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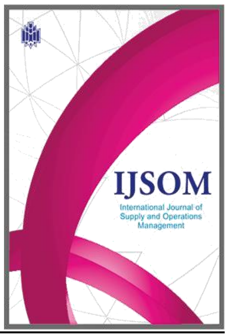

Dual-channel Supply Chain Synchronization with Deterministic and Stochastic Demand under Cost-sharing Contract

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Abstract
Due to the globalization of markets and the advancements in communications, such as the Internet and e-commerce, consumers are directly linked to manufacturers. Consumers can search a product in an offline store, and then they buy it in an online store. This paper investigates the influence of channel synchronization on the supplier, the retailer and the entire supply chain in the dual-channel supply chains. Contracts are valuable tools used in both theory and practice to coordinate various supply chains among synchronization mechanisms. This study presents a hybrid model with a new demand function and surveys the effects of free riding on sales effort in a dual-channel supply chain comprising of one manufacturer and one offline store. To achieve beneficial outcomes, this paper considers a cost-sharing contract to synchronize a dual-channel supply chain. Finally, we focus on the expansion of the model in the stochastic demand solved by the genetic algorithm.

Keywords: Dual-channel supply chain; Sales effort; free riding; Cost-sharing contract.

1. Introduction

In today's global markets, supply chains play a fundamental role in the development of an organization and profit maximization. In today's business environment, competitive forces need organized method-oriented structures and companies to manage their processes. Supply Chain Management (SCM) manages controls and coordinates supply and demand planning, preparing materials, producing and planning products, controlling the stock, distributing, delivering and serving. Therefore, the customers can receive reliable and fast services and high-quality products in a low cost (Tavana, Kaviani, Caprio and Rahpeyma (2016)).

Due to the prevalent e-commerce and the increasing competition all over the world, more manufacturers and customary retail firms tend to the Internet supply channel as an important means to extend the market. It means that consumers can attain product from dual-channel which is the traditional retail channel and online direct channel in the market (Li, Ma and Sun (2015) and Ma, Li and Ren (2017)).

Many enterprises can improve the profit through the supply chain by expanding cooperation and synchronization. In general, enterprises with an independent system obtain less profit than an integrated supply chain so that the objective is to maximize the total profit through the chain. However, there are some positions which the members of a supply chain system can obtain a Pareto improvement.

Majority of retailers should synthesize sales efforts into their operations to attract customers. Sales efforts may include advertisement, services or sales promotions such as free gifts, discounts or loyalty club rewards. Many researchers inspect the effects of sales efforts of operational strategies in a firm (Gilbert and Cvsa, (2003)). Anyway, the improvement of

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Corresponding author email address: mfakhrzad@yazd.ac.ir the Internet and e-commerce caused online stores and changed sales channels from single- to two or more channels. Consequently, researchers shifted their attention from single to dual channels. For example, Chen (2010) built a price-service competitive model with a retailer implementing sales efforts. When consumers admit the online channel in a certain region, both the manufacturer and retailer get profit.

Among different channels, researchers have notable insight into free riding. Customers may employ free riding when they use an offline store to obtain information or try products, then they shift to an online store to buy the products. Free riding is one of the most important issues in the dual-channel system and reduces the profit. Consequently, researchers investigate this issue in marketing and industrial organization extensively (for discussions in a retail channel environment, see Antia et al. (2004)). For example, Chiu et al. (2011) studied the most popular type of free riding: searching for product information in an online store and then purchasing it in an offline store. Sandrine (2013) investigated realizing free riding from a consumer-enabling perspective by considering shopping motives and sociodemographic covariates. Via a survey of decision-making behavior, she understood that free riders mainly seek appropriate price, comparisons and flexibility. When consumers accept cross-channel rather than single-channel behavior, free riding is more plausible, which is eventuated in a negative outcome of multi-channel retailing (Zhou, Guo and Zhou (2018)).

The following are the summary of contributions of this paper: (1) the manufacturer can design an incentive contract to encourage offline retailers to increase their sales efforts levels. (2) There exists an optimal level that will maximize the expected profit of the common retailer. (3) Approaches that the manufacturer can take to improve the efficiency of the dual-channel supply chain in free riding behavior in point of view. (it should be a complete sentence)

The remainder of this paper is organized as follows: Section 2 reviews the related literature; Section 3 formulates demand functions for offline store and online store; section 4 introduces the decision making structures; Section 5 indicates the model extension in a stochastic setting and the genetic algorithm is applied for optimizing the model; and Section 6 draws the conclusion.

2. Literature review

Channel competition has attracted much attention in the supply chain and marketing literature in the last two decades. Channel competition can be classified into upstream and downstream competition the prior meaning the competition among suppliers/manufacturers, whereas the former refers to the competition among retailers. Table 1 refers to papers about channel competition under deterministic and stochastic demand settings.

Dong and Zhu (2006), Sabbaghi, Sheffi and Tsitsiklis (2007), Chen and Li (2007) implemented wholesale price contract to coordinate the supply chain. Also Donohue (2000), He et al (2006), Hou et al (2010), and Höhn (2010) focused on buying back contract in audio and book industries. Revenue sharing contract was introduced by Giannoccaro and Pontrandolfo (2004), Dong and Li (2009), Van der Rhee et al. (2010) .An overview of the extensive models of synchronization by contracts can also be found in Lariviere (1999), Cachon (2003), Höhn (2010), Wang (2002), Gomez-Padilla et al. (2005) Albrecht (2010). Yang and Xiao (2017) developed three game models of a green supply chain with governmental interventions under fuzzy uncertainties of both manufacturing cost and consumer demand, and studied how prices, green levels and expected profits are influenced by channel leadership and governmental interventions. A robust multi-trip vehicle routing problem of perishable products with intermediate depots and time windows is formulated to deal with the uncertain nature of demand parameter (Tirkolaee, Goli, Bakhsi and Mahdavi (2017)). A fuzzy closed-loop supply chain with one manufacturer, one retailor and one collector is investigated (Rabbani, Alamdar and Heydari (2018)). Tirkolaee, Mahdavi, and Esfahani (2018) investigated a novel mathematical model that is developed for robust periodic capacitated arc routing problem (PCARP) considering multiple trips and drivers and crew’s working time to study the uncertain nature of demand parameter.

Table1. Literature on upstream and downstream competition

Competition type demand	**Deterministic demand**	**Stochastic**
Downstream competition	Ingene and Parry (1995), padmanabhan and Png (1997), and Cachon and Lariviere (2005)	Yao et al. (2008)
Upstream competition	Choi (1991) and Pan et al. (2010) Our model	TulikaChakraborty, SatyaveerS.Chauhan, NavneetVidyarthi(2015) Our model

Table2 lists papers on supply chain contracts under deterministic and stochastic demand settings.

Table2. Literature on supply chain contracts with deterministic and stochastic demand

Contract	**Deterministic demand**	**Stochastic demand**
Wholesale price/price-only contract	Shugan and Jeuland (1988), Choi (1991, 1996)	Petruzzi and Dada (1999) and Pan et al. (2009)
Return policy/buy-back contract	Pa dmanabhan and Png (1997)	Pasternack (1985) Emmons and Gilbert (1998) , Donohue (2000), Bose and Anand (2007), Yao et al (2008)
Revenue sharing contract	Giannoccaro and Pontrandolfo (2004)	Cachon et al. (2001), Wang et al. (2004), Q in and Yang(2008) , Pang et al.(2014), Tang and Kouvelis (2014)
cost-sharing contract	Xujin Pu, Lei Gong, Xiaohua Han (2016), Our model	Our model

In this paper, the supply chain synchronization approach is presented by a new hybrid model (Supply Chain Includes demand function, order, total residual value and maintenance cost of surplus products at the end of the period) considering the cost-sharing contract which has not been addressed in the literature of the subject. We also develop this model as a stochastic.

3. The model

We consider a supply chain consisting of one manufacturer who produces a particular product by cost c, which is distributed by both its own online store and an offline store as in Figure 1.The related market includes two channel levels: manufacturer level and retailer level. Consumers can also buy products either at the online price p_d or offline pricep_r. Manufacturer sells orders of the offline stores by wholesale price w. The offline store can exert sales effort levels to attract consumers. Therefore, market demand will increase for the retail channel.

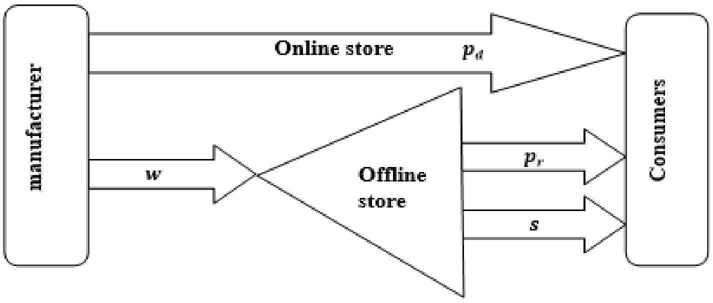

Figure 1. Dual-channel supply chain structure

Before formulating this study, we define the parameters and variables as follows:

Parameters
a_r: Base demand for the offline store ($a_r = \theta a$)
a_d: Base demand for the online store($a_d = (1-\theta)a$)
a: The base-level demand ($a > 0$)
θ: Consumers' preference to use the offline store
$1\text{-}\theta$: Consumers' preference to use the online store
q: Order quantity of retailer from the supplier
w: Wholesale price per unit
c: The cost of producing per unit according to the supplier
l: Scratch value of each remaining product according to the retailer at the end of period
h: Maintaining cost of each remaining product according to the retailer at the end of period
b: Cost of each unit shortage according to the retailer
μ: Demand mean
σ: Deviation of demand
x_r: Demand in the offline store
x_d: Demand in the online store

β_1: Coefficient of price elasticity of x_r
β_2: Coefficient of price elasticity of x_d
γ_1 and γ_2 : degree to which the goods sold via the two channels are substitutes

Variables
p_r: Offline price
p_d: Online price
s: Sale effort
τ: Free riding rate
t: Total sales effort cost $(0 \leq t \leq 1)$
$1 - t$: Offline store sales effort cost

Indices
r: offline store
d: online store
o: coordinated set

Our basic model uses demand functions (McGuire and Staelin and Huang and Swaminathan (2009)) given by:

$$x_r = a_r - \beta_1 p_r + \gamma_1 p_d + (1 - \tau)s \tag{1}$$

$$x_d = a_d - \beta_2 p_d + \gamma_2 p_r + \tau s \tag{2}$$

Assuming that the cross-price effects are symmetric, we have $\gamma_1 = \gamma_2 = e$. Thus:

$$x_r = a_r - \beta_1 p_r + e p_d + (1 - \tau)s \tag{3}$$

$$x_d = a_d - \beta_2 p_d + e p_r + \tau s \tag{4}$$

Relating above Eqn. (3) and Eqn. (4), total demand of the dual-channel supply chain is:

$$x_t = x_r + x_d \tag{5}$$

4. Decision-making structures

Free riding reduces offline retailer's motivation to grow sales efforts, so total sales will be decreased. They worry about free riding while selling their products via a dual-channel. Therefore, the manufacturer should create an incentive contract to persuade offline retailers to increase sales effort levels. Accordingly, we present a cost-sharing contract in which the manufacturer supposes a fraction $t(0 \leq t \leq 1)$ of the total sales-effort cost and the offline store supposes$1 - t$. Thus, the profits of manufacturer and offline store are given as follows:

$$\pi_m^o = (w - c)q_r + (p_d - c)x_d - \frac{t\eta s^2}{2} \tag{6}$$

$$\pi_r^o = \begin{cases} -wq_r + p_r x_r + (l - h)(q_r - x_r) - \frac{(1-t)\eta s^2}{2} & x_r < q_r \\ -wq_r + p_r q_r - b(x_r - q_r) - \frac{(1-t)\eta s^2}{2} & x_r > q_r \end{cases} \tag{7}$$

And the total profit is:

$$\pi_t^o = \begin{cases} -cq_r + (p_d - c)x_d + p_r x_r + (l - h)(q_r - x_r) - \dfrac{\eta s^2}{2} & x_r < q_r \\ -cq_r + (p_d - c)x_d + p_r q_r - b(x_r - q_r) - \dfrac{\eta s^2}{2} & x_r > q_r \end{cases} \tag{8}$$

Thus, by substituting (3) and (4) into π_t^o we will have:

$$\pi_t^o = \begin{cases} -cq_r + (p_d - c)(a_d - \beta_2 p_d + e p_r + \tau s) + p_r(a_r - \beta_1 p_r + e p_d + (1 - \tau)s) & \\ +(l - h)(q_r - a_r + \beta_1 p_r - e p_d - (1 - \tau)s_r) - \dfrac{\eta s^2}{2} & x_r < q_r \\ -cq_r + (p_d - c)(a_d - \beta_2 p_d + e p_r + \tau s) + p_r q_r & \\ -b(a_r - \beta_1 p_r + e p_d + (1 - \tau)s - q_r) - \dfrac{\eta s^2}{2} & x_r > q_r \end{cases} \tag{9}$$

Following backward induction, the central decision-maker maximizes profit by maximizing π_t^o over p_r and p_d:

$$\frac{\partial \pi_t^o}{\partial p_r} = \begin{cases} (p_d - c)e + (a_r - \beta_1 p_r + e p_d + (1-\tau)s) - \beta_1 p_r + (l-h)\beta_1 = 0 & x_r < q_r \\ (p_d - c)e + b\beta_1 = 0 & x_r > q_r \end{cases} \tag{10}$$

$$\frac{\partial \pi_t^o}{\partial p_d} = \begin{cases} a_d - \beta_2 p_d + e p_r + \tau s - \beta_2 (p_d - c) + e p_r - (l-h)e = 0 & x_r < q_r \\ a_d - \beta_2 p_d + e p_r + \tau s - be = 0 & x_r > q_r \end{cases} \tag{11}$$

By solving Eqn. (10) and Eqn. (11) we can yield the following optimal solutions for a given s:

when $x_r < q_r$:

$$p_r^{o^*} = \frac{\beta_2 a_r + e a_d + ((1-\tau)\beta_2 + e\tau)s + (\beta_1\beta_2 - e^2)(l-h)}{2(\beta_1\beta_2 - e^2)} \tag{12}$$

$$p_d^{o^*} = \frac{-e a_r + \beta_1 a_d + ((1-\tau)e + \beta_1\tau)s + (\beta_1\beta_2 - e^2)c}{2(\beta_1\beta_2 - e^2)} \tag{13}$$

and when $x_r > q_r$:

$$p_r^{o^*} = \frac{-e a_d - \beta_1\beta_2 b + ce\beta_2 - e\tau s + be^2}{e^2} \tag{14}$$

$$p_d^{o^*} = \frac{ce - b\beta_1}{e} \tag{15}$$

We use the following terms to simplify:

$$A = \frac{\beta_2 a_r + e a_d + (\beta_1\beta_2 - e^2)(l-h)}{2(\beta_1\beta_2 - e^2)} \qquad B = \frac{(1-\tau)\beta_2 + e\tau}{2(\beta_1\beta_2 - e^2)}$$

$$D = \frac{-e a_r + \beta_1 a_d + (\beta_1\beta_2 - e^2)c}{2(\beta_1\beta_2 - e^2)} \qquad E = \frac{(1-\tau)e + \beta_1\tau}{2(\beta_1\beta_2 - e^2)}$$

$$F = \frac{-e a_d - \beta_1\beta_2 b + ce\beta_2 + be^2}{e^2} \qquad G = \frac{-\tau}{e}$$

$$H = \frac{ce - b\beta_1}{e}$$

Therefore, for $x_r < q_r$:

$$p_r^{o^*} = A + Bs \tag{16}$$

$$p_d^{o^*} = D + Es \tag{17}$$

and for $x_r > q_r$:

$$p_r^{o^*} = F + Gs \tag{18}$$

$$p_d^{o^*} = H \tag{19}$$

Substituting (16), (17), (18) and (19) into π_t^o, we have:

$$\pi_t^o = \begin{cases} -cq_r + (D + Es - c)[a_d - \beta_2(D + Es) + e(A + Bs) + \tau s] \\ \quad +(A + Bs)[a_r - \beta_1(A + Bs) + e(D + Es) + (1 - \tau)s] \\ +(l - h)[q_r - a_r + \beta_1(A + Bs) - e(D + Es) - (1 - \tau)s] - \dfrac{\eta s^2}{2} & x_r < q_r \\ -cq_r + (H - c)[a_d - \beta_2 H + e(F + Gs) + \tau s] + (F + Gs)q_r \\ \quad -b[a_r - \beta_1(F + Gs) + eH + (1 - \tau)s - q_r] - \dfrac{\eta s^2}{2} & x_r > q_r \end{cases} \tag{20}$$

Finding the optimal s to maximize π_t^o, we differentiate π_t^o with respect to s and let the derivative be zero. Therefore, we yield the optimal sales effort level for the offline store:
For $x_r < q_r$:

$$s^{o^*} = \frac{-Ea_d + 2\beta_2 ED - 2eEA + 2\beta_1 BA - \beta_2 Ec + (eB + \tau)(c - D) - Ba_r - eD - (1 - \tau)A - (l - h)(\beta_1 B - eE - 1 + \tau)}{(1 - \beta_2)E^2 + 2eEB + 2\tau E - 2\beta_1 B^2 + eE + (1 - \tau)(1 + B) - \eta}$$

And also for $x_r > q_r$:

$$s^{o^*} = \frac{(H - c)(eG + \tau) + Gq_r + b\beta_1 G - b(1 - \tau)}{\eta}$$

We present the results for the numerical examples, in which $a = 40$, $\Theta = 0.7$, $\eta = 4$, $\beta_1 = 1$, $\beta_2 = 1.1$, $e = 0.2$, $c = 30$, $b = 3$, $h = 4$, $l = 3$ and $q_r = 100$.

Figure 2 indicates the effect of τ on s in the centralized supply chain. It demonstrates that the offline store has less of the incentive to apply sales effort when the free-riding rate is high in any supply chain cooperation patterns. This finding matches the real world because offline stores prevent to invest in non-distribution of fair profit. (I don't understand this part)

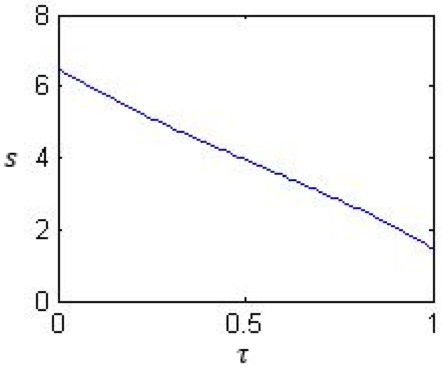

Figure 2. The effect of τ on s.

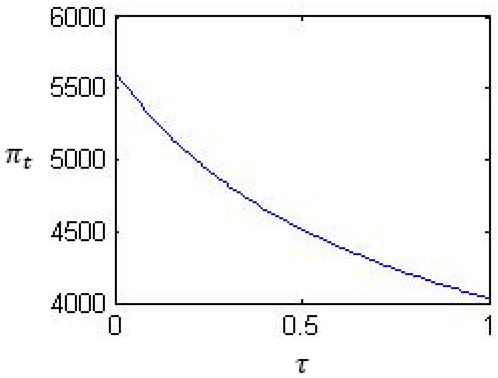

Figure 3. The effect of τ on π_t

We understand from Figure 3 that while the free-riding rate is growing, the supply chain profit decreases. Although the manufacturer may benefit from the offline store, the entire supply chain harm. In other words, the offline stores expected harm of free riding is more than the manufacturer's expected profit. To improve the supply chain performance and increase profits, both members will benefit from a synchronized supply chain mechanism.

Figure 4 shows that the retailer demand decreases by τ rate in the market. Moreover, the total market demand will increase in the coordinated system against the decentralized system.

Figure 5 indicates that the offline price reduces while the rate of free riding increases because the offline store must decrease its retail price to make consumer interested. Therefore, when abundant of free-riding consumers shop in the market, the offline store can conceal the sales effort cost.

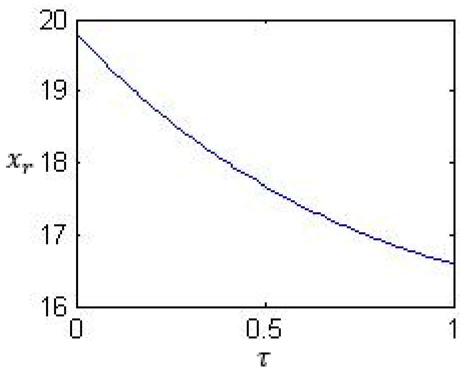

Figure 4. The effect of τ on x_r

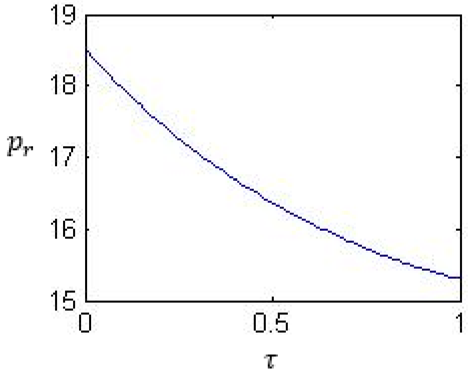

Figure 5. The effect of on p_r

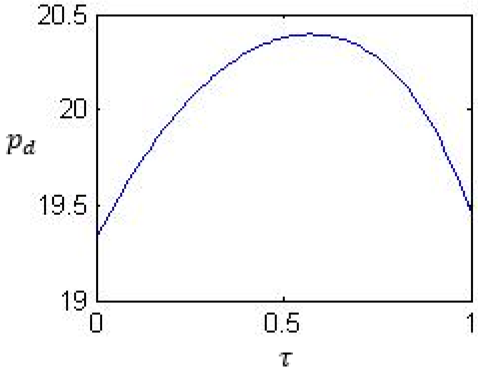

Figure 6. The effect of on p_d

Figure 6 illustrates the effect of rate τ on the online price. When τ is below a threshold, the online price increases by τ. However, when τ is the above the threshold, the online price decreases as τ increases. Although analyzing the threshold is complicated in a mathematical projection, we can numerically suppose the manufacturer's threshold which can be used to s the optimal cost sharing rate.

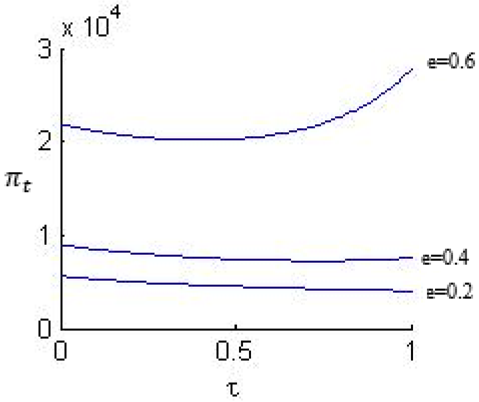

Figure 7. The effect of s on π_t with different levels of e

Figure 7 shows that as e increases,the profit of the dual-channel supply chain will increase in the coordinated setting. Additionally, the manufacturer is more desired to suggest a coordination contract when e is rather high. If the free-riding rate achieves 100%, the offline store's sales effort will not attract consumers, so the offline store will be reluctant to apply any sales effort, even if the manufacturer proposes a cost-sharing contract.

The left gragh in figure 8 shows that the price of the offline store decreases byτ. When channel competition increases, the offline store is more inclined to reduce its price; by the same way, when e increases, the offline price will increase too. The graph on the right illustrates that as e increases, the online price will increase.

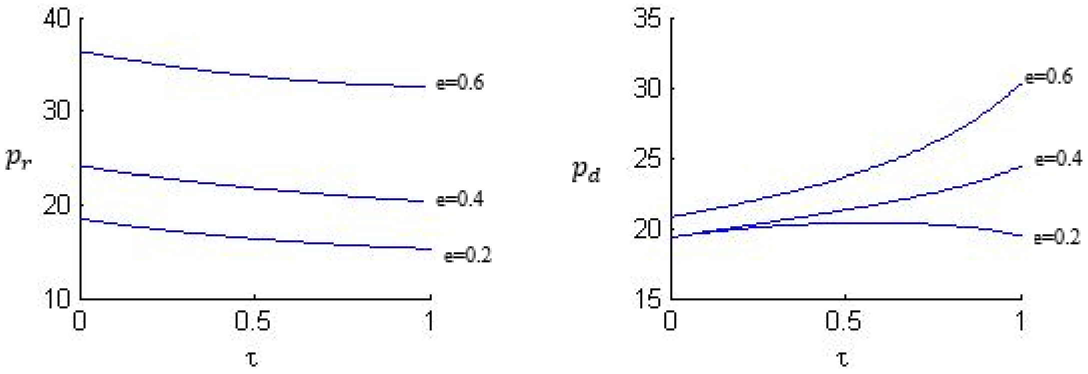

Figure 8. The effect of τ on p_r and p_d with different levels of e

5. Model extension in a stochastic setting

While we exert the deterministic demand in previous section, we consider stochastic demand in this section. Anyway, the analytical results are rather difficult. Thus, we concentrate on extending the model and employ a numerical survey although the main contributions of the cost sharing contract remain reliable in the previous sections.

By developing the demand function in previous section, we suppose the offline store demand $\tilde{x}_r$ and the online store demand $\tilde{x}_d$ are as follows:

$$\tilde{a} = a + \varepsilon$$

$$\tilde{x}_r = \Theta(a+\varepsilon) - \beta_1 p_r + e p_d + (1-\tau)s \qquad (21)$$

$$\tilde{x}_d = (1-\Theta)(a+\varepsilon) - \beta_2 p_d + e p_r + \tau s \qquad (22)$$

Here ε represents a normal distribution such that $E(\varepsilon) = 0$ and $var(\varepsilon) = \sigma^2$. Based on the conclusion above, we can calculate the random profit of centralized dual channel supply chain as follows:

$$\pi_t^o = \begin{cases} -cq_r + (p_d - c)((1-\Theta)(a+\varepsilon) - \beta_2 p_d + e p_r + \tau s) + p_r(\Theta(a+\varepsilon) - \beta_1 p_r + e p_d + (1-\tau)s) & \\ +(l-h)(q_r - \Theta(a+\varepsilon) + \beta_1 p_r - e p_d - (1-\tau)s_r) - \dfrac{\eta s^2}{2} & \tilde{x}_r < q_r \\ -cq_r + (p_d - c)((1-\Theta)(a+\varepsilon) - \beta_2 p_d + e p_r + \tau s) + p_r q_r & \\ -b(\Theta(a+\varepsilon) - \beta_1 p_r + e p_d + (1-\tau)s - q_r) - \dfrac{\eta s^2}{2} & \tilde{x}_r > q_r \end{cases}$$

So, the expected profit of the retailer is expressed as follows:

$$E(\pi_t^o(\varepsilon)) = -cq_r + (p_d - c)\big((1-\Theta)(a+\varepsilon) - \beta_2 p_d + e p_r + \tau s\big) - \frac{\eta s^2}{2}$$
$$+ \int_{-\infty}^{T} [p_r(\Theta(a+\varepsilon) - \beta_1 p_r + e p_d + (1-\tau)s) + (l-h)(q_r - \Theta(a+\varepsilon) + \beta_1 p_r - e p_d - (1-\tau)s_r)] f(\varepsilon) d\varepsilon$$
$$+ \int_{T}^{+\infty} [\, p_r q_r - b(\Theta(a+\varepsilon) - \beta_1 p_r + e p_d + (1-\tau)s - q_r)] f(\varepsilon) d\varepsilon$$

Where $T = \frac{1}{\Theta}(q_r - \Theta(a+\varepsilon) + \beta_1 p_r - e p_d - (1-\tau)s)$.
Finding the optimal p_r , p_d and s to maximize $E(\pi_t^o(\varepsilon))$, we differentiate $E(\pi_t^o(\varepsilon))$ with respect to p_r , p_d and s :

$$\frac{\partial E(\pi_t^o(\varepsilon))}{\partial p_r} = (l-h+b)\beta_1 T f(T) + [\Theta a + e p_d + (1-\tau)s + (l-h)\beta_1][\tfrac{\beta_1 p_r}{\Theta} f(T) + \textstyle\int_{-\infty}^{T} f(\varepsilon) d\varepsilon]$$
$$-2\beta_1 p_r \int_{-\infty}^{T} f(\varepsilon) d\varepsilon + \Theta \int_{-\infty}^{T} \varepsilon f(\varepsilon) d\varepsilon - \frac{{\beta_1}^2 {p_r}^2}{\Theta} f(T) + \beta_1 p_r T f(T) - (q_r + b\beta_1)(\frac{\beta_1 p_r}{\Theta} f(T)$$

$$-\int_{T}^{+\infty} f(\varepsilon)d\varepsilon) + (l-h-b)[q_r + \Theta a - ep_d - (1-\tau)s]\frac{\beta_1}{\Theta}f(T) + ep_d - ce$$

$$\frac{\partial E(\pi_t^o(\varepsilon))}{\partial p_d} = (p_r + l - h)eTf(T) + (1-\Theta)\int_{-\infty}^{+\infty} \varepsilon f(\varepsilon)d\varepsilon - e(p_r - (l-h))(\frac{ep_d}{\Theta}f(T) - \int_{-\infty}^{T} f(\varepsilon)d\varepsilon)$$
$$-eb\int_{T}^{+\infty} f(\varepsilon)d\varepsilon + \frac{e^2bp_d}{\Theta}f(T) + [-\Theta p_r a + \beta_1 {p_r}^2 - (1-\tau)sp_r - (l-h)(q_r - \Theta a + \beta_1 p_r - (1-\tau)s)$$
$$+p_r q_r - \Theta ba + b\beta_1 p_r - b(1-\tau)s + bq_r]\frac{e}{\Theta}f(T) + (1-\Theta)a - 2\beta_2 p_d + ep_r + \tau s + c\beta_2$$

$$\frac{\partial E(\pi_t^o(\varepsilon))}{\partial s} = \frac{1}{\Theta}(\tau-1)(p_r\Theta + (l-h)\Theta + b)Tf(T) + (1-\tau)(p_r + h - l)[\frac{\tau-1}{\Theta}f(T) + \int_{-\infty}^{T} f(\varepsilon)d\varepsilon]$$
$$+b(1-\tau)[s(\tau-1)f(T) - \int_{T}^{+\infty} f(\varepsilon)d\varepsilon] + \frac{1}{\Theta}(\tau-1)[\ \Theta p_r a - \beta_1 {p_r}^2 + ep_r p_d$$
$$+(l-h)(q_r - \Theta a + \beta_1 p_r - ep_d) + p_r q_r - \Theta ba + b\beta_1 p_r - ebp_d + bq_r] + ep_d - \eta s$$

Solving these equations using numerical methods we can find the optimal p_r , p_d and s. But we use the genetic algorithm because of the complexity of computations in numerical methods.

5.1. Genetic algorithm methodology

The genetic algorithm is a method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution (Abarghoei, Hosaini-Nasab and Fakhrzad, (2012)).

The following outline reviews the genetic algorithm steps (Fakhrzad and Esfahanib, (2013)):

1. The first step is creating a random initial population.

2. Then a sequence of new populations will be created by the algorithm. At each step, to create the next population, the algorithm is used by people in the current generation. To create the new population, the following steps are implemented:

    - Each member of the current population is graded by computing its suitability of value.
    - The crude appropriate scores are scaled to modify them into a more benefit of values.
    - Selects members, called parents, based on their suitability. (it should be a complete sentence)
    - The members of the current populations that have lower suitability are chosen as upper class so that these elite individuals are selected to the next population.
    - The parents give birth to children. Children are produced either by making random alternations to one parent or by connecting the vector entrance of a pair of parents.
    - The algorithm construct the next generation by substituting the current population with the children.

3. When we face to the stopping criteria, the algorithm will stop.

5.2. Genetic algorithm implementation

Genetic algorithm is implied for the following fitness function:

$$E(\pi_t^o(\varepsilon)) = -cq_r + (p_d - c)\big((1-\Theta)(a+\varepsilon) - \beta_2 p_d + ep_r + \tau s\big) - \frac{\eta s^2}{2}$$
$$+\int_{-\infty}^{T} [p_r(\Theta(a+\varepsilon) - \beta_1 p_r + ep_d + (1-\tau)s) + (l-h)(q_r - \Theta(a+\varepsilon) + \beta_1 p_r - ep_d - (1-\tau)s_r)]f(\varepsilon)d\varepsilon$$
$$+\int_{T}^{+\infty} [p_r q_r - b(\Theta(a+\varepsilon) - \beta_1 p_r + ep_d + (1-\tau)s - q_r)]f(\varepsilon)d\varepsilon$$

Optimizing the above suitable scaling function, we obtain$s$, p_r and p_d.

Initialization

A binary description is required to describe each person in the population**.** The size of the population determines how many people are in each generation. The create function determines the function that develops the initial population. A random initial population is produced by a uniform distribution, so we use a uniform function. Via the scaled values from the suitable scaling function, the admiration function selects parents for the next generation. We can choose stochastic uniform function for it. The first step is a uniform random number.

Reproduction

Creating children is decided by reproduction options at each new generation. The numbers of individuals that are outliving to the next generation are upper classes. We set upper class as a positive integer less than or equal to the population size.

The fraction of the next generation is related to Crossover fraction. Mutation produces the outstanding individuals in the next generation. Set Crossover fraction to be a fraction between 0 and 1, either by entering the fraction in the text box or by moving the slider.

Mutation

Mutation functions provide genetic variety and enable the genetic algorithm to be more expanded by making small random alternations in the individuals in the population. We can choose the Uniform function that is a two-step procedure. Since each entrance has the same probability as the mutation rate, first, the algorithm chooses a fraction of the vector entrances of an individual for mutation. In the second step, each selected entrance is substituted by a random number which is selected uniformly.

Crossover

Connecting two individuals or parents to configure a new individual or child for the next generation is Crossover. For producing a random binary vector, we use the scattered function. When the vector is 1, it chooses the genes from the first parent and when the vector is 0, it chooses the genes from the second parent, then it blends the genes to form a child. For example:

p1 = [a b c d e f g h]
p2 = [1 2 3 4 5 6 7 8]
Random crossover vector = [1 1 0 0 1 0 0 0]
Child = [a b 3 4 e 6 7 8]

Stopping criteria

What causes the algorithm to finish is stopping criteria. The maximum number of iterations is determined by generations. Table 3 presents the results for the numerical examples with Matlab software, where $a = 400$, $\Theta = 0.7$, $\eta = 4$, $\beta_1 = 1$, $\beta_2 = 1.1$, $e = 0.2$, $c = 30$, $b = 9$, $h = 5$, $l = 1.5$ and $q_r = 14$. Also, we use population size=20, elite count=2, crossover fraction=0.8, mutation rate=0.1 and generations=100 (stopping criteria).

Table 3. The results by different τ

τ	π	s	p_d	p_r
0	522.831	3.447	20.848	18.506
0.1	475.888	5.401	21.944	17.563
0.2	441.188	7.164	24.500	15.696
0.3	402.684	7.065	20.281	18.975
0.4	380.862	4.315	21.161	18.472
0.5	374.991	6.203	20.499	18.648
0.6	364.149	3.039	19.304	20.151
0.7	300.410	3.088	19.269	20.237
0.8	272.476	8.952	18.721	20.255
0.9	220.832	4.896	18.907	20.561
1	164.571	2.734	22.643	18.209

We understand from Figure 9 that while the free-riding rate is growing, the supply chain profit decreases. Although the manufacturer may benefit, the offline store and the entire supply chain is harmed. In other words, the offline stores expected harm from free riding is more than the manufacturer's expected profit. It also shows the coordination contract improves supply chain performance and the expected utilities of both member. Although uncertain demand may be inconvenient for the supply chain decision makers because of the complex mathematical computing process, the proposed coordination mechanism via a cost-sharing contract still works effectively.

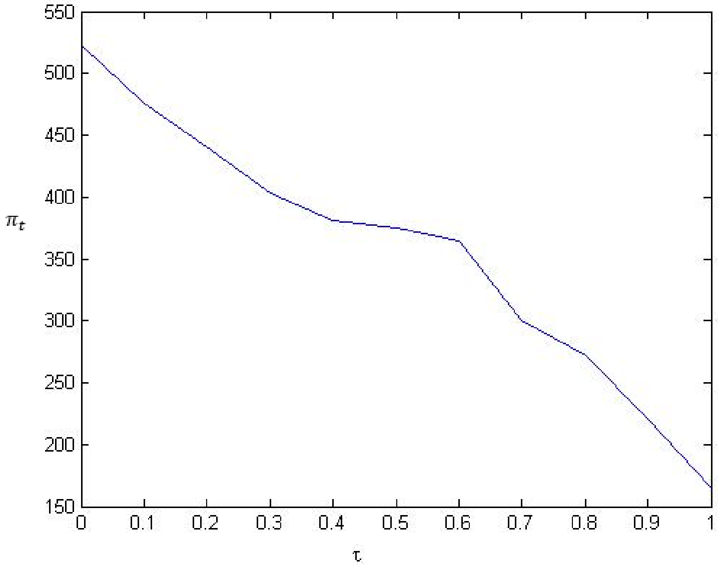

Figure 9. The effect of τ on π_t

6. Conclusion

This paper presented the coordination of a dual channel supply chain including a retailer and a supplier. The main innovation of this study is to present a new model and implement a synchronization approach using the cost-sharing contract in relation to previous studies. The most important management application of this research is to deal with stochastic demand through cost-sharing contract.

The consequences indicate that the offline store will decline its sales efforts by its Internet-based competitors. Therefore, offline store decreases the manufacturer's qualification to contest effectively in the market. Furthermore, the results confirm that manufacturers should propose an impressive cost-sharing contract to convince offline stores to spend more on sales efforts and create a win–win situation. The models comprise horizontal contestation between the offline store and the online store and vertical competition between the offline store and the manufacturer. This study expands the previous research by offering a cost-sharing contract in both deterministic and stochastic settings. Under this contract, the manufacturer provides partial reparation to the offline store to compensate free riding harms; therefore, it enhances the sales effort of the offline store and increases the efficiency of supply chain.

Further research may be conducted considering a demand depending on retail prices and even suppliers, incentive contracts and the inclusion of risk aversion.

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