function is very difficult. Similarly the total shortage cost per unit time in fuzzy sense is given by,

$$\varsigma_{Ts}=\frac{1}{T}\varsigma\left(\frac{ae^{b(T-t_1)}}{b^2}\right) \qquad (26)$$

Defuzzified value of fuzzy number ς_{Ts} by using signed distance method is given by,

$$(C_{Ts})_{ds}=\frac{1}{T}\left(\frac{ae^{b(T-t_1)}}{b^2}\right)\left[C_{s1}+C_{s2}+C_{s3}+C_{s4}\right] \qquad (27)$$

5. Algorithm

Step 1: Enter the demand (here demand is power demand), purchasing costs, holding costs and deterioration costs for all products.

Step 2: Define fuzzy trapezoidal number for the demand (here demand is power demand), purchasing costs, holding costs and deterioration costs for all products.

Step 3: We determine the total cost for crisp model,

$$=Tc(t_1,T)$$

$$Tc(t_1,T)=\frac{1}{T}\left[C_0+C_h\left[\frac{t_1ae^{bt_1}}{(b+a_1)}-\frac{ae^{bt_1}}{b(b+a_1)}\right]+C_p\left(C_0\left[\frac{a^2e^{b(t_1+T)}}{b(b+a_1)}-\frac{a^2e^{2bT}}{2b(b+a_1)}\right]\right)+C_s\left[\frac{ae^{b(T-t_1)}}{b^2}\right]\right].$$

Step 4: From equation (22), we determine the total cost for fuzzy model.

Step 5: Defuzzified value of fuzzy number ς_{Ts} by using signed distance method.

Step 6: Compared the total inventory cost for crisp model and fuzzy model.

Step 7: Print the comparison between the crisp model and the fuzzy model.

6. Numerical Analysis

To find the planned method, let us consider the following given data:

6.1 Crisp Model

a=110 per year, b=0.522 per unit, C_0 =Rs. 200 per order,

Table1. Illustration of the solution procedure for the Numerical Model

Changing Parameters	Values of the Parameters (Per Years)	T(Year)	t_1 (Year)	TC (Rs.)	C_{TS}
Ch	5	0.8215	0.6792	390.642	50.765
Cs	15				
Cp	20				
a_1	0.012				

6.2 Numerical Analysis for Fuzzy Model

Input Data

Let a = (80, 100, 120, 140), b = (0.452, 0.55, 0.623, 0.685),

$\varsigma h = (\varsigma h_1, \varsigma h_2, \varsigma h_3, \varsigma h_4)$ = (2, 4, 6, 8),

$\varsigma s = (\varsigma s_1, \varsigma s_2, \varsigma s_3, \varsigma s_4)$ = (12, 14,16,18),

$\varsigma p = (\varsigma p_1, \varsigma p_2, \varsigma p_3, \varsigma p_4)$ = (14, 18, 22, 26)

And $a_1 = (a_{11}, a_{12}, a_{13}, a_{14})$ = (0.004, 0.008, 0.012, 0.016). Then by using signed distance method, we obtain:

Case 1:

When ςh, ςs, ςp and a_1 are fuzzy trapezoidal numbers. The solution of fuzzy model is:

t_1= 0.6203 year, $T\varsigma_{ds}(t_1,T)$= Rs. 415.532, T= 0.8021year, ς_{Ts} = 55.445

Case 2:

When ςs , ςp and a_1 are fuzzy trapezoidal numbers. The solution of fuzzy model is:

t_1= 0.6313 year, $T\varsigma_{ds}(t_1,T)$ =Rs. 405.012, T= 0.8246 year, ς_{Ts} = 53.175

Case 3:

When ςp and a_1 are fuzzy trapezoidal numbers. The solution of fuzzy model is: t_1= 0.6589 year, 6=Rs. 406.182, T= 0.8279 year, ς_{Ts} = 51.635.

$T\varsigma_{ds}(t_1,T)$ =Rs. 406.182

Case 4:

When a_1 is fuzzy trapezoidal number. The solution of fuzzy model is: t_1= 0.6614 year, $T\varsigma_{ds}(t_1,T)$ =Rs. 400.019, T= 0.8389 year, ς_{Ts} = 50.425

Case 5:

When none of ςh , ςs , ςp and a_1 is fuzzy trapezoidal numbers. The solution of fuzzy model is: t_1= **0.6792 year**, $T\varsigma_{ds}(t_1,T)$ =**Rs. 390.642**, T= **0.8215 year**, ς_{Ts} = **50.765**

Comparison Table for Optimal Results

Table 2. Comparison between Crisp and Fuzzy

Model	**Optimal Value Of t_1 (Yrs)**	**Optimal Value of T (Yrs)**	**Optimal Value of TC(Rs.)**	**Optimal Value Of $T\varsigma(t_1,T)$ (Rs.)**	**Optimal Value of C_{TS}**	**Optimal Value of ς_{Ts}**
Crisp	0.6792	0.8215	390.642		50.765	
Fuzzy	0.6203	0.8021		415.532		55.445

6.3 Sensitivity Analysis

The results are shown in below tables: The change in the values of parameters may happen due to uncertainties in any decision- making situation. In order to examine the implications of these changes, the sensitivity analysis will be of great help in decision- making. We now study the effects of changes in the values of the system parameters Ch , Cp , Cs and a_1 on the optimal replenishment policy of the Crisp model.

Table 3. Sensitivity analysis on Ch

Defuzzified value of Ch (Rs. Per unit per year)	**Fuzzify value of parameter Ch**	**T(Year)**	**t_1 (Year)**	**TC (Rs.)**
3	(0, 2, 4, 6)	1.325	0.7652	348.23
4	(1, 3, 5, 7)	1.026	0.7321	368.14
5	(2, 4, 6, 8)	0.9456	0.6548	398.25
6	(3, 5, 7, 9)	0.8752	0.6376	412.56
7	(4, 6, 8, 10)	0.8496	0.5931	423.45

Table 4. Sensitivity analysis on Cp

Defuzzified value of Cp (Rs. Per unit per year)	**Fuzzify value of parameter Cp**	**T(Year)**	**t_1 (Year)**	**TC (Rs.)**
16	(10, 14, 18, 22)	0.9561	0.6376	423.45
18	(12, 16, 20, 24)	0.9263	0.6341	425.32
20	(14, 18, 22, 26)	0.9248	0.6301	427.64
22	(16, 20, 24, 28)	0.9215	0.6102	429.71
24	(18, 22, 26, 30)	0.9200	0.6096	431.09

Table 5. Sensitivity analysis on Cs

Defuzzified value of Cs (Rs. Per unit per year)	**Fuzzify value of parameter Cs**	**T(Year)**	**t_1 (Year)**	**TC (Rs.)**
11	(8, 10, 12, 14)	0.9621	0.6456	410.15
13	(10, 12, 14, 16)	0.9153	0.6541	423.12
15	(12, 14, 16, 18)	0.9018	0.6661	436.74
17	(14, 16, 18, 20)	0.8915	0.6792	442.91
19	(16, 18, 20, 22)	0.8768	0.6863	449.53

Table 6. Sensitivity analysis on a_1

Defuzzified value of a_1 **(Rs. Per unit per year)**	**Fuzzify value of parameter** a_1	**T(Year)**	t_1 **(Year)**	**TC (Rs.)**
0.006	(0.000, 0.004, 0.008, 0.012)	0.8931	0.6846	450.55
0.008	(0.002, 0.006, 0.010, 0.014)	0.8903	0.6714	456.12
0.010	(0.004, 0.008, 0.012, 0.016)	0.8896	0.6604	461.47
0.012	(0.006, 0.010, 0.014, 0.018)	0.8815	0.6563	468.09
0.014	(0.008, 0.012, 0.016, 0.020)	0.8754	0.6428	473.30

6.4 Managerial implications

The following interesting observations are made regards managerial implication

1. From table 6.3.1, as we increase the holding cost Ch, the optimum values of t_1 and T decreases. By this fact the total cost increases.
2. From table 6.3.2, as increase the purchasing cost Cp, the optimum values of t_1 and T decreases, by this effect the total cost increases.
3. From the table 6.3.3, as we increases the value of shortage cost Cs, the optimum value of t_1 increases and the optimum value of T decreases and finally the total cost increases.
4. From the table 6.3.4, as we increase the deterioration cost a_1, the optimum values of t_1 and T decreases, from these the total cost increases.
5. The changes in holding cost the total cost function becomes more sensitive (Table 6.3.1).
6. The changes in purchasing cost and deterioration cost the total cost becomes less sensitive (Table 6.3.2 & 6.3.4).
7. The comparison of optimal results obtained in crisp model and fuzzy model, we observe that the optimal values shortage cost and the total cost increases in fuzzy model (Table 6.2.1).

8. In Case 5. None of ςh, ςs, ςp and a_1 is fuzzy trapezoidal numbers. Then the solution of fuzzy model is similar to the crisp model. So we change the nature of ςh, ςs, ςp and a_1

From fuzziness to crispiness, the results immediately turned fuzzy model into crisp model.

7. Conclusion

We presented fuzzy inventory model for deteriorating items with shortages under fully backlogged condition. Naturally the inventory model consists of the shortage cost and deterioration cost. Here we used the power demand and the deterioration rate was constant. In fuzzy environment, all related inventory parameters were assumed to be trapezoidal fuzzy numbers.

The optimum results of fuzzy model were defuzzified into signed distance method. This will increase the total profit. A numerical analysis was illustrating the total cost. Sensitivity analysis indicates the total cost function was more sensitive to change the value of holding cost. For other related parameters we can decide the optimum value of total cost. For further studies we are planning to extent the mathematical models to consider more factors related to supply chain performance.

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