The Optimization of a Multi-Product Three-Echelon Supply Chain

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Abstract

This paper aims at the single-objective optimization of multi-product for three-echelon supply chain architecture consisting of production plants, distribution centers (DCs), and customer zones (CZs). The key design decisions considered in the study include the quantity of products to be shipped from plants to DCs, from DCs to CZs, cycle length, and the production quantity so as to minimize the total cost. To optimize the objective, three-echelon network model is mathematically represented considering the associated constraints, production, capacity and shipment costs and solved using genetic algorithm (GA) and Simulated Annealing (SA). Some numerical illustrations are provided at the end to not only show the applicability of the proposed methodology, but also to select the best method using a t-test along with the simple additive weighting (SAW) method.

Keywords: Three echelon supply chain; Genetic algorithm; Simulated annealing Algorithm.

1. Introduction

Supply chain is a system of facilities and activities that functions to procure, produce, and distribute goods to the customers. ‘Supply chain management is basically a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores so that the merchandise is produced and distributed at the right quantities, to the right locations, and at the right time in order to minimize system-wide costs (or maximize profits) while satisfying service-level requirements’ (Simchi et al., 2000, p. ?). The above definition reveals that there are many independent entities in a supply chain, each of which tries to maximize its own inherent objective functions in business transactions. This is a complicated problem as too many factors are involved and need more than one objective to be satisfied simultaneously. Since today, although the success measures for the companies are determined by lesser costs, shorter production time, shorter lead time, less stock, larger product range, more reliable delivery time, higher quality, better customer services, and efficient coordination between the customer demand, supply, and production, the trade-off between cost investment and service levels may change over time. Hence the supply chain performance needs to be evaluated continuously and the supply chain managers should make timely and right decisions (Shen, 2007).

The key issues in supply chain management can broadly be divided into three main categories: (i) supply chain design, (ii) supply chain planning, and (iii) supply chain control. In the supply chain design phase, strategic decisions, like facility location decisions and technology selection decisions play major roles. It is very important to design an efficient supply chain to facilitate the transfer of goods. Environmental changes during the facility’s lifetime can drastically alter the appeal of a particular site, turning today’s optimal location into tomorrow’s investment blunder. Assigning the best locations for the new facilities is thus an important strategic challenge (Owen and Daskin, 1998).

In traditional supply chain management, the focus of the supply chain network designs is usually on single objective,
minimum cost, or maximum profit. But the design, planning, and scheduling projects usually involve the trade-offs among different incompatible goals, such as fair profit distribution among all members, customer service levels, fill rates, safety stocks, etc. (Chen & Lee, 2004). Therefore, real supply chains are to be optimized simultaneously by considering more than a single objective.

Many of the problems that occur in supply chain optimization are combinatorial in nature and picking a set of optimal solutions in case of multi-objective formulations requires a algorithm that can efficiently search the entire objective space using small amounts of computation time. The literature shows that evolutionary algorithms perform well in this respect and give optimal results when applied to many combinatorial problems. This work proposes the utility of genetic algorithm for the simultaneous optimization of one objective, minimizing total supply chain cost for a three-echelon supply chain architecture so as to arrive at an efficient supply chain design and optimal shipment plan which can be used as decision support system. Weber et al. (2000) proposed a method for suppliers’ selection which involves developing the supplier order quantity solutions using a SA method and evaluating these solutions using data envelopment analysis (DEA).

Proposing the idea that supply chain design should be considered in terms of cooperating systems involved in networks, Chandra and Kumar (2001) developed two single-objective linear models for SCN design: (1) a decomposition model identifying common constraints and (2) a dynamic process flow model using the flow of components in a network.

In most of the classical supply chain network designs, the aim has been to send products from one layer to another in order to supply demands such that the sum of strategic and tactical/operational cost is minimized. For instance, Amiri (2006) developed a SC model to obtain the best strategic decisions on locating production plants and distribution warehouses in order to dispatch the products from plants to customers with the goal of minimizing the total costs of the distribution network. Gebennini et al. (2009) suggested a three-stage production-distribution system to minimize costs.

Intricacy involved in mutual relations between various supply chain components together with risks and uncertainties throughout the chain have turned the SC decision-making process into a challenging problem, where newer? Aims are propounded. The uncertainties in supply chain networks are divided into three classes based on the supplier layer, receiver layer, and in the middle layers. Because reversing the decisions in relation to the SC network configuration is very costly and difficult, the importance of the interactions between these decisions is largely enhanced under uncertainty.

Bidhandi and Yusuff (2011) modeled a stochastic supply chain network as a two-stage program under strategic and tactical decisions. They also investigated how the demands of customer, operational costs, and the capacity of the facilities might be highly uncertain as all of them can severely affect the strategic decisions. In the strategic level, Snyder (2006) investigated a problem called “reliable facility location problem (RFLP)” to locate the facilities at the distributor level of a SC under uncertainty when the facilities were subject to random failures. Murthy et al. (2004) indicated that the uncertainty at the strategic level is the most important and difficult issue to be considered. In the tactical level, Van Landeghem and Vannaele (2002) worked on a supply chain planning problem that involved the distribution of raw materials and products. Moreover, Jamshidi et al. (2012) suggested a bi-objective multi-echelon SCN design model considering several transportation options at each level of the chain with different costs and a capacity constraint.

The SC network design under demand uncertainty has received significant attention in the last decade. For instance, Cardona-Valdés et al.(2011) purposed the design of a bi-objective two-echelon production distribution network under demand uncertainty to minimize both the total cost and the total service time. El-Sayed et al.(2010) developed a multi-period three-echelon forward-reverse logistics network design under demand uncertainty in the forward direction and deterministic customer demand in the reverse direction in order to maximize the total expected profits. In the strategic level, Schüt et al. (2009) presented another SC problem as a two-stage stochastic program under the short-term operations and demand uncertainty to minimize the total expected costs. Georgiadis et al. (2011) investigated a supply chain network design problem under time-varying uncertain demand in terms of a number of probable scenarios.

Wang et al. (2011) studied a two-echelon SC with stochastic demand to make decisions at both strategic and operational levels to maximize profit, where a genetic algorithm (GA) with efficient greedy heuristics was employed to solve the problem. Moreover, the optimization of a bi-criteria multi-echelon SC in the presence of demand uncertainty with the goals of maximizing the net present value and minimizing the expected lead time was investigated by You and Grossmann (2008). They proposed a ε-constraint method to solve the multi-period mixed-integer nonlinear programming (MINLP) problem.

Furthermore, Olivares-Benitez et al. (2012) modeled a two-echelon single-product SC design problem as a bi-objective mixed-integer programming and studied the three variations of the classical ε-constraint methods to generate Pareto-optimal solutions. Ruiz-Femenia et al. (2013) analyzed the influence of demand uncertainty on the multi-objective
optimization of chemical supply chains, simultaneously considering their economic and environmental performance. Moreover, Rodriguez et al. (2014) proposed an optimization model to redesign the supply chain of spare part delivery under demand uncertainty from strategic and tactical perspectives in planning horizon consisting of multiple periods. They addressed the uncertain demand by defining the optimal amount of safety stock that would guarantee a certain service level at a customer site. Moreover, the risk-pooling effect was taken into account when defining inventory levels in distribution centers and customer zones.

Mirzapour et al. (2011) addressed a multi-site, multi-period, multi-product three-echelon SC under uncertainties of cost parameters and demand fluctuations. They utilized the LP-metric method to solve the proposed bi-objective problem as a single-objective mixed integer programming. Azaron et al. (2008) proposed a multi-objective stochastic programming approach for the supply chain network design problem in which demands, supplies, processing time, shortage, transportation, and capacity expansion costs were purposed uncertain. They applied the goal attainment method to obtain the minimum values of the financial risk, the total expected costs, and the variance of the total cost as Pareto-optimal solutions.

Mele et al. (2007) proposed an agent-based approach to solve problems involved in SCs that are either driven by pull strategies or operate under uncertain environments. Song et al. (2014) considered a manufacturing supply chain problem with multiple suppliers in the presence of multiple uncertainties, such as uncertain material supplies, random customer demands, and stochastic production times. Pishvae et al. (2014) developed a multi-objective possibilistic programming model to design a sustainable medical supply chain network under uncertainty by considering the conflicting economic, environmental, and social objectives. Moreover, Wu et al. (2013) developed a stochastic fuzzy multi-objective programming model for supply chains that would outsource risk in the presence of both random and fuzzy uncertainties.

Many researchers have widely applied GA to solve SCM (stands for?) problems. Tsai and Chao (2009) developed a dynamic adaptive GA using a chromosome refinement procedure designed to adapt the ordinal structure of the genes within a chromosome. Prakash et al. (2012) provided a knowledge-based GA (KBGA) to optimize a SC network. Bandyopadhyay and Bhattacharya fbu(2014) proposed a tri-objective optimization problem for a two echelon serial supply chain. They considered the modification of nondominated sorting genetic algorithm-II (NSGA-II) with a mutation algorithm that has been embedded into the modified NSGA-II to solve the problem. A Lagrangian heuristic approach was also utilized in their research to compare the obtained results.

Zegordi et al. (2010) used a GA to solve a mixed integer programming problem developed for a two-stage SC problem that involved the scheduling of products and vehicles. Costa et al. (2010) presented a new efficient encoding/decoding procedure used within a GA to minimize the total current cost of a single-product three-stage SCN design under strategic decisions. Moreover, Wang and Hsu (2010) considered a spanning-tree-based GA to minimize the relevant cost associated with strategic decisions in a closed-loop logistic network problem formulated into an integer linear programming model.

Naimi Sadigh et al. (2012) investigated a multi-product manufacturer–retailer supply chain, where demand of each product is jointly influenced by price and advertising expenditure. They proposed several solution procedures including imperialist competitive algorithm, modified imperialist competitive algorithm, and evolution strategy. Pasandideh et al. (2010) considered a multi-product economic production quantity problem with limited warehouse-space in which the orders are delivered discretely in form of multiple pallets and the shortages are completely backlogged. Baykasoglu and Gocken (2010) presented a direct solution method that is based on ranking methods of fuzzy numbers and Tabu search to solve fuzzy multi-objective aggregate production planning problem.

There are other approaches in the literature to solve different SC problems. For instance, Cardona-Valdés et al. (2011) investigated the design of a two-echelon SCN with uncertain demand. An important contribution of their study was the development of a Tabu search within the framework of multi-objective adaptive memory programming to find optimal Pareto fronts of a two-stage stochastic bi-objective programming problem.

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The current study addresses single-objective optimization of a multi-product three-echelon supply chain network. The network consists of certain manufacturing plants, distribution centers (DCs), and customer nodes.

The remainder of this paper is organized as follows. In Section 2, the problem is stated and the assumptions are explained in more details. Section 3 provides the mathematical model of the problem at hand. In Section 4, the solution approaches are described. Section 5 contains numerical examples for a set of designed problems of different sizes. In this section, we apply the t-test and the simple additive weighting (SAW) method, as a statistical and a multi-attribute decision-making (MADM) approach respectively, to compare the algorithms. Finally, the conclusion and suggestions for future studies are presented Section 6.

2. The problem

Consider a three-echelon supply chain network shown in Fig. 1. This network consists of manufacturing plants to the left, distribution centers (DCs) in the middle of the figure, and customer nodes to the right.

The main assumptions of this problem are:
- The supply chain has an integrated structure consisting of manufacturing plants and potential DCs designed to supply customer demands for several products in single period.
- Shortage can not occur at customer nodes.
- More than one plant can replenish the demand of a given warehouse.
- More than one distributor center can replenish the demand of each customer.
- All the internal parameters, such as production and transportation costs, and holding costs of inventory for products are considered as the uniform random variables.
- The capacities of the manufacturers, the distributors, and the retailers are known.
- The warehouses operate perfectly.
- All of the products are manufactured by one machine.
- A definite cycle length for all items is perceived: \( T_1 = T_2 = \ldots = T_n = T \).
- It is assumed that the production rate of the \( i \)th item is \( p_i \) per cycle, and the quantity of the good items produced, denoted by \( d_i \), in every cycle.

![Figure 1. A three-echelon supply chain net](image-url)
We assume that the entire quantity of imperfect quality items will be reworked, and at the end of the rework period, a \( m_i \) portion as scrap will be left.

The manufacturer at the same time utilizes the same resource for production and rework processes.

For the common production system, budget and capacity are finite.

In this research, the following basic assumption of EPQ inventory model with the rework process is that the rate of production minus defectives must always be greater than or equal to the demand.

The inspection process for the products is perfect, and the inspection time is zero.

The locations of plants, distributors, retailers and suppliers are fixed.

Products are temporarily stored at the distributors before delivery to the retailers.

There is no inventory at the distributor centers and customer zones at the beginning or end of the cycle.

Under the above-mentioned conditions, the present study attempts to make decisions so as to determine a proper SCN configuration in order to minimize the total cost.

The proposed mathematical model can determine the economic production quantity and the quantity of products produced by each plant in a period, and assign products to the transportation channels between the SC entities in a period.

Furthermore, several limitations were considered in the proposed model as follows:

- The capacities of the manufacturer, distributor, and retailers are limited.
- The storage capacities for each perfect product are limited.
- All demands must be satisfied during the planning horizon.
- The production and reworking times are limited.

### 3. Mathematical model

A single-objective mixed-integer non-linear mathematical formulation of the problem at hand is derived in this section using the following notations including indices, parameters, and decision variables.

#### Sets of indices:
- \( i \) The set of indices \( \{i = 1, 2, \ldots, n_i\} \) used for manufacturing plants
- \( j \) The set of potential distribution centers (DCs) or warehouses \( \{j = 1, 2, \ldots, n_j\} \)
- \( k \) The set of customer nodes \( \{k = 1, 2, \ldots, n_k\} \)
- \( l \) The set of finished products \( \{l = 1, 2, \ldots, n_l\} \)

#### Parameters:
- \( c_{il} \) Unit production cost of product \( l \) produced by manufacturing plant \( i \)
- \( p_{il} \) The production rate of \( i \)th product at manufacturing plant \( i \), units/unit time
- \( d_{bl} \) The demand rate of \( i \)th product by all distribution center, units/unit time
- \( d_{rk} \) Demand of product \( l \) by customer \( k \), units
- \( r_{el} \) Rework cost per defective good \( l \) by manufacturer \( i \)
- \( h_{mi} \) Holding cost of products \( l \) for defective goods stored at manufacturer \( i \)
- \( u_{il} \) Percentage of defective goods \( l \) produced by manufacturer \( i \)
- \( s_{ci} \) Percentage of scrap products\( l \) produced by factory \( i \)
- \( f_l \) Volume of one unit of product \( l \) \( (m^3) \)
- \( m_{il} \) The proportion of scrap of \( i \)th product in rework at manufacturing plant \( i \)
- \( h_{il} \) Unit inventory holding cost of product \( l \) by plant \( i \)
- \( ic_{il} \) Unit inspection cost of product \( l \) by plant \( i \)
- \( A_{il} \) Setup cost of producing product \( l \) by manufacturing plant \( i \)
- \( CM_{ijl} \) Shipping cost of each product \( l \) from manufacturer \( i \) to distributor \( j \)
- \( CD_{jkl} \) Shipping cost of each product \( l \) from distributor \( j \) to retailer \( k \)
- \( T_{dif} \) Shipping cost of each defective goods \( l \) at manufacturer \( i \) (from production to defective goods storage and vice versa).
- \( dc_{il} \) Discount cost per scrap products \( l \) for sale by manufacturing plant \( i \)
- \( S_i \) Total storage capacity of plant \( i \)
- \( W_i \) The total available budget of manufacturing plant \( i \) per period
- \( E_j \) Total storage capacity of distributor \( j \)
- \( ES_{jl} \) Storage capacity of distributor \( j \) for product \( l \)
- \( R_k \) Total storage capacity of retailer \( k \)
- \( RS_{kl} \) Storage capacity of retailer \( k \) for product \( l \)
- \( G_{il} \) Production capacity of factory \( i \) for product \( l \)

#### Decision variables:

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\( Q_{il} \)  EPQ of products \( l \) by manufacturing plant \( i \)  
\( Q_i \)  Quantity of all products produced by manufacturing plant \( i \)  
\( INV_{il} \)  Inventory of product \( l \) at manufacturing plant \( i \) per period  
\( \text{perfect}_{il} \)  Quantity of perfect products \( l \) produced by factory \( i \) per period  
\( T \)  Cycle length  
\( X_{ijl} \)  Quantity of products \( l \) transported from factory \( i \) to distributor \( j \)  
\( Y_{jkl} \)  Quantity of products \( l \) transported from distributor \( j \) to retailer \( k \)

The production cycle length is the sum of the production uptimes for the good and incomplete items, \( t_1^i \) and \( t_3^i \), respectively, the reworking time, \( t_2^i \), and the production downtimes, \( t_3^i \) and \( t_4^i \). Hence, one has a total production cycle length of:

\[
T = \sum_{j=1}^{3} t_{il}^j
\]

Since all products are produced on a single machine with a limited capacity, the cycle length for all products is shown in Figure 2. Therefore, we have:

\[
\begin{align*}
H_{il}^{\text{max}} &= H_i + u_il(1 - m_il)Q_{il} - (1 + u_il)db_{il} Q_{il} \\
H_{il}^{\text{max}} &= H_i + u_il(1 - m_il)Q_{il} - (1 + u_il)db_{il} Q_{il} \\
H_{il}^{\text{max}} &= H_i + u_il(1 - m_il)Q_{il} - (1 + u_il)db_{il} Q_{il}
\end{align*}
\]

Hence, from Eq. (1), the cycle length for a single product is:

\[
T = \frac{1 - sc_{il} - m_{il}u_{il})Q_{il}}{db_{il}}
\]

\[
Q_{il} = \frac{T \times db_{il}}{(1 - sc_{il} - m_{il}u_{il})}
\]

\[
TC = \sum_{i=1}^{ni} \sum_{l=1}^{nl} c_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{l=1}^{nl} hm_{il}u_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{j=1}^{nj} \sum_{l=1}^{nl} X_{ijl}(CM_{jil}) + \sum_{j=1}^{nj} \sum_{k=1}^{nk} \sum_{l=1}^{nl} Y_{jkl}(CD_{jkl}) + \sum_{i=1}^{ni} \sum_{l=1}^{nl} r_{il}u_{il}Q_{il}
\]

\[
+ \sum_{i=1}^{ni} \sum_{l=1}^{nl} Td_{il}u_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{l=1}^{nl} d_{il}(sc_{il} + m_{il}u_{il})Q_{il}
\]
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\[ + \sum_{i=1}^{n_l} \sum_{i=1}^{n_l} A_{il} + \sum_{i=1}^{n_l} \sum_{i=1}^{n_l} c_{il} Q_{il} + \sum_{i=1}^{n_l} \sum_{i=1}^{n_l} h_{il} \text{INV}_{il} \]  

Constraints:

\[ Q_{il} \leq G_{il} \] (10)
\[ \sum_{i=1}^{n_l} X_{ijl} \leq E_j \] (11)
\[ \sum_{i=1}^{n_l} X_{ijl} \leq E_{sl} \] (12)
\[ \sum_{j=1}^{n_j} Y_{jkl} \leq R_k \] (13)
\[ \sum_{j=1}^{n_j} Y_{jkl} \leq R_{sl} \] (14)
\[ \text{perfect}_{il} = Q_{il}(1 - sc_{il} - (u_{il} \times m_{il})) \] (15)
\[ \sum_{i=1}^{n_i} X_{ijl} = \sum_{k=1}^{n_k} Y_{jkl} \] (16)
\[ \sum_{j=1}^{n_j} Y_{jkl} = dr_{kl} \] (17)
\[ \sum_{i=1}^{n_i} X_{ijl} = \text{perfect}_{il} \] (18)
\[ \sum_{i=1}^{n_i} (t_{il}^1 + t_{il}^2) + \sum_{i=1}^{n_i} ts_{il} \leq T \] (19)
\[ \sum_{i=1}^{n_i} (c_{il} + re_{il} \times u_{il})Q_{il} \leq W_i \] (20)
\[ \sum_{i=1}^{n_i} X_{ijl} = T \times db_{il} \] (21)
\[ \sum_{i=1}^{n_i} f_i((1 - sc_{il} - m_{il}u_{il})p_{il} - (1 + u_{il})db_{il}) \frac{Q_{il}}{p_{il}} \leq S_i \] (22)
\[ Q_{il}, Q_{i}, T, X_{ijl}, Y_{jkl} \geq 0, \text{integer} \] (23)

Eq. (9) represents an objective function that minimizes the total costs of the supply chain, including the costs of production, holding at the distributor, defective good storage, rework of defective goods, transportation from the manufacturers to the distributors, transportation from the distributors to the retailers, from the manufacturers to defective good storage and vice versa, the discount cost of scrap goods for sale, and the retailer shortages due to defective goods production. Eq. (10) states the restriction of production capacity. Eqs. (11) and (12) denote the delivery capacity limitations of the distributors for all types of products and each type of product, respectively. Eqs. (13) and (14) consider the delivery capacity limitations of the retailers for all product types and each product type, respectively. Eq. (15) expresses the total amount of perfect goods after reworking. Eq. (16) shows the balance between the total inputs and outputs of goods moving to and from the distributors during the cycle. Eq. (17) explains how the total demands during the cycle are supplied. Eq. (18) assures that the total goods shipped from the manufacturers to the distributors are of perfect quality. In other words, the scrap goods are removed from the system. In the joint production, systems having rework, the overall production, rework, and setup times ought to be smaller than the cycle length.

Eq. (19) assures \( \sum_{i=1}^{n_l}(t_{il}^1 + t_{il}^2) + \sum_{i=1}^{n_l} ts_{il} \) must be less than or equal to \( T \). Since the production quantity \( isq_i^x \), the total available budget is \( W \), and \( u_i q_i^x \) is the number of the \( i \)-th product which requires rework. Eq. (20) shows the budget constraint.

Eq. (21) expresses the total goods shipped from the manufacturers to the distributors that are equal to the product of the length of each period and the demand rate of \( i \)-th product by all distribution center. Eq. (22) shows the space of the
warehouse to store the products at manufacturing plants is limited. Eq.(23) indicates that the production amount, deliveries to warehouses and retailers, and perfect goods after reworking should all have positive values.

4. Solution algorithms

4.1. GA

Genetic algorithm (GA) is introduced first by Holland (1975). The formulation of the model is a non-linear integer-programming (NIP) model. This characteristic causes the model to be hard enough to be solved by an exact method (Gen, 1997). Genetic algorithms are stochastic search techniques based on the mechanism of natural selection and natural genetics. Each individual in the population is called a chromosome, deputizing a solution to the problem at hand. The chromosomes evolve through consecutive iterations, called generations.

During every generation, the chromosomes are evaluated, utilizing some measures of fitness. To create the subsequent generation, new chromosomes, called offspring, are built by either crossover operator or mutation operator. A new generation is built according to the fitness values of the chromosomes. After some generations, the algorithm converges to the best chromosome (Pasandideh and Niaki, 2008).

The usual form of GA was described by Goldberg (1989). GA is a stochastic search technique whose solution process mimics natural evolutionary phenomena: genetic inheritance and Darwinian strife for survival (Gen, 2000). Recently, the GA has been receiving great attention and it has successfully been applied to other problems in the supply chain environment (Chen et al., 1998; Park, 2001).

GA is known as a problem-independent approach; however, the chromosome representation is one of the critical issues when applying it to optimization problems.

4.1.1. GA algorithm in initial and general conditions

The required initial information to start a GA is:

(i) Population size (nPop): It is the number of the chromosomes or scenarios that are kept in each generation.
(ii) Crossover rate (Pc): This is the probability of performing a crossover in the GA method.
(iii) Mutation rate (Pm): This is the probability of performing mutation in the GA method.
(iv) maximum number of iterations (N)
(v) The general steps involved in a GA algorithm are as follows:

1. Initialization.
   1.1. Set the parameters (N, Pc, Pm, stopping criteria, selection strategy, crossover operation, mutation operation, and number of generation).
   1.2. Initialize the population (randomly).
2. Evaluate the fitness. Repeat
3. Select individuals for the mating pool (size of mating pool = N)
4. For each consecutive pair, apply crossover with probability Pc.
5. For each new generation, apply mutation with probability Pm.
6. Replace the current population by the resulting mating pool.
7. Evaluate the fitness.
Until stopping criteria is met.

4.1.2. The chromosomes

One of the important components of the GA is the selection of chromosomes. We attempted to select the best chromosomes in the proposed GAs that would give us good results and require a low run-time. In the studied mathematical model, the variable \( Y_{jk} \) has both direct and indirect relationships with the variables \( S_{cl} \), \( X_{jl} \), and \( Q_{il} \). Therefore, any change in variable \( Y_{jk} \) leads to certain changes in other variables, and thus, the variable \( Y_{jk} \) was chosen as the chromosome. The chromosomes are displayed in a matrix with the columns denoting the number of products (l) and rows (j * k). Each gene in the matrix was randomly created by Matlab software, as shown in Eq. (24).

\[
\begin{pmatrix}
Y_{111} & \cdots & Y_{11l} \\
\vdots & \ddots & \vdots \\
Y_{1k1} & \cdots & Y_{1kl}
\end{pmatrix}_{(j \times k) \times l}
\] \hspace{1cm} (24)

4.1.3. Evaluation

A fitness function is required to evaluate the chromosomes of each generation. In most GA applications, the objective function of the optimization model at hand is considered a fitness function. However, as explained before, the inventory model of this research has some constraints. These characteristics make the probability of a generated chromosome being feasible very low. In order to promote this chance, a penalty function is defined to be a positive and known sum of
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squared violation of each constraint. As a result, the penalty and the fitness function in which violation per constraint is denoted by $E_i$ is defined as follows:

Penalty function = \[ \begin{cases} \sum_{i=1}^{4} E_i / 4 & \text{If the chromosome is infeasible region} \\ 0 & \text{If the chromosome is feasible region} \end{cases} \]

Fitness function = Penalty function + Objective function

(25)

4.1.4. Initial population
In this step, a collection of chromosomes is randomly generated.

4.1.5. Crossover
To perform the crossover, two chromosomes must be merged. First, the chromosomes to be combined should be identified and allowed to mix. The columns will be combined for each chromosome selected for the crossover, and the intersection point will be used to combine the chromosomes.

For example, if we work with two chromosomes with four periods and three rows, the combination of the chromosomes will proceed as follows. First, the intersection point is chosen. Thereafter, the values of both sides of the matrix are exchanged. The intersection point shown in this example denotes the first period (Figure 3).

\[
\begin{array}{cccc}
Y_{111} & Y_{112} & Y_{113} & Y_{114} \\
Y_{211} & Y_{212} & Y_{213} & Y_{214} \\
Y_{311} & Y_{312} & Y_{313} & Y_{314} \\
\end{array}
\quad
\begin{array}{cccc}
Y'_{111} & Y'_{112} & Y'_{113} & Y'_{114} \\
Y'_{211} & Y'_{212} & Y'_{213} & Y'_{214} \\
Y'_{311} & Y'_{312} & Y'_{313} & Y'_{314} \\
\end{array}
\]

\begin{array}{cccc}
Selected Parent 1 \\
\end{array}
\quad
\begin{array}{cccc}
Selected Parent 2 \\
\end{array}

\[
\begin{array}{cccc}
Y_{111} & Y'_{112} & Y_{113} & Y_{114} \\
Y'_{211} & Y'_{212} & Y_{213} & Y'_{214} \\
Y_{211} & Y'_{212} & Y_{213} & Y'_{214} \\
\end{array}
\quad
\begin{array}{cccc}
Y'_{111} & Y_{112} & Y_{113} & Y_{114} \\
Y'_{211} & Y'_{212} & Y_{213} & Y'_{214} \\
Y_{211} & Y'_{212} & Y_{213} & Y'_{214} \\
\end{array}
\]

\begin{array}{cccc}
Produced Child 1 \\
\end{array}
\quad
\begin{array}{cccc}
Produced Child 2 \\
\end{array}

Figure 3. Display of the cross over operation.

4.1.6. Mutation
The mutation probability refers to the probability of change in any gene. In this research, each chromosome receives a certain number of genes that are assumed to change, and this number is derived by multiplying the total number of genes by the mutation probability. Accordingly, a number of genes must be selected to undergo mutation. The resulting value is rounded off to the nearest whole number. If four genes should undergo mutation, these four genes are randomly selected, and their values are changed (Figure 4).

\[
\begin{array}{cccc}
Y_{111} & Y_{112} & Y_{113} & Y_{114} \\
Y_{211} & Y_{212} & Y_{213} & Y_{214} \\
Y_{311} & Y_{312} & Y_{313} & Y_{314} \\
\end{array}
\quad
\begin{array}{cccc}
Y'_{111} & Y_{112} & Y_{113} & Y'_{114} \\
Y_{211} & Y'_{212} & Y_{213} & Y_{214} \\
Y_{311} & Y_{312} & Y_{313} & Y_{314} \\
\end{array}
\]

\[
\begin{array}{cccc}
Parent and selected genes \\
\end{array}
\quad
\begin{array}{cccc}
Produced child \\
\end{array}
\]

Figure 4. Display of the mutation operation.
4.1.7. Chromosomes selection
In genetic algorithms, the selection operator is used to guide the search process towards more promising regions in a search space. Some selection methods, like roulette wheel, tournament, ranking, and elitist are discussed in Michalewicz (1996). Furthermore, a detailed explanation of the operation of roulette wheel selection can be found in Goldberg (1989). Since in the roulette wheel selection, better solutions get higher chance to become parents of the next generation, it is used to select the chromosomes of this research. The selection in this method is based on the fitness value of the chromosomes. We select N chromosomes among the parents and the offspring with the best fitness values.

4.1.8. Stopping criterion
The last step in a GA methodology is to check if the method has found a solution that is good enough to meet a stopping criterion. Stopping criteria is a set of conditions such that when the method satisfies them, a good solution is obtained. In this study, a pre-specified maximum number of iterations (N) is considered to stop the algorithms.

4.2. SA
Simulated annealing (SA) is a generic probabilistic meta-heuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space (Kirkpatrick et al 1983). The method was independently described by Kirkpatrick et al (1983) and Cerny (1985). The title and the inspiration come from annealing in metallurgy, a technique that involves heating and controlled cooling of a material to increase the size of its crystals and reduce their deficiencies.

Both of them are attributes of the material that depend on its thermodynamic free energy. Heating and cooling the material impress both the temperature and the thermodynamic free energy. While the very amount of cooling brings the same amount of decrease in temperature, it will afford a bigger or smaller decrease in the thermodynamic free energy depending on the rate that it occurs, with a slower rate generating a bigger decrease.

This notion of slow cooling is implemented in the Simulated Annealing algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space. Accepting worse solutions is a basic property of meta-heuristics because it allows for a more extensive search for the optimal solution.

The procedure of simulated annealing algorithm is given below for the minimization problem:

- Get an initial temperature $T > 0$.
- Get an initial solution $S$.
- Initial solution ($S$) is considered as the best solution.
- The following loop repeats until stopping criteria is met.
- Perform the following loop L times.
- Generate solution $S'$ in neighborhood of $S$.
- Let $\Delta = \text{cost}(S') - \text{cost}(S)$.
- If ($\Delta < 0$) (downhill move), set $S = S'$.
- If ($\Delta > 0$) (uphill move), Create a random number ($x$), $x \in [0, 1]$
- If $x < \exp(-\Delta/T)$, set $S = S'$.
- End loop
- Set $T = \alpha \times T$ (reduce temperature).
- If stopping criteria is met, the algorithm is terminated.

The required initial information to start a SA is:

- Population size ($nPop$): It is the number of the initial solutions
- Final Temperature ($TF$)
- Temperature Decrement Rate ($\alpha$)
- Iterations at each temperature ($IT$)

4.2.1. Determining starting temperature
There are two key parameters in the cooling process that determine how firm or amorphous the result of the metal in its frozen state will be. The first one is the temperature starting point from which the cooling begins, and the second is the rate by which the temperature is falling. Concerning the rate of decay, it should be low enough to allow the atoms in the molten metal to line themselves up and give enough time to form a crystal lattice with the minimum internal energy. Evidently, a slow decay will lead to a long-time solidifying process.
To reduce the run time, one may think of a low starting temperature. However, on the other hand, if the initial temperature is not copious enough, atoms of the molten metal would not have enough freedom to rearrange their positions in a very regular minimum energy structure.

In this algorithm, starting temperature is calculated based on Equation (26).

\[
\text{temperature} = \frac{\text{TF}}{a^{\text{IT}}} \tag{26}
\]

4.2.2. Initial solution structure
The quality of the initial solution affects the performance of the meta-heuristics. To reduce this dependency, several methods start their search from multiple primary solutions. The structure presentation of the chromosome in this algorithm is similar to GA algorithm that was explained previously.

In this step, a chromosomes is generated. Equation (25) is considered as the fitness function.

4.2.3. Neighbourhood search
The way in which a meta-heuristic moves from one solution to its neighbor is another critical component in SA algorithm. For every iteration, the neighbourhood solution is generated using the mutation operator used in GA algorithm. This process is called neighbourhood generation.

4.2.4. Temperature Decrement
There are some rules for temperature decrement, where the most common method is classic temperature decrement \((T = T \times \alpha)\). The temperature decrement rate is \(\alpha\).

4.2.5. Stopping criteria
The assumed criteria in this algorithm is reaching to Final Temperature (TF).

4.3. Tuning the parameters of the two solution algorithms
In this paper, a regression approach is utilized for calibrating the main parameters of the algorithms. First, each algorithm is employed 30 times. Each time their parameters are changed in their relevant intervals. The values of the related responses are first normalized by the linear norm.

Then, a quadratic regression function for each measure is estimated using the Design Expert software to find the significant relationships between the parameters and their responses. Next, the estimation of the results utilizing the estimated regression function is taken to be optimized by the GAMS software in order to find the optimal combinations of the parameters. The quadratic regression function consists of interaction, linear, and quadratic coefficients shown in Equation (27).

\[
E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2 + \beta_{44} X_4^2 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \tag{27}
\]

Where \(E(Y)\) is the expected value of a given response, \(\beta_0\) is a constant coefficient representing the intercept, \(\beta_i\), \(i = 1,2,3,4\) are the linear coefficients, \(\beta_{ii}\), \(i = 1,2,3,4\) are the quadratic coefficients, \(\beta_{ij}\), \(i \neq j\) are the interaction coefficients, and \(X_i\), \(i = 1,2,3,4\) are the four parameters of the algorithms. This process is explained in the next section, where input data of the problem instances are given. Furthermore, the results obtained using the parameter-tuned algorithms are analyzed.

Numerical examples evaluate the performances of the two meta-heuristic algorithms in the next section. Then, they are solved employing GA and SA. Numerical examples are provided in the next section to evaluate the performances of the two single-objective meta-heuristic algorithms in the next section.

5. Numerical results
Consider a multi-product, multi-period, three-echelon SC problem, in which the internal parameters are randomly generated based on Uniform distributions in their corresponding ranges for different problem instances shown in Table 1. And the ranges of the parameters are given in Table 2.
Table 1. Generated problem instances

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing plants</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>9</td>
<td>13</td>
</tr>
<tr>
<td>Distributor centers</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>Customers</td>
<td>3</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>17</td>
</tr>
<tr>
<td>Products</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2. Parameter range

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Uniform Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>$[500, 700]$</td>
</tr>
<tr>
<td>$pc$</td>
<td>$[4, 8]$</td>
</tr>
<tr>
<td>$u$</td>
<td>$[0.0, 0.09]$</td>
</tr>
<tr>
<td>$A$</td>
<td>$[1, 4]$</td>
</tr>
<tr>
<td>$ts$</td>
<td>$[0.0025, 0.0045]$</td>
</tr>
<tr>
<td>$re$</td>
<td>$[4, 8]$</td>
</tr>
<tr>
<td>$h$</td>
<td>$[2, 30]$</td>
</tr>
<tr>
<td>$f$</td>
<td>$[0.1, 3]$</td>
</tr>
<tr>
<td>$hd$</td>
<td>$[7, 13]$</td>
</tr>
<tr>
<td>$T_{cm}$</td>
<td>$[9, 15]$</td>
</tr>
<tr>
<td>$T_{cd}$</td>
<td>$[6, 10]$</td>
</tr>
<tr>
<td>$T_{cdef}$</td>
<td>$[3, 7]$</td>
</tr>
<tr>
<td>$disc$</td>
<td>$[8, 14]$</td>
</tr>
<tr>
<td>$cb$</td>
<td>$[2, 3]$</td>
</tr>
<tr>
<td>$sc$</td>
<td>$[0.04, 0.06]$</td>
</tr>
<tr>
<td>$m$</td>
<td>$[0, 0.9]$</td>
</tr>
<tr>
<td>$ci$</td>
<td>$[1, 3]$</td>
</tr>
<tr>
<td>$Inv$</td>
<td>$[2800, 3900]$</td>
</tr>
<tr>
<td>$hm$</td>
<td>$[4, 8]$</td>
</tr>
<tr>
<td>$S$</td>
<td>$[2000, 3000]$</td>
</tr>
<tr>
<td>$p$</td>
<td>$[5000, 12000]$</td>
</tr>
<tr>
<td>$db$</td>
<td>$[4000, 5000]$</td>
</tr>
<tr>
<td>$scd$</td>
<td>$[10000, 18000]$</td>
</tr>
<tr>
<td>$tscr$</td>
<td>$[15500, 28900]$</td>
</tr>
<tr>
<td>$sto$</td>
<td>$[4000, 8500]$</td>
</tr>
<tr>
<td>$W$</td>
<td>$[1000000000, 4000000000]$</td>
</tr>
</tbody>
</table>

The parameter ranges of both algorithms along with their levels are shown in Table 3.

Table 3. Algorithm parameter ranges along with their levels.

<table>
<thead>
<tr>
<th>Single-objective algorithms</th>
<th>Algorithm parameter</th>
<th>Parameter range</th>
<th>Low(-1)</th>
<th>Medium (0)</th>
<th>High (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>nPop(A)</td>
<td>10-50</td>
<td>10</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Pc (B)</td>
<td>0.6-0.8</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Pm (C)</td>
<td>0.01-0.1</td>
<td>0.01</td>
<td>0.055</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Ngen (D)</td>
<td>5-15</td>
<td>5</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>SA</td>
<td>nPop (A)</td>
<td>1-11</td>
<td>1</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>IT (B)</td>
<td>31-59</td>
<td>31</td>
<td>45</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>Alpha (C)</td>
<td>0.8-0.9</td>
<td>0.8</td>
<td>0.85</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>TF (D)</td>
<td>0.1-0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

As mentioned in the previous section, the parameters of the algorithms is first tuned to obtain better solutions. Therefore, each algorithm is employed 30 times, each time their parameters change in their corresponding ranges. Then, the fitness function is recorded in each run. As an example, Table 4 contains the experimental results of employing GA.

Table 4. Experimental results of employing GA.

<table>
<thead>
<tr>
<th>No.</th>
<th>nPop</th>
<th>N</th>
<th>Pc</th>
<th>Pm</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>15</td>
<td>0.8</td>
<td>0.01</td>
<td>741571</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>5</td>
<td>0.6</td>
<td>0.1</td>
<td>729150</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>10</td>
<td>0.7</td>
<td>0.055</td>
<td>724177</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>10</td>
<td>0.7</td>
<td>0.055</td>
<td>730092</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
<td>10</td>
<td>0.7</td>
<td>0.055</td>
<td>725932</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>15</td>
<td>0.7</td>
<td>0.055</td>
<td>736817</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>10</td>
<td>0.6</td>
<td>0.055</td>
<td>719168</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>5</td>
<td>0.8</td>
<td>0.01</td>
<td>719685</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>15</td>
<td>0.6</td>
<td>0.1</td>
<td>714431</td>
</tr>
</tbody>
</table>
Table 4. Continued

<table>
<thead>
<tr>
<th>No.</th>
<th>nPop</th>
<th>N</th>
<th>Pc</th>
<th>Pm</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<td>0.055</td>
<td>737146</td>
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<td>15</td>
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<td>0.01</td>
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</tr>
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<td>15</td>
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<td>0.1</td>
<td>736120</td>
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<td>10</td>
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<td>0.055</td>
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<td>0.055</td>
<td>725094</td>
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<td>50</td>
<td>5</td>
<td>0.6</td>
<td>0.01</td>
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<td>0.055</td>
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<tr>
<td>22</td>
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<td>15</td>
<td>0.6</td>
<td>0.01</td>
<td>712495</td>
</tr>
<tr>
<td>23</td>
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<td>15</td>
<td>0.8</td>
<td>0.1</td>
<td>733302</td>
</tr>
<tr>
<td>24</td>
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<td>5</td>
<td>0.8</td>
<td>0.01</td>
<td>733738</td>
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<tr>
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<td>15</td>
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<td>0.01</td>
<td>729551</td>
</tr>
<tr>
<td>26</td>
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<td>0.6</td>
<td>0.1</td>
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</tr>
<tr>
<td>27</td>
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<td>0.8</td>
<td>0.1</td>
<td>729045</td>
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<tr>
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<td>0.8</td>
<td>0.1</td>
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</tr>
<tr>
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<td>30</td>
<td>10</td>
<td>0.7</td>
<td>0.055</td>
<td>712495</td>
</tr>
<tr>
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<td>5</td>
<td>0.8</td>
<td>0.1</td>
<td>740144</td>
</tr>
</tbody>
</table>

To investigate the relationship between the fitness value and the parameters, a regression analysis using the Design Expert software is then employed. The quadratic regression coefficients of the fitness function of the GA are given in Table 5.

Table 5. The Quadratic regression coefficients of the fitness function of the GA

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient(β)</th>
<th>Standard error</th>
<th>F-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>-</td>
<td>-</td>
<td>3.21</td>
<td>0.0161</td>
</tr>
<tr>
<td>Constant</td>
<td>7.283E+005</td>
<td>1895.34</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>nPop</td>
<td>-6524.19</td>
<td>1438.16</td>
<td>20.58</td>
<td>0.0004</td>
</tr>
<tr>
<td>N</td>
<td>-1943.33</td>
<td>1438.16</td>
<td>1.83</td>
<td>0.1966</td>
</tr>
<tr>
<td>Pc</td>
<td>3151.17</td>
<td>1438.16</td>
<td>4.80</td>
<td>0.0446</td>
</tr>
<tr>
<td>Pm</td>
<td>-602.20</td>
<td>1438.16</td>
<td>0.18</td>
<td>0.6813</td>
</tr>
<tr>
<td>nPop × N</td>
<td>-1909.55</td>
<td>1525.40</td>
<td>1.57</td>
<td>0.2298</td>
</tr>
<tr>
<td>nPop × Pc</td>
<td>2088.34</td>
<td>1525.40</td>
<td>1.87</td>
<td>0.1911</td>
</tr>
<tr>
<td>nPop × Pm</td>
<td>2521.94</td>
<td>1525.40</td>
<td>0.18</td>
<td>0.6813</td>
</tr>
<tr>
<td>N × Pc</td>
<td>634.84</td>
<td>1525.40</td>
<td>0.17</td>
<td>0.6832</td>
</tr>
<tr>
<td>N × Pm</td>
<td>-2431.72</td>
<td>1525.40</td>
<td>2.54</td>
<td>0.1318</td>
</tr>
<tr>
<td>Pc × Pm</td>
<td>1954.59</td>
<td>1525.40</td>
<td>1.64</td>
<td>0.2195</td>
</tr>
<tr>
<td>nPop × nPop</td>
<td>-3074.92</td>
<td>3790.68</td>
<td>0.66</td>
<td>0.4299</td>
</tr>
<tr>
<td>N × N</td>
<td>7805.23</td>
<td>3790.68</td>
<td>4.24</td>
<td>0.0573</td>
</tr>
<tr>
<td>Pc × Pm</td>
<td>-6028.67</td>
<td>3790.68</td>
<td>2.53</td>
<td>0.1326</td>
</tr>
<tr>
<td>Pm × Pm</td>
<td>3411.03</td>
<td>3790.68</td>
<td>0.81</td>
<td>0.3824</td>
</tr>
</tbody>
</table>

This relationship is obtained as:

\[ E(GA) = E(Cost)_{GA} = 728300 - 6524.18735 \times nPop - 1943.33149 \times IT + 3151.16663 \times Pc - 602.20327 \times Pm - 1909.54674 \times nPop \times IT + 2088.34096 \times nPop \times Pc + 2521.94017 \times nPop \times Pm + 634.84159 \times IT \times Pm - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times IT^2 - 6028.67381 \times Pc^2 + 3411.02944 \times Pm^2 \]

(28)
Afterwards, the regression function is optimized using the GAMS software to obtain the best values of the parameters in their corresponding ranges. The results of parameter setting process of GA and SA are presented in Table 6 and 7, respectively.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>nPop</th>
<th>N</th>
<th>Pc</th>
<th>Pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>50</td>
<td>6</td>
<td>0.6</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>nPop</th>
<th>Alpha</th>
<th>TF</th>
<th>IT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SA</td>
<td>4</td>
<td>0.8</td>
<td>0.3</td>
<td>59</td>
</tr>
</tbody>
</table>

In order to validate the results obtained using GA, not only a $t$-test is used, but also a multiple attribute decision-making (MADM) approach is employed. The $t$ test is employed using the SPSS software. The null hypothesis in this test is the equality of the means, i.e., $H_0: \mu_1=\mu_2$, where $\mu_1$ and $\mu_2$ are the response means. To do this, both algorithms are first implemented $n=30$ times, each time with their optimal parameter setting.

Then, the values of objective functions and the CPU time of problem instances are obtained. The means of the 30 runs for each of the 5 problem instances solved by GA and SA are provided in Tables 8 and 9, respectively.

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>Cost</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>114032.0426</td>
<td>1.5871297</td>
</tr>
<tr>
<td>2</td>
<td>548930.3058</td>
<td>15.8196912</td>
</tr>
<tr>
<td>3</td>
<td>722018.301</td>
<td>35.924424</td>
</tr>
<tr>
<td>4</td>
<td>930298.327</td>
<td>48.297758</td>
</tr>
<tr>
<td>5</td>
<td>1642840.303</td>
<td>207.424508</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>Objective function</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>106836.7178</td>
<td>1.059103</td>
</tr>
<tr>
<td>2</td>
<td>532068.1821</td>
<td>9.838433</td>
</tr>
<tr>
<td>3</td>
<td>717157.165</td>
<td>22.81341</td>
</tr>
<tr>
<td>4</td>
<td>922899.1441</td>
<td>48.1613</td>
</tr>
<tr>
<td>5</td>
<td>1611441.6</td>
<td>222.3112</td>
</tr>
</tbody>
</table>

Based on 95% confidence level, i.e., $a=0.05$, the results of the $t$-test on the means of fitness function and CPU time obtained using the two algorithms on the first problem are shown in Table 10.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
<th>95% Confidence Interval of the Difference</th>
<th>t</th>
<th>df</th>
<th>Sig (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective function</td>
<td>-7399.18286</td>
<td>11484.72311</td>
<td>2096.81397</td>
<td>-11687.64895 to -3110.71677</td>
<td>-3.529</td>
<td>29</td>
<td>0.001</td>
</tr>
<tr>
<td>CPU Time</td>
<td>-0.13645</td>
<td>13.85549</td>
<td>2.52965</td>
<td>-5.31018 to 5.03727</td>
<td>-0.054</td>
<td>29</td>
<td>0.957</td>
</tr>
</tbody>
</table>

As seen in this table, there are significant differences between the fitness function means of the two algorithms, since the related value of significance (two-tailed) are less than 0.05.

The values of significance (two-tailed) of the measure comparisons in all problem instances are shown in Table 11.
Table 11. The p-values of the t-tests on the equality of the means of the measures in all problem instances

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>Objective function</th>
<th>CPU Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
<td>0.957</td>
</tr>
<tr>
<td>5</td>
<td>0.000</td>
<td>0.108</td>
</tr>
</tbody>
</table>

As seen in this table, there are significant differences between the objective function means of the two algorithms obtained in all problem instances and there are significant differences between the CPU time means of the two algorithms obtained in fourth and fifth problem instances.

To select the better algorithm, the SAW method (Hwang and Yoon, 1981), as one of the multiple attribute decision-making approach, is utilized here. As the SAW method starts with normalizing a decision matrix based on the linear method, a decision matrix $D = [d_{ij}]_{m \times n}$ is first organized for $m$ (here 2) alternatives and $n$ indices (here 2) based on the desirability of alternative $i$ towards index $j$ denoted by $d_{ij}$. In the next step, an un-scaled weight matrix is made the weights obtained of Entropy method. The weights are shown in Table 12.

Table 12. The weight vector of the means of the two measures in all problem instances

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>Objective function</th>
<th>CPU TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02579788</td>
<td>0.9742021</td>
</tr>
<tr>
<td>2</td>
<td>0.004417</td>
<td>0.99558296</td>
</tr>
<tr>
<td>3</td>
<td>0.00022701</td>
<td>0.999773</td>
</tr>
<tr>
<td>4</td>
<td>0.8884662</td>
<td>0.111533849</td>
</tr>
<tr>
<td>5</td>
<td>0.071977388</td>
<td>0.928023</td>
</tr>
</tbody>
</table>

Then, the total sum of each row is computed. At the end, an algorithm with the largest total sum of weights (TSW) is selected for each of the problem instances. Table 13 shows TSWs of each algorithm in all problem instances.

Table 13. Comparison results using the SAW method

<table>
<thead>
<tr>
<th>Problem No.</th>
<th>GA</th>
<th>SA</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.674262303</td>
<td>1</td>
<td>SA</td>
</tr>
<tr>
<td>2</td>
<td>0.623444931</td>
<td>1</td>
<td>SA</td>
</tr>
<tr>
<td>3</td>
<td>0.635120081</td>
<td>1</td>
<td>SA</td>
</tr>
<tr>
<td>4</td>
<td>0.992618417</td>
<td>1</td>
<td>SA</td>
</tr>
<tr>
<td>5</td>
<td>0.998624336</td>
<td>0.937856412</td>
<td>GA</td>
</tr>
</tbody>
</table>

As seen in this table, SA is the better algorithm for problem instances with small and medium sizes, while GA works satisfactorily. In short, based on the results obtained using the t-test and the SAW method, it can be concluded that both algorithms work satisfactorily to optimize the SCN problem at hand.

Both algorithms are coded and implemented in MATLAB 8.20 and run on a computer with core (TM) i3-CPU 2.40 GHz, RAM 4.00 GB.

7. Conclusion

In this paper, we have proposed a new mathematical model for a three-echelon defective goods supply chain network (DGSCN) that not only minimizes the costs of production, holding, transport, defective goods, and scrap products, but also determines the economic production quantity (EPQ) and the appropriate cycle length. The GA and SA methods were used to solve the proposed mathematical model. To model the real states accurately, the values of the parameters were treated as probabilities with uniform distributions.

Five problem instances of small, medium, and large were used to demonstrate the application of the proposed methodology as well as to compare the performances in terms of the means of two performance measures. After the parameter calibration process of both algorithms based on a statistical method using 30 samples, the algorithms were employed to solve the problem instances, each 30 times. The results of t-test and the SAW method at the end showed SA acted better than GA for most of problem instances.

For future researches in this area, we recommend the followings:
In addition to the storage capacity limitation, we may consider other constraints too.

Other meta-heuristic search algorithms may also be employed and a comparison may be made among the algorithms.

We can consider backorder or lost sale for shortages.

Some parameter of the model may be either fuzzy or random variable. This being the case, the model has either fuzzy or stochastic nature.

Researchers could apply other mutation and crossover operators.

For the further remarks, discounts or inflation for system costs can be taken into account.

A distance parameter between the layers as well as travel times to transport the products to customer nodes can appropriately be considered.

The SCN using the De-Novo programming approach can also be designed.

References


The Optimization of a Multi-Product Three-Echelon Supply Chain


The Optimization of a Multi-Product Three-Echelon Supply Chain


