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# A New Robust Mathematical Model for the Multi-product Capacitated Single Allocation Hub Location Problem with Maximum Covering Radius

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**Abstract**

This paper presents a new robust mathematical model for the multi-product capacitated single allocation hub location problem with maximum covering radius. The objective function of the proposed model minimizes the cost of establishing hubs, the expected cost of preparing hubs for handling products, shipping and transportation in all scenarios, and the cost variations over different scenarios. In the proposed model, a single product of a single node cannot be allocated to more than one hub, but different products of one node can be allocated to different hubs. Also, a product can be allocated to a hub only if equipment related to that product is installed on that hub. Considering the NP-Hard complexity of this problem, a GA-based meta-heuristic algorithm is developed to solve the large-scale variants of the problem. To evaluate the performance of the proposed algorithm, its results are compared with the results of the exact method and simulated annealing algorithm. These results show the good performance of the proposed algorithm.

**Keywords:** Multi-product; Hub location; Single allocation; Robust optimization; Genetic algorithm; Simulated annealing algorithm.

## 1. Introduction

The flow of shipment from the origin to the destination is the most important issue in transportation, logistics and communication networks. Shipment flow systems can be divided into two general types: Direct shipment, and Hub-Spoke network. Hubs are special facilities that act as intermediaries facilitating the flow in the distribution networks. The hub location problem consists of the location of indirect transportation points in a distribution network and allocation of points of origin and destination (Alumar, S., Kara, B.Y., (2008)). In the hub location problem, which is a variant of optimization problems, the goal is generally to find a suitable location for hubs and to determine the best routes through which cargo can be transferred such that the function expressing cost or time of collection/distribution would be minimized. The aim of this approach is to reduce the costs (cost of establishing hubs and costs of transportation) and to gain desired economic benefits by establishing transport links through the hubs. The hub location problems can be divided into two groups: The first group is single-allocation problems, where a single hub alone deals with the entire flow of the destination node. In the second group called multiple-allocation problems, the flow of each node can be processed by more than one hub. In the hub location problem, decisions regarding hub location must be made for a long-term outlook, and for this to be realized, parameters such as transportation cost and demand must be estimated for the span of years. This issue can cause some degree of uncertainty in parameters, which if remain unaddressed, can lead to ineffective decisions. On the other hand, the classic models are focused on the total cost and may allocate a customer to a very far hub, reducing in effect the customer satisfaction. Adding a neighborhood radius to formulations allows us to set a minimum level of satisfaction for all customers. This paper presents a robust model for multi-product hub location problem with covering radius. In summary, innovations of this study include incorporation of covering radius into multi-product capacitated single-allocation hub location problem, incorporation of data uncertainty and development of a robust model for the proposed

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problem, development of two meta-heuristic methods for solving the proposed problem, and evaluation of the performance of the proposed methods.
In the remainder of this paper, Section 2 reviews the previous studies on hub location problem, Section 3 describes the proposed robust model, Section 4 presents the proposed solution methods, Section 5 evaluates the performance of the proposed algorithms, and Section 6 concludes the paper.

## 2. Review of Literature

The first mathematical formulation of hub problem was presented in 1987 by O'kelly, who explored the concept of the hub in the context of air passenger transportation (O'kelly, M.E., (1987)). Applications of hub location and allocation problem include land cargo transportation (Cunha, C. B., & Silva, M. R. (2007)), air passenger transportation (Aykin, T. (1995a)), air cargo transportation (Kara, B. Y., & Tansel, B. C. (2000)), maritime transportation (Chou, C. C. (2010)), postal services (Ernst, A. T., & Krishnamoorthy, M. (1998)), communication networks (Monma, C.L., Sheng, D.D., (1986)), computer systems (Gavish, B., & Suh, M. (1992)) and emergency services (Berman, O., Drezner, Z., & Wesolowsky, G. O. (2007)). Labbe´et al. studied the capacitated single allocation hub location problem with the branch and bound method (Labbe´,M.,Yaman,H.,Gourdin,E., (2005) ). Every et al. explored the capacitated multiple allocation hub location problems (Ebery, J., Krishnamoorthy, M., Ernst, A., Boland, N., (2000)). Boland et al. introduced new properties for both capacitated and uncapacitated hub location problems, which led to better results with less computational time (Boland, N., Krishnamoorthy, M., Ernst, A.T., Ebery, J., (2004)). Campbell proposed a linear formulation for the problem (Campbell, J.F., (1994)). Costa et al. developed a bi-criteria formulation aimed at minimizing the costs and time of service (Costa, M. G., Captivo, M. E., & Climaco, J. (2008)). Kara and Tanse developed several linear formulations for single-allocation hub covering problem and proved the NP-hard complexity of this hub covering problem (Kara, B., & Tansel, B. (2003)). Ernst used the concept of covering radius to provide improved formulations for the single allocation hub covering problem (Ernst, A., Jiang, H., & Krishnamoorthy, M. (2005)). Wagner provided improved formulations for both groups of single and multiple allocation hub covering problems (Wagner, B. (2008)). Correia and Nickel studied the classic capacitated single allocation hub location problem with multiple products (I.Correia · S.Nickel ·F.Saldanha,(2014)). The importance of addressing the uncertainty issue in facility location problems have drawn more attention to this issues in that context. Snyder stated that stochastic optimization depends on distributing uncertain parameters and developed a robust programming method in which the goal was to find the worst case scenario based on a set of predefined uncertainty variables (Snyder, L.V., (2006)). However, the literature on the incorporation of uncertainty in the hub location context is scarce. Alumur expressed the effect of uncertainty in demand and preparation cost through a scenario-based stochastic model providing sub-optimal solutions of the problem under uncertainty (Alumur, S. A., Nickel, S., & Saldanha-da-Gama, F. (2012)). Contreras studied the incapacitated multiple allocation hub location problem with uncertain shipping costs and demand with a known probability distribution (Contreras, I., Cordeau, J. F., & Laporte, G. (2011a)). Canovas and Hamacher developed robust uncapacitated hub location formulations using exact models. Because of the complexity of multi-stage robust optimization, the theory of most works is focused on two stages (Canovas, L., García, S., & Marín, A. (2007))( Hamacher, H. W., Labbé, M., Nickel, S., & Sonneborn, T. (2004)). Assavapokee et al. proposed an algorithm that minimizes the maximum regret (T . Assavapokee, M.J. Realff, J.C. Ammons.(2008)). Contreras and Cordeau studied stochastic capacitated hub location problems for random demand and shipping cost (Contreras, I., Cordeau, J. F., & Laporte, G. (2011c)). Merakli et al. studied a multiple allocation P-hub median problem with uncertain demand, developed a mixed integer programming model for this problem, and then solved it with Benders decomposition (Meraklı, Merve, and Hande Yaman (2016)). Ghaderi and Rahmaniani developed a variable neighborhood search algorithm for uncapacitated multiple allocation p-hub median problem (Ghaderi, Abdolsalam, and Ragheb Rahmaniani (2015)).

Ghaffari–Nasab et al. considered that the quantity of the commodity flows between pairs of customer nodes are of stochastic nature and employed a robust optimization approach to model the problem. The numerical experiments showed the capability of the presented robust model to immunize the system against violation of capacity constraints with a relatively small cost increase, known as the robustness cost (Ghaffari–Nasab, N., Ghazanfari, M., Saboury, A., & Fathollah, M. (2015)). Ghodratnama et al. presented a novel multi-objective mathematical model to solve a capacitated single-allocation hub location problem with a supply chain overview wherein some of the parameters of the proposed mathematical model were regarded as uncertain parameters, and a robust approach used to solve the given problem (Ghodratnama, A., Tavakkoli-Moghaddam, R., & Azaron, A. (2015)). Boukani et al. developed a robust optimization approach to capacitated single and multiple allocation hub location problems. They found that ignoring uncertainty in the supply chain network design sometimes causes large losses and expenses (Boukani, F. H., Moghaddam, B. F., & Pishvaee, M. S. (2016)). Meraklı, M., & Yaman developed a linear mixed integer programming formulations using a minimax criteria for the robust uncapacitated multiple allocation p-hub median problem under polyhedral demand uncertainty and utilized two Benders decomposition based exact solution algorithms in order to solve large-scale instances of the problems (Meraklı, M., & Yaman, H. (2016)). Meraklı, M., & Yaman in another research presented a mathematical formulation of a capacitated multiple allocation hub location problem with hose demand uncertainty and utilized an algorithm to solve the dual sub problem using complementary slackness (Meraklı, M., & Yaman, H. (2017)). Madani et al. Considered the probability of the availability of the hubs and routes, and developed a multi-objective mathematical to tackle the reliable p-hub maximal covering problem (Madani, S.R., Nookabadi, A.S., Hejazi, S.R. (2017)). de Sá et al. considered a multiple allocation incomplete hub location problem in which a hub network can be partially interconnected by hub arcs, direct connections between non-hub nodes are allowed, and uncertainty is assumed for the data of origin-destination demands and hub fixed costs and employed a robust optimization approach to address the demand flows and fixed setup costs uncertainty (de Sá, E. M., Morabito, R., & de Camargo, R. S. (2018)).

In summary, innovations of this study include: Incorporation of covering radius into multi-product capacitated single-allocation hub location problem. In the hub location problem models, classic models are focused on the total costs and may allocate hubs very far from a customer that affects on customer satisfaction. Adding a neighborhood radius to formulations allows us to set a minimum level of satisfaction for all customers. Also this paper presents Incorporation of data uncertainty and developing a robust model for the proposed problem. According to the authors knowledge this is the first search on robust multi-product hub location problem. Moreover presenting of two meta-heuristic methods for solving the proposed problem, and evaluation of the performance of the proposed methods are other innovations of this research.

## 3. Robust Programming Model

Robust optimization is one of the best-known approaches to deal with uncertainty in elements of optimization problems and has attracted increasing attention since the late 1990s. The three famous approaches to robust optimization have been provided by Mulvey et al. [36], Ben-Tal et al. (Ben-Tal , A.Nemirovski (2000)), (Ben-Tal , A.Nemirovski (1998)) and Bertsimas (Bertsimas, D., Sim, M., (2004)). In the robust model provided by Mulvey, the goal is to find a solution that not only would be valid for all scenarios but also would minimize the variations of the objective function in those scenarios. To control the variations of the objective function, in 1995 Mulvey created a robust model through the development of a two-stage programming model by replacing the old cost-minimization goal with another one containing the variations of the objective function in different scenarios. Equation 1 shows the resulting objective function.

$$Min\ z=\sum_s P_s F_s + \lambda \sum_s P_s \left( F_s - \sum_s P_s F_s \right)^2 \tag{1}$$

In this equation, $P_s$ is the Probability of scenarios. This equation includes the average value of the objective function in different scenarios plus the weighted variance of objective function value in different scenarios ( $\lambda$ is the weight expressing the significance of variations of the objective function in different scenarios). the non-linearity of the provided objective function increases the computational complexity, so Yu and Li (Yu CS, Li HL (2000)) suggested the use of absolute value instead of second power and linearized the objective function with a non-negative variable $\theta_s$ .

$$Min\ z=\sum_s P_s F_s + \lambda \sum_s P_s \left[ \left( F_s - \sum_s P_s F_s \right) + 2\theta_s \right] \tag{2}$$

$$F_s - \sum_s P_s F_s + \theta_s \geq 0 \tag{3}$$

### 3.1. Problem Definition

In this study, the aim is to develop a robust model for capacitated single allocation hub location problem with covering radius. The objective function of proposed model seeks to minimize the costs associated with establishing hubs, preparation costs associated with product handling, the expected cost of shipping and transportation in all scenarios, and cost variations over different scenarios. In the proposed model, a single product of a single node cannot be allocated to more than one hub, but different products of one node can be allocated to different hubs. Also, a product can be allocated to a hub only if equipment related to that product is installed in that hub. The assumptions of the proposed model are as follows:

- There are multiple types of products that need to be shipped through the network.
- For each hub node (potential hub), there is a limitation on the maximum number of product types. Also, handling each product type requires special equipment installed in that hub.
- Product allocation is based on single allocation rule. Therefore, a single product of a single node must be allocated to only one hub, but different products of one node can be allocated to different hubs.
- Each hub has a limited capacity.
- Each hub has a covering radius, meaning that it can only support the nodes located within that radius.
- Amount of product p to be shipped from i to j is uncertain and change over the scenarios.
- Cost per unit and per unit of distance for shipment of all products are uncertain.
- Fixed costs of establishing a hub at node k for handling products are uncertain.

A schematic of the problem is shown in figure 1.

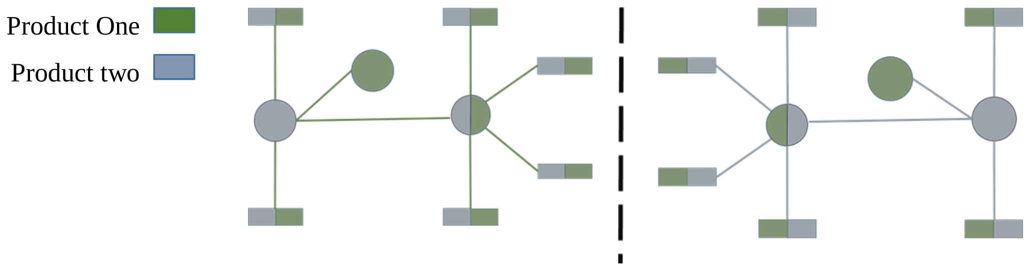


**Figure 1.** A schematic of the problem

**Sets and Parameters**

| | |
|---|---|
| S | set of scenarios; denoted by subscript s |
| $N$ | set of nodes; denoted by subscripts i, j , and k |
| $P$ | set of products; denoted by subscript p |
| $\Pr^s$ | Probability of scenario s |
| $d_{ij}$ | the distance between nodes i and j |
| $\omega_{ij}^{ps}$ | Amount of product p to be shipped from i to j under scenario s |
| $O_i^{ps}=\sum_{j\in N}\omega_{ij}^{ps}$ | Amount of product p to be shipped from node i under scenario s |
| $A_i^{ps}=\sum_{j\in N}\omega_{ji}^{ps}$ | Amount of product p to be shipped to node i under scenario s |
| $C_k^p$ | Capacity of hub established at node k for product p |
| $M_k$ | maximum number of product types that can be handled by a hub located at node k |
| $\delta^{ps}$ | Cost per unit and per unit of distance for shipment of product p between a non-hub node and a hub under scenario s |
| $\alpha^{ps}$ | Cost per unit and per unit of distance for shipment of product p between two hubs nodes under scenario s |
| $F_k$ | Fixed cost of establishing a hub at node k, independent of the product(s) to be handled |
| $f_k^{ps}$ | Fixed cost of establishing a hub at node k for handling product p under scenario s |
| $R_{ik}$ | =1 if node i is within covering radius of the hub established at node k; =0 otherwise |

**Variables**

| | |
|---|---|
| $x_{ik}^{ps}$ | A binary variable; =1 if in scenario s, product p with the source node i is to be allocated to hub k ; =0 otherwise |
| $ST_k^{ps}$ | A binary variable; =1 if in scenario s, equipment required for handling product p is to be installed at hub k ; =0 otherwise |
| $z_k$ | A binary variable; =1 if a hub is to be established at node k ; =0 otherwise |
| $y_{ikl}^{ps}$ | Fraction of the flow of product p coming from node i which is to be shipped via hubs k and l |
| $av$ | mathematical expectation of transportation cost in different scenarios |

### 3.2. The proposed robust model

$$Min \sum_{k\in N} F_k z_k + av + \lambda\sum_{s'\in S}\Pr^{s'}\left[\begin{array}{l}\sum_{p\in P}\sum_{k\in N} f_k^{ps'} ST_k^{ps'} + \sum_{p\in P}\sum_{i\in N}\sum_{k\in N} d_{ik}\delta^{ps'}\left(O_i^{ps'}+A_i^{ps'}\right)x_{ik}^{ps'} \\ +\sum_{p\in P}\sum_{i\in N}\sum_{k\in N}\sum_{l\in N}\alpha^{ps'} d_{kl} y_{ikl}^{ps'} - av + 2\theta_{s'}\end{array}\right] \tag{4}$$

s.t:

$$av = \sum_{s \in S} \Pr^s \left[ \begin{array}{l} \sum_{p \in P} \sum_{k \in N} f_k^{ps} ST_k^{ps} + \sum_{p \in P} \sum_{i \in N} \sum_{k \in N} d_{ik} \delta^{ps} \left( O_i^{ps} + A_i^{ps} \right) x_{ik}^{ps} \\ + \sum_{p \in P} \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \alpha^{ps} d_{kl} y_{ikl}^{ps} \end{array} \right] \quad (5)$$

$$\sum_{p \in P} \sum_{k \in N} f_k^{ps'} ST_k^{ps'} + \sum_{p \in P} \sum_{i \in N} \sum_{k \in N} d_{ik} \delta^{ps'} \left( O_i^{ps'} + A_i^{ps'} \right) x_{ik}^{ps'} \quad (6)$$

$$+ \sum_{p \in P} \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \alpha^{ps'} d_{kl} y_{ikl}^{ps'} - av + \theta_{s'} \geq 0 \, \forall s' \in S$$

$$x_{ik}^{ps} \leq R_{ik} ST_k^{ps} \qquad \forall i, k \in N, p \in P, s \in S \quad (7)$$

$$\sum_{i \in N} O_i^{ps} x_{ik}^{ps} \leq C_k^p \qquad \forall k \in N, p \in P, s \in S, s \in S \quad (8)$$

$$\sum_{p \in P} ST_k^{ps} \leq M_k z_k \qquad \forall k \in N, s \in S \quad (9)$$

$$\sum_{k \in N} x_{ik}^{ps} = 1 \qquad \forall i \in N, p \in P, s \in S \quad (10)$$

$$O_i^{ps} x_{ik}^{ps} + \sum_{l \in N} y_{ilk}^{ps} = \sum_{j \in N} \omega_{ij}^{ps} x_{jk}^{ps} + \sum_{l \in N} y_{ikl}^{ps} \qquad \forall i, k \in N, p \in P, s \in S \quad (11)$$

$$\sum_{l \in N} y_{ikl}^{ps} \leq O_i^{ps} x_{ik}^{ps} \qquad \forall i, k \in N, p \in P \quad (12)$$

$$y_{ikl}^{ps} \geq 0 \,,\, x_{ik}^{ps} \,,\, ST_k^{ps} \,,\, z_k \in \{0,1\} \qquad \forall i, k, l \in N, p \in P, s \in S \quad (13)$$

Equation (4) is the objective function consisting the cost of hub establishing and preparation, the average cost of transport in different scenarios, and objective function variations in different scenarios. Constraint (5) defines the average cost of transportation, and Constraint (6) is related to the linearization of the model. Constraint (7) states that in a given scenario, product p of node i will be allocated to hub k only if that hub is equipped to handle product p, and node i is within covering radius of that hub. Constraint (8) limits to the capacity of a hub in relation to a product. Constraint (9) defines a limit for the number of the product that can be allocated to a hub. Constraint (10) ensures that in each scenario, every node is allocated to at least one hub. Constraint (11) ensures the balance of flow in each scenario. Constraint (12) states that in each scenario, the maximum amount of product p to be transported from node i via hub k is equal to the total outgoing flow of product p from node i. Constraint (13) define the domains of variables.

## 4. Solution Method

In the NP-Hard problems, it is impossible to solve the large size instances using exact methods in the appropriate computational time, so heuristic methods [41] and metaheuristic methods [42-43] are applied to solve the problems. According to the NP-Hardness of the proposed model, to solve the proposed model in large scale, two meta-heuristic algorithms based on the simulated annealing algorithm and the genetic algorithm are proposed. The following subsections first describe the solution representation, and then the method of generating initial solution, and finally the procedure of proposed algorithms.

### 4.1. Solution Representation

In this paper, each solution is represented by a string of length 2N where N is the total number of nodes. The first N cells pertain to the position of hubs. In this part of the string, value 1 means a hub is established at the node corresponding to the cell. The second N cells pertain to the priority of node in hub allocation process. In this part of the string, cells can take values in the interval [0, 1]. Figure 2 shows the solution of a 10-node problem.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0.15 | 0.17 | 0.25 | 0.12 | 0.08 | 0.94 | 0.09 | 0.23 | 0.46 | 0.66 |

**Figure 2.** solution's representation for a problem with 10 nodes

In Figure 2, the first 10 cells indicate that hubs will be located at nodes 1, 4, and 9, and values of the second 10 cells show the priority of hub allocation. In this example, node 6 has the highest priority. Products of node 6 will be sorted by their shipping cost in a non-ascending order, and if possible, products of this node will be allocated to the nearest hub. The selected hub will be then equipped to handle the products allocated to it. When a hub lacks the equipment installation capacity required to handle a certain product, that equipment will be installed at next nearest hub. This process will repeat for all hubs according to nodes to which they are allocated. It is worth mentioning that to solve the robust model, for each scenario one string of length N specifying the priorities of nodes in that scenario will be added to the solution.

### 4.2. The proposed simulated annealing algorithm

First introduced by Kirkpatrick (Kirkpatrick, S. (1984)), Simulated Annealing (SA) meta-heuristic algorithm has found extensive application in discrete optimization problems. This algorithm starts the search with an initial solution and at each step generates a neighboring solution seeking an improvement. If the incumbent solution is worse than the obtained solution, it will be replaced with the new one. When the found solution is not as good as an incumbent solution, a replacement can still occur but only with a pre-specified probability. Algorithm steps continues until reaching a stop condition. The stop condition is usually a predetermined number of iteration or lack of any improvement in several successive iterations. The probability of accepting a weaker solution during the search is expressed by equations (14) and (15).

$$exp\left(-\Delta E / T\right) \tag{14}$$

$$\Delta E = f_{new} - f_{current} \tag{15}$$

In these equations, $f_{new}$ and $f_{current}$ are the new and incumbent solutions, and T is the temperature. Temperature is an algorithm parameter that determines the probability of accepting weaker solutions (the higher the temperature, the higher the probability). In this study, the initial temperature for each problem was determined by producing several random solutions and averaging their objective function values. In the proposed algorithm, as algorithm continues to run, temperature decreases according to the equation $T_{new} = DC.T_{current}$, where DC is a temperature reduction coefficient determined by trial and error to be 0.9. The flowchart of proposed SA algorithm is shown in Figure 3.

**Steps of the proposed simulated annealing algorithm**

1.1. Generate a random initial solution and calculate its objective function value. Set the algorithm parameters including initial temperature and temperature reduction coefficient.
1.2. Use one of the following methods to generate a neighborhood for the current solution.
1.3. Flip (0 to 1 and 1 to 0) the values of the cells selected randomly from the first N cells of solution string.
1.4. Swap the values of two cells selected randomly from the second N cells to alter the Node's priorities.
1.5. Calculate the value of the objective function for the new neighboring solution. In case of improvement, replace the incumbent solution. Otherwise, calculate the acceptation probability and generate a random number. If the random number is greater than acceptance probability, replace the incumbent solution with the new (weaker) one, otherwise, discard the new solution.
1.6. After producing 10 consecutive neighbors (the value 10 was obtained by trial and error) reduce the temperature.
1.7. Repeat the steps 2, 3 and 4 until the temperature becomes 0.05 of its initial value.

### 3.1. The proposed Genetic Algorithm

The main concept of evolutionary algorithms was introduced in 1975 by Holland (Holland, J. H. (1975)). Genetic algorithms (GA) are inspired by Darwin's evolution theory and is based on the principle of survival of the fittest. One of the main advantages of the genetic algorithm over older optimization methods is that at each given moment GA deals with a population or a set of points while older methods deal with only one point. This means that GA processes a large number of solutions at a time. This algorithm first finds a set of random solutions (chromosomes) called the initial population and produce then determines the fitness value of each chromosome according to a fitness function. It then selects the chromosomes having the best fitness for generating the next generation. The algorithm then uses an operator called crossover to produce the next generation and then subjects some of the new members of the population to another operator called mutation. Finally, the best members of the current and previous generations form the next generation and this process continues until reaching a stop condition (Jolai, F., Amalnick, M. S., Alinaghian, M., Shakhsi-Niaei, M., & Omrani, H. (2011)). The flowchart of proposed genetic algorithm is shown in figure 4.

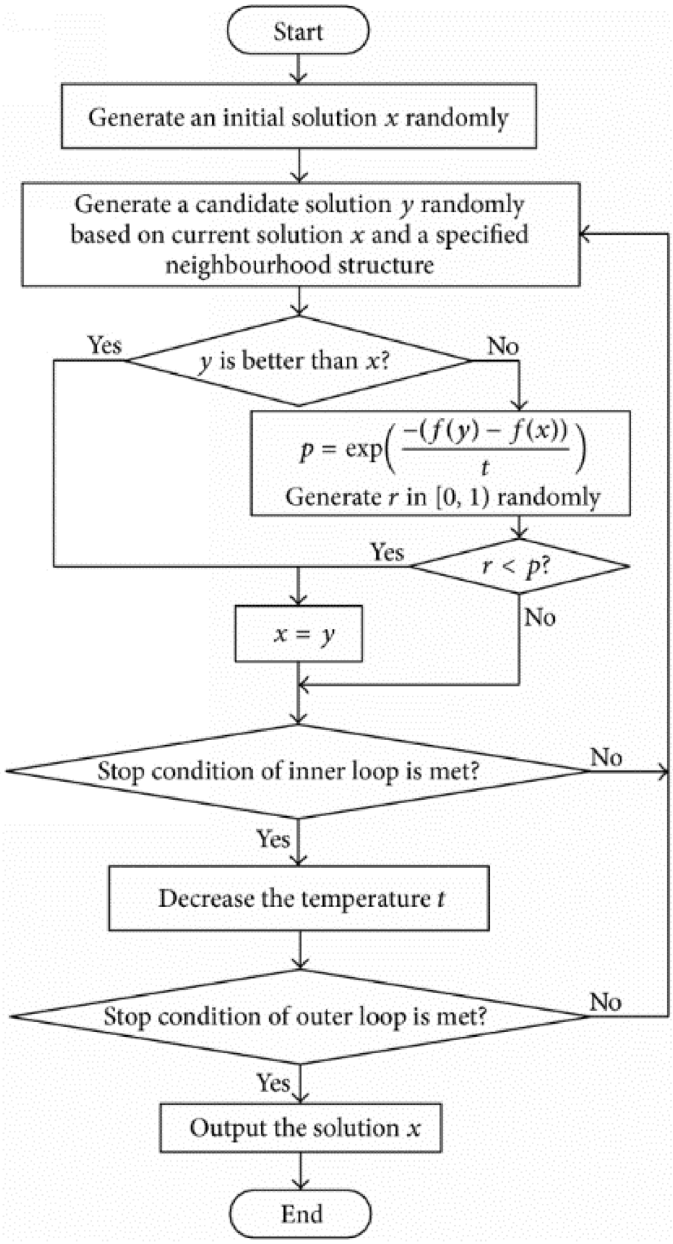


**Figure 3.** The flowchart of proposed SA algorithm

**Steps of the proposed Genetic Algorithm**

1. Generating initial solutions: randomly generate an initial population. Set the number of generations to 50 (obtained by trial and error). Calculate the objective function values of the solutions.
2. Generating children: produce a generation of offspring as large as initial generation by selecting two parents and using crossover operator to produce two children.
3. Parent selection: select parents using simple tournament method. For this purpose, randomly select two solutions from the population, and select the better solution; if solutions are equally fit make a random selection between the two.
4. Crossover: select parents using simple tournament method; generate two random numbers in the interval [0,2N] and use them to divide each selected parent string to three sections. Swap the middle sections of the parents to generate two children.
5. Mutation: After generating all children, apply mutation operator on 0.05 of all cells pertaining to children.
6. Mutation operator: randomly select a child, and then randomly select one cell of its string. If the selected cell is in the first part of the string (first N cells) flip its value; if the selected cell is in the second part of the string (second N cells) assign it with a randomly generated number in the interval [0,1].
7. Forming the next generation: After generating the children and applying the mutation operator, calculate their objective function values. Select 50 of the fittest children and parents to form the next generation.
8. Stop condition: repeat the algorithm steps for 100 iterations.

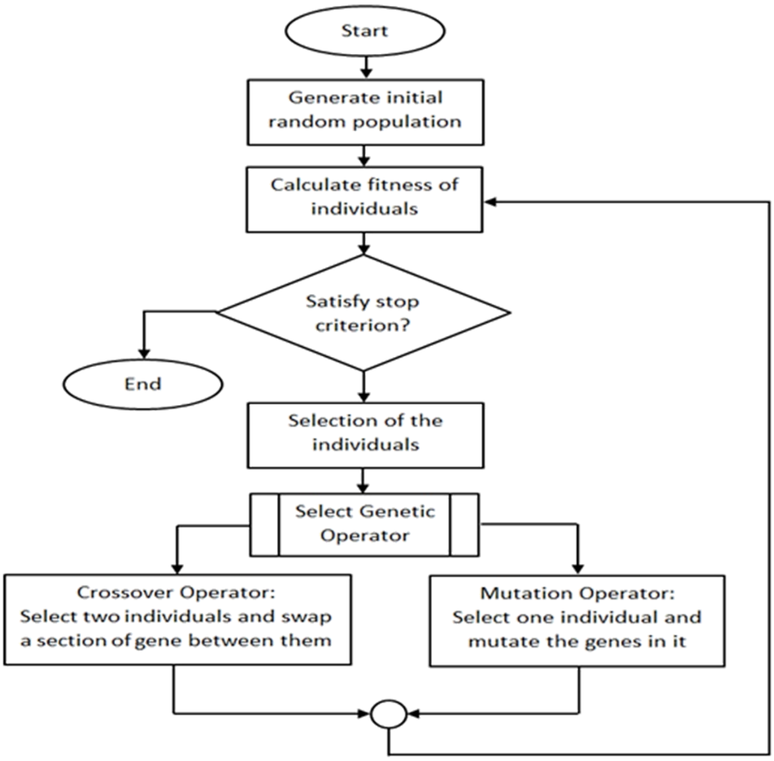


**Figure 4:** The flowchart of proposed genetic algorithm

## 4. Results

To evaluate the performance of the proposed meta-heuristic algorithms, 10 small scale and 12 large scale problems were generated. For small-scale problems, the results of the algorithms were compared with the exact solutions. For large scale problems, algorithms were compared with each other. Table 1 shows the ranges of generated parameters. It should be mentioned that the proposed algorithms were coded in MATLAB (2013), and were executed on a computer with Core i5@ 2.7 GHz CPU and 6GB RAM.

Table 1: parameters Intervals in sample problem generation

| parameters | Intervals |
|---|---|
| N | {6,10,20,30,40} |
| P | {2,4,6} |
| S | 4 |
| $\omega_{ij}$ | [5,15] |
| $\chi^{p}$ | [2,4] |
| $\delta^{p}$ | [2,4] |
| $\alpha^{p}$ | [0,1] |
| $g_{k}$ | [2E+6, 3E+6] |
| $f_{k}^{p}$ | [2E+5,3.5E+5] |

### 4.1. Small-scale problems

To evaluate the performance of the proposed meta-heuristic algorithms in solving small-scale problems, 36 small problems were generated. The results obtained by solving these problems provided in Table 2. It should be noted that the exact solution of small-scale problems was obtained via Branch and Bound method for GAMS software. To calculate the quality of two metaheuristic algorithms answers, in the small size problems the PRE index (percentage relative error) is used [44] (equation 16).

$$PRE = \frac{al - opt}{opt} \times 100 \tag{16}$$

In this formula $al$ is the answer obtained by metaheuristic algorithm and $opt$ is the optimal value obtained by the exact method. For better analyzing the performance of proposed metaheuristic algorithms, each test problem is solved 30 times by each algorithm and the best, and average of PRE index is reported along with average of CPU time. The results are presented in table 2. In this table, N and P are respectively the number of points and the number of products. Also for each algorithm, MPRE and BPRE are an average value and Best value of PRE index, respectively. Also, T is the time to reach to the optimal solution using gams software and the VT is the average of CPU time in 30 times of solving problem, using metaheuristic algorithm.

**Table 2.** Results in small sizes

| # | N | P | Gams | | GA | | | SA | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | obj(10^6) | T | VT | MPRE | BPRE | VT | MPRE | BPRE |
| 1 | 10 | 2 | 9.17 | 92 | 11 | 0.04 | 0 | 9 | 0.08 | 0 |
| 2 | 10 | 2 | 5.2 | 93 | 12 | 0.05 | 0 | 10 | 0.06 | 0 |
| 3 | 10 | 2 | 4.59 | 95 | 12 | 0.05 | 0 | 8 | 0.06 | 0 |
| 4 | 10 | 3 | 12.54 | 101 | 14 | 0.04 | 0 | 10 | 0.16 | 0 |
| 5 | 10 | 3 | 8.25 | 99 | 11 | 0.01 | 0 | 9 | 0.1 | 0 |
| 6 | 10 | 3 | 4.8 | 105 | 14 | 0.03 | 0 | 11 | 0.07 | 0 |
| 7 | 10 | 4 | 14.11 | 107 | 13 | 0.05 | 0 | 11 | 0.12 | 0 |
| 8 | 10 | 4 | 10.46 | 112 | 12 | 0.04 | 0 | 10 | 0.04 | 0 |
| 9 | 10 | 4 | 7.35 | 107 | 12 | 0.05 | 0 | 8 | 0.06 | 0 |
| 10 | 15 | 2 | 27.08 | 444 | 38 | 0.05 | 0 | 30 | 0.19 | 0 |
| 11 | 15 | 2 | 21.98 | 457 | 41 | 0.03 | 0 | 33 | 0.07 | 0 |
| 12 | 15 | 2 | 18.17 | 442 | 38 | 0.07 | 0 | 35 | 0.05 | 0 |
| 13 | 15 | 3 | 42.83 | 485 | 41 | 0.05 | 0 | 28 | 0.14 | 0 |
| 14 | 15 | 3 | 31.17 | 472 | 36 | 0.07 | 0 | 33 | 0.21 | 0 |
| 15 | 15 | 3 | 18.31 | 483 | 45 | 0.04 | 0 | 31 | 0.06 | 0 |
| 16 | 15 | 4 | 51.26 | 499 | 48 | 0.04 | 0 | 34 | 0.26 | 0 |
| 17 | 15 | 4 | 35.66 | 507 | 38 | 0.08 | 0 | 34 | 0.09 | 0 |
| 18 | 15 | 4 | 26.07 | 503 | 44 | 0.07 | 0 | 32 | 0.01 | 0 |
| 19 | 20 | 2 | 72.73 | 1374 | 88 | 0.1 | 0 | 80 | 0.01 | 0 |
| 20 | 20 | 2 | 42.74 | 1361 | 113 | 0.05 | 0 | 66 | 0.22 | 0 |
| 21 | 20 | 2 | 26.07 | 1370 | 104 | 0.09 | 0 | 70 | 0.13 | 0 |
| 22 | 20 | 3 | 91.21 | 1438 | 84 | 0.07 | 0 | 71 | 0.02 | 0 |
| 23 | 20 | 3 | 67.41 | 1448 | 95 | 0.06 | 0 | 64 | 0.18 | 0 |
| 24 | 20 | 3 | 43.55 | 1429 | 89 | 0.04 | 0 | 89 | 0.01 | 0 |
| 25 | 20 | 4 | 115.39 | 1488 | 96 | 0.06 | 0 | 76 | 0.25 | 0 |
| 26 | 20 | 4 | 99.02 | 1468 | 111 | 0.08 | 0 | 83 | 0.19 | 0 |
| 27 | 20 | 4 | 53.04 | 1501 | 92 | 0.05 | 0 | 70 | 0.13 | 0 |
| 28 | 25 | 2 | 129.54 | 3342 | 192 | 0.02 | 0 | 136 | 0.08 | 0 |
| 29 | 25 | 2 | 110.9 | 3333 | 219 | 0.09 | 0 | 133 | 0.25 | 0 |
| 30 | 25 | 2 | 84.34 | 3352 | 201 | 0.08 | 0 | 139 | 0.06 | 0 |
| 31 | 25 | 3 | 182.99 | 3369 | 181 | 0.11 | 0 | 171 | 0.27 | 0 |
| 32 | 25 | 3 | 125.64 | 3441 | 186 | 0 | 0 | 174 | 0.16 | 0 |
| 33 | 25 | 3 | 82.11 | 3372 | 214 | 0.01 | 0 | 169 | 0.2 | 0 |
| 34 | 25 | 4 | 256.23 | 3565 | 161 | 0.01 | 0 | 125 | 0.13 | 0 |
| 35 | 25 | 4 | 176.47 | 3548 | 214 | 0.09 | 0 | 159 | 0.05 | 0 |
| 36 | 25 | 4 | 103.12 | 3478 | 165 | 0.13 | 0 | 167 | 0.08 | 0 |
| **average** | | | | 1357.8 | 85.6 | 0.06 | 0 | 67 | 0.12 | 0 |

As Table 2 shows, in term of BPRE the proposed GA and SA show almost equal performance in solving small-scale problem instances, however, GA is better in terms of MPRE. The MPRE of proposed GA and SA are, respectively 6% and 12%. Figure 3 shows the CPU time of to metaheuristic algorithms.

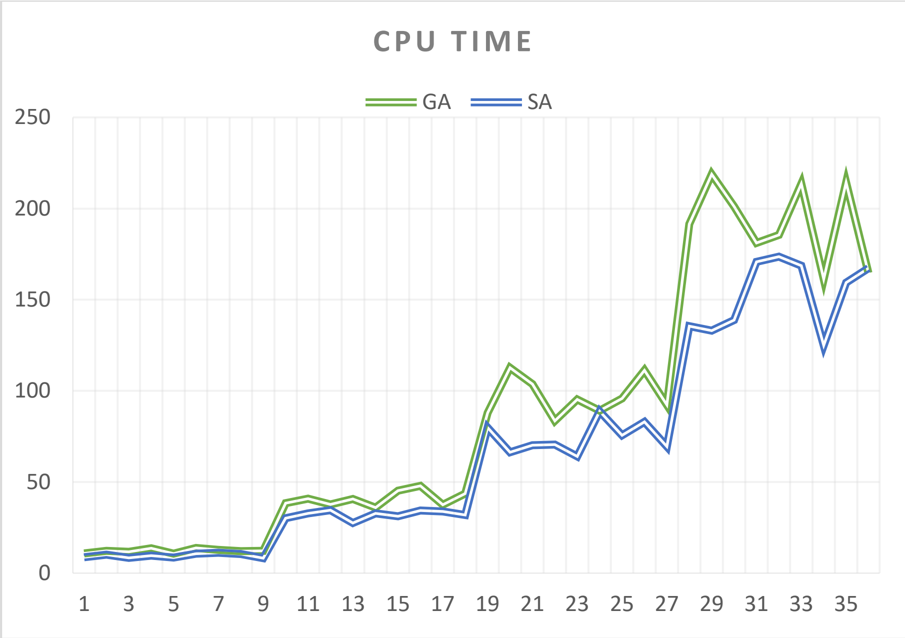


**Figure 3:** MT index of GA and SA algorithm

As can be seen in figure 3, the average solution time of, the proposed SA, and the proposed GA are almost equal. The average value of CPU time are, 85 and 67 seconds for GA and SA respectively.

### 4.2. Large-scale problems

Performance of the proposed meta-heuristic algorithms in solving large-scale problems was evaluated by generating and solving 36 large instance problems. Table 3 presents the results obtained by solving these problems. In this set of test problems reaching to the optimal solution in reasonable time is impossible, so to calculate the quality of two metaheuristic algorithms answers, the RPD index is used [47] (equation 17).

$$RPD = \frac{al - best}{best} \times 100 \qquad (17)$$

In this formula, the $al$ al is the answer obtained by metaheuristic algorithm and $best$ is the best answer obtained by two metaheuristic algorithm.

**Table 3.** Results in large size problems

| # | N | P | GA | | | SA | | |
|---|---|---|---|---|---|---|---|---|
| | | | VT | MRPD | BRPD | VT | MRPD | BRPD |
| 1 | 30 | 2 | 345.6 | 0.003 | 0 | 269.2 | 0.018 | 0 |
| 2 | 30 | 2 | 288.9 | 0 | 0 | 228.8 | 0.034 | 0 |
| 3 | 30 | 2 | 281.1 | 0.006 | 0 | 298.4 | 0 | 0 |
| 4 | 30 | 3 | 295.2 | 0.027 | 0 | 261.4 | 0.063 | 0.036 |
| 5 | 30 | 3 | 383.7 | 0.001 | 0 | 238.6 | 0.049 | 0.041 |
| 6 | 30 | 3 | 295.8 | 0.014 | 0 | 285.4 | 0.039 | 0 |
| 7 | 30 | 4 | 339.3 | 0.036 | 0 | 262.9 | 0.081 | 0 |
| 8 | 30 | 4 | 367.9 | 0.035 | 0 | 281.3 | 0.012 | 0 |
| 9 | 30 | 4 | 345.2 | 0.01 | 0.008 | 289.2 | 0.047 | 0 |
| 10 | 40 | 2 | 765.9 | 0.018 | 0.004 | 655.3 | 0.002 | 0 |
| 11 | 40 | 2 | 800 | 0.037 | 0 | 585.7 | 0.005 | 0 |
| 12 | 40 | 2 | 667.4 | 0.05 | 0.007 | 653.3 | 0.004 | 0 |
| 13 | 40 | 3 | 730.9 | 0.024 | 0.009 | 558.4 | 0.008 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 14 | 40 | 3 | 891.1 | 0.039 | 0 | 560.2 | 0.064 | 0 |
| 15 | 40 | 3 | 767 | 0.01 | 0 | 642.4 | 0.028 | 0.027 |
| 16 | 40 | 4 | 913.2 | 0.091 | 0 | 653.4 | 0.003 | 0 |
| 17 | 40 | 4 | 879.2 | 0.014 | 0 | 640.8 | 0.096 | 0.047 |
| 18 | 40 | 4 | 676.3 | 0.112 | 0 | 540.2 | 0.063 | 0 |
| 19 | 50 | 2 | 1662.4 | 0.001 | 0 | 1304 | 0.078 | 0.056 |
| 20 | 50 | 2 | 1314.3 | 0.044 | 0 | 1002.3 | 0.004 | 0 |
| 21 | 50 | 2 | 1421.7 | 0.055 | 0.016 | 1236.2 | 0.041 | 0 |
| 22 | 50 | 3 | 1555.7 | 0.044 | 0.007 | 1005 | 0 | 0 |
| 23 | 50 | 3 | 1564.8 | 0.02 | 0 | 1235.6 | 0.125 | 0.098 |
| 24 | 50 | 3 | 1359.7 | 0.006 | 0 | 1003.1 | 0.065 | 0.055 |
| 25 | 50 | 4 | 1315.4 | 0.008 | 0.007 | 1260.9 | 0.19 | 0 |
| 26 | 50 | 4 | 1523 | 0.112 | 0 | 1135.1 | 0.024 | 0 |
| 27 | 50 | 4 | 1254.9 | 0.062 | 0 | 1188.9 | 0.01 | 0 |
| 28 | 60 | 2 | 2924.9 | 0.01 | 0 | 1701 | 0.018 | 0.012 |
| 29 | 60 | 2 | 2192 | 0.01 | 0 | 1702 | 0.003 | 0 |
| 30 | 60 | 2 | 3067.2 | 0.046 | 0 | 2368.9 | 0.006 | 0 |
| 31 | 60 | 3 | 2224.4 | 0.053 | 0 | 1955.4 | 0.05 | 0 |
| 32 | 60 | 3 | 2250.6 | 0.023 | 0 | 1703.9 | 0.112 | 0 |
| 33 | 60 | 3 | 2539.9 | 0.02 | 0 | 1779.6 | 0.058 | 0 |
| 34 | 60 | 4 | 2235.6 | 0.024 | 0 | 2066.3 | 0.127 | 0.108 |
| 35 | 60 | 4 | 3023.4 | 0.009 | 0 | 1772.4 | 0.007 | 0 |
| 36 | 60 | 4 | 2229.3 | 0.04 | 0 | 2065.8 | 0.046 | 0 |
| | | | 1269.3 | 0.031 | 0.002 | 983.1 | 0.044 | 0.013 |

As Table 3 shows, for large-scale problems, the proposed GA has a better performance than the proposed SA in view of BRPD and MRPD indices. For problems of this size, BRPD index of proposed SA and GA are, respectively, 0.2% and 1.3%. The proposed GA obtained the best solution for 29 of 36 problem instances, while the proposed SA obtained the best solution for 27 problems. Figure 4 shows the MRPD index of the proposed algorithm.

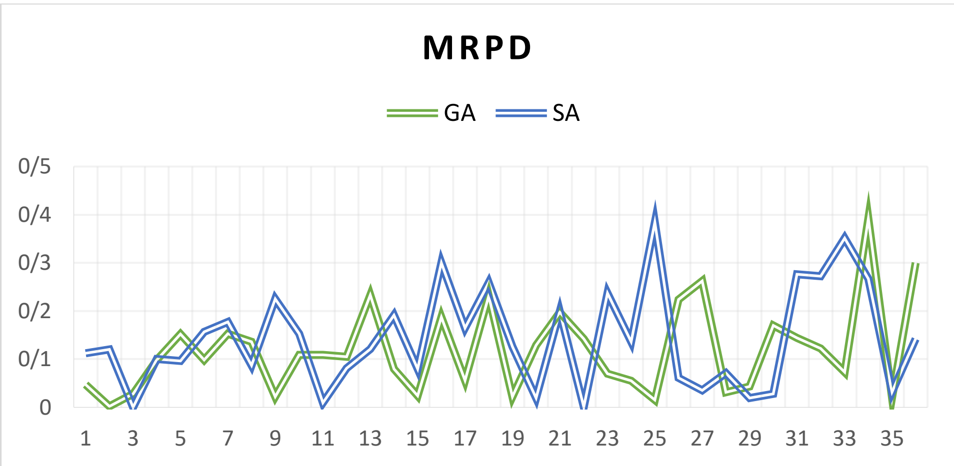

**Figure 4:** MRPD index of GA and SA algorithm

As can be seen in Figure 3, according to the MRPD, two algorithms almost have same performance, the average value of this index for GA and SA algorithm are 3.1% and 4.4% respectively. For better analyzing the results, the statistical analysis is performed on the results.

The Statistical test is performed for processing time, mean PRE, best PRE, worst PRE, mean RPD, best RPD, worst RPD indices. In order to compare the performance of each algorithm statistically, the obtained results are analyzed using T-test. Firstly, a “Two-Sample For Variances F-Test” is done to determine whether the variances are equal or not, then, if the variances are equal, a “Two-Sample Assuming Equal Variances T-test” is done and otherwise, a “Two-Sample Assuming Unequal Variances T-test” is applied to the obtained results(p<5%). All tests are one-tail tests, and the null hypothesis is that the variances or averages are equal and the other hypothesis is that one is more than the other.

**Table 4.** Statistical Analysis of results in small and large size problems

| size | index | Variance | | | | Averages | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | F | P(F<=f) one-tail | F Critical one-tail | result | t Stat | P(T<=t) two-tail | t Critical two-tail | result |
| Small | time | 1.584 | 0.092 | 1.772 | Accept | 1.247 | 0.217 | 1.995 | Accept |
| | Mean PRE | 0.145 | 8.22E-08 | 0.564 | Accept | -4.468 | 3.07E-05 | 1.995 | Reject |
| | Best PRE | - | - | 0.564 | Accept | - | - | 1.995 | Accept |
| | Worst PRE | 0.145 | 8.2E-08 | 0.564 | Accept | -4.580 | 2.04E-05 | 1.995 | Reject |
| Large | time | 1.815 | 0.043 | 1.772 | Reject | 1.620 | 0.110 | 1.995 | Accept |
| | Mean RPD | 0.408 | 0.005 | 0.564 | Accept | -1.434 | 0.156 | 1.995 | Accept |
| | Best RPD | 0.017 | 0 | 0.564 | Accept | -2.518 | 0.014 | 1.995 | Reject |
| | Worst RPD | 0.150 | 1.2E-07 | 0.564 | Accept | -0.935 | 0.353 | 1.995 | Accept |

As can be seen in table 4 Except for the large size problems’ processing time criterion, in the other criteria, There is not enough evidence to reject the equality of variances (p<5%). According to the statistical results, in the small size problems, in the Mean PRE and Worst PRE the performance of GA is better than SA algorithm but in other criteria, both algorithms have the same performance. Also in the large size problems GA algorithm, reach to the better result in the Best RPD, but in the other criteria, the performance of two algorithms was same.

### 4.3. Price of robustness

Solutions obtained by a robust counterpart of a mathematical model remains feasible with little changes in the value of objective function against the extreme changes in the problem’s parameters. But in the robust models, the optimal solution has more costs as compared to the mathematical model. increasing Costs in the robust model is due to increase in the number facilities constructed to deal with uncertainty. In order to show the effect of considering the robust counterpart of the model, a sample instance of the problem is taken and solved using the Cplex solver in different conditions. Firstly, the problem is solved by considering each scenario separately, and five different sets of solutions obtained. Then, each solution set is tested in the problem with all scenarios (and their related probabilities, it is worth mentioning that the probability of All scenarios were considered to be equal) and the obtained objective values are reported in Table 5. In this table, SOBJ is the objective value of model when it contains only one scenario. To calculate the AOBJ, first, the model is run only with one scenario and then the first stage variables are fixed in the model and model is run again considering all scenarios. And finally, ROBJ is the objective value of the robust model.

**Table 5**: price of robustness

| Scenario | SOBJ | AOBJ | ROBJ |
|---|---|---|---|
| Scenario #1 | 28,562 | 36,296 | 32,620 |
| Scenario #2 | 30,921 | 35,124 | |
| Scenario #3 | 32,035 | 33,808 | |
| Scenario #4 | 35,739 | 35,296 | |
| Scenario #5 | 39,601 | 37,007 | |
| average | 33,372 | 35,506 | 32,620 |

As the results show, although the total costs are lower when there is only one scenario when there are multiple scenarios similar to the real world, it would be much better to take all scenarios and their probabilities into consideration and solve the robust model.

## 5. Conclusion

This paper presented a new robust mathematical model for the classic multi-product capacitated single allocation hub location problem with covering radius. Considering the NP-Hard complexity of this problem, two meta-heuristic algorithms were developed to solve the large-scale variant of the problem. To evaluate the performance of these algorithms, a number of large and small-scale problems were generated and the results of these two algorithms were compared with the results of the exact solution. For small-scale problems, the two proposed algorithms show almost equal performance. Average of PRE index of the proposed SA and GA were, respectively, 5% and 12%. For the large-scale problems, the proposed GA obtained the best solutions of 29 out of 36 problem instances. These results show the good performance of the proposed algorithms. Considering covering radius of a customer as a factor related to the customer satisfaction, and presenting a bi-objective mathematical model in which one objective function tries to minimize all costs and other tries to maximize the customer satisfaction by minimizing the maximum covering radius of customers, is an interesting subject that can be considered in the future research.

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