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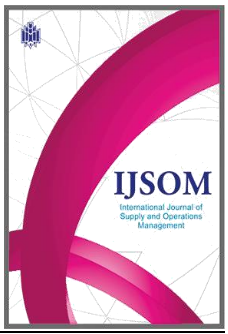

Performance Measurement and Productivity Management in Production Units with Network Structure by Identification of the Most Productive Scale Size Pattern

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Abstract
Managers tend to improve resource (input) utilization in organizations to obtain the highest level of productivity. Additionally, many industrial units have multi-stage structure in which the output of one stage is the input of the next one. This paper, for the first time, presents data envelopment analysis (DEA) approaches to achieve the most productivity in two-stage decision making units (DMUs). By considering internal activities in system, radial and non-radial models are proposed to evaluate network DMUs and radial model is developed to identify the most productive scale size (MPSS) pattern. Proposed models are applied to optimize the performance of bank branches as units with two-stage structure. Results show that efficiency scores and improvements needed in costs and paid interests (inputs) to get more incomes and facilities (outputs). This study provides managers with information to propose better strategies to improve not only the overall performance but also the efficiency of each stage.

Keywords: Data envelopment analysis (DEA); Network DEA; Most productive scale size (MPSS); Scale efficient target.

1. Introduction

Identification of the most productive scale size (MPSS) patterns, which are called scale efficient targets, shows the needed improvements of resources (as inputs of organization) to obtain the highest possible level of productivity. Besides, in multi-stage production systems, the performance of each stage and overall performance must be improved. In production and industrial units with network structure, the outputs of one stage are the inputs of the next. For example, in Banks, as two-stage systems, labor, physical capital, and financial equity capital are the inputs of the first stage to produce deposits as the output of this stage. In the second stage, the inputs are the deposits raised from the first stage and the outputs are loans and security investments. Assessing multi-stage systems needs special considerations including how each stage performs, what the efficiency score of each score is, how the stages are related to one another, etc. Investigations to achieve the answers, especially in managerial decisions, identify inefficiencies which may exist in internal activities and provide managers with useful insights to optimize overall performance of system.

As a nonparametric technique, data envelopment analysis (DEA) is mathematical programming to evaluate the performance of homogenous decision-making units (DMUs). Traditional DEA models evaluate two-stage DMU as a black box and neglect the connectivity which may exist among the stages. We look inside the system and introduce models to optimize two-stage DMU by considering the intermediate activities between the stages. Furthermore, in network DEA models which will be mentioned in next section, constraints related to intermediate activities are considered as inequalities which, as will be shown in this paper, result in contradictions in optimality. In this paper, this point is taken into consideration. Additionally, each stage of network structure production systems may consist of parallel parts in which the inputs and the outputs of the entire stage are separated for each part. This paper addresses units with such a two-stage structure and presents radial and non-radial models to measure efficiency scores.

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An important issue here is the identification of MPSS pattern for each stage and for overall DMU that can help managers to improve the scale size of inputs and outputs values to obtain the highest level of productivity. Banker (1984) and Banker and Thrall (1992), noted that the production possibility (X_o, Y_o) in production possibility set T represents MPSS for its specific mix of inputs/outputs if and only if for all $(\alpha X_o, \beta Y_o) \in T$ we have $\alpha \geq \beta$. In other words, a production possibility is not MPSS when either (a) all outputs can be increased in proportions that are at least as great as the corresponding proportional increases in all inputs, or (b) all inputs can be decreased in proportions that are at least as great as the accompanying proportional reduction in all outputs. (Banker, Cooper, Seiford & Zhu (2004)) In this paper, for the first time, models are proposed to identify MPSS pattern for two-stage-structure DMUs. Setting scale efficient target to a system aids managers in optimal use of existing resources and gives information about whether increments in resources are profitable or not.

The Next section reviews studies which have been conducted about network DEA models. Section 3 covers the approaches to address the problem of optimizing two-stage DMU's. The application of proposed models in banking system is provided in section 4.

2. Literature review

For the first time, Färe and Primont (1984) applied DEA, as a network DEA to evaluate the performance of multi-plant firms. Färe and Grosskopf (2000) suggested models to measure efficiency score of multi-stage DMUs in static and dynamic cases. By considering the intermediate products Wang, Gopal & Zionts (1997) proposed a DEA model to measure the efficiency score of two-stage structure DMUs, then Seiford and Zhu (1999) extended their approach. By projecting two-stage structure DMUs on efficient frontier, Chen and Zhu (2004) improved the models presented by Seiford and Zhu. Kao and Hwang (2008) and Chen, Cook, Li, & Zhu (2009) evaluated efficiency score of two-stage DMU. The former introduced efficiency of the DMU as the product of efficiencies of its stages and the other introduced it as a weighted mean of efficiency scores of stages. Yang, Wu, Liang, Bi & Wu (2011) presented a non-linear model in order to measure the efficiency of two-member supply chains, as two-stage DMUs. Paradi, Rouatt & Zhu (2011) developed a two-stage DEA approach by consideration geographic location and market size and applied it to assess bank branches. Fukuyama and Mirdehghan (2012) proposed slack-based network approach for identifying the efficiency status of each DMU and its divisions. Amirteimoori (2013) and Liu (2014) proposed DEA approaches for measuring the efficiency of two-stage decision process in the presence of imperfect outputs and Fuzzy data respectively. Wang, Huang, Wu & Liu (2014) utilized the network DEA approach to evaluate the efficiencies of major Chinese commercial banks. Sahoo, Zhu, Tone & Klemen (2014) proposed and developed two approaches for evaluating two sub-division production systems: a single network technology for two interdependent sub-technologies and the second method, which is due to Kao and Hwang (2011), assumes two independent sub-technology frontiers, one for each sub-technology. Lu, Kweh & Huang (2014) developed a network DEA approach to evaluate the national innovation system (NIS) and examined the effect of intellectual capital (IC) on NIS performance through truncated regression. Liu, Zhoua, Maa, Liu & Shen (2015) proposed models to measure the efficiencies of China's listed banks with undesirable inputs, intermediates and outputs. Barros and Wanke (2015) presented an efficiency assessment of African airlines using the TOPSIS technique for order preference by comparing with the ideal solution. TOPSIS is a multi-criteria decision making technique, which, similar to DEA, ranks a finite set of units based on the minimization of distance from an ideal point, and the maximization of distance from an anti-ideal point. In this research, TOPSIS was initially used in a two-stage approach, in order to assess the relative efficiency of African airlines using the most frequent indicators adopted by the literature on airlines. Chao, Yu & Wu (2015) applied a dynamic network slack-based measure data envelopment analysis model (DNSBM) to measure the efficiency of Taiwanese banks during the period 2005–2011. Hu and Yu (2015) used a two-stage least-square approach to investigate the relationships among risk, capital, and operating efficiency for Taiwanese life insurance companies from 2004 to 2009. Chen, Chiu, Jan, Chen & Liu (2015) used the hybrid DEA model, evaluating the proportionate inputs with a radial measure and the non-proportionate inputs with a non-radial measure, in order to examine the impact of non-performing loans (as bank risks) on the efficiency of Taiwan's banking sector from 2006 to 2010. The automatic service input, which is defined as non-radial input, is assumed to not increase or decrease proportionally with other inputs which are defined as radial inputs. Chao, Yu, Lee & Hsiao (2016) developed a dynamic multi-activity network DEA model to measure the efficiency of twenty-seven Taiwanese banks under a multi-stage and multi-activity production process during the sample period of 2006–2012. Fukuyama and Matousek (2016) developed a bank network revenue function to evaluate banks' network revenue performance. The bank network revenue function is constructed as the difference between total revenue and the reserves for possible loan losses to incorporate the roles played by non-performing loans in bank production. They applied this function to evaluate Japanese banks from September 2000 to March 2013. Wankea, Maredzab & Guptac (2017) proposed a strategic fit assessment of mergers and acquisitions (M & A) in South African banks in which a network DEA approach is adopted to compute the impact of contextual variables on several types of efficiency scores of the resulting virtual merged banks: global (merger), technical (learning), harmony (scope), and scale (size) efficiencies. Koushki (2017) presented a dynamic DEA network approach to evaluate two-stage structure DMUs where the activity and the performance of DMU in one period affected its efficiency in the next. According to the results of proposed dynamic model, the inefficiencies of DMU's improve considerably. Lin, Chen, Hu & Li (2017) decomposed the overall efficiency of a mutual fund in the whole investment interval into efficiencies at individual periods. Efficiency decomposition reveals the time at which the inefficiency occurs. Their multi-period network DEA approach provides expected inputs, outputs and intermediate variables at individual periods, which are helpful for managers to find factors causing the overall inefficiency of a fund. They applied proposed approach to assess the relative performance of funds in Chinese security market and European security market, respectively. Shokri Kahi, Yousefi, Shaban pour & Farzipoor Saen (2017) improved the dynamic DEA model proposed by Tone and Tsutsui (2014) and developed an additive network DEA model to evaluate sustainability of supply chains in several periods. In their method, the carry-overs in period (t) enter period (t+1). They defined activities among periods as desirable and undesirable carry-overs. They used all the links in assessing the sustainability of supply chains (DMUs). A stochastic two-stage network DEA model was introduced by Zhou, Lin, Xiao, Ma & Wu (2017) as a deterministic linear programming model under the assumption that components of inputs, outputs and intermediate products are related with some basic stochastic factors ,and was applied to evaluate 16 commercial banks in China. Galagedera, Roshdi, Fukuyama & Zhu (2018) developed a network DEA model to assess overall and stage-level performance of fund management function as a three-stage production process. The stage-level processes operate under environmental conditions-levels of risk exposure which are treated as conditions imposed on the intermediate measures (products). Li, Chen, Cook, Zhang & Zhu (2018) presented two-stage DEA model to identify the leader (dominant) stage related to non-cooperative game. (stackelberg game or leader-follower)

3. Methodology

3.1 Network DEA

Consider DMU $_j$ (j=1,…,n) with two- stage structure. Let X_{1j}, and Z_{1j} be vectors of inputs and outputs of part 1 of stage 1. Let X_{2j}, and Z_{2j} be vectors inputs and outputs of part 2 of stage 1 as well,. Consider Z_{1j} and Z_{2j} as the vectors of inputs of stage 2 with Y_j as output vector. A network with two stages connected in series structure is depicted in Figure1, in which the first stage consists of two stages connected with parallel structure.

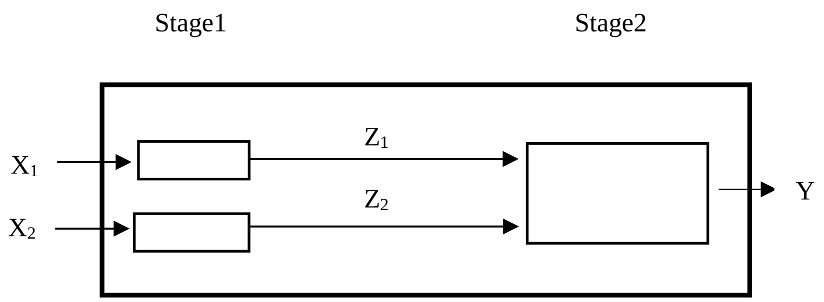

Figure1. DMU with special two-stage structure

Without considering intermediate products Z1j and Z_{2j}, the radial input-oriented CCR model (Charnes, Cooper & Rhodes (1978)) to measure the efficiency score of DMU_0 is defined as follow:

$$
\begin{aligned}
&\theta^* = \min \theta \\
&s.t. \sum_{j=1}^{n} \lambda_j X_j \le \theta X_o \\
&\sum_{j=1}^{n} \lambda_j Y_j \ge Y_o \\
&\lambda \ge 0
\end{aligned} \tag{1}
$$

This model considers only the overall input and output (X_j, Y_j) of system. But by considering network structure of DMU, the production possibility set (PPS)T_N is defined. PPS contains all DMU_j for j=1,…,n and their positive combinations as follows:

$$
T_N = \begin{Bmatrix} (X_1, X_2, Z_1, Z_2, Y) \mid \sum_{j=1}^{n} \lambda_j^1 X_{1j} \le X_1, \sum_{j=1}^{n} \lambda_j^2 X_{2j} \le X_2, \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = Z_1, \sum_{j=1}^{n} \lambda_j^3 Z_{1j} = Z_1, \sum_{j=1}^{n} \lambda_j^2 Z_{2j} = Z_2, \sum_{j=1}^{n} \lambda_j^3 Z_{2j} = Z_2, \sum_{j=1}^{n} \lambda_j^3 Y_j \ge Y \\ \lambda^k \ge 0, k = 1,2,3 \end{Bmatrix}
$$

Note that $Z_l = \sum_{j=1}^{n} \lambda_j^1 Z_{lj}$, $Z_l = \sum_{j=1}^{n} \lambda_j^2 Z_{lj}$, $Z_l = \sum_{j=1}^{n} \lambda_j^3 Z_{lj}$ for $l = 1,2$ show the series relationship and connectivity between stages, since outputs of stage1 are the same as inputs of stage2.

Many previous suggested models to evaluate DMUs with network structure considered inequalities $\sum_{j=1}^{n} z_j \lambda_j \geq z_0$, $\sum_{j=1}^{n} z_j \lambda_j \leq z_0$ instead of the above equalities. However, according to these inequalities, in optimality we will have $\sum_{j=1}^{n} z_j \lambda^*_j - S^{*+} = z_0$, $\sum_{j=1}^{n} z_j \lambda^*_j + S^{*-} = z_0$ .Therefore, in optimality equalities $\sum_{j=1}^{n} z_j \lambda^*_j = z_0 + S^{*+}$, $\sum_{j=1}^{n} z_j \lambda^*_j = z_0 - S^{*-}$ are contradictions when the slack variables are non-zero.

3.2 Performance evaluation

Radial and non-radial models will be presented to measure the efficiency scores of DMU with the structure shown in Figure.1. In the radial model, the objective is minimizing the multiplier of inputs to reduce them (or maximizing multiplier of outputs to expand them) in order to obtain the target through a radius crossing DMU. In the non-radial case, decrements in inputs' values and increments in outputs' values are calculated based on slack variables related to the constraints in inputs and outputs. Constraints of models are determined according to set T_N .

1) Radial models

The efficiency score of DMU_o (DMU under evaluation) will be obtained by solving the following model. Note that in the proposed models, the radial reduction of inputs and the radial increment of outputs are denoted by γ_1, γ_2 respectively. α_1, α_2 represent the possible change of intermediate activities when optimizing DMU_o .

$$
\begin{aligned}
& Min \quad \frac{\gamma_1}{\gamma_2} \\
& s.t. (\gamma_1 X_{1o}, \gamma_1 X_{2o}, \alpha_1 Z_{1o}, \alpha_2 Z_{2o}, \gamma_2 Y_o) \in T_N
\end{aligned}
\tag{2}
$$

Model (2) is follow linear programming:

$$
\begin{aligned}
& Min \quad \frac{\gamma_1}{\gamma_2} \\
& s.t. \sum_{j=1}^{n} \lambda_j^1 X_{1j} \leq \gamma_1 X_{1o} \\
& \quad \sum_{j=1}^{n} \lambda_j^2 X_{2j} \leq \gamma_1 X_{2o} \\
& \quad \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = \alpha_1 Z_{1o} \\
& \quad \sum_{j=1}^{n} \lambda_j^3 Z_{1j} = \alpha_1 Z_{1o} \\
& \quad \sum_{j=1}^{n} \lambda_j^2 Z_{2j} = \alpha_2 Z_{2o} \\
& \quad \sum_{j=1}^{n} \lambda_j^3 Z_{2j} = \alpha_2 Z_{2o} \\
& \quad \sum_{j=1}^{n} \lambda_j^3 Y_j \geq \gamma_2 Y_o
\end{aligned}
$$

$$\alpha_1 > 0, \alpha_2 > 0, \gamma_1 \le 1, \gamma_2 \ge 1, \lambda^k \ge 0 \qquad k = 1,2,3 \tag{3}$$

Let $\left\{\lambda_o^k = 1, \lambda_j^k = 0, \gamma_1 = 1, \gamma_2 = 1, \alpha_1 = 1, \alpha_2 = 1\right\}$, $k = 1,2,3,\ j = 1,..,n,\ j \ne o$. Thus, a feasible solution of model (3) is obtained.

Definition1. A DMU_o is efficient if and only if we have $\gamma_1^* = 1, \gamma_2^* = 1$ in the optimal solution of model (3)

Theorem1. Any DMU with the input/output vectors as $(\gamma_1^* X_{1o}, \gamma_1^* X_{2o}, \alpha_1^* Z_{1o}, \alpha_2^* Z_{2o}, \gamma_2^* Y_o)$ is efficient.

Proof. For assessing DMU $(\gamma_1^* X_{1o}, \gamma_1^* X_{2o}, \alpha_1^* Z_{1o}, \alpha_2^* Z_{2o}, \gamma_2^* Y_o)$ by replacing the coordinate of the unit we have the following model. After solving this model, let $\bar{\gamma}_1, \bar{\gamma}_2$ be the optimal values of γ_1, γ_2 .

$$\begin{aligned}
Min \quad & \frac{\gamma_1}{\gamma_2} \\
s.t. & \sum_{j=1}^{n} \lambda_j^1 X_{1j} \le \gamma_1(\gamma_1^* X_{1o}) \\
& \sum_{j=1}^{n} \lambda_j^2 X_{2j} \le \gamma_1(\gamma_1^* X_{2o}) \\
& \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = \alpha_1 Z_{1o} \\
& \sum_{j=1}^{n} \lambda_j^3 Z_{1j} = \alpha_1 Z_{1o} \\
& \sum_{j=1}^{n} \lambda_j^2 Z_{2j} = \alpha_2 Z_{2o} \\
& \sum_{j=1}^{n} \lambda_j^3 Z_{2j} = \alpha_2 Z_{2o} \\
& \sum_{j=1}^{n} \lambda_j^3 Y_j \ge \gamma_2(\gamma_2^* Y_o) \\
& \gamma_1 \le 1, \gamma_2 \ge 1, \lambda^k \ge 0 \qquad k = 1,2,3
\end{aligned} \tag{4}$$

If $\bar{\gamma}_1 < 1$, from $\bar{\gamma}_2 \ge 1$ we have $\frac{\bar{\gamma}_1 \gamma_1^*}{\bar{\gamma}_2 \gamma_2^*} < \frac{\gamma_1^*}{\gamma_2^*}$; which is a contradiction. Similar contradiction is obtained by assuming $\bar{\gamma}_2 > 1$.■Linear counterpart of model (3) will be obtained by dividing all constraints by γ_2 :

$$\begin{aligned}
Min \quad & \omega \\
s.t. & \sum_{j=1}^{n} \lambda_j'^1 X_{1j} \le \omega X_{1o} \\
& \sum_{j=1}^{n} \lambda_j'^2 X_{2j} \le \omega X_{2o} \\
& \sum_{j=1}^{n} \lambda_j'^1 Z_{1j} = \alpha_1' Z_{1o} \\
& \sum_{j=1}^{n} \lambda_j'^3 Z_{1j} = \alpha_1' Z_{1o} \\
& \sum_{j=1}^{n} \lambda_j'^2 Z_{2j} = \alpha_2' Z_{2o}
\end{aligned}$$

$$\sum_{j=1}^{n}\lambda_j'^{3}Z_{2j}=\alpha_2'Z_{2o}$$

$$\sum_{j=1}^{n}\lambda_j'^{3}Y_j\geq Y_o$$

$$\alpha_1'>0,\alpha_2'>0,\lambda'^{k}\geq 0 \qquad k=1,2,3 \tag{5}$$

Where $\lambda_j'^{k}=\dfrac{1}{\gamma_2}\lambda_j^{k},\omega=\dfrac{\gamma_1}{\gamma_2},\ \alpha_1'=\dfrac{1}{\gamma_2}\alpha_1,\alpha_2'=\dfrac{1}{\gamma_2}\alpha_2,\ k=1,2,3$.

To determine the values of γ_1,γ_2, a method in the next section will be presented.

2) Non-radial model

Reductions in input values (and increments in output values) based on slack variables related to inputs (and outputs) constraints is another approach to measure efficiency score of DMU_o, which is called non-radial slack-based model,. Objective function is defined according to slacks to obtain the maximum improvements of input and output values. Moreover, connectivity between the stages results in equalities $\sum_{j=1}^{n}\lambda_j^{1}Z_{1j}=\sum_{j=1}^{n}\lambda_j^{3}Z_{1j}$, $\sum_{j=1}^{n}\lambda_j^{2}Z_{2j}=\sum_{j=1}^{n}\lambda_j^{3}Z_{2j}$. The mentioned model is as follows:

$$Min\quad \rho=\frac{1-\frac{1}{2}\left(\frac{1}{m1}\sum_{i=1}^{m1}\frac{s_{1i}^{-}}{x_{i1o}}+\frac{1}{m2}\sum_{i=1}^{m2}\frac{s_{2i}^{-}}{x_{i2o}}\right)}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^{+}}{y_{ro}}}$$

$$s.t.\sum_{j=1}^{n}\lambda_j^{1}X_{1j}+S_1^{-}=X_{1o}$$

$$\sum_{j=1}^{n}\lambda_j^{2}X_{2j}+S_2^{-}=X_{2o}$$

$$\sum_{j=1}^{n}\lambda_j^{1}Z_{1j}=\sum_{j=1}^{n}\lambda_j^{3}Z_{1j}$$

$$\sum_{j=1}^{n}\lambda_j^{2}Z_{2j}=\sum_{j=1}^{n}\lambda_j^{3}Z_{2j}$$

$$\sum_{j=1}^{n}\lambda_j^{3}Y_j-S^{+}=Y_o$$

$$S_1^{-},S_2^{-},S^{+},\lambda^1,\lambda^2,\lambda^3\geq 0 \tag{6}$$

Definition2. A DMU with coordination of $(X_{1o},X_{2o},Z_{1o},Z_{2o},Y_o)$ is efficient if and only if in optimality $S_1^{-^*}=0,S_2^{-^*}=0,S^{+^*}=0$.

Theorem2. A DMU with coordination of $(X_{1o}-S_1^{-^*},X_{2o}-S_2^{-^*},Z_{1o},Z_{2o},Y_o+S^{+^*})$ is efficient.

Proof. We apply model (6) for evaluating the efficiency of $(X_{1o}-S_1^{-^*},X_{2o}-S_2^{-^*},Z_{1o},Z_{2o},Y_o+S^{+^*})$ as follows:

$$Min\quad \rho=\frac{1-\frac{1}{2}\left(\frac{1}{m1}\sum_{i=1}^{m1}\frac{s_{1i}^{-}}{x_{i1o}}+\frac{1}{m2}\sum_{i=1}^{m2}\frac{s_{2i}^{-}}{x_{i2o}}\right)}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^{+}}{y_{ro}}}$$

$$
\begin{aligned}
s.t. & \sum_{j=1}^{n} \lambda_j^1 X_{1j} = X_{1o} - S_1^{-*} - S_1^{-} \\
& \sum_{j=1}^{n} \lambda_j^2 X_{2j} = X_{2o} - S_2^{-*} - S_2^{-} \\
& \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = \sum_{j=1}^{n} \lambda_j^3 Z_{1j} \\
& \sum_{j=1}^{n} \lambda_j^2 Z_{2j} = \sum_{j=1}^{n} \lambda_j^3 Z_{2j} \\
& \sum_{j=1}^{n} \lambda_j^3 Y_j = Y_o + S^{+*} + S^{+} \\
& \lambda^k \geq 0 \qquad\qquad k = 1,2,3
\end{aligned} \tag{7}
$$

Let $\overline{S}_1^-$, $\overline{S}_2^-$, $\overline{S}^+$ be optimal vectors in model (7). If there exists $i' \in \{1,\ldots,m1\}$ satisfying $\overline{S}_{1i'}^- > 0$ then by considering vectors $S_1^{-*} + \overline{S}_1^-, S_2^{-*} + \overline{S}_2^-, S^{+*} + \overline{S}^+$ the objective value is less than that obtained from $S_1^{-*} + \overline{S}_1^-, S_2^{-*} + \overline{S}_2^-, S^{+*} + \overline{S}^+$, this is a contradiction with the optimality of the vectors $S_1^{-*}, S_2^{-*}, S^{+*}$.■

3.3 Scale efficient targets

In this section, the definition of MPSS will be presented, according to the input and output vectors in the special structure network. Also, a new approach will be provided for identifying the MPSS and the way for projecting DMUs on MPSS points, as scale efficient target.

Definition3. DMU $(X_1, X_2, Z_1, Z_2, Y) \in T_{NV}$ is MPSS if and only if for every DMU $(\mu X_1, \mu X_2, \eta_1 Z_1, \eta_2 Z_2, \beta Y) \in T_{NV}$ we have $\mu \geq \beta$ where

$$
T_{NV} = \left\{ \begin{array}{l} (X_1, X_2, Z_1, Z_2, Y) \mid \sum_{j=1}^{n} \lambda_j^1 X_{1j} \leq X_1, \sum_{j=1}^{n} \lambda_j^2 X_{2j} \leq X_2, \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = Z_1, \sum_{j=1}^{n} \lambda_j^3 Z_{1j} = Z_1, \sum_{j=1}^{n} \lambda_j^2 Z_{2j} = Z_2, \sum_{j=1}^{n} \lambda_j^3 Z_{2j} = Z_2, \sum_{j=1}^{n} \lambda_j^3 Y_j \geq Y \\ \lambda^k \geq 0, 1\lambda^k = 1, k = 1,2,3 \end{array} \right\}
$$

Equalities $\sum_{j=1}^{n} \lambda_j^l = \sum_{j=1}^{n} \lambda_j^{l+1}, \quad l = 1,2$ are taken into account to hold connectivity between two stages in scale efficient target for each DMU. Therefore, model (3) is written as follows:

$$
\begin{aligned}
Min \quad & \frac{\gamma_1}{\gamma_2} \\
s.t. & \sum_{j=1}^{n} \lambda_j^1 X_{1j} \leq \gamma_1 X_{1o} \\
& \sum_{j=1}^{n} \lambda_j^2 X_{2j} \leq \gamma_1 X_{2o} \\
& \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = \alpha_1 Z_{1o} \\
& \sum_{j=1}^{n} \lambda_j^3 Z_{1j} = \alpha_1 Z_{1o} \\
& \sum_{j=1}^{n} \lambda_j^2 Z_{2j} = \alpha_2 Z_{2o} \\
& \sum_{j=1}^{n} \lambda_j^3 Z_{2j} = \alpha_2 Z_{2o}
\end{aligned}
$$

$$\sum_{j=1}^{n}\lambda_j^3 Y_j \geq \gamma_2 Y_o$$

$$\sum_{j=1}^{n}\lambda_j^l = \sum_{j=1}^{n}\lambda_j^{l+1} \qquad l=1,2,\ \alpha_1>0,\alpha_2>0,\gamma_1\leq 1,\gamma_2\geq 1,\lambda^k\geq 0 \qquad k=1,2,3 \tag{8}$$

Theorem 3. Let γ_1^*,γ_2^* be the optimal values of γ_1,γ_2 obtained by solving model (8), while DMU $(X_1,X_2,Z_1,Z_2,Y)\in T_{NV}$ is evaluated. DMU (X_1,X_2,Z_1,Z_2,Y) is MPSS if and only if in optimality we have $\gamma_1^*=1,\gamma_2^*=1$.

Proof. Assume $\left\{\lambda^{k*}\right\}_{k=1}^{3}$ satisfies the following constraints:

$$\sum_{j=1}^{n}\lambda_j^{1*}X_{1j} \leq \gamma_1^* X_1$$

$$\sum_{j=1}^{n}\lambda_j^{2*}X_{2j} \leq \gamma_1^* X_2$$

$$\sum_{j=1}^{n}\lambda_j^{1*}Z_{1j} = \alpha_1^* Z_1$$

$$\sum_{j=1}^{n}\lambda_j^{3*}Z_{1j} = \alpha_1^* Z_1$$

$$\sum_{j=1}^{n}\lambda_j^{2*}Z_{2j} = \alpha_2^* Z_2$$

$$\sum_{j=1}^{n}\lambda_j^{3*}Z_{2j} = \alpha_2^* Z_2$$

$$\sum_{j=1}^{n}\lambda_j^{3*}Y_j \geq \gamma_2^* Y$$

$$\sum_{j=1}^{n}\lambda_j^{l*} = \sum_{j=1}^{n}\lambda_j^{l+1*} \qquad l=1,2$$

$$\alpha_1^*>0,\alpha_2^*>0,\gamma_1^*\leq 1,\gamma_2^*\geq 1,\lambda^{k*}\geq 0 \qquad k=1,2,3 \tag{9}$$

Consider variable transformation as $\sum_{j=1}^{n}\lambda_j^{k*}=\delta,\quad k=1,2,3$. Dividing all of the above-mentioned constraints by δ, we will arrive at the following results:

$$\sum_{j=1}^{n}\lambda_j^{1''}X_{1j} \leq \frac{\gamma_1^*}{\delta}X_1$$

$$\sum_{j=1}^{n}\lambda_j^{2''}X_{2j} \leq \frac{\gamma_1^*}{\delta}X_2$$

$$\sum_{j=1}^{n}\lambda_j^{1''}Z_{1j} = \frac{\alpha_1^*}{\delta}Z_1$$

$$\sum_{j=1}^{n}\lambda_j^{3''}Z_{1j} = \frac{\alpha_1^*}{\delta}Z_1$$

$$\sum_{j=1}^{n}\lambda_j^{2''}Z_{2j} = \frac{\alpha_2^*}{\delta}Z_2$$

$$\sum_{j=1}^{n}\lambda_j^{3''} Z_{2j} = \frac{\alpha_2^*}{\delta} Z_2$$

$$\sum_{j=1}^{n}\lambda_j^{2''} Y_j \geq \frac{\gamma_2^*}{\delta} Y \qquad \lambda''^k \geq 0, 1\lambda''^k = 1 \qquad k = 1,2,3 \tag{10}$$

In the model above variable transformations $\lambda_j^{k''} = \frac{1}{\delta}\lambda_j^{k*}, \quad k = 1,2,3, \; j = 1,\dots,n$ have been used.

In the case that $\gamma_1^* = 1, \gamma_2^* > 1$, it is rational that $\frac{\gamma_2^*}{\delta} > \frac{\gamma_1^*}{\delta}$. In addition, it can also be concluded that $(\frac{\gamma_1^*}{\delta}X_1, \frac{\gamma_1^*}{\delta}X_2, \frac{\alpha_1^*}{\delta}Z_1, \frac{\alpha_2^*}{\delta}Z_2,, \frac{\gamma_2^*}{\delta}Y) \in T_{NV}$. Thus, (X_1, X_2, Z_1, Z_2, Y) cannot be a MPSS point. Similar result can be achieved by assuming that $\gamma_1^* < 1$. If $(X_1, X_2, Z_1, Z_2, Y) \in T_{NV}$ is not a MPSS, then there exists $(\mu X_1, \mu X_2, \eta_1 Z_1, \eta_2 Z_2, \beta Y) \in T_{NV}$ satisfying $\beta > \mu$. Therefore we have

$$\sum_{j=1}^{n}\lambda_j^1 X_{1j} \leq \mu X_1$$

$$\sum_{j=1}^{n}\lambda_j^2 X_{2j} \leq \mu X_2$$

$$\sum_{j=1}^{n}\lambda_j^1 Z_{1j} = \eta_1 Z_1$$

$$\sum_{j=1}^{n}\lambda_j^3 Z_{1j} = \eta_1 Z_1$$

$$\sum_{j=1}^{n}\lambda_j^2 Z_{2j} = \eta_2 Z_2$$

$$\sum_{j=1}^{n}\lambda_j^3 Z_{2j} = \eta_2 Z_2$$

$$\sum_{j=1}^{n}\lambda_j^3 Y_j \geq \beta Y$$

$$\lambda^k \geq 0, \; 1\lambda^k = 1 \qquad k = 1,2,3 \tag{11}$$

Dividing the aforementioned constraints by β, the following constraints will be acquired:

$$\sum_{j=1}^{n}\hat{\lambda}_j^1 X_{1j} \leq \frac{\mu}{\beta} X_1$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^2 X_{2j} \leq \frac{\mu}{\beta} X_2$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^1 Z_{1j} = \frac{\eta_1}{\beta} Z_1$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^3 Z_{1j} = \frac{\eta_1}{\beta} Z_1$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^2 Z_{2j} = \frac{\eta_2}{\beta} Z_2$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^3 Z_{2j} = \frac{\eta_2}{\beta} Z_2$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^2 Y_j \geq Y$$

$$\sum_{j=1}^{n}\hat{\lambda}_j^1 = \sum_{j=1}^{n}\hat{\lambda}_j^2 = \sum_{j=1}^{n}\hat{\lambda}_j^3 = \frac{1}{\beta} \qquad \hat{\lambda}^k \geq 0 \qquad k = 1,2,3 \tag{12}$$

Where $\hat{\lambda}_j^k = \frac{1}{\beta}\lambda_j^k,\ k = 1,2,3,\ \ j = 1,\dots,n$. If $\beta > \mu$, then we can find a solution for model (8) with $\gamma_1^* = \frac{\mu}{\beta} < 1, \gamma_2^* = 1$.■ Considering vectors S^{-1}, S^{-2}, S^{+} model (8) can be written as follows:

$$Min \quad \frac{\gamma_1}{\gamma_2} - \varepsilon(1S^{-1} + 1S^{-2} + 1S^{+})$$

$$s.t. \sum_{j=1}^{n}\lambda_j^1 X_{1j} = \gamma_1 X_{1o} - S^{-1}$$

$$\sum_{j=1}^{n}\lambda_j^2 X_{2j} = \gamma_1 X_{2o} - S^{-2}$$

$$\sum_{j=1}^{n}\lambda_j^1 Z_{1j} = \alpha_1 Z_{1o}$$

$$\sum_{j=1}^{n}\lambda_j^3 Z_{1j} = \alpha_1 Z_{1o}$$

$$\sum_{j=1}^{n}\lambda_j^2 Z_{2j} = \alpha_2 Z_{2o}$$

$$\sum_{j=1}^{n}\lambda_j^3 Z_{2j} = \alpha_2 Z_{2o}$$

$$\sum_{j=1}^{n}\lambda_j^3 Y_j = \gamma_2 Y_o + S^{+}$$

$$\alpha_1 > 0,\ \alpha_2 > 0, \gamma_1 \leq 1, \gamma_2 \geq 1, S^{-1} \geq 0, S^{-2} \geq 0, S^{+} \geq 0, \lambda^k \geq 0 \qquad k = 1,2,3$$

$$1\lambda^l = 1\lambda^{l+1} \qquad l = 1,2 \tag{13}$$

Dividing the constraints by γ_2 , the following model will be obtained:

$$Min \quad \omega - \varepsilon(1\bar{S}^{-1} + 1\bar{S}^{-2} + 1\bar{S}^{+})$$

$$s.t. \sum_{j=1}^{n}\bar{\lambda}_j^1 X_{1j} = \omega X_{1o} - \bar{S}^{-1}$$

$$\sum_{j=1}^{n}\bar{\lambda}_j^2 X_{2j} = \omega X_{2o} - \bar{S}^{-2}$$

$$\sum_{j=1}^{n}\bar{\lambda}_j^1 Z_{1j} = \bar{\alpha}_1 Z_{1o}$$

$$\sum_{j=1}^{n}\bar{\lambda}_j^3 Z_{1j} = \bar{\alpha}_1 Z_{1o}$$

$$\sum_{j=1}^{n}\bar{\lambda}_j^2 Z_{2j} = \bar{\alpha}_2 Z_{2o}$$

$$\sum_{j=1}^{n}\bar{\lambda}_j^3 Z_{2j}=\bar{\alpha}_2 Z_{2o}$$
$$\sum_{j=1}^{n}\bar{\lambda}_j^3 Y_j=Y_o+\bar{S}^{+}$$
$$\bar{\alpha}_1>0,\bar{\alpha}_2>0,\bar{S}^{-1}\geq 0,\bar{S}^{-2}\geq 0,\bar{S}^{+}\geq 0,\bar{\lambda}^k\geq 0 \qquad k=1,2,3$$
$$1\bar{\lambda}^l=1\bar{\lambda}^{l+1} \qquad l=1,2 \tag{14}$$

Where $\bar{\lambda}_j^k=\frac{1}{\gamma_2}\lambda_j^k \quad k=1,2,3,\omega=\frac{\gamma_1}{\gamma_2},\ \bar{S}^{-1}=\frac{1}{\gamma_2}S^{-1},\bar{S}^{-2}=\frac{1}{\gamma_2}S^{-2},\ \bar{S}^{+}=\frac{1}{\gamma_2}S^{+}$.

Theorem4. Let $\lambda^{k*},\quad k=1,2,3$, $S^{-1*},S^{-2*},S^{+*},\alpha_1^*,\alpha_2^*$ be an optimal solution of model (13) and $\sum_{j=1}^{n}\lambda_j^{k^*}=\delta$. DMU $\frac{1}{\delta}(\gamma_1^* X_{1o}-S^{-1^*},\gamma_1^* X_{2o}-S^{-2^*},\alpha_1^* Z_{1o},\alpha_2^* Z_{2o},\gamma_2^* Y_o+S^{+^*})$ is MPSS.

Proof: The proof is similar to the case in theorem3.■

Theorem4 gives a MPSS project for $(X_{1o},X_{2o},Z_{1o},Z_{2o},Y_o)$. Similar to the proof of theorem2, by considering definition 1, it can be easily proved that $(\gamma_1^* X_{1o}-S^{-1^*},\gamma_1^* X_{1o}-S^{-2^*},\alpha_1^* Z_{1o},\alpha_2^* Z_{2o},\gamma_2^* Y_o+S^{+^*})$ is efficient. Therefore, $\frac{1}{\delta}(\gamma_1^* X_{1o}-S^{-1^*},\gamma_1^* X_{2o}-S^{-2^*},\alpha_1^* Z_{1o},\alpha_2^* Z_{2o},\gamma_2^* Y_o+S^{+^*})\in T_{NV}$ is called a scale efficient target of $(X_{1o},X_{2o},Z_{1o},Z_{2o},Y_o)$. Model (13) may have more than one optimal solution. Thus, let $\{\lambda^k\}_{k=1}^3$ satisfy the following constraints (as an optimal solution of model (13)):

$$\sum_{j=1}^{n}\lambda_j^1 X_{1j}=\gamma_1^* X_{1o}-S^{-1*}$$
$$\sum_{j=1}^{n}\lambda_j^2 X_{2j}=\gamma_1^* X_{2o}-S^{-2*}$$
$$\sum_{j=1}^{n}\lambda_j^1 Z_{1j}=\alpha_1^* Z_{1o}$$
$$\sum_{j=1}^{n}\lambda_j^3 Z_{1j}=\alpha_1^* Z_{1o}$$
$$\sum_{j=1}^{n}\lambda_j^2 Z_{2j}=\alpha_2^* Z_{2o}$$
$$\sum_{j=1}^{n}\lambda_j^3 Z_{2j}=\alpha_2^* Z_{2o}$$
$$\sum_{j=1}^{n}\lambda_j^3 Y_j=\gamma_2^* Y_o+S^{+*}$$
$$\lambda^k\geq 0,1\lambda^l=1\lambda^{l+1} \qquad k=1,2,3,\ l=1,2 \tag{15}$$

Then let $\sum_{j=1}^{n}\lambda_j^k=\delta,\quad k=1,2,3$ and $\tau=\frac{1}{\delta}$.

Definition 4. DMU $(\tau(\gamma_1^* X_{1o}-S^{-1*},\gamma_1^* X_{2o}-S^{-2*},\alpha^* Z_{1o},\alpha^* Z_{2o},\gamma_2^* Y_o+S^{+*}))$ is the largest MPSS project of $(X_{1o},X_{2o},Z_{1o},Z_{2o},Y_o)$ if and only if for each of $\bar{\tau}>\tau$:

$\overline{\tau}(\gamma_1^* X_{1o} - S^{-1*}, \gamma_1^* X_{2o} - S^{-2*}, \alpha^* Z_{1o}, \alpha^* Z_{2o}, \gamma_2^* Y_o + S^{+*}) \notin T_{NV}$. In a similar manner, the smallest MPSS by projection of $(X_{1o}, X_{2o}, Z_{1o}, Z_{2o}, Y_o)$ can also be defined.

According to the definition above, the largest MPSS project of $(X_{1o}, X_{2o}, Z_{1o}, Z_{2o}, Y_o)$ is $\frac{1}{\delta}(\gamma_1^* X_{1o} - S^{-1^*}, \gamma_1^* X_{2o} - S^{-2^*}, \alpha^* Z_{1o}, \alpha^* Z_{2o}, \gamma_2^* Y_o + S^{+^*})$ when the minimum value of δ is taken into account. Therefore, dividing constraints (15) by δ , the largest MPSS project of $(X_{1o}, X_{2o}, Z_{1o}, Z_{2o}, Y_o)$ can be acquired by solving the following model:

$$
\begin{aligned}
& Max\ \tau \\
& s.t. \sum_{j=1}^{n} \tilde{\lambda}_j^1 X_{1j} = \tau(\gamma_1^* X_{1o} - S^{-1*}) \\
& \sum_{j=1}^{n} \tilde{\lambda}_j^2 X_{2j} = \tau(\gamma_1^* X_{2o} - S^{-2*}) \\
& \sum_{j=1}^{n} \tilde{\lambda}_j^1 Z_{1j} = \tau(\alpha^* Z_{1o}) \\
& \sum_{j=1}^{n} \tilde{\lambda}_j^3 Z_{1j} = \tau(\alpha^* Z_{1o}) \\
& \sum_{j=1}^{n} \tilde{\lambda}_j^2 Z_{2j} = \tau(\alpha^* Z_{2o}) \\
& \sum_{j=1}^{n} \tilde{\lambda}_j^3 Z_{2j} = \tau(\alpha^* Z_{2o}) \\
& \sum_{j=1}^{n} \tilde{\lambda}_j^3 Y_j = \tau(\gamma_2^* Y_o + S^{+*}) \\
& \tilde{\lambda}^k \geq 0, 1\tilde{\lambda}^k = 1 \qquad k = 1,2,3
\end{aligned} \tag{16}
$$

By changing the Max in the above model to Min the smallest MPSS project of $(X_{1o}, X_{2o}, Z_{1o}, Z_{2o}, Y_o)$ will be obtained. The largest and the smallest MPSS projects of $(X_{1o}, X_{2o}, Z_{1o}, Z_{2o}, Y_o)$ are respectively shown by (X_1, X_2, Z_1, Z_2, Y) , $(X_1', X_2', Z_1', Z_2', Y')$. The mentioned MPSS's are determined according to the following formulas:

$$X_1 = \tau^*(\gamma_1^* X_{1o} - S^{-1*}) \quad X_2 = \tau^*(\gamma_1^* X_{2o} - S^{-2*}) \quad Z_k = \tau^*(\alpha^* Z_{ko}) \quad k = 1,2 \quad Y = \tau^*(\gamma_2^* Y_o + S^{+*}) \tag{17}$$

$$X_1' = \tau'(\gamma_1^* X_{1o} - S^{-1*}) \quad X_2' = \tau'(\gamma_1^* X_{2o} - S^{-2*}) \quad Z_k' = \tau'(\alpha^* Z_{ko}) \quad k = 1,2 \quad Y' = \tau'(\gamma_2^* Y_o + S^{+*}) \tag{18}$$

Note that τ^* and τ' are considered to be the maximum and the minimum values of τ subject to constraints of model (16). Dividing constraints of model (16) by γ_2^* , the results are obtained as following:

$$
\begin{aligned}
& \sum_{j=1}^{n} \lambda_j^1 X_{1j} = \tau(\omega^* X_{1o} - \overline{S}^{-1*}) \\
& \sum_{j=1}^{n} \lambda_j^2 X_{2j} = \tau(\omega^* X_{2o} - \overline{S}^{-2*}) \\
& \sum_{j=1}^{n} \lambda_j^1 Z_{1j} = \tau(\overline{\alpha}_1^* Z_{1o}) \\
& \sum_{j=1}^{n} \lambda_j^3 Z_{1j} = \tau(\overline{\alpha}_1^* Z_{1o})
\end{aligned}
$$

$$\sum_{j=1}^{n} \lambda_j^2 Z_{2j} = \tau(\overline{\alpha}_2^* Z_{2o})$$

$$\sum_{j=1}^{n} \lambda_j^3 Z_{2j} = \tau(\overline{\alpha}_2^* Z_{2o})$$

$$\sum_{j=1}^{n} \lambda_j^3 Y_j = \tau(Y_o + \overline{S}^{+*})$$

$$1\lambda^k = \frac{1}{\gamma_2^*} \qquad k = 1,2,3 \tag{19}$$

In the above constraints, variable transformations $\lambda_j^k = \frac{1}{\gamma_2^*}\tilde{\lambda}_j^k, \overline{\alpha}^* = \frac{1}{\gamma_2^*}\alpha^* \quad k = 1,2$ are considered. Values of $\omega^*, \overline{\alpha}^*,\ \overline{S}^{-1^*}, \overline{S}^{-2^*}, \overline{S}^{+^*}$ are determined by solving model (14) and are then utilized in constraints (19). Optimal value of γ_2^* is obtained by solving model (16) (subject to constraints (19)) and finally the value of γ_1^* is determined by using ω^* and γ_2^*.

4. Results

Input and output data of 21 branches of the Melat bank in Iran, is gathered in Table 1. For performance evaluation of these branches, a special network is required as depicted in Figure 1, which simultaneously contains parallel and series stages. Inputs of the first part of stage 1 are personnel costs (x_{11}) and paid interests (x_{12}). Inputs of the second part of the stage 1 are personnel costs (x_{21}) and paid interests (x_{22}) related to the foreign currency transactions. Outputs of the first part are raised funds (z_1) and outputs of the second part are raised funds (z_2) related to the foreign currency transactions. In stage 2, outputs are loans (y_1) and common incomes (y_2). Non-performing loans are considered as bad (undesirable) outputs since in the second stage, some loans might become non-performing. This means that borrowers are unable to make full or even partial repayment. As discussed in the literature on DEA, the inverse value of this bad output can be considered as good output (y_3). Data in Table 1 is represented in million dollars.

Table 1. Input and output Data

DMU	x_{j11}	x_{j12}	x_{j21}	x_{j22}	y_{j1}	y_{j2}	z_{j1}	z_{j2}
1	402	3279	86	142	389339	2436	32e-4	461036
2	400	7867	81	260	233480	4615	42e-4	273318
3	1259	5469	257	440	386164	7895	32e-4	471235
4	12.3	457	2.5	1.2	28596	424	61e-4	59290
5	81.3	700	10	13	57264	305	76e-4	91725
6	429	2518	54	14.5	28654	3299	9e-4	49697
7	277	1678	57	65.4	223004	3416	42e-4	233577
8	22.1	617	4.2	4.24	42399	1070	9e-4	74880
9	305	7396	51	405	252491	11613	35e-5	261356
10	713	2741	142	109	224689	3152	36e-4	248816

Table 1. Continued

DMU	x_{j11}	x_{j12}	x_{j21}	x_{j22}	y_{j1}	y_{j2}	z_{j1}	z_{j2}
11	30	578	3.2	2.5	50728	387	7e-4	73706
12	15	623	2.4	3.02	42167	575	11e-4	76245
13	34.1	777	6.3	16.2	43836	1228	9e-4	73260
14	23.6	554	2.3	1.2	52001	400	7e-4	72641
15	73.2	3052	9.7	7.7	52599	3096	6e-4	84311
16	225	2411	45	74.5	282270	3582	3e-6	380434
17	679	3563	115	459	409531	8140	2e-6	515388
18	274	5491	150	51	228087	566	3.5e-6	251519
19	288	3630	172	55.5	231330	712	3.5e-6	262947
20	402	3293	109	87.5	387381	5230	2.3e-6	485744
21	648	2840	104.5	317	406597	4464	1.8e-6	491195

Table 2. Results of models (3), (6), (16) and formula (17)

DMU	ρ^*	γ_1^*	γ_2^*	τ^*	x_{11}	x_{12}	x_{21}	x_{22}	y_1	y_2
1	0.303	0.441	1.116	0.065	2.5	1.2	12.3	457	28596	424
2	0.291	0.399	1.250	0.058	2.5	1.2	12.3	457	28596	424
3	0.144	0.205	1.134	0.050	2.5	1.2	12.3	457	28596	424
4	1	1	1	1	2.5	1.2	12.3	457	28596	424
5	0.535	0.633	1.664	0.299	2.5	1.2	12.3	457	28596	424
6	0.136	0.214	1.250	0.726	2.5	1.2	12.3	457	28596	424
7	0.320	0.469	1.327	0.093	2.5	1.2	12.3	457	28596	424
8	0.865	1	1.333	0.794	2.5	1.2	12.3	457	28596	424
9	0.300	0.398	1	0.050	2.5	1.2	12.3	457	28596	424
10	0.201	0.387	1.354	0.093	2.5	1.2	12.3	457	28596	424
11	0.988	1	1.428	0.922	2.5	1.2	12.3	457	28596	424
12	0.800	1	1.200	0.542	2.5	1.2	12.3	457	28596	424
13	0.751	0.875	1.250	0.276	2.5	1.2	12.3	457	28596	424
14	0.809	0.812	1	1.060	2.5	1.2	12.3	457	28596	424
15	0.342	0.400	1	0.136	2.5	1.2	12.3	457	28596	424
16	0.714	1	1.666	0.071	2.5	1.2	12.3	457	28596	424
17	0.217	0.434	1.158	0.050	2.5	1.2	12.3	457	28596	424
18	0.212	0.260	1.250	0.062	2.5	1.2	12.3	457	28596	424
19	0.145	0.194	1.111	0.111	2.5	1.2	12.3	457	28596	424
20	0.288	0.383	1.233	0.059	2.5	1.2	12.3	457	28596	424
21	0.300	0.404	1.190	0.059	2.5	1.2	12.3	457	28596	424

Table 2 contains results of models (3), (6), (16) and formula (17). Efficiency scores based on slacks show that only DMU No.4 is efficient and DMU11 is the nearest in ranking. DMUs 8, 11, 12 and 16 are efficient in stage 1, which means that raised funds of these bank branches are in acceptable levels compared with their personal costs and paid interests. Among bank branches 9, 14 and 15 which are efficient in stage 2, branch No.14 has considerable raised funds and notable loans. Except DMUs 5 and 16, output values have not been increased considerably to improve overall performance; whereas reductions which are to be done in input values (costs and paid interests) are noticeable. It means that these branches would provide the same facilities at lower costs. Branch No.6 consumes much resource but has low raised funds and has the worst performance in stage1 among others; but in stage2, it provided acceptable loans from its raised funds. Therefore, to improve the performance of this branch the inefficiency of stage1 must be improved only.

The Above discussion implies that inspecting the multi-stage system, and analyzing intermediate activities and the performance of each sub-system give useful managerial insights and determine the stage inefficiencies which show how to optimize overall performance by improving the performance of only one stage.

As mentioned earlier, one of the important concerns of senior managers is finding the most productive scale size pattern for the firms under their control. Thus, in this application we tried to find the suitable MPSS point, as well as the largest MPSS point and scale efficient target according to the proposed models.

Considering methods discussed in this paper, DMU4 is MPSS and is benchmarked by other branches. Furthermore, to project DMUs on MPSS, except DMUs 4, 8, 11 and 14, , the input values of stage 1 and the outputs values of stage 2 have first been scaled so that all their values have decreased because of the inequalities $\gamma_1^*\tau^* < 1$, $\gamma_2^*\tau^* < 1$; Then, by considering suitable values of slacks, the input and output values have been adjusted. In accordance with what has been mentioned, DMU4 is determined as scale efficient target of other DMUs, which is also the only MPSS among all observations. Therefore, it gives information about how much increments in resources are profitable and guides managers to improve the scale size of input and output values to obtain the highest level of productivity.

5. Conclusion

To evaluate a production system with multi-stage structure, the connectivity and continuity between the stages must be considered; otherwise the model does not reflect true production process. We inspect the system and propose methods to improve overall efficiency and efficiency of the stages. We tried to set target to each of stages in a way that a unique target is set for outputs and inputs of the first and second stages, respectively. Our models measure efficiency scores and determine changes needed in input and output values to improve inefficiencies, which are useful for managers to make operational decisions. As the concept of MPSS is important for senior managers of different organizations, we discussed this concept in our special two-stage network. Thus, according to obtained largest and smallest MPPS points, it is possible to find the extent of increase or decrease in input values to acquire the highest level of productivity. In addition, radial and non-radial models have also been proposed in order to measure the efficiency of such network. Moreover, models for setting scale efficient targets have been presented to each stage and to overall two-stage DMU. Finally, proposed models have been applied to evaluate the performance and determine the scale efficient targets of 21 branches of the Melat bank. We found MPSS point, the largest MPSS and scale efficient target.

6. Contributions and suggestions

To evaluate network systems, the main point is that the links between the stages are to be taken into consideration. In traditional DEA models, internal activities are neglected and the model does not reflect true production process. Network DEA measures efficiency scores of multi-stage DMUs which guide managers to make better decisions to improve overall performance and efficiency of each sub-division. In modeling network system considering inequalities related to intermediate activities results in contradictions in optimality. It has been modified in modeling the system in this paper. Furthermore, we consider two-stage structure where the first stage consists of parallel parts. This structure is more applicable than the common one-part stage in the real world. In addition, it is important to know if output values increase by increasing input values (resources) in a certain proportion, then what is the maximum value of this proportion so that no more increment is profitable. The answer lies in the identification of MPSS pattern which have been discussed in this paper. Since the activities of DMU in one period affect its performance in the next period, identifying MPSS pattern by dynamic DEA models can guide future research by considering links among periods. Other research strands include ranking DMU's according to distance from MPSS point and identifying MPSS pattern under uncertain data assumption.

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