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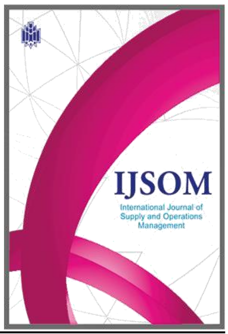

Development of an Integrated Model for Maintenance Planning and Statistical Process Control

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Abstract
An integrated model of maintenance planning and statistical process control is developed for a production process. The process has two operational states including an in-control state and an out-of-control state, where the process failure mechanism is supposed as a general continuous distribution with non-decreasing failure rate. Based on the information obtained from the control chart, three types of maintenance actions may be implemented in the process. The integrated model optimally determines the parameters of the control chart and maintenance actions so that the expected cost per time unit is minimized. To evaluate the performance of the integrated model, a stand-alone model is developed. In the stand-alone model, only maintenance planning is considered. Finally, a real case study is presented to clarify the performances of these models.

Keywords: Maintenance; Control chart; Statistical process control; Process failure mechanism; Integrated model.

1. Introduction

Maintenance management (MM) and statistical process control (SPC) are two key tools for the management and control of production processes. Although for several years, from the academic and practical point of view, these two key tools have been considered and analyzed separately, some integrated models have been recently developed to consider MM and SPC jointly. Many researchers (Liu, Jiang, and Zhang 2017),(Jianlan Zhong & Yizhong Ma 2017),(Xiang 2013),(H. Rasay, Fallahnezhad, and Zaremehrjerdi 2018) mentioned that there are many interactions and interrelations between MM and SPC that verify the development of the integrated models.

Integrated models of MM and SPC can be classified based on different criteria, such as type of the control chart employed for the process monitoring, process failure mechanism, number of the process states, inspection policy applied to the process monitoring, impact of the maintenance on the process, and maintenance policy in different situations. Different types of control charts are employed in the integrated models of MM and SPC, such as $\bar{x}$ control chart ((Zhou and Zhu 2008),(S. Panagiotidou and Tagaras 2012),(Xiang 2013)), Shewhart chart with variable parameters(Sofia Panagiotidou and Nenes 2009), Bayesian control chart(Khaleghei et al. 2014), chi-square chart(Wu and Makis 2008) (Hasan Rasay, Fallahnezhad, and Zare Mehrjerdi 2018), cause-selecting control chart (Jianlan Zhong & Yizhong Ma 2017), and exponential weighted moving average (EWMA) chart ((Charongrattanasakul and Pongpullponsak 2011),(Abouei Ardakan et al. 2016)). From the perspective of process failure mechanism, in some integrated models, it is assumed that the probabilities of process transitions between different states are based on an exponential distribution ((Liu et al. 2013),(Wu and Makis 2008)). Some models are developed based on the Weibull distribution ((Zhou and Zhu 2008),(Linderman, McKone-Sweet, and Anderson 2005)), and in some research studies, it is supposed that the failure mechanism follows a general distribution ((S. Panagiotidou and Tagaras 2012),(Hassan Rasay, Fallahnezhad, and Zaremehrjerdi 2018)). In some models, the number of the process states is assumed to be two-state including an in-control state and an out-of-control state ((Linderman, McKone-Sweet, and Anderson 2005),(Zhou and Zhu 2008)).

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Some integrated models are three- state in a system including an in-control state, an out-of-control state, and a failure state ((S. Panagiotidou and Tagaras 2012),(Yin et al. 2015)). Also in some studies, a system has several operational states plus a failure state ((Xiang 2013),(Tagaras 1988)).

Different inspection policies are applied to monitor processes, such as equidistance interval policy ((Yin et al. 2015),(Jianlan Zhong & Yizhong Ma 2017)) and constant hazard policy ((Ben Daya.M and Rahim.M.A 2000),(S. Panagiotidou and Tagaras 2012)). In some integrated models, the effect of maintenance on systems is supposed to be perfect ((Linderman, McKone-Sweet, and Anderson 2005),(Sofia Panagiotidou and Tagaras 2010),(Abouei Ardakan et al. 2016)), while in some models, it is assumed that the maintenance effect is imperfect ((S. Panagiotidou and Tagaras 2012),(Xiang 2013),(Ben Daya.M and Rahim.M.A 2000)). While a perfect maintenance restores the system to the best-as-new state, an imperfect maintenance renews the system to the state between "as-good-as-new" state and the current state ((Xiang 2013), (S. Panagiotidou and Tagaras 2012)). Based on the process state, different maintenance policies are implemented in the process. A compensatory maintenance is applied when a false alarm is issued from the control chart, a reactive maintenance is implemented when facing the out-of-control state, and a corrective maintenance is applied in the state of complete process failure.

In this paper, a process that has two operational states (an in-control state and an out-of-control state) is considered. The process failure mechanism follows a general continues distribution with non-decreasing failure rate. Based on the information obtained from the control chart, three types of maintenance actions are possible to be conducted on the process, and four scenarios are possible for the evolution of the process in a production cycle. An integrated model of MM and SPC is presented for the process. To evaluate the performance of the integrated model, a stand-alone maintenance model is also developed. Thus, the methodology of the paper is based on a stochastic mathematical model. The model is derived according to the possible scenarios in each inspection period and renewal reward process.

The rest of the paper is organized as follows: in section 2, the general structure of the problem is described. Derivation of the integrated model is described in section 3. In section 4, a stand-alone maintenance model is developed. Section 5 elaborates the inspection policy applied in the integrated model. In section 6, details about the optimization of the models are presented. Section 7 presents a real case study. Also, some sensitivity analyses are conducted in section 7, and finally in section 8, the concluding remarks are presented.

2. Problem description

Consider a production process that has two operational states: an in-control state denoted as state 0 and an out-of-control state denoted as state 1. The operation of the process in state 1 is undesirable because in comparison with state 0, it leads to much more operational cost and also yields higher quality costs. The time that the process spends in state 0 before transition to state 1, the process failure mechanism, follows a general continues distribution function with non-decreasing failure rate.

The process is monitored as follows: at specific time points, such as $(t_1,t_2,\dots,t_{m-1})$, these time pointes are the decision variables of the model, n units of the produced items of the process are selected, and suitable quality characteristic(s) are measured and then, a proper statistical procedure is calculated. The results of the statistical analysis are plotted on a desired control chart. If they fall within the control limits of the control chart, the process continues its operation without any interruption. If they fall outside the control limits, an alarm is issued from the control chart. After that, an investigation is performed on the system to verify this alarm. If the investigation concludes that the chart signal is incorrect (i.e., the process is in state 0), a compensatory maintenance (CM) is conducted on the process, but if the investigation concludes that the chart signal is correct, a reactive maintenance (RM) is implemented on the system. Henceforth, we call the investigation performed after releasing the alarm of the control chart as the maintenance inspection to distinguish it from the sampling inspection.

At the end of the production cycle (at time point t_m), there is no sampling from the produced items, but only the maintenance inspection is applied to determine the true state of the process. If the maintenance inspection indicates that the system is in the in-control state at t_m , then a preventive maintenance (PM) is conducted, but if the maintenance inspection indicates that the system state is out-of-control at t_m ,then RM is applied. Hence, a production cycle of the process starts in state 0 and is terminated due to implement one type of the maintenance actions (RM, PM or CM).

Based on the descriptions presented so far, four scenarios are possible for the evolution of the process in a production cycle. These scenarios are illustrated in Figure 1 and elaborated as follows:

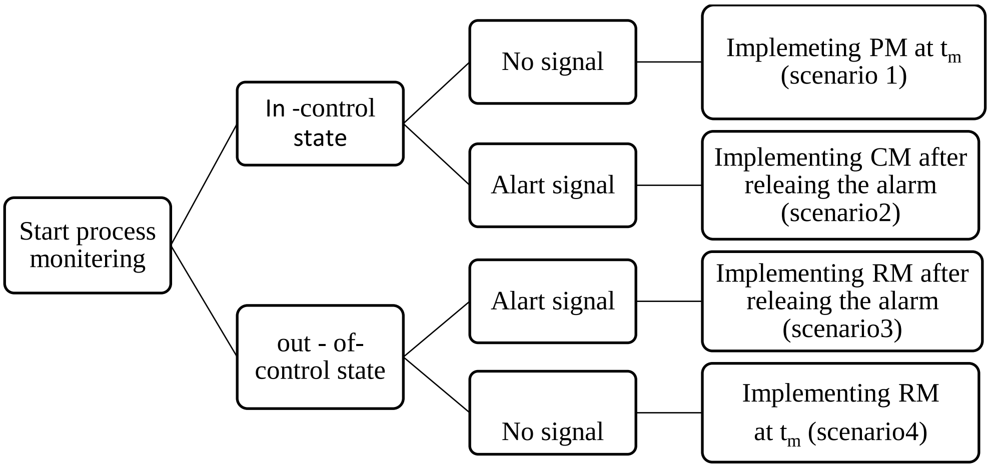


Figure 1. Different scenarios for the process evolution in a production cycle

Scenario 1: The process remains in state 0 until t_m and no alarm is released from the control chart in the previous inspection periods. Hence, PM is conducted on the process at t_m.
Scenario 2: While the process is operating in state 0, a false alarm is released from the control chart. Hence, CM is implemented, and the process is renewed.
Scenario 3: The Process shifts to state 1 before t_{m-1}, and an alarm is released from the control chart in one of the remaining inspection periods. Thus, RM is implemented and the process is renewed.
Scenario 4: The process shifts to state 1 before t_m , but the control chart cannot release this state. In other words, no alarm indicating the out-of-control state of the process is issued by the control chart in the remaining inspection periods. Hence, at t_m, after the maintenance inspection, the true state of the process is identified, and RM is conducted.

3. Derivation of the integrated model

The notations:

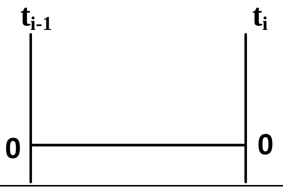
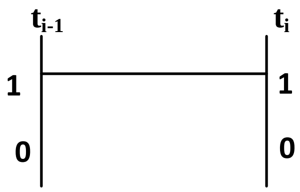
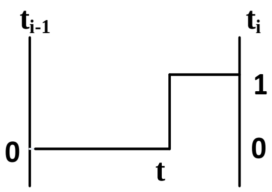
Notation	**Description**
C_i	Expected cost of the process in time unit when the process is in state i (i=0,1 and $C_1>C_0$)
W_{QC}	Cost of the sampling inspection
W_{PM}	Cost of the preventive maintenance
W_{RM}	Cost of the reactive maintenance
W_{CM}	Cost of the compensatory maintenance
W_I	Cost of the maintenance inspection
Z_{PM}	Expected time required for the preventive maintenance
Z_{RM}	Expected time required for the reactive maintenance
Z_{CM}	Expected time required for the compensatory maintenance
Z_I	Expected time required for the maintenance inspection
$f(t)$	Probability density function of the process failure mechanism
$F(t)$	Cumulative distribution function (c.d.f) of the process failure mechanism ($\overline{F}(x)=1-F(x)$)
X_{t_i}	A variable indicating the state of the process at time point t_i ($X_{t_i}=0$ *or* 1)
t_i	Time points of the sampling inspection (they are decision variables of the integrated models) (i=1,..,m-1)
α	Probability of type I error
β	Probability of type II error
n	Size of the samples in each sampling inspection (it is a decision variable of the model)
k	Control limit parameter (decision variable)
t_m	Scheduled time to perform the preventive maintenance (decision variable)
m	Maximum number of the inspection periods (decision variable)
$E[T_0]$	Expected time that the process operates in state 0 in a production cycle
$E[T_1]$	Expected time that the process operates in state 1 in a production cycle
$E[QC]$	Expected numbers of the sampling inspection in a production cycle
P_{PM}	Probability of termination of a production cycle due to the preventive maintenance
P_{CM}	Probability of termination of a production cycle due to the compensatory maintenance
P_{RM}	Probability of termination of a production cycle due to the reactive maintenance

The process monitoring described in the previous section can be considered as a renewal reward process consisting of the stochastic and independent identical cycles. *E[T]* and *E[C]* are defined as the expected time length of a production cycle and the expected cost of a production cycle, respectively. Hence, the expected cost of the process per time unit (*ECT*) can be obtained using the following formula:

$$ECT = \frac{E[\mathrm{C}]}{E[\mathrm{T}]} \tag{1}$$

Now, consider a single arbitrary inspection interval, such as (t_{i-1}, t_i). Given the state of the process immediately after the inspection at t_{i-1}, three different cases can be considered for the evolution of the system in this interval. Table 1 shows these cases along with the occurrence probability of the cases. Also, for these three cases, duration of time that the process operates in each state, i.e., in state 0 or in state 1, is shown in Table 1.

Table 1. Different scenarios for the evolution of the process in an inspection interval

Case	Figure	Occurrence probability	Duration of time that the process operates in state 1	Duration of time that the process operates in state 0
a		$P(a_{t_{i-1}}) = \frac{\overline{F}(t_i)}{\overline{F}(t_{i-1})}$	0	$t_i - t_{i-1}$
b		$P(b_{t_{i-1}}) = 1$	$t_i - t_{i-1}$	0
c		$P(c_{t_{i-1}}) = \int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\overline{F}(t_{i-1})}$ $= 1 - P(\mathrm{a}_{t_{i-1}})$	$t_i - t$	$t - t_{i-1}$

The cases are elaborated as follows:

Case a: In this case, the process is in state 0 at t_{i-1} and remains in this state until t_i. The occurrence probability of this case is as follows:

$$P(a_{t_{i-1}}) = P(t > t_i \mid \mathrm{X}_{\mathrm{t}_{i-1}} = 0) = \frac{\overline{F}(t_i)}{\overline{F}(t_{i-1})} \tag{2}$$

Case b: In this case, the process is in state 1 at t_{i-1}, hence remains in this state until t_i. The occurrence probability of this case is as follows:

$$P(b_{t_{i-1}}) = P(t > t_i \mid X_{t_{i-1}} = 1) = 1 \tag{3}$$

Case c: In this case, the process is in state 0 at t_{i-1} but at time point t ($t_{i-1} < t < t_i$), the process transits to stat 1. The occurrence probability of this case is as follows:

$$P(c_{t_{i-1}}) = P(\mathrm{t}_{i-1} < t < t_i \mid X_{\mathrm{t}_{i-1}} = 0) = \int_{t_{i-1}}^{t_i} \frac{f(t)dt}{\overline{F}(t_{i-1})} \tag{4}$$

Let define $P_{t_i}^0, P_{t_i}^1$ as the probability of process operation in state 0 or 1 immediately after the inspection at t_i , respectively.

$P_{t_i}^0$ is computed as follows:

$$P_{t_i}^0 = \overline{F}(t_i)(1-\alpha)^{\mathrm{i}};\; i = 1,2,...,m-1 \tag{5}$$

Equation 5 is derived based on the fact that the process is in state 0 at t_i if the process failure mechanism be more than t_i, and no false alarm is released from the control chart in the previous inspection periods.

$P_{t_i}^1$ is given by this recursive formula:

$$\begin{aligned} p_{t_i}^1 &= \beta\left[p_{t_{i-1}}^1 \times p(b_{t_{i-1}}) + p_{t_{i-1}}^0 \times p(\mathrm{c}_{t_{i-1}})\right] \\ &= \beta\left[p_{t_{i-1}}^1 + p_{t_{i-1}}^0 \times p(c_{t_{i-1}})\right]; \qquad i = 1,2,...,m-1 \end{aligned} \tag{6}$$

The sum of the two terms inside the square brackets is the probability of the process operation in state 1 just before the inspection at t_i. Also, if the process is in state 1 before the inspection at t_i ,with the probability of β, the control chart cannot detect the out-of-control state, and the process continues its operation in state 1 after the inspection performed at t_i.

As, it is assumed that all the maintenance actions are perfect; thus, the following equation holds true at the start of each production cycle:

$$P_0^0 = 1 ; P_0^1 = 0 \tag{7}$$

Equation 7 states that the process is always in the in-control state at the start of a production cycle. Based on the notations and assumptions introduced so far, *E[C]* and *E[T]* in Equation 1 are given by the following two equations:

$$\begin{aligned} E[C] &= C_0\,\mathrm{E}[\mathrm{T}_0] + C_1\,\mathrm{E}[\mathrm{T}_1] + \mathrm{W}_{\mathrm{QC}}\mathrm{E}[\mathrm{QC}] \\ &\quad + W_{PM}P_{PM} + W_{RM}\,\mathrm{P}_{RM} + W_{CM}P_{CM} + W_I \end{aligned} \tag{8}$$

$$E[T] = \mathrm{E}[\mathrm{T}_0] + \mathrm{E}[\mathrm{T}_1] + Z_{PM}P_{PM} + W_{RM}\,\mathrm{P}_{RM} + Z_{CM}P_{CM} + Z_I \tag{9}$$

Now, we proceed to compute each term in Equation 8 and 9. If T_0^i is defined as the expected time length that the process operates in state 0 at interval (t_{i-1},t_i), then *E[T₀]* can be computed as follows:

$$E[T_0] = \sum_{i=1}^{m} T_0^i \tag{10}$$

while T_0^{i} is obtained as follows:

$$T_0^i = P_{t_{i-1}}^0\left[\frac{\overline{F}(t_i)}{\overline{F}(\mathrm{t}_{\mathrm{i-1}})}(t_i - t_{i-1}) + \int_{t_{i-1}}^{t_i}\frac{f(t)}{\overline{F}(\mathrm{t}_{\mathrm{i-1}})}(t - t_{i-1})\mathrm{dt}\right];\quad i = 1,2,...,m \tag{11}$$

Considering Table 1, it can be seen that if the process be in state 0 at t_{i-1}, and scenario *a* or *c* occurs for the evolution of the process, then the process operates in state 0 in one part of the interval (t_{i-1},t_i). If scenario *a* occurs, the process operates in state 0 at interval, while if scenario *c* occurs, the time length that the process operates in state 0 is $t - t_{i-1}$.

If T_1^i is defined as the expected time length that the process operates in state 1 in interval (t_{i-1},t_i), then $E[T_1]$ is computed as follows:

$$E[T_1] = \sum_{i=1}^{m} T_1^i \tag{12}$$

while T_1^{i} is obtained as follows:

$$T_1^i = P_{t_{i-1}}^0\left[\int_{t_{i-1}}^{t_i}\frac{f(t)}{\overline{F}(\mathrm{t}_{i-1})}(\mathrm{t_i} - \mathrm{t})\mathrm{dt}\right] + P_{t_{i-1}}^1(t_i - t_{i-1});\qquad i = 1,2,...,m \tag{13}$$

Equation 13 is derived in the same manner as equation 11 is derived.

If P_{CM}^{i} is defined as the probability of performing CM just after the inspection at t_i , then P_{CM} is given by this equation:

$$P_{CM} = \sum_{i=1}^{m-1} P_{CM}^{i} \tag{14}$$

while P_{CM}^{i} is computed as follows:

$$P_{CM}^{i} = \overline{F}(t_i)(1-\alpha)^{i-1}\alpha; \quad i = 1,2,...,m-1 \tag{15}$$

Equation 15 is derived based on the fact that CM is implemented if a false alarm is issued from the control chart. At t_i , a false alarm is issued from the control chart if the process operates in state 0 , no false alarm is released in the previous inspection periods, and a false alarm is issued in the current inspection period (i.e., at t_i).

If P_{RM}^{i} is defined as the probability of conducting RM just after the inspection at t_i, the following formula is obtained to compute P_{RM} :

$$P_{RM} = \sum_{i=1}^{m} P_{RM}^{i} \tag{16}$$

while P_{RM}^{i} is computed as follows:

$$P_{RM}^{i} = (1-\beta)\left[P_{t_{i-1}}^{1} + P_{t_{i-1}}^{0}\int_{t_{i-1}}^{t_i}\frac{f(t)dt}{\overline{F}(t_{i-1})}\right]; \qquad i = 1,2,...,m-1 \tag{17}$$

The sum of the two terms inside the square brackets is the probability of the process operation in state 1 just before the inspection at t_i. Also, if the process is in state 1 before the inspection at t_i , with the probability of $1-\beta$, the control chart detects the out-of-control state and RM is implemented.

Since, there is no sampling inspection in the last inspection period, and only the maintenance inspection is performed at time point t_m, P_{RM}^{m} can be computed based on this formula:

$$P_{RM}^{m} = \left[P_{t_{m-1}}^{1} + P_{t_{m-1}}^{0}\int_{t_{i-1}}^{t_i}\frac{f(t)dt}{\overline{F}(t_{i-1})}\right], \tag{18}$$

Based on the assumptions explained about the system, a production cycle is terminated due to the performance of one of the following maintenance actions: RM, CM, or PM. The probabilities of termination of a production cycle due to the performance of RM and CM were computed so far. Hence, the probability of termination of a production cycle due to the performance of PM is as follows:

$$P_{PM} = 1 - P_{RM} - P_{CM} \tag{19}$$

If P_{QC}^{i} is defined as the probability of performing the sampling inspection at the end of interval (t_{i-1},t_i), then E[QC] is given by the following equation:

$$E[\text{QC}] = \sum_{i=1}^{m-1} P_{QC}^{i} \tag{20}$$

while P_{QC}^{i} is given by the following equation:

$$P_{QC}^{i} = P_{t_{i-1}}^{0} + P_{t_{i-1}}^{1}; \quad i = 1,2,...,m-1 \tag{21}$$

Note that, the sum of the two terms in the right side of Equation 21 not necessarily equals one because it is possible that a production cycle is terminated before reaching the time point t_{i-1} due to performance of CM or RM. Now, each term in equation 8 and 9 is computed and derivation of the integrated model is completed.

4. Derivation of the stand-alone model

In this model, it is assumed that the process starts its operation in the in-control state and at time point t_m, and the maintenance inspection is conducted on the process to clarify the true state of the process. In the stand-alone model, only maintenance planning is considered and t_m is the only decision variable. If the maintenance inspection indicates that the process state is 1 at t_m, then RM is applied. Otherwise, PM is implemented on the process. Hence, in this model, there is no sampling inspection. Similar to the integrated model, the production cycles can be considered as a renewal reward process consisting of stochastic and independent identical cycles. Thus, *ECT* can be obtained based on equation 1, while *E[T]* and *E[C]* are computed as follows:

$$E[\mathrm{T}] = \mathrm{t_m} + \mathrm{Z_{PM}}\overline{F}(t_m) + Z_{RM}[1-\overline{F}(t_m)] \tag{22}$$

$$E[\mathrm{C}] = F(t_m)\left[C_0\int_0^{t_m} tf(t)\,|\,t<t_m)dt + C_1\left(t_m - \int_0^{t_m} tf(t\,|\,t<t_m)dt\right) + W_{RM}\right] + \overline{F}(t_m)\left[C_0 t_m + W_{PM}\right] \tag{23}$$

5. Inspection policy

Manford (Munford 1981) presented different types of inspection policies to monitor processes. From a theoretical point of view, the inspection times, t_i (i=1,2,...,m), can be any arbitrary values; however, in practice, the inspection frequency should be designed based on a simple rule such that it can be applied in practice. “Constant hazard policy” is one of the commonly applied rules in practice to determine inspection times when the time of quality shift does not follow the exponential distribution(S. Panagiotidou and Tagaras 2012). Based on this rule, the probability of quality shift remains constant in each inspection interval, given that at the start of that interval, the process still operates in the in - control state. According to this rule, the inspection times are obtained based on this formula:

$$\int_{t_{i-1}}^{t_i} h(t)dt = \int_{t_i}^{t_{i+1}} h(t)dt \qquad i = 1,2,...,m-1 \tag{24}$$

By assigning an arbitrarily value to the first inspection time point ,t_1, the other inspection time points can be determined using Equation (24). In this formula, h(t) is the hazard rate function of the process failure mechanism, and it is obtained as follows: $\frac{f(t)}{\bar{F}(t)}$. It is worth noting that if the process failure mechanism obeys the exponential distribution, Equation (24) leads to the fixed sampling frequency. Another simple rule to determine the inspection time points is a constant inspection periods rule that conducts the inspection in the equidistance interval.

6. Optimization of the models

The aim of development of the integrated model is to optimally determine the parameters associated with the SPC and MM so that *ECT* of the process is minimized. Thus, in the integrated model, Equation 1 should be optimized, while *E[C]* and *E[T]* are computed based on equation 8 and 9, respectively. Optimization of the integrated model determines the parameters of the used control chart (i.e., the sample size, the control limit parameter, and the inspection time points) along with the maximum duration of the production cycle, t_m, and the maximum numbers of inspection periods, *m*. On the other hand, to optimize the *ECT* of the stand-alone maintenance model, Equation 1 should be optimized, while *E[T]* and *E[C]* are computed based on Equation 22 and 23, respectively. The optimization of *ECT* in the maintenance model determines the optimal value of t_m .

A grid search algorithm is used to optimize the models. In the algorithm, the continued variables (i.e., t_m, t_1 and *k*) are discretized in reasonable ranges. The algorithm is coded in Matlab software and it could be made available upon request by the first author of the paper.

7. An illustrative example and sensitivity analysis

In this section, a real example is presented to clarify the performances of the models. This example is selected from Zhou and Zhu’s article (Zhou and Zhu 2008). It is about a manufacturer producing nonreturnable glass bottles which are designed to package a carbonated soft drink beverage. The manufacturer used $\bar{x}$ control chart to monitor the process. When the process is in the in-control state, the quality characteristic follows a normal distribution with the mean and variance of μ_0, σ^2, respectively. In the out-of-control state, the mean of the process shifts to $\mu_1 = \mu_0 + \delta\sigma$, while the variance of the process remains unchanged. Also, δ indicates the magnitude of the shift, and it is assumed to be constant. The thickness of the bottles is an important quality characteristic. Suppose that the thickness of the bottle in the in-control state is 10mm and a single assignable cause leads to a shift in the mean of the process with the magnitude of $\delta = 1$.

For the $\overline{X}$ chart, the probability of type I error and type II error are given by:

$$\alpha = 2\Phi(-k) \quad and \quad \beta = \Phi(k - \delta\sqrt{n}) - \Phi(-k - \delta\sqrt{n}) \tag{25}$$

Where $\Phi(\bullet)$ indicates the cumulative distribution function (c.d.f) of the standard normal distribution. Also, the process failure mechanism is based on a Weibull distribution as the following form:

$$f\ (t) = \nu\lambda^{\nu} t^{\nu-1} \exp\{-(\lambda t)^{\nu}\}\ ; \qquad \theta, \lambda, \nu, t \geq 0 \tag{26}$$

where, v is the shape parameter, and λ is the scale parameter. For the considered process in this section, the value of the shape parameter is 2 and the value of the mean of the Weibull distribution ,μ, is 17.5 hours. The other parameters of the process are illustrated in Table 2. In this table, C_f and C_v are the fixed and variable sampling cost respectively. Thus, W_{QC} for n units is C_f+n×C_v.

Table 2. The parameters of the process

parameter	δ	C_f	C_v	W_I	C_0	C_1	W_{RM}	W_{PM}	W_{CM}	Z_I	Z_{RM}	Z_{PM}	Z_{CM}
value	1	10	0.1	100	10	200	2000	3000	1000	0.3	1	0.8	0.6

The results of the optimization of the two models are elaborated as follows. In the integrated model, the values of the decision variables are: *ETC=130, t_1=2.3, k=3.5, n=9, m=53* and *t_m=121.9*. These results indicate that the process monitoring should start at time point 2.3. The other time points of inspection are computed using Equation 24. At each time point of the inspection, a sample with size 9 is taken from the bottles produced by the process and the thickness of the bottles is measured as a critical quality characteristic. The control limit parameter of the $\bar{X}$ chart is 3.5, and the maximum duration of a production cycle is 121.9.

For the maintenance model, the results of the optimization are *ECT=157.31, t_m=28.5*. These results indicate that, based on the value of *ECT*, the integrated model has a better performance. Table 3 shows the results of a sensitivity analysis for some important parameters of the integrated model.

Table 3. The result of a sensitivity analysis for some important factors of the integrated model

μ	δ=1		δ=2	
	C_0=10	C_0=20	C_0=10	C_0=20
17.5	ECT=130.9 t_1=2.5 k=3.1 n=27 m=48 t_m=120	ETC=139.7 t_1=2.7 k=3.1 n=27 m=45 t_m=118.8	ETC=130.1 t_1=2.3 k=3.5 n=9 m=53 t_m=121.9	ECT=138.8 t_1=2.5 k=3.5 n=9 m=50 t_m=122.5
25	ECT=99.6 t_1=2.5 k=3.2 n=28 m=69 t_m=172	ECT=108.5 t_1=2.7 k=3.1 n=28 m=63 t_m=170	ECT=98.6 t_1=2.3 k=3.5 n=9 m=75 t_m=172	ECT=107.7 t_1=2.5 k=3.5 n=9 m=69 t_m=172

As the results of Table 3 denote, increasing the value of δ leads to an increase in the value of k and a decrease in the value of n. This trend can be justified based on the fact that it is easier for the control chart to release a bigger shift in the mean of the quality characteristic. Also, the increase of δ decreases the value of ETC to the limited extent, while the effect of this change on the other variables is insignificant. Increasing the value of C_0 leads to an increase in the value of ETC and t_1, while the effect of this change on the other variables is negligible. Finally, changing the mean of the process failure mechanism from 17.5 to 25 leads to a decrease in the value of ECT and an increase in the value of m and t_m. The same analysis is conducted about the stand-alone maintenance model. The results of the analyses are shown in Table 4.

Table 4. Sensitivity analysis of the stand-alone model

μ	δ=1		δ=2	
	C_0=10	C_0=20	C_0=10	C_0=20
17.5	ECT=157 t_m=28.5	ETC=162.9 t_m=29.3	ETC=157 t_m=28.5	ECT=162.9 t_m=28.5
25	ECT=134 t_m=33.9	ECT=141.9 t_m=34.8	ECT=134 t_m=33.9	ECT=141.9 t_m=34.8

As the results of Table 4 show, the integrated model in all cases leads to less values of *ECT* in comparison with the maintenance model. Also, in the maintenance model, the length of the production cycle, t_m, is much less than the corresponding value in the integrated model. Since, there is no sampling inspection in the maintenance model, changing the magnitude of the process shift, δ, has no effect on the decision variable of this model. The effects of change in the value of C_0 and in the mean of the process failure mechanism are similar to the integrated model.

8. Conclusion

In this paper, a production process that has two operational states, i.e., an in-control state and an out-of-control state, is studied. Two models are developed for the process. The first model is an integrated model of maintenance planning and statistical process control, while the second model is a stand-alone model that only considers maintenance planning. In the integrated model, based on the information obtained from the control chart, different types of maintenance actions are possible to be implemented in the process. The integrated model determines the parameters related to the control chart and the maintenance measures so that the expected cost per time unit is optimized. The validity of the model is checked according to real data from an industrial problem. The result of the example indicates that the integrated model leads to the less expected cost per unit time in comparison with the stand-alone model. Finally, a sensitivity analysis is conducted for the three key parameters of the models.

The proposed integrated model has a general structure because a general form is considered for the process failure mechanism. Moreover, different types of inspection policies can be applied to the model. Thus, the main novelty of the paper is the development of an integrated model with a general structure and a wide application domain. According to the results of the paper, in a production system, integration of the decisions related to maintenance and process control can lead to a significant decrease in operational costs. This research can be developed in several directions including 1- considering more complex production systems, 2- using multivariate control charts or profile monitoring approaches in the model, and 3- integration of the decisions related to the production planning in the model.

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