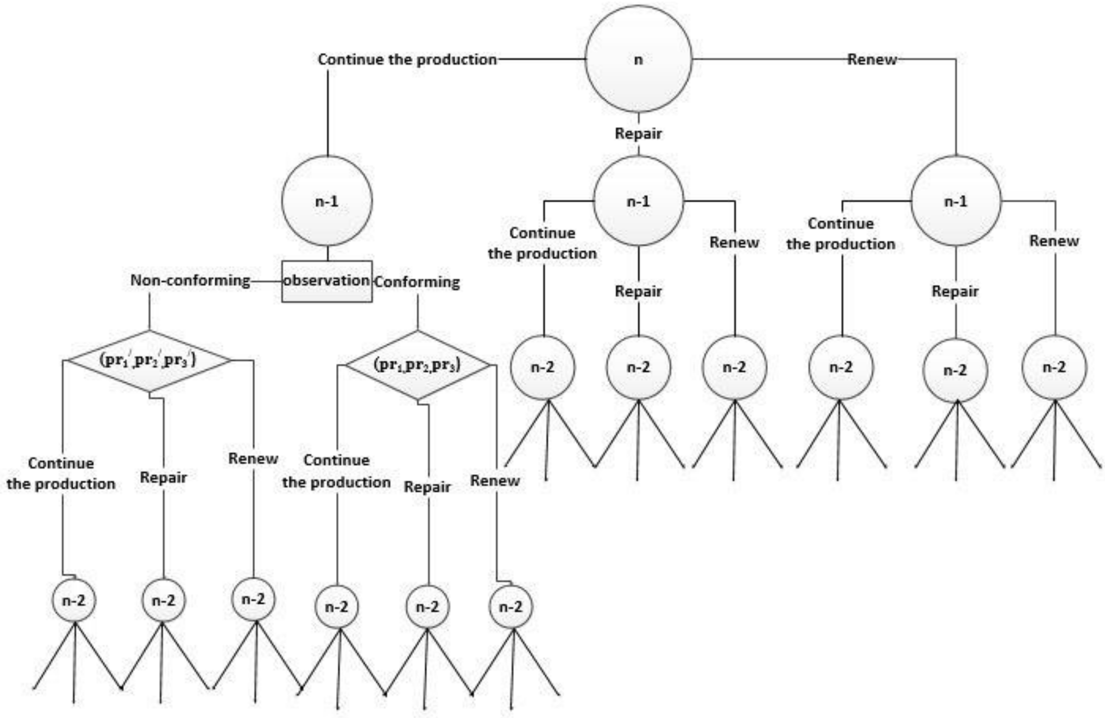

Figure1. Decision tree that includes the decisions of renew, repair or do nothing and continue

Based on the statistical quality control techniques and partially observable Markov decision processes (POMDP), it is proposed that the probability of producing a defective product is determined based on the machine state. The statistical quality control is used to determine the probability distribution of defective items. If the machine is in bad state, the defective observation distribution follows Bernoulli distribution with parameter p_1 and if the machine is in medium state, the defective observation distribution follows Bernoulli distribution with parameter p_2. If the machine is in good state, the defective observation distribution follows Bernoulli distribution with parameter p_3. This assumption is shown in Figure 2.

To illustrate the model, some assumptions should be considered. It is assumed that the machine can be placed in three states: good, Medium and bad. The backward dynamic programming is used; π (the probability that the machine is in bad state) is considered as state variable and the number of the programming periods equals stage variable; and programming is done in finite time horizon.

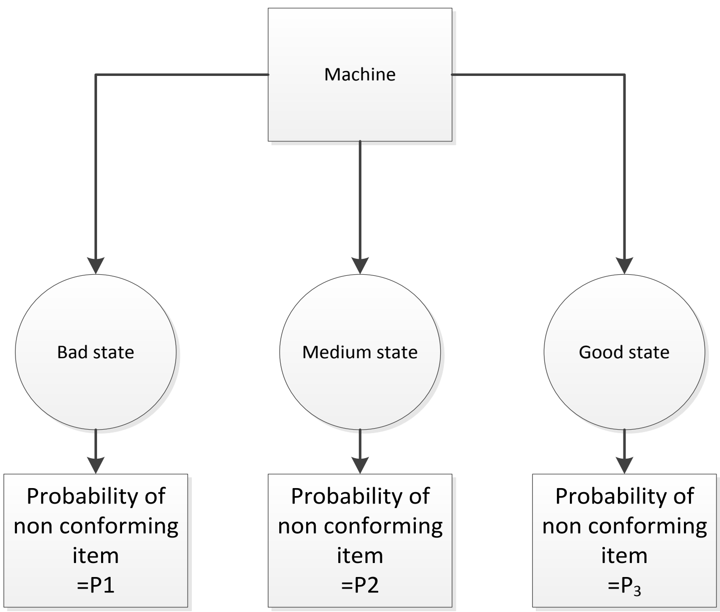

Figure 2. Determining the parameters of the Bernoulli distribution for producing one item

Parameters and formulas of the model are illustrated in the following; some of them are obtained using the bayesian inference method.

3. Notations

The notations required to model the problem at hand are given as:

π: probability of machine state (machine states includes bad, medium, good state)
Pr: Posterior probability of machine state when a conforming item is produced.
$Pr^{'}$: Posterior probability of machine state when a non-conforming item is produced.
π_1: Probabilty that the machine is in bad state; $\{s_t=0\}$.
π_2: Probability that the machine is in medium state; $\{s_t=1\}$.
π_3: Probability that the machine is in good state; $\{s_t=2\}$.
α: Discount factor; $\alpha \in [0,1]$
p_1: Probability that the observation is defective if the machine is in the bad state.
p_2: Probability that the observation is defective if the machine is in the medium state.
P_3: Probability that the observation is defective if the machine is in the good state.
z: Probability that the observation is defective.
L: The defective observation.
pr_1: The posterior probability that the machine is in the bad state when a conforming item is produced.
pr_2: The posterior probability that the machine is in the medium state when a conforming item is produced.
pr_3: The posterior probability that the machine is in the good state when a conforming item is produced.
$pr_1^{'}$: The posterior probability that the machine is in the bad state when a non-conforming item is produced.
$pr_2^{'}$: The posterior probability that the machine is in the medium state when a non-conforming item is produced.
$pr_3^{'}$: The posterior probability that the machine is in the good state when a non-conforming item is produced.
π_0: Probability of the machine state for new machine.
π_{01}: Probabilty that the machine is in the bad state after the machine is renewed.
π_{02}: Probabilty that the machine is in the medium state after the machine is renewed.
π_{03}: Probabilty that the machine is in the bad state after the machine is renewed.
n: The number of remained stages (the stage variable).
T: The coefficient for the cost of repair decision in different states.
T_1: The coefficient for the cost of repair decision when the machine is in the bad state.
T_2: The coefficient for the cost of repair decision when the machine is in the medium state.
T_3: The coefficient for the cost of repair decision when the machine is in the good state.
π_{11}: Probability that the machine is in the bad state after the machine is repaired.
π_{12}: Probability that the machine is in the medium state after the machine is repaired.
π_{13}: Probability that the machine is in the good state after the machine is repaired.
R: The fixed cost for renew decision
A: Profit of a conforming item.
C: Cost of one non-conforming item.
M: The coefficient for the salvage value of machine (the value of the machine when no stage is remaining and the process terminates.)
M_1: The coefficient for the salvage value when the machine is in the bad state.
M_2: The coefficient for the salvage value when the machine is in the medium state.
M_3: The coefficient for the salvage value when the machine is in the good state.
$V_0(\pi)$: The salvage value of the machine (the value of the machine when no stage is remaining and the process terminates).

The optimality equation is illustrated as following:

$$V_n(\pi) = V_n(\pi_1, \pi_2, \pi_3) = \min\{\mathrm{R} + \alpha \, \mathrm{V}_{n-1}(\pi_{01}, \pi_{02}, \pi_{03}), \\ T_1\pi_1 + T_2\pi_2 + T_3\pi_3 + \alpha V_{n-1}(\pi_{11}, \pi_{12}, \pi_{13}), ZC - (1-Z)A + \alpha V_{n-1}(pr_1, pr_2, pr_3)\,\mathrm{Z} \\ + \alpha \, \mathrm{V}_{n-1}(pr_1^{'}, pr_2^{'}, pr_3^{'})(1-Z)\} \tag{1}$$

where

$$\pi = (\pi_1, \pi_2, \pi_3) \tag{2}$$

$$\pi_0 = (\pi_{01}, \pi_{02}, \pi_{03}) \tag{3}$$

$$M = (M_1, M_2, M_3) \tag{4}$$

$$V_0(\pi) = M\pi \tag{5}$$

$$T = (T_1, T_2, T_3) \tag{6}$$

(7)

$$pr = (pr_1, pr_2, pr_3)$$

(8)

$$pr' = (pr_1', pr_2', pr_3') \tag{9}$$

(10)

$$\pi_1 = (\pi_{11}, \pi_{12}, \pi_{13})$$

$$Z = \mathrm{P}(L \mid s_t = 0)\mathrm{P}(s_t = 0) + \mathrm{P}(L \mid s_t = 1)\mathrm{P}(s_t = 1) + \mathrm{P}(L \mid s_t = 2)\mathrm{P}(s_t = 2)$$
$$= \pi_1 p_1 + \pi_2 p_2 + \pi_3 p_3 = \pi_1 p_1 + \pi_2 p_2 + (1 - \pi_1 - \pi_2) p_3$$

$$pr_1 = \mathrm{P}(s_t = 0 \mid L) = \frac{\mathrm{P}(L \mid s_t = 0)\mathrm{P}(s_t = 0)}{\mathrm{P}(L)} = \frac{p_1 \pi_1}{Z} \tag{11}$$

$$pr_2 = \mathrm{P}(s_t = 1 \mid L) = \frac{\mathrm{P}(L \mid s_t = 1)\mathrm{P}(s_t = 1)}{\mathrm{P}(L)} = \frac{p_2 \pi_2}{Z} \tag{12}$$

$$pr_3 = \mathrm{P}(s_t = 2 \mid L) = \frac{\mathrm{P}(L \mid s_t = 2)\mathrm{P}(s_t = 2)}{\mathrm{P}(L)} = \frac{p_3 \pi_3}{Z} \tag{13}$$

$$Pr_1' = P(s_t = 0 \mid L^c) = \frac{P(L^c \mid s_t = 0)P(s_t = 0)}{P(L^c)} = \frac{(1 - p_1)\pi_1}{1 - Z} \tag{14}$$

$$Pr_2' = P(s_t = 1 \mid L^c) = \frac{P(L^c \mid s_t = 1)P(s_t = 1)}{P(L^c)} = \frac{(1 - p_2)\pi_2}{1 - Z} \tag{15}$$

$$Pr_3' = P(s_t = 2 \mid L^c) = \frac{P(L^c \mid s_t = 2)P(s_t = 2)}{P(L^c)} = \frac{(1 - p_3)\pi_3}{1 - Z} \tag{16}$$

The condition-based maintenance (CBM) and sequential sampling plan are used to illustrate the model proposed. CBM is used so that the point is placed in continue sampling area then the decisions of repairing the machine or continuing sampling can be chosen until the point is placed in rejection area and decisions of the renew are selected; if the point is placed in accept area then the decisions of do-nothing is selected. Figure. 3 clearly shows sequential sampling method in machine replacement problem.

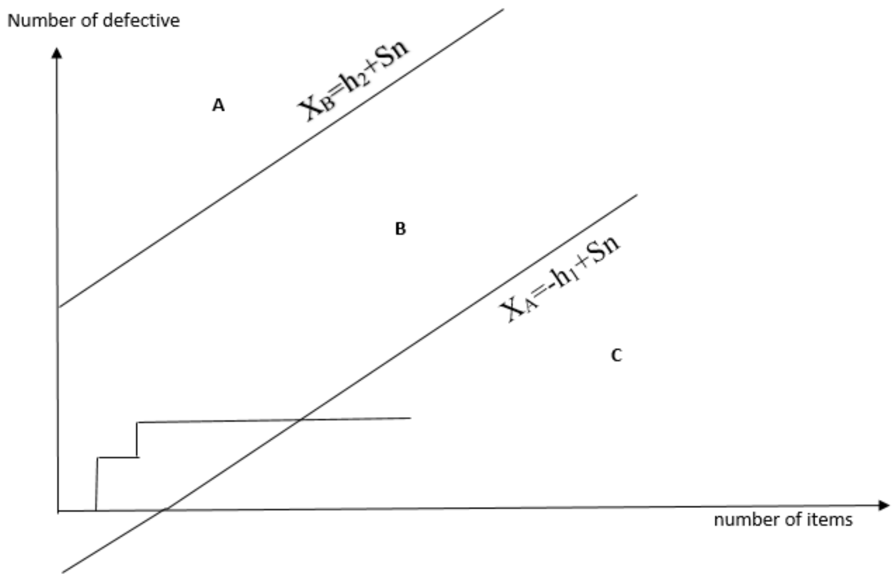

Figure 3. Sequential sampling plan for machine replacement problem

- A: Renew machine.
- B: Repair machine
- C: Continue the production without any maintenance action.

4. Numerical example

A numerical example is solved for illustrating the application of proposed methodology. Input data of the problem is as following:

$$\begin{pmatrix} R=30, \alpha=0.95, \pi_{01}=0.03, \\ \pi_{02}=0.27, \pi_{03}=0.7, \\ T_1=15, T_2=10, T_3=8, \\ \pi_{11}=0.2, \pi_{12}=0.3, \pi_{13}=0.5 \\ p_1=0.8, p_2=0.1, p_3=0.1, \\ C=15, A=5, M_1=2, M_2=6 \\ M_3=8 \end{pmatrix}$$

Assumptions and equations used in this model are simulated by MATLAB software.
For example, if n=5; decision making stages are available then the results for different values of state variable (π_1, π_2, π_3) are reported in Table 1.

Table 1. Total expected costs for each (π_1, π_2, π_3) and its optimal decision

(π_1, π_2, π_3)	**Cost(Renew)**	**Cost(Repair)**	**Cost(Continue the production)**	$V_n(\pi_1, \pi_2, \pi_3)$	**Decision**
(0,0,1)	26.54022	11.35199	-7.3829	-7.3829	Continue the production
(0,0.1,0.9)	26.54022	11.55199	-7.53765	-7.53765	Continue the production
(0,0.2,0.8)	26.54022	11.75199	-7.69241	-7.69241	Continue the production
(0,0.3,0.7)	26.54022	11.95199	-7.84716	-7.84716	Continue the production
(0,0.4,0.6)	26.54022	12.15199	-8.00192	-8.00192	Continue the production
(0,0.5,0.5)	26.54022	12.35199	-8.15668	-8.15668	Continue the production
(0,0.6,0.4)	26.54022	12.55199	-8.31143	-8.31143	Continue the production
(0,0.7,0.3)	26.54022	12.75199	-8.46619	-8.46619	Continue the production
(0,0.8,0.2)	26.54022	12.95199	-8.62095	-8.62095	Continue the production
(0,0.9,0.1)	26.54022	13.15199	-8.7757	-8.7757	Continue the production
(0,1,0)	26.54022	13.35199	-8.93046	-8.93046	Continue the production
(0.1,0,0.9)	26.54022	12.05199	-2.15346	-2.15346	Continue the production
(0.1,0.1,0.8)	26.54022	12.25199	-2.30486	-2.30486	Continue the production
(0.1,0.2,0.7)	26.54022	12.45199	-2.45627	-2.45627	Continue the production
(0.1,0.3,0.6)	26.54022	12.65199	-2.60767	-2.60767	Continue the production
(0.1,0.4,0.5)	26.54022	12.85199	-2.75907	-2.75907	Continue the production
(0.1,0.5,0.4)	26.54022	13.05199	-2.91048	-2.91048	Continue the production
(0.1,0.6,0.3)	26.54022	13.25199	-3.06188	-3.06188	Continue the production
(0.1,0.7,0.2)	26.54022	13.45199	-3.21328	-3.21328	Continue the production
(0,1,0.8,0.1)	26.54022	13.65199	-3.36469	-3.36469	Continue the production
(0.1,0.9,0)	26.54022	13.85199	-3.51609	-3.51609	Continue the production

Table 1. Continued

(π_1, π_2, π_3)	Cost(Renew)	Cost(Repair)	Cost(Continue the production)	$V_n(\pi_1, \pi_2, \pi_3)$	Decision
(0.2,0,0.8)	26.54022	12.75199	2.172761	2.172761	Continue the production
(0.2,0.1,0.7)	26.54022	12.95199	2.05248	2.05248	Continue the production
(0.2,0.2,0.6)	26.54022	13.15199	1.9322	1.9322	Continue the production
(0.2,0.3,0.5)	26.54022	13.35199	1.811919	1.811919	Continue the production
(0.2,0.4,0.4)	26.54022	13.55199	1.691638	1.691638	Continue the production
(0.2,0.5,0.3)	26.54022	13.75199	1.571358	1.571358	Continue the production
(0.2,0.6,0.2)	26.54022	13.95199	1.451077	1.451077	Continue the production
(0.2,0.7,0.1)	26.54022	14.15199	1.330797	1.330797	Continue the production
(0.2,0.8,0)	26.54022	14.35199	1.210516	1.210516	Continue the production
(0.3,0,0.7)	26.54022	13.45199	6.108511	6.108511	Continue the production
(0.3,0.1,0.6)	26.54022	13.65199	5.991166	5.991166	Continue the production
(0.3,0.2,0.5)	26.54022	13.85199	5.873822	5.873822	Continue the production
(0.3,0.3,0.4)	26.54022	14.05199	5.756477	5.756477	Continue the production
(0.3,0.4,0.3)	26.54022	14.25199	5.639133	5.639133	Continue the production
(0.3,0.5,0.2)	26.54022	14.45199	5.521788	5.521788	Continue the production
(0.3,0.6,0.1)	26.54022	14.65199	5.404444	5.404444	Continue the production
(0.4,0,0.6)	26.54022	14.15199	10.03251	10.03251	Continue the production
(0.4,0.1,0.5)	26.54022	14.35199	9.915163	9.915163	Continue the production
(0.4,0.2,0.4)	26.54022	14.55199	9.797818	9.797818	Continue the production
(0.4,0.3,0.3)	26.54022	14.75199	9.680474	9.680474	Continue the production
(0.4,0.4,0.2)	26.54022	14.95199	9.563129	9.563129	Continue the production

Table 1. Continued

(π_1, π_2, π_3)	**Cost(Renew)**	**Cost(Repair)**	**Cost(Continue the production)**	**$V_n(\pi_1, \pi_2, \pi_3)$**	**Decision**
(0.4,0.5,0.1)	26.54022	15.15199	9.445785	9.445785	Continue the production
(0.5,0,0.5)	26.54022	14.85199	13.71193	13.71193	Continue the production
(0.5,0.1,0.4)	26.54022	15.05199	13.62182	13.62182	Continue the production
(0.5,0.2,0.3)	26.54022	15.25199	13.53171	13.53171	Continue the production
(0.5,0.3,0.2)	26.54022	15.45199	13.44161	13.44161	Continue the production
(0.5,0.4,0.1)	26.54022	15.65199	13.3515	13.3515	Continue the production
(0.5,0.5,0)	26.54022	15.85199	13.26139	13.26139	Continue the production
(0.6,0,0.4)	**26.54022**	**15.55199**	**17.3677**	**15.55199**	**Repair**
(0.6,0.1,0.3)	26.54022	15.75199	17.27759	15.75199	Repair
(0.6,0.2,0.2)	26.54022	15.95199	17.18748	15.95199	Repair
(0.6,0.3,0.1)	26.54022	16.15199	17.09738	16.15199	Repair
(0.7,0,0.3)	26.54022	16.25199	21.02347	16.25199	Repair
(0.7,0.1,0.2)	26.54022	16.45199	20.93336	16.45199	Repair
(0.7,0.2,0.1)	26.54022	16.65199	20.84325	16.65199	Repair
(0.8,0,0.2)	26.54022	16.95199	24.67924	16.95199	Repair
(0.8,0.1,0.1)	26.54022	17.15199	24.58913	17.15199	Repair
(0.9,0,0.1)	26.54022	17.65199	27.46757	17.65199	Repair
(1,0,0)	26.54022	18.35199	29.53257	18.35199	Repair

As can be seen, the optimal policy is in the form of a control threshold policy. When (π_1^*=0.6, π_2^*=0, π_3^*=0.4), then optimal decision is changed from continue-the-production decision to the repair decision; Fig 4 clearly shows this issue. According to input data, the renew decision is not recommended. By changing input data, the renew decision is applied which is further illustrated in the subsequent section.

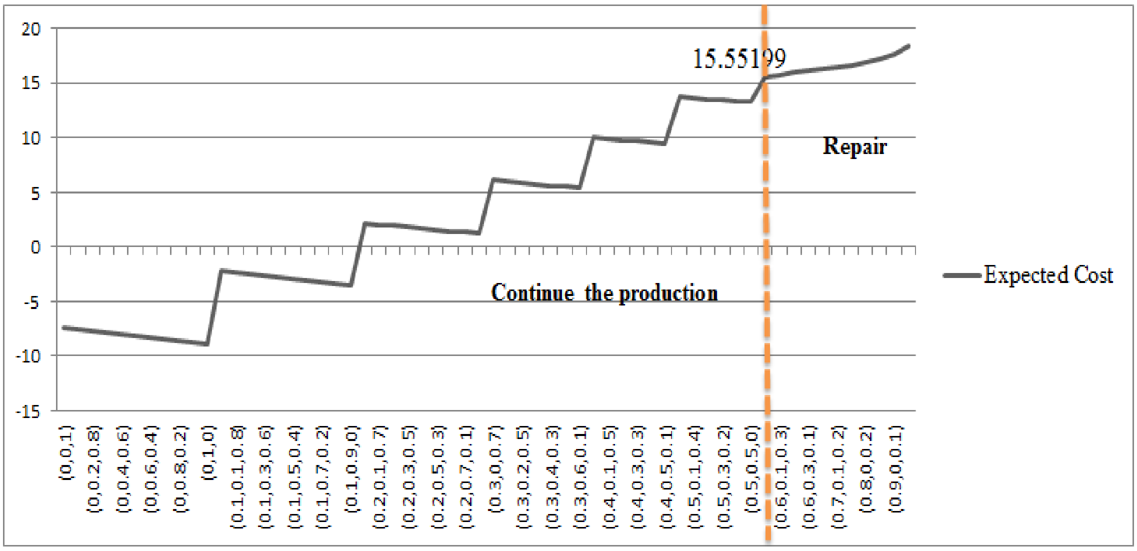

Figure 4. Diagram of the expected costs for each (π_1, π_2, π_3)

5. Sensitivity Analysis

A sensitivity analysis is used to analyze the effects of changing parameters on the optimal solution. In each case of sensitivity analysis, one parameter of the model is altered. It is necessary to adjust the parameter value in a level so that one can easily interpret its behavior. The decision numbers for decisions of Renew, Repair and continue the production are 1, 2 and 3 respectively. For example, 3→2 means that the optimal decisions change from continue- the production-decision to repair decision based on the cost objective function. Also 3→1 means that the optimal decisions changed from continue-the production-decision to renew decisions in the optimal threshold policy. The results are shown in Table 2.

Table 2 shows that the optimal threshold changes by changing parameters of the model. For example, by increasing R and C, the repair decision area decreases. In other words, the optimal threshold shifts to the right, as shown more clearly in Fig 5 and Fig 6. The result of sensitivity analysis for parametes π_{01}, π_{02}, π_{03}, M_1, M_2, M_3 shows that changing of these parameters does not affect optimal threshold. In addition, changing parameters p_2, p_3 does not follow a regular pattern.

Table 2. The results of sensitivity analysis for the proposed sampling plan

Parameters	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**
R	0	(0.2,0,0.8) 3→2	10	(0.5,0,0.5) 3→2	40	**(0.6,0,0.4) 3→2**
α	0.8	(0.7,0,0.3) 3→2	0.9	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
p_1	0	(1,0,0) 3	0.5	(0.6,0,0.4) 3→2	1	**(0.5,0,0.5) 3→2 (0.5,0.3,0.2) 2→3 (0.6,0,0.4) 3→2**
P_2	0	(0.5,0,0.5) 3→2 (0.5,0.3,0.2) 2→3 (0.6,0,0.4) 3→2	0.5	irregular	1	**Irregular**

Table 2. Continued

Parameters	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**	**Changed value**	**(π_1^*, π_2^*, π_3^*)**
P_3	0	(0.5,0.3,0.2) 3→2	0.5	irregular	1	**Irregular**
T_1	0	(0.5,0,0.5) 3→2	20	(0.6,0,0.4) 3→2	40	**(0.7,0,0.3) 3→1**
T_2	0	(0.5,0.2,0.3) 3→2	20	(0.6,0,0.4) 3→2 (0.6,0.2,0.2) 2→3 (0.7,0,0.3) 3→2	40	**(0.8,0,0.2) 3→2**
T_3	0	(0.5,0,0.5) 3→2 (0.5,0.3,0.2) 2→3 (0.6,0,0.4) 3→2	20	(0.7,0,0.3) 3→2	40	**(0.7,0.2,0.1) 3→2**
π_{01}	0	(0.6,0,0.4) 3→2	0.5	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
π_{02}	0	(0.6,0,0.4) 3→2	0.5	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
π_{03}	0	(0.6,0,0.4) 3→2	0.5	(0.6,0,0.4) 3→2	1	**(0.6,0,0.4) 3→2**
π_{11}	0	(0.3,0,0.7) 3→2 (0.3,0.2,0.5) 2→3 (0.4,0,0.6) 3→2	0.5	(0.7,0,0.3) 3→1	1	**(0.7,0,0.3) 3→1**
π_{12}	0	(0.5,0,0.5) 3→1	0.5	(0.7,0,0.3) 3→1	1	**(0.7,0,0.3) 3→1**
π_{13}	0	(0.5,0,0.5) 3→1	0.5	(0.6,0,0.4) 3→1	1	**(0.7,0,0.3) 3→1**
A	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**
C	0	(1,0,0) 3	10	(0.7,0,0.3) 3→2	20	**(0.5,0,0.5) 3→2**
M_1	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**
M_2	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**
M_3	0	(0.6,0,0.4) 3→2	10	(0.6,0,0.4) 3→2	20	**(0.6,0,0.4) 3→2**

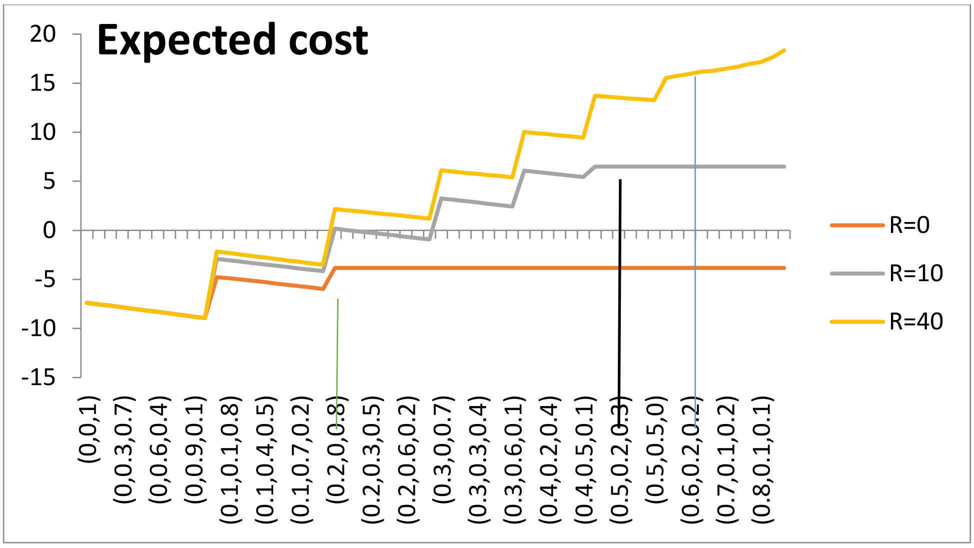

Figure 5. The results of sensitivity analysis for parameters R

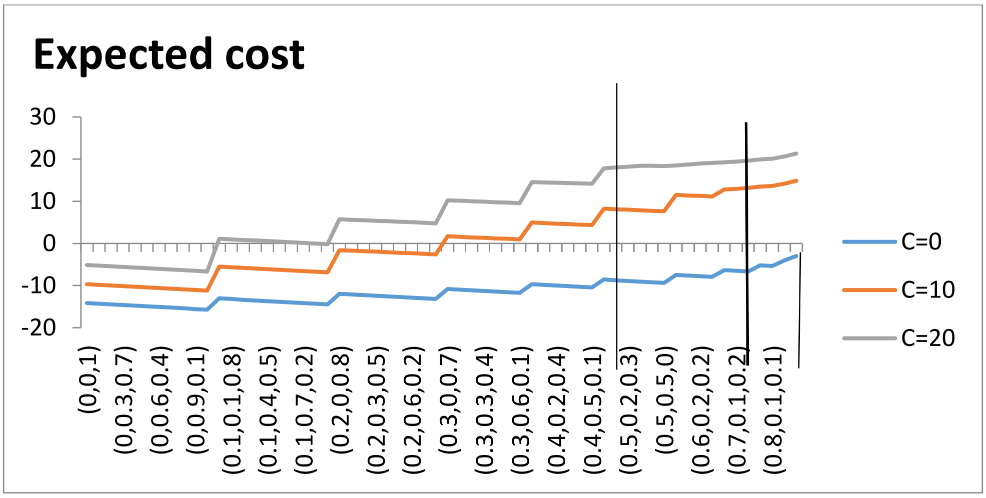

Figure 6. The results of sensitivity analysis for parameters C

6. Conclusion

In this article, we presented a backward dynamic programming model for three-state machine replacement problems in a finite time horizon in order to determine a control threshold policy using POMDP technique and sequential sampling plan. This model is applied for optimizing expected cost in machine replacement problem based on the methods of sequential sampling and Bayesian inferences. A decision tree is implemented to determine which decision can be chosen; if each decision is chosen the related cost is applied. A cost objective function including the costs of replacement and repair, and the cost of defectives. The presented model can be used in the production departments in which machine deterioration is monitored using the quality of produced items. In this paper, the medium state of machine is considered and the results show that the proposed model presents an exact and optimal maintenance policy and develops prior researches. In addition, the sensitivity analysis demonstrates that changing the input data significantly influences the optimal solution.

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