



Integrating DEA and Group AHP for Efficiency Evaluation and the Identification of the Most Efficient DMU

Amir Amini ^{a,*}, Alireza Alinezhad^b

^a Department of Industrial Engineering, Urmia University of Technology, Urmia, Iran.

^b Faculty of Industrial and Mechanical Engineering, Qazvin branch, Islamic Azad University, Qazvin, Iran.

Abstract

Selection problems which contain many criteria are important and complex problems that involve different approaches have been proposed to fulfill this job. The Analytic Hierarchy Process (AHP) can be very useful in obtaining a likely result which can consider the decision maker's subjective ideas. On the other hand, the Data Envelopment Analysis (DEA) has been a popular method for measuring the relative efficiency of decision making units (DMUs) and ranking them objectively in quantitative data. In this paper, a three-step procedure based on both DEA and AHP was formulated and applied to a case study. The procedure maintained the philosophy inherent in DEA by allowing each DMU to generate its own vector of weights. These vectors of weights were used to construct a group of pairwise comparison matrices which were perfectly consistent. Then, we utilized group AHP method to produce the best common weights compatible with the DMUs judgments. Using the proposed approach can give precise evaluation, combining the subjective opinion with the objective data of the relevant factors. The applicability of the proposed integrated model was illustrated using a real data set of a case study, which consisted of 19 facility layout alternatives.

Keywords: Data envelopment analysis; Group analytic hierarchy process; Common weights; Efficiency evaluation; Most efficient decision-making unit

1. Introduction

In order to survive the increasingly intense competitions, companies are currently trying to find better locations, system designs, materials, and so on to satisfy their customers' needs?. Therefore, selection problems are of the most challenging decision-making areas the manager of a company encounters. There are many research subjects within the research field of selection problems, such as portfolio selection, supplier selection, technology selection, material selection, and so forth. That is why so many approaches have been suggested for selection problems and this problem has found a significant number of applications in various fields.

Even though a good amount of research work carried out on selection problems, there is still a need for simple and systematic scientific methods or mathematical tools to guide user organizations in taking a proper selection decision. Making decision in presence of multiple conflicting criteria is known as multiple criteria decision-making (MCDM) process, and MCDM approaches like AHP and DEA methods are the most common approaches, which have been used in selection problems.

DEA is a non-parametric method for measuring the efficiency of a set of decision-making units (DMUs), such as firms or public sector agencies (Azadi, Jafarian, Farzipoor Saen and Mirhedayatian, 2015). Inherent philosophy of DEA approach is allowing each DMU to have the most favorable weights as long as the efficiency scores of all DMUs calculated from the same set of weights do not exceed one. This flexibility in selecting the weights deters the comparison among DMUs on a common base. Furthermore, it has some drawbacks such as unrealistic input/output weights, lack of discrimination among efficient DMUs, and finding the most efficient DMU.

AHP is widely used in multiple criteria decision analysis methodology. It operates by structuring a decision problem as a hierarchical model consisting of criteria and alternatives. A very important step in an AHP application is the need to estimate weights of decision entries (which can be criteria or alternatives). The flexibility of AHP has allowed its use in group decision making. Group decision making process is strongly evident in many organizations in today's highly competitive business environment, where most decisions are usually made after extensive studies and consultation, either internal or external (Dong and Cooper, 2015).

This paper proposes an integration of DEA and group AHP methods for efficiency evaluation. The procedure maintained the philosophy inherent in DEA, allowing each DMU to produce its own vector of weights which maximized the efficiency score of that DMU as long as the efficiency scores of all DMUs calculated from the same set of weights did not exceed one. These vectors of weights were used to construct a group of pairwise comparison matrices and check whether they were perfectly consistent. In other words, each DMU was asked (as a decision maker) to compare the relative importance of inputs/outputs and a pairwise comparison matrix was developed using the efficiency judgments (by solving one of the DEA models). Then, we utilized group AHP method to produce the best common weights which were consistent with DMUs judgments. Based on these common weights, the efficiency score of DMUs can be calculated and using them for ranking and finding the most efficient DMU which was a desirable goal in many applications of DEA.

The remainder of this paper is organized as follows: Section 2 briefly discusses DEA and group AHP. In section 3, the Group DEAHP model which combines DEA and AHP is presented. The applicability of the proposed integrated model is illustrated using a real data set of a case study consisting of 19 facility layout alternatives in section 4, and finally, conclusions are presented in section 5.

2. Literature of the related review

The complexity of the decisions that the manager faces makes it difficult to rely to a single decision maker's knowledge and capabilities to obtain a meaningful and reliable solution. Therefore, group decision making has received significant attention in both research and practice. Group decision making (GDM) is a procedure that combines the individuals' judgments and a common opinion on behalf of a whole group. To express the judgments of individuals, several formats are usually used in GDM, such as fuzzy preference relations (Tanino, 1984; Cabrerizo, Moreno, Perez and Herrera-Viedma, 2010; Xu, Li, and Wang, 2013), linguistic preference relations (Herrera, Herrera-Viedma and Verdegay, 1995; Herrera, Herrera-Viedma and verdegay, 1996; Wu and Xu, 2012; Alonso, Pérez, Cabrerizo and Herrera-Viedma., 2013), utility functions (Brock, 1980; Keeney and Kirkwood, 1975; Greco, Kadziński, Mousseau and Słowiński, 2012; Huang, Chang, Li and Lin, 2013), and the AHP (Dyer and Forman, 1992; Van Den Honert and Lootsma, 1997; Chiclana, Herrera and Herrera-Viedma, 2001; Altuzarra Moreno-Jimenez and Salvador, 2010). Our method integrates two well-known models, DEA and group AHP. Both DEA and AHP are commonly used in practice and many researchers highlight the relationship between DEA and AHP techniques.

First of all, Shang and Sueyoshi (1995) used a combination of DEA and AHP approaches for the selection of a flexible manufacturing system. Sinuany-Stern, Mehrez and Hadad (2000) derived the AHP pairwise comparison matrices mathematically from the input/output data by running pairwise DEA runs. Yang and Kuo (2003) proposed an AHP process and DEA approach to solve a plant layout design problem. Ertay, Ruan and Tuzkaya (2006) addressed the evaluation of the facility layout design by developing a robust layout framework based on the DEA/AHP methodology. Azadeh, Ghaderi and Izadbakhsh (2008) proposed the integration of DEA and AHP with computer simulation for railway system improvement and optimization. Wang, Liu and Elhag (2008) proposed an integrated AHP-DEA methodology. Tseng Chiu and Chen (2009) measured business performance in the high-tech manufacturing industry by using DEA, AHP, and a fuzzy MCDM approach. Recently, Yousefi and Hadi-Vencheh (2010) proposed a decision-making model in automobile industry by integration of AHP, TOPSIS, and DEA. Contreras (2011) proposed a new model consisting of two stages. First, a DEA-inspired model for the aggregation of preferences is applied, wherein the objective is not the maximization of the aggregated value, but rather the ordinal position induced by these values. Second, in order to obtain a group solution, the procedure derives a compromise solution by determining a social vector of weights for evaluating the complete set of alternatives.

HakimiAsl, Amalnicka Zorriassatineb and HakimiAsl (2016) integrated Fuzzy AHP and VIKOR methodologies assess, and select green suppliers of a solar power plant. To evaluate the effectiveness of multi-stage units in presence of undesirable elements, a new model in the DEA by network structure is offered by Amini, Alinezhad and Salmanian (2016) that can analyze the performance considering undesirable factors.

Li, Liu, Wang and Gao (2016) presented an enhanced DEA model, which modified conventional DEA model by adding the constraint cones generated from the Fuzzy- AHP model to evaluate the transit operator's efficiency. The proposed model aimed at including preference information over indicators in DEA process.

Although all these efforts developed their methods for selecting or evaluating DMUs, some requirements cannot be satisfied. At first, the simple implementation of the method is of prime importance. Moreover, most methods are qualitative and the usual way they make their evaluations is to list all the criteria in a form and ask the decision makers to give their evaluations for each criterion. In this paper, a quantitative method with a simple implementation is presented to solve this problem. At first, the following two subsections describe DEA and AHP methods briefly, after which, in section 3, a new hybrid model is described.

2.1. DEA preliminaries

DEA was first proposed by Charnes, Cooper and Rhodes (1978) and during the past two decades, it has emerged as an important tool in the field of efficiency measurement. DEA is a nonparametric approach that does not require any assumption about the functional form of production. The authors proposed a data-oriented approach to measure the performances of decision-making units (DMUs). The method converts multiple inputs into multiple outputs. DMUs can be manufacturing units, universities, schools, bank branches, hospitals, power plants, etc. (Wang Nguyen and Nguyen, 2015). DEA is a quantitative method, which can avoid the subjective factors of decision makers (Dobos and Vörösmarty, 2018).

Assume that there are n DMUs, (DMU $_j$: $j = 1, \dots, n$) which consume m inputs (x_{ij} : $i = 1, \dots, m$) to produce s outputs (y_{rj} : $r = 1, \dots, s$). A standard formulation of DEA creates a separate linear program for each DMU. It is instructive to apply the output-oriented version of the multiplier BCC model as follows:

$$\begin{aligned}
 & \text{Min} \quad \sum_{i=1}^m v_i x_{io} - \gamma_o \\
 & \text{s.t.} \quad \sum_{r=1}^s u_r y_{ro} = 1 \\
 & \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + \gamma_o \leq 0 \quad j = 1, \dots, n \\
 & \quad u_r, v_i \geq \varepsilon \quad r = 1, \dots, s, \quad i = 1, \dots, m
 \end{aligned} \tag{1}$$

Where x_{ij} and y_{rj} are the inputs/outputs of the j -th DMU, v_i and u_r are the inputs/outputs weights of the DMU $_o$ which are under evaluation, and optimal values of v_i and u_r are the best vectors of weights which maximize the efficiency score of DMU $_o$.

2.2. Group AHP preliminaries

The AHP (Saaty, 1980) is a multi-criteria, decision-making method that has been used in many applications related to decision-making problems (Ho, 2008) and is applicable to both individual and group decision making situations. To obtain group priorities in the AHP, the two mostly used procedures are the aggregation of individual judgments (AIJ) and the aggregation of individual priorities (AIP) (Ramanathan and Ganesh, 1994).

AHP was designed to solve complex problems involving multiple criteria. It allows decision makers to specify their preference using a simple scale, which can be very useful in helping a group or an individual to make a synthetic decision. Many authors suggested AHP as a proper approach to selection problems because of its inherent capacity to handle qualitative and quantitative criteria. The hierarchical structure used in formulating the AHP model can enable all the members of the evaluation team to visualize the problem systematically in terms of relevant criteria and sub-criteria. Because of the importance of the views given by the decision makers, however, the AHP model may produce the result greatly affected by the subjective attitudes of the decision makers greatly the quantitative data.

AHP, which is a comprehensive tool developed by Saaty (1977) for constructing decision models and establishing the decision priorities with respect to a finite set of alternatives, has been widely applied to group decisions because of the flexible structure and our innate ability to make relative comparisons. Allocating the weight or importance to each individual within a group is an important component in the decision process and plays a key role in obtaining the final solution in an AHP model. In the past three decades, multiple methods have been proposed to determine the weights of individuals (Ramanathan and Ganesh, 1994; Saaty, 1994; Forman and Peniwati, 1998; Bolloju, 2001; Van den Honert, 2001). However, these methods suffer from several drawbacks. First, most of these methods assign the weights according to subjective judgments. Thus, at least one individual must serve as a judge of the judges to provide this subjective weighting for the preferences of the decision makers. In practice, this potential for bias is a significant obstacle to overcome. Furthermore, it could be more reasonable to assign the weights of importance to each decision maker according to how compatible their judgments are with those of others (Xu and Cai, 2011; Xu, Li, and Wang., 2013). Therefore, we develop a dynamic method using the opinion transition probabilities, which serves as a way to measure the compatibility between decision makers, to allocate the weights to the decision makers in place of needing a judge.

AHP was first proposed by Saaty (1980), AHP follows four steps, the first two of which incorporate the individual preferences (judgments) that reflect the relative importance of the alternatives through a pairwise comparison judgment matrix A. Here, if $a_{ij} \cdot a_{jk} = a_{ik}$ ($i, j, k=1, \dots, n$), we said that the pairwise comparison matrix A is consistent. For using AHP, it is necessary that decision maker's judgment be consistent or near consistent, which is evaluated based on the definition of consistency ratio. However, since decision maker cannot estimate precisely measurement values, the pairwise comparison matrices are more likely to be cardinally inconsistent.

The flexibility of AHP has allowed its use in group decision making. The AHP literature describes two different ways of approaching group decision making in order to obtain group priorities. These are (i) Aggregation of individual judgments (AIJ) and (ii) Aggregation of individual priorities (AIP). In AIJ procedure, a new judgment matrix for the group as a whole is constructed on the basis of individual judgments using the weighted geometric mean method (WGMM), and then the group's priorities are drawn from this group judgment matrix. Using the row geometric mean method (RGMM), the individual priorities are to obtain in AHP, and the group's priorities are established on the basis of the individual priorities using the weighted geometric mean method (Blagojevic, Srdjevic, Srdjevic and Zoranovic, 2015).

3. Group DEAHP method

Because of its great flexibility and wide applicability, integrated AHP approaches have been studied extensively for the last 20 years (Ho and Ma, 2017). Real life applications of AHP show that in many cases, some decision makers do not want to change their individual judgments hoping to obtain stronger consensus for the group decision. Therefore, this research has focused on evidencing the practicality and usefulness of establishing a group aggregation procedure which would return a more concise group priority vector corresponding with the consensus degree (consensus level) principle between decision makers. As a result of this focus, we developed the procedure described in this paper and tested it along with several other procedures presented in the literature. The core of our approach holds that the group Euclidean distance should be used to measure the consensus level among decision makers while the simulated annealing (SA) algorithm is used to maximize consensus level. One important reason for such an approach is that the group Euclidean distance is a universal cardinal error measure which in many cases perfectly follows the purpose of the AHP to calculate cardinal information (weights) and not only ranks of alternatives like many other multi-criteria methods do (Azadeh, Ghaderi and Izadbakhsh, 2008; Alonso et al., 2013; Dong and Cooper, 2015; Blagojevic et al., 2015).

There have been several previous attempts in the literature to tie AHP and DEA. In this section, we use group AHP to make an aggregated weight vector of DMUs input/output weights. This weight vector can be used as common weights for efficiency evaluation, ranking, and finding the most efficient DMU.

Whenever using the AHP within a group decision making context, the optimal outcome is interpreted as achieving the highest degree of consensus among decision makers while deriving the group priority vector.

1. At first, we solve output-oriented BCC model to find optimal weights. Suppose that optimal solution of model (1) be as follows: $W_o = (v_{1o}, \dots, v_{mo}, u_{1o}, \dots, u_{so})$ $o=1, \dots, n$. Note that, we only need the weights vector assigned to the inputs/outputs of DMU_o . In other words, we can use any other DEA models with or without consideration of slack and surplus variables. These vectors of inputs/outputs weights are used to construct a group of pairwise comparison matrices A_j ($j = 1, \dots, n$) for each DMU_j , where they are perfectly consistent, e.g. for DMU_o we have:

$$A_o = \begin{bmatrix} a^j_{11} & \dots & a^j_{1(m+s)} \\ \vdots & \ddots & \vdots \\ a^j_{(m+s)1} & \dots & a^j_{(m+s)(m+s)} \end{bmatrix} = \begin{bmatrix} v_{1o}/v_{1o} & v_{1o}/v_{2o} & \dots & v_{1o}/v_{mo} & v_{1o}/u_{1o} & v_{1o}/u_{2o} & \dots & v_{1o}/u_{so} \\ v_{2o}/v_{1o} & v_{2o}/v_{2o} & \dots & v_{2o}/v_{mo} & v_{2o}/u_{1o} & v_{2o}/u_{2o} & \dots & v_{2o}/u_{so} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ v_{mo}/v_{1o} & v_{mo}/v_{2o} & \dots & v_{mo}/v_{mo} & v_{mo}/u_{1o} & v_{mo}/u_{2o} & \dots & v_{mo}/u_{so} \\ u_{1o}/v_{1o} & u_{1o}/v_{2o} & \dots & u_{1o}/v_{mo} & u_{1o}/u_{1o} & u_{1o}/u_{2o} & \dots & u_{1o}/u_{so} \\ u_{2o}/v_{1o} & u_{2o}/v_{2o} & \dots & u_{2o}/v_{mo} & u_{2o}/u_{1o} & u_{2o}/u_{2o} & \dots & u_{2o}/u_{so} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{so}/v_{1o} & u_{so}/v_{2o} & \dots & u_{so}/v_{mo} & u_{so}/u_{1o} & u_{so}/u_{2o} & \dots & u_{so}/u_{so} \end{bmatrix}$$

2. All relevant pairwise comparisons can be calculated this way. In other words, we ask each DMU (as a decision maker) to compare the relative importance of inputs/outputs and each DMU constructs its pairwise comparison judgment matrix based on the best weights produced according to a DEA model. Based on these pairwise comparison matrices, we can use one of the AIP or AIJ methods to aggregate the n set of inputs/outputs weights. Here, we use the AIP method. Then, the aggregated matrix of all of these pairwise matrices, A^G , will produce as follows:

$$A^G=(a^G)_{(m+s)(m+s)} \text{ where } a^G_{(i+r)(i+r)} = \left(\prod_{j=1}^n a^j_{(i+r)(i+r)}\right)^{1/n} \quad i = 1, \dots, m, \quad r = 1, \dots, s$$

1. When the RGMM prioritization procedure is applied, and after normalization, $W^G=(v^*_1, \dots, v^*_m, u^*_1, \dots, u^*_s)$ is a common set of weights.

Finally, for efficiency evaluation in BCC model, in addition to inputs/outputs weights, we need the value of γ^* . Note that, normalization of these common weights is associated with the coefficients of a supporting hyper plane that contain production possibility set (PPS) of BCC model in only one of the half spaces and pass among at least one of its points. Therefore, we can find the value of γ^* based on the value of W^G accompanying the input/output values of the observed DMUs. To this end, it is sufficient to solve the following model which can be performed based on simple comparisons.

$$\begin{aligned} & \text{Max } \gamma_o \\ & \text{s.t. } \sum_{r=1}^s u_r^* y_{rj} - \sum_{i=1}^m v_i^* x_{ij} + \gamma_o \leq 0 \quad j = 1, \dots, n \end{aligned}$$

2. In other words, it is sufficient to compute γ^* as:

$$3. \gamma^* = \text{Min } \left\{ \sum_{i=1}^m v_i^* x_{ij} - \sum_{r=1}^s u_r^* y_{rj}, \quad j = 1, \dots, n \right\} \tag{2}$$

In this manner, the output oriented efficiency score of $DMU_j, j=1, \dots, n$, can be obtained by using these common weights as follows:

$$Eff(DMU_j) = \frac{\sum_{i=1}^m v_i^* x_{ij} - \gamma^*}{\sum_{r=1}^s u_r^* y_{rj}} \quad j = 1, \dots, n \tag{3}$$

4. Numerical example

Table 1, which is provided by Ertay, Ruan and Tuzkaya (2006), shows the real data of 19 DMUs that consume two inputs to produce four outputs. In this section, efficiency score of these 19 DMUs is evaluated using the suggested method in section 3. The optimal variables of the output-oriented BCC model 1 are depicted in table 2.

Table 1. Inputs/outputs of DMUs

DMU	Inputs		Outputs			
	Cost	Adjacency	Shape ratio	Flexibility	Quality	Hand-carry utility
1	20309.56	6405.00	0.4697	0.0113	0.0410	30.89
2	20411.22	5393.00	0.4380	0.0337	0.0484	31.34
3	20280.28	5294.00	0.4392	0.0308	0.0653	30.26
4	20053.20	4450.00	0.3776	0.0245	0.0638	28.03
5	19998.75	4370.00	0.3526	0.0856	0.0484	25.43
6	20193.68	4393.00	0.3674	0.0717	0.0361	29.11
7	19779.73	2862.00	0.2854	0.0245	0.0846	25.29
8	19831.00	5473.00	0.4398	0.0113	0.0125	24.80
9	19608.43	5161.00	0.2868	0.0674	0.0724	24.45
10	20038.10	6078.00	0.6624	0.0856	0.0653	26.45
11	20330.68	4516.00	0.3437	0.0856	0.0638	29.46
12	20155.09	3702.00	0.3526	0.0856	0.0846	28.07
13	19641.86	5726.00	0.2690	0.0337	0.0361	24.58

Table 1. Continued

DMU	Inputs		Outputs			
	Cost	Adjacency	Shape ratio	Flexibility	Quality	Hand-carry utility
14	20575.67	4639.00	0.3441	0.0856	0.0638	32.20
15	20687.50	5646.00	0.4326	0.0337	0.0452	33.21
16	20779.75	5507.00	0.3312	0.0856	0.0653	33.60
17	19853.38	3912.00	0.2847	0.0245	0.0638	31.29
18	19853.38	5974.00	0.4398	0.0337	0.0179	25.12
19	20355.00	17402.00	0.4421	0.0856	0.0217	30.02

Table 2. Results of efficiency evaluation of DMUs in output oriented BCC model

DMU	V ₁	V ₂	U ₁	U ₂	U ₃	U ₄	Γ
1	0.000100	0.000100	0.868057	0.000100	0.000100	0.019173	1.52945564
2	0.000100	0.000100	0.874607	0.000100	0.000100	0.019685	1.51159883
3	0.000100	0.000100	0.881823	0.000100	0.000100	0.020248	1.49192528
4	0.000100	0.000098	0.882244	0.000100	0.937710	0.021657	1.37840852
5	0.000814	0.000172	1.033410	6.003779	0.000100	0.004785	16.0251
6	0.000100	0.000100	0.847696	0.402349	0.000100	0.022662	1.41622636
7	0.000100	0.000141	1.183573	0.068757	0.000100	0.026118	1.38276956
8	0.001372	0.000100	1.765472	0.000100	0.000100	0.009014	26.6891
9	0.001426	0.000100	1.768615	0.000100	0.000100	0.020153	27.4864
10	0.000953	0.000100	1.505646	0.000100	0.000100	0.000100	18.6981
11	0.000100	0.000100	0.000100	1.681307	0.000100	0.029058	1.42609621
12	0.000100	0.000100	0.000100	1.746634	0.000100	0.030297	1.38570900
13	0.001646	0.000100	0.000100	0.000100	0.000100	0.040682	31.7947
14	0.000100	0.000100	0.000100	1.567012	0.000100	0.026889	1.49675728
15	0.000100	0.000100	0.863997	0.000100	0.000100	0.018857	1.54052683
16	0.000100	0.000100	0.000100	1.514958	0.000100	0.025901	1.52893850
17	0.000100	0.000100	0.000100	1.761468	0.000100	0.030579	1.37653800
18	0.001371	0.000100	1.765440	0.000100	0.000100	0.008899	26.6809
19	0.000100	0.000100	0.834814	0.286838	0.000100	0.020199	1.49980655

Based on these set of weights, we construct the pairwise comparison matrices A_j ($j=1, \dots, 19$). For example, based on the preferred set of weights for DMU₅, pairwise comparison matrix A_5 is as follows:

$$A_5 = \begin{bmatrix} .000814 & .000814 & .000814 & .000814 & .000814 & .000814 \\ .000814 & .000172 & 1.03341 & 6.003979 & .0001 & .004785 \\ .000172 & .000172 & .000172 & .000172 & .000172 & .000172 \\ .000814 & .000172 & 1.03341 & 6.003979 & .0001 & .004785 \\ 1.03341 & 1.03341 & 1.03341 & 1.03341 & 1.03341 & 1.03341 \\ .000814 & .000172 & 1.03341 & 6.003979 & .0001 & .004785 \\ 6.003979 & 6.003979 & 6.003979 & 6.003979 & 6.003979 & 6.003979 \\ .000814 & .000172 & 1.03341 & 6.003979 & .0001 & .004785 \\ .0001 & .0001 & .0001 & .0001 & .0001 & .0001 \\ .000814 & .000172 & 1.03341 & 6.003979 & .0001 & .004785 \\ .004785 & .004785 & .004785 & .004785 & .004785 & .004785 \\ .000814 & .000172 & 1.03341 & 6.003979 & .0001 & .004785 \end{bmatrix}$$

Finally, by using these pairwise comparison matrices, the integrated weigh vector is produced as:

$$W^*=(v^*_1, v^*_2, u^*_1, u^*_2, u^*_3, u^*_4)=(.003562, .001539, .713029, .074878, .002656, .204343).$$

Moreover, based on formula (2), γ^* is obtained as 69.496388. Then, the efficiency scores of DMUs, based on these common weights and formula (3) are computed, which are depicted in Table 3. Note that, by using our proposed technique, DMU₇ is identified as the most efficient DMU and we have a full ranking of DMUs.

Table 3. Comparison of BCC output oriented efficiency scores and Group DEAHP scores

GAHP-BCC Model			BCC Model		
Rank	DMU	Efficiency score	Rank	DMU	Efficiency score
1	7	1	1	7	1
2	17	0.909981	1	17	1
3	12	0.748931	1	12	1
4	4	0.682338	1	10	1
5	6	0.675324	1	9	1
6	5	0.643589	1	5	1
7	11	0.634673	2	14	0.975886
8	9	0.627127	3	6	0.959286
9	14	0.624267	4	11	0.944669
10	2	0.583343	5	4	0.942403
11	3	0.564242	6	8	0.940494
12	8	0.562149	7	3	0.938524
13	13	0.561600	8	2	0.935608
14	15	0.550519	9	15	0.915061
15	16	0.546546	10	16	0.909308
16	15	0.523314	11	13	0.903961
17	1	0.522911	12	18	0.876324
18	18	0.522759	13	1	0.875656
19	19	0.216651	14	19	0.439388

5. Conclusion

DEA has been broadly used to take into account multiple criteria in decision making problems. DEA is a non-parametric linear programming technique for evaluating the relative efficiency of DMUs. Over the past three decades, a variety of DEA models have been used to evaluate the technical efficiency or technical effectiveness of DMUs in different settings. However, most of these works evaluate the performance from the perspective of technical efficiency or technical effectiveness.

We believe that the main advantage of integrating DEA and group AHP is that it is independent of the used prioritization method, while other tested group aggregation procedures are not. By Integrating DEA and group AHP, minimum values of the group Euclidean distance are computed and the highest degree of consensus is achieved without changing any of the individual judgments of decision makers participating in the group. Based on the results presented in this paper, we think that the proposed approach within the AHP group decision making framework could be extended to situations which decision makers do not have equal weights, unlike the examples we used in this paper, and also for cases when other cardinal error measures (e.g. Manhattan distance) are used.

As explained in Section 1, the selection problem is a very important problem in many organizations. There are some disadvantages in some approaches which are used to solve this problem. For example, AHP which is based on the corresponding pairwise comparison judgment matrices made by relevant decision makers contains much subjective opinion. On the other hand, DEA, which is based on the objective quantitative data of the selected input/output factors, has no subjective views of the decision makers.

This paper introduced a three-step approach, which combines both DEA and group AHP to solve this problem, which can find a balance between subjectivity and objectivity. The aim of the DEA model was to construct the weights for the management (input) and the efficiency evaluation and the identification of most efficient DMU (output). As soon as the DEA evaluations are gathered to formulate the pairwise comparison judgment matrices, the normalized weights are calculated to be used to synthesize the final evaluations.

The proposed model which is based on the integration of DEA and group AHP models takes the best advantages of both models and be computationally efficient. It gives a full ranking of DMUs and is suitable for situations in which return to scale is constant or variable.

References

- Alonso, S., Pérez, I. J., Cabrerizo, F. J. and Herrera-Viedma, E. (2013). A linguistic consensus model for Web 2.0 communities. *Applied Soft Computing*, Vol. 13, pp. 149-157.
- Altuzarra, A., Moreno-Jimenez, J. M. and Salvador, M. (2010). Consensus Building in AHP-Group Decision Making: A Bayesian Approach, *Operations Research*, Vol. 58, pp. 1755-1773.
- Amini, A., Alinezhad, A. and Salmanian, S. (2016). Development of Data Envelopment Analysis for the Performance Evaluation of Green Supply Chain with Undesirable Outputs, *International Journal of Supply and Operations Management*, Vol. 3(2), pp. 1267-1283.
- Azadeh A., Ghaderi S. F. and Izadbakhsh, H. (2008). Integration of DEA and AHP with computer simulation for railway system improvement and optimization, *Applied Mathematics and Computation*, Vol. 195(2), pp. 755-785.
- Azadi, M., Jafarian, M., Farzipoor Saen, R. and Mirhedayatian, S.M. (2015). A new fuzzy DEA model for evaluation of efficiency and effectiveness of suppliers in sustainable supply chain management context, *Computers & Operations Research*, Vol. 54, pp. 274–285.
- Blagojevic, B., Srdjevic, B., Srdjevic, Z. and Zoranovic, T. (2015). Heuristic aggregation of individual judgments in AHP group decision making using simulated annealing algorithm, *Information Sciences* (Article in press).
- Bolloju, N. (2001). Aggregation of analytic hierarchy process models based on similarities in decision makers' preferences, *European Journal of Operational Research*, Vol. 128, pp. 499-508.
- Brock, H. W. (1980). The Problem of " Utility Weights" in Group Preference Aggregation, *Operations Research*, Vol. 28, pp. 176-187.
- Cabrerizo, F. J., Moreno, J. M., Perez, I. J. and Herrera-Viedma, E. (2010). Analyzing consensus approaches in fuzzy group decision making: advantages and drawbacks, *Soft Computing*, Vol. 14, pp. 451-463.
- Charnes A., Cooper W.W. and Rhodes E. (1978). Measuring the efficiency of decision making units, *European Journal of Operational Research*, Vol. 2, pp. 429-444.
- Chiclana, F., Herrera, F. and Herrera-Viedma, E. (2001). Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations, *Fuzzy Sets and Systems*, Vol. 122, pp. 277-291.
- Contreras I. (2011). A DEA-inspired procedure for the aggregation of preferences, *Expert Systems with Applications*, Vol. 38, pp. 564-570.
- Dobos, I. and Vörösmarty, G. (2018). Inventory-related costs in green supplier selection problems with Data Envelopment Analysis (DEA), *International Journal of Production Economics* (Article in press).
- Dong, Q. and Cooper, O. (2015). A peer-to-peer dynamic adaptive consensus reaching model for the group AHP decision making, *European Journal of Operational Research* (Article in press).
- Dyer, R. F. and Forman, E. H. (1992). Group decision support with the Analytic Hierarchy Process, *Decision Support Systems*, Vol. 8, pp. 99-124.
- Ertay T., Ruan D. and Tuzkaya U. R. (2006). Integrating data envelopment analysis and analytic hierarchy for the facility layout design in manufacturing systems, *Information Sciences*, Vol. 176, pp. 237–262.
- Forman, E. and Peniwati, K. (1998). Aggregating individual judgments and priorities with the analytic hierarchy process, *European Journal of Operational Research*, Vol. 108, pp. 165-169.

- Greco, S., Kadziński, M., Mousseau, V. and Słowiński, R. (2012). Robust ordinal regression for multiple criteria group decision: UTAGMS-GROUP and UTADISGMS-GROUP, *Decision Support Systems*, Vol. 52, pp. 549-561.
- HakimiAsl, M., Sadegh Amalnicka, M., Zorriassatineb, F. and HakimiAsl, A. (2016). Green Supplier Evaluation by Using an Integrated Fuzzy AHP- VIKOR Approach, *International Journal of Supply and Operations Management*, Vol. 3(2), pp. 1284-1300.
- Herrera, F., Herrera-Viedma, E. and Verdegay, J. L. (1995). A sequential selection process in group decision making with a linguistic assessment approach, *Information Sciences*, Vol. 85, pp. 223-239.
- Herrera, F., Herrera-Viedma, E. and verdegay, J. L. (1996). A model of consensus in group decision making under linguistic assessments, *Fuzzy Sets and Systems*, Vol. 78, pp. 73-87.
- Ho, W. and Ma, X. (2017). The state-of-the-art integrations and applications of the analytic hierarchy process, *European Journal of Operational Research* (Article in press).
- Ho, W. (2008). Integrated analytic hierarchy process and its applications – A literature review, *European. Journal of Operational. Research*. Vol. 186, pp. 211–228.
- Huang, Y.-S., Chang, W.-C., Li, W.-H. and Lin, Z.-L. (2013). Aggregation of utility-based individual preferences for group decision-making, *European Journal of Operational Research*, Vol. 229, pp. 462-469.
- Keeney, R. L. and Kirkwood, C. W. (1975). Group Decision Making Using Cardinal Social Welfare Functions, *Management Science*, Vol. 22, pp. 430-437.
- Li, X., Liu, Y., Wang, Y. and Gao, Z. (2016). Evaluating transit operator efficiency: An enhanced DEA model with constrained fuzzy-AHP cones, *Journal of traffic and transportation engineering*, Vol. 3(3), pp. 215-225.
- Shang J. and Sueyoshi T. (1995). A Unified Framework for the Selection of a Flexible Manufacturing System, *European Journal of Operational Research*, Vol. 85(2), pp. 297-315.
- Sinuany-Stern Z., Mehrez A. and Hadad Y. (2000). An AHP/DEA methodology for ranking decision making units, *International Transactions in Operational Research*, Vol. 7, pp. 109-124.
- Ramanathan, R. & Ganesh, L.S. (1994). Group preference aggregation methods employed in AHP: An evaluation and an intrinsic process for deriving members' weight ages, *Eur. J. Oper. Res.*, Vol. 79, pp. 249–265.
- Saaty T.L. (1980). *The Analytic Hierarchy Process*, McGraw-Hill: New York.
- Saaty, T. L. (1994). *Fundamentals of decision making and Priority Theory with The Analytic Hierarchy Process*, RWS Publications, Pittsburgh.
- Saaty, T. L. (1977). A scaling method for priorities in hierarchical structures, *Journal of Mathematical Psychology*, Vol. 15, pp. 234-281.
- Tanino, T. (1984). Fuzzy preference orderings in group decision making, *Fuzzy Sets and Systems*, Vol. 12, pp. 117-131.
- Tseng F., Chiu Y. and Chen J. (2009). Measuring business performance in the high-tech manufacturing industry: A case study of Taiwan's large-sized TFT-LCD panel companies, *Omega*, Vol. 37(3), pp. 686-697.
- Van Den Honert, R. C. and Lootsma, F. A. (1997). Group preference aggregation in the multiplicative AHP The model of the group decision process and Pareto optimality, *European Journal of Operational Research*, Vol. 96, pp. 363-370.
- Van den Honert, R. C. (2001). Decisional power in group decision making: A note on the allocation of group members' weights in the multiplicative AHP and SMART. *Group Decision and Negotiation* 10, 275-286. *Information Sciences*, Vol. 181, pp. 150-162.

- Wang , C. N., Nguyen, X. T. and Nguyen, X. H. (2015). Strategic Alliance Decision-making for the Auto Industry base on an Integrate DEA and GM (1,1) Approach, *International Journal of Supply and Operations Management*, Vol. 2, No. 3, pp. 856-870.
- Wang Y., Liu J. and Elhag T. (2008). An integrated AHP–DEA methodology for bridge risk assessment, *Computers and Industrial Engineering*, Vol. 54(3), pp. 513-525.
- Wu, Z. and Xu, J. (2012). Consensus reaching models of linguistic preference relations based on distance functions. *Soft Computing*, Vol. 16, pp. 577-589.
- Xu, Z. (2009). An automatic approach to reaching consensus in multiple attribute group decision making, *Computers & Industrial Engineering*, Vol. 56, pp. 1369-1374.
- Xu, Z. and Cai, X. (2011). Group consensus algorithms based on preference relations. *Information Sciences*, Vol. 181, pp. 150-162.
- Xu, Y., Li, K. W. and Wang, H. (2013). Distance-based consensus models for fuzzy and multiplicative preference relations, *Information Sciences*, Vol. 253, pp. 56-73.
- Yang T. and Kuo C.A. (2003). A hierarchical AHP/DEA methodology for the facilities layout design problem, *European Journal of Operational Research*, Vol. 147, pp. 128–136.
- Yousefi A. and Hadi-Vencheh A. (2010). An integrated group decision making model and its evaluation by DEA for DEA for automobile industry, *Expert Systems with Applications*, Vol. 37(12), pp. 8543-8556.