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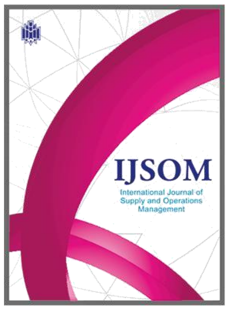

On Solutions of Possibilistic Multi- objective Quadratic Programming Problems

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Abstract
In this paper, a multi- objective quadratic programming (Poss- MOQP) problem with possibilistic variables coefficients matrix in the objective functions is studied. Through the use of $\beta-$level sets the Poss- MOQP problem is converted into the corresponding deterministic multi- objective quadratic programming ($\beta-$ MOQP) problem and hence into the single parametric quadratic programming problem using the weighting method. An extended $\beta-$ possibly efficient solution is specified. A necessary and sufficient condition for finding such a solution is established. A relationship between the solutions of possibilistic levels is constructed. Numerical example is given to clarify the obtained results.

Keywords: Multi- objective quadratic programming; Possibilistic variables; Possibilistic efficient solution; $\beta-$ Level set; $\beta-$ Possibly efficient solution; $\beta-$Possibly optimal solution.

1. Introduction

In many scientific areas, such as system analysis and operations research, a model has to be set up based on data which are only approximately known. Fuzzy Sets Theory, introduced by Zadeh (1965), makes it possible to incorporate the unknown and approximate data to the mathematical models . Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers by the use of a fuzzification principle. Luhandjula (1987) introduced the multi- objective programming problems with possibilistic data. Hussein (1992) presented the vector optimization problems with possibilistic weights. Sakawa and Yano (1989) introduced the concept of $\alpha-$Pareto optimality of fuzzy parametric program. Kassem (1998) studied the nonlinear programming problems with possibilistic variables in the objective functions parameters and introduced the parametric study corresponding to the resulted $\alpha-$ possibly optimal solution. Tanaka et al. (2000) proposed algorithms to solve linear programming problems for obtaining center vectors and distribution matrices in sequence based on different kinds of possibility distributions of fuzzy variables.

Quadratic optimization is considered as one of the most important areas of non- linear programming. Numerous problems in real world applications, including problems in planning, scheduling, economics, engineering design and control etc…, are naturally expressed as quadratic problems .The quadratic problem is the most interesting class of the optimization problems, and it is known as an NP-hard problem. There are several method and algorithms for solving the quadratic programming problem introduced by Pardalos and Rosen (1987) and Horst and Tuy (1993). Ammar (2008) studied multiobjective quadratic programming problem with fuzzy random coefficients matrix in the objective functions and a decision vector as fuzzy variables. Jana et al. (2009) added an entropy objective function to the multiobjective portfolio selection model that is to generate a well-diversified asset portfolio based on the possibilities mean value and variance of constraints distribution. Inuiguchi and Ramik (2000) introduced a review on some fuzzy linear programming methods and techniques in practical from your point of view. Khalifa and ZeinEldein (2014) presented portfolio selection problem as multi- objective quadratic- linear programming problem with fuzzy objective functions coefficients and applied fuzzy programming approach for solving the problem. Ammar and Khalifa (2003) introduced fuzzy portfolio selection problem as quadratic programming approach. Ammar and Khalifa (2015) introduced portfolio selection problem as quadratic

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programming problem with inexact rough interval in the objective function and constraints. Kheirfam (2011) used a fuzzy ranking and arithmetic operations to transform the quadratic programming problem with fuzzy numbers in the coefficients and variables into the corresponding deterministic one and solved it to obtain a fuzzy optimal solution. Based on Zadeh (1978), he transformed the fuzzy quadratic programming problem into a pair of two- level mathematical programs, regarding the extension principle. Khalifa (2016) proposed an interactive approach for solving multi- objective nonlinear programming problem. The approach is based on the Reference Direction (RD) introduced by Narula et al. (1993) and the Attainable Reference Point (ARP) method introduced by Wang et al. (2001). Canestrelli et al. (1996) studied possibilistic quadratic programming.

This paper deals with a multi- objective quadratic programming (MOQP) problem with possibilistic variables coefficients matrix in the objective functions. The (Poss- MOQP) is formulated by incorporating possibilistic data in the objective functions' coefficients. Through the use of the α - level sets, the considered problem is converted into the corresponding deterministic quadratic problem and hence the concept of α - efficient solution is introduced instead of efficient solution. A necessary and sufficient condition for such solution is established and hence the relationship between solutions of possibilistic levels is constructed.

The paper is divided to the following 6 sections: In section2, some preliminaries needed for the paper are introduced. In section3, the possibilistic multiobjective quadratic programming problem and its specific definition and properties is introduced. In section4, the $\beta-$ possibly efficient solution of the Poss- MOQP problem will be characterized. In section 5, a numerical example is given for illustration. Finally, some concluding remarks are reported in section6.

2. Preliminaries

In this section, the possibilistic variable, its $\beta-$ level set, support, and convexity definitions which are necessary in this paper are introduced (see, Kassem (1998), and Hussein (1998)).

Definition1. A possibilistic variable $u \in V$ is a variable characterized by a possibility distribution $\varphi_u(v)$, which means that, a possibility distribution φ_v associated with v may be viewed as fuzzy constraints on the values which may be assigned to v. Such distribution is characterized by a possibility distribution function $\varphi_u : V \rightarrow [0, 1]$ which is associated with each $u \in V$, the degree of compatibility of the variable u with the realization $v \in V$.
If V is Cartesian product of $V_1, V_2, \ldots, V_n$, then $\varphi_u(v_1, v_2, \ldots, v_n)$ is an $n-$ ary possibility distribution, i.e., $\varphi_u(v) = (\varphi_{u_1}(v_1), \varphi_{u_2}(v_2), \ldots, \varphi_{u_n}(v_n))$.

Definition2. The $\beta-$ level set of a possibilistic variable $u \in V$ is defined as follows:
$u_\beta = \{u \in V : \varphi_u(v) \geq \beta\}$.

Definition3. A possibility distribution φ_u on V is called convex if
$$\varphi_u(wv^1 + (1-w)v^2) \geq \min(\varphi_u(v^1), \varphi_u(v^2)); \forall v^1, v^2 \in V, 0 \leq w \leq 1.$$

Definition4. The support of a possibilistic variable u is denoted by $Supp(u)$ and is defined as:
$$Supp(u) = \left\{ v \in V : \sup_{t \in N_\varepsilon(v)} (\varphi_u(t) > 0; \forall \varepsilon > 0 \right\}, \text{ where } N_\varepsilon(v) = \{t \in V : \|t - v\| < \varepsilon\}.$$

3. Problem definition and solution concept

Consider the following possibilistic multi- objective quadratic programming problem:

(Poss- MOQP) $$Min\, Z^{(k)}(x, \tilde{C}^{(k)}, \tilde{Q}^{(k)}) = \left(\tilde{C}^{(k)}\right)^T x + \frac{1}{2} x^T \tilde{Q}^{(k)} x$$
$$= \sum_{j=1}^{n} \left(\tilde{c}_j^{(k)}\right)^T x_j + \frac{1}{2} \sum_{j=1}^{n} \sum_{j=1}^{n} x_j^T \left(\tilde{q}_{jj}^{(k)}\right) x_j, \; k = 1, 2, \ldots, K$$
Subject to

$x \in M = \{x \in R^n : Ax \le B, x \ge 0\}.$

Where, $\tilde{Q}^{(k)} = \left(\tilde{q}_{jj}^{(k)}\right)_{n \times n}, \ k = 1, 2, \ldots, K;, \ \tilde{C}^{(k)} = \left(\tilde{c}_j^{(k)}\right)_{1 \times n}, \ k = 1, 2, \ldots, K,$ are possibilistic variables on R characterized by possibility distributions $\varphi_{\tilde{q}_{jj}^{(k)}}, \ k = 1, 2, \ldots, K; \ j = 1, 2, \ldots, n;$ and $\varphi_{\tilde{c}_j^{(k)}}, \ j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, K; \ A = \left(a_{ij}\right)_{m \times n}, \ B = \left(b_i\right)_{m \times 1}$, and $x = \left(x_j\right)$. It is assumed that all possibilistic variables inserted in Poss-MOQP problem are convex with bounded and closed supports and $u_0 = Supp(u)$.

Definition5. A point x^* is said to be a possibly efficient solution of Poss- MOQP problem if $\tilde{Z}^{(k)}(x^*, \tilde{C}^{(k)}, \tilde{Q}^{(k)}) (\le) \tilde{Z}^{(k)}(x, \tilde{C}^{(k)}, \tilde{Q}^{(k)})$ with $\tilde{Z}^{(k)}(x^*, \tilde{C}^{(k)}, \tilde{Q}^{(k)}) (<) \tilde{Z}^{(k)}(x, \tilde{C}^{(k)}, \tilde{Q}^{(k)})$ holds for at least one $k = 1, 2, \ldots, K.$

Definition5. $x^* \in X$ is called an $\beta -$possibly efficient solution for Poss- MOQP problem if

$$Poss\begin{pmatrix} Z^{(1)}\left(x^*, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right) \le Z^{(1)}\left(x, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right), Z^{(2)}\left(x^*, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right) \le Z^{(2)}\left(x, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right), \ldots, \\ Z^{(k-1)}\left(x^*, C^{(k-1)}, \tilde{Q}^{(k-1)}\right) \le Z^{(k-1)}\left(x, \tilde{C}^{(k-1)}, \tilde{Q}^{(k-1)}\right), Z^{(k)}\left(x^*, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right) \le Z^{(k)}\left(x, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right), \\ Z^{(k+1)}\left(x^*, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right) \le Z^{(k+1)}\left(x, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right), \ldots, Z^{(K)}\left(x^*, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \le Z^{(K)}\left(x, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \end{pmatrix} \ge \beta \tag{1}$$

On account of the extension principle,

$$Poss\begin{pmatrix} Z^{(1)}\left(x^*, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right) \le Z^{(1)}\left(x, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right), Z^{(2)}\left(x^*, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right) \le Z^{(2)}\left(x, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right), \ldots, \\ Z^{(k-1)}\left(x^*, C^{(k-1)}, \tilde{Q}^{(k-1)}\right) \le Z^{(k-1)}\left(x, \tilde{C}^{(k-1)}, \tilde{Q}^{(k-1)}\right), Z^{(k)}\left(x^*, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right) \le Z^{(k)}\left(x, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right), \\ Z^{(k+1)}\left(x^*, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right) \le Z^{(k+1)}\left(x, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right), \ldots, Z^{(K)}\left(x^*, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \le Z^{(K)}\left(x, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \end{pmatrix} \ge \beta$$

$$= \underset{C, Q}{Sup} \quad \min\begin{pmatrix} \varphi_{\tilde{C}^{(1)}}(C^{(1)}), \ \varphi_{\tilde{C}^{(2)}}(C^{(2)}), \ldots, \varphi_{\tilde{C}^{(k-1)}}(C^{(k-1)}), \varphi_{\tilde{C}^{(k)}}(C^{(k)}), \varphi_{\tilde{C}^{(k+1)}}(C^{(k+1)}), \ldots, \varphi_{\tilde{C}^{(K)}}(C^{(K)}); \\ \varphi_{\tilde{Q}^{(1)}}(Q^{(1)}), \ \varphi_{\tilde{Q}^{(2)}}(Q^{(2)}), \ldots, \varphi_{\tilde{Q}^{(k-1)}}(Q^{(k-1)}), \varphi_{\tilde{Q}^{(k)}}(Q^{(k)}), \varphi_{\tilde{Q}^{(k+1)}}(Q^{(k+1)}), \ldots, \varphi_{\tilde{Q}^{(K)}}(Q^{(K)}); \end{pmatrix} \tag{2}$$

Where,

$$(C, Q) = \left\{ \begin{array}{l} \left(\left(C^{(1)}, Q^{(1)}\right), \left(C^{(2)}, Q^{(2)}\right), \ldots, \left(C^{(K)}, Q^{(K)}\right)\right) \in R^{K(n \times 1)} \times R^{K(n \times n)} : Z^{(1)}\left(x^*, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right) \le Z^{(1)}\left(x, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right), \\ Z^{(2)}\left(x^*, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right) \le Z^{(2)}\left(x, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right), \ldots, Z^{(k-1)}\left(x^*, \tilde{C}^{(k-1)}, \tilde{Q}^{(k-1)}\right) \le Z^{(k-1)}\left(x, \tilde{C}^{(k-1)}, \tilde{Q}^{(k-1)}\right), \\ Z^{(k)}\left(x^*, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right) \le Z^{(k)}\left(x, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right), Z^{(k+1)}\left(x^*, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right) \le Z^{(k+1)}\left(x, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right), \ldots, \\ Z^{(K)}\left(x^*, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \le Z^{(K)}\left(x, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \end{array} \right\} \tag{3}$$

And $\varphi_{\tilde{C}^{(k)}}(k = 1, 2, \ldots, K)$, and $\varphi_{\tilde{Q}^{(k)}}(1, 2, \ldots, K)$ are $(n \times 1)$- ary and $(n \times n)$- ary possibility distributions.

4. Characterization of $\beta -$ possibly efficient solution of Poss- MOQP problem

T0 characterize the $\beta -$ possibly efficient solution of the Poss- MOQP problem; let us consider $\beta -$parametric multi-objective quadratic programming ($\beta -$ MOQP) problem as follows:

($\beta -$MOQP) $\quad Min\, Z^{(k)}(x, C^{(k)}, Q^{(k)}) = \left(C^{(k)}\right)^T x + \frac{1}{2} x^T Q^{(k)} x$

$\sum_{j=1}^{n}\left(c_j^{(k)}\right)^T x_j + \frac{1}{2}\sum_{j=1}^{n}\sum_{j=1}^{n} x_j^T \left(q_{jj}^{(k)}\right) x_j,\ k = 1, 2, \ldots, K$

Subject to

$x \in M, C^{(k)} \in \left(\tilde{C}^{(k)}\right)_\beta, Q^{(k)} \in \left(\tilde{Q}^{(k)}\right)_\beta.$

Where, $\left(\tilde{c}_j^{(k)}\right)_\beta$, and $\left(\tilde{q}_{jj}^{(k)}\right)_\beta$ are $\beta -$ level sets of the possibilistic variables $\tilde{c}_j^{(k)}$, and $\tilde{q}_{jj}^{(k)}$. For any $\beta \in (0, 1]$, and based on the convexity assumption, $\varphi_{\tilde{c}_j^{(k)}}, \left(\tilde{c}_j^{(k)}\right)_\beta ; \varphi_{\tilde{q}_{jj}^{(k)}}, \left(\tilde{q}_{jj}^{(k)}\right)_\beta,\ j = 1, 2, \ldots, n;$ $k = 1, 2, \ldots, K$, are real intervals which are denoted by $\left[\left(c_j^{(k)}\right)_\beta^-, \left(c_j^{(k)}\right)_\beta^+\right]$, and $\left[\left(q_{jj}^{(k)}\right)_\beta^-, \left(q_{jj}^{(k)}\right)_\beta^+\right]$. Let $\pi_\beta^{(k)}$ be the set of $n \times 1$ matrices $C^{(k)} = \left(c_j^{(k)}\right)$ with $c_j^{(k)} \in \left[\left(c_j^{(k)}\right)_\beta^-, \left(c_j^{(k)}\right)_\beta^+\right]$, $k = 1, 2, \ldots, K;$ and $\psi_\beta^{(k)}$ be the sets of $n \times n$ matrices $Q^{(k)} = \left(q_{jj}^{(k)}\right)$ with $q_{jj}^{(k)} \in \left[\left(q_{jj}^{(k)}\right)_\beta^-, \left(q_{jj}^{(k)}\right)_\beta^+\right]$, $k = 1, 2, \ldots, K.$ It is obvious that the $\beta -$MOQP problem may be rewritten as the following form:

$$Min\ Z^{(k)}\left(x, C^{(k)}, Q^{(k)}\right), k = 1, 2, \ldots, K$$

Subject to

$$x \in M, C^{(k)} \in \pi_\beta^{(k)},\ \text{and}\ Q^{(k)} \in \psi_\beta^{(k)}, k = 1, 2, \ldots, K. \tag{4}$$

Definition6. $x^* \in M$ is said to be $\beta -$parametric optimal efficient solution of $\beta -$MOQP problem if and only if there does not exist another $x \in M, C^{(k)} \in \pi_\beta^{(k)}$, and $Q^{(k)} \in \psi_\beta^{(k)}$ such that $Z^{(k)}\left(x, C^{(k)}, Q^{(k)}\right) \le Z^{(k)}\left(x^*, C^{(k)}, Q^{(k)}\right)$ for all $k = 1, 2, \ldots, K$ and strict inequality holds for at least one k.

Theorem1. $x^* \in M$ is $\beta -$ possibly efficient solution for Poss- MOQP problem if and only if $x^* \in M$ is $\beta -$ parametric efficient solution for $\beta -$PMOQP problem.

Proof. (The proof will be in contra positive direction)

Necessity: Assume that $x^* \in M$ is the $\beta -$ possibly efficient solution for Poss- MOQP problem and $x^* \in M$ is not $\beta -$parametric efficient solution for $\beta -$PMOQP problem. Then there are x°, $H^{(k)} \in \pi_\beta^{(k)}$, and $T^{(k)} \in \psi_\beta^{(k)}$, $k = 1, 2, \ldots, K$ such that $Z^{(r)}\left(x^\circ, H^{(k)}, T^{(k)}\right) \le Z^{(r)}\left(x^*, H^{(k)}, T^{(k)}\right)$, for all $r \in \{1, 2, \ldots, K\}$ and $k \in \{1, 2, \ldots, K\}$, such that $Z^{(k)}\left(x^\circ, H^{(k)}, T^{(k)}\right) < Z^{(k)}\left(x^*, H^{(k)}, T^{(k)}\right)$. For $H^{(k)} \in \pi_\beta^{(k)}$, and $T^{(k)} \in \psi_\beta^{(k)}, k = 1, 2, \ldots, K$, we have

$$Poss\begin{pmatrix} Z^{(1)}\left(x^\circ, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right) \le Z^{(1)}\left(x^*, \tilde{C}^{(1)}, \tilde{Q}^{(1)}\right), Z^{(2)}\left(x^\circ, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right) \le Z^{(2)}\left(x^*, \tilde{C}^{(2)}, \tilde{Q}^{(2)}\right), \ldots, \\ Z^{(k-1)}\left(x^\circ, C^{(k-1)}, \tilde{Q}^{(k-1)}\right) \le Z^{(k-1)}\left(x^*, \tilde{C}^{(k-1)}, \tilde{Q}^{(k-1)}\right), Z^{(k)}\left(x^\circ, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right) \le Z^{(k)}\left(x^*, \tilde{C}^{(k)}, \tilde{Q}^{(k)}\right), \\ Z^{(k+1)}\left(x^\circ, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right) \le Z^{(k+1)}\left(x^*, \tilde{C}^{(k+1)}, \tilde{Q}^{(k+1)}\right), \ldots, Z^{(K)}\left(x^\circ, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \le Z^{(K)}\left(x^*, \tilde{C}^{(K)}, \tilde{Q}^{(K)}\right) \end{pmatrix} \ge \beta$$

This contradicts the $\beta -$ possibly efficient of $x^* \in M$ for Poss- MOQP and the necessity part is hold.

Sufficiency: Let $x^* \in M$ to be the $\beta-$ parametric efficient solution for $\beta-$ PMOQP problem and $x^* \in M$ not to be $\beta-$ possibly efficient solution for Poss- MOQP problem.Then $x^\wedge \in M, k=1,2,\ldots,K$ such that

$$Poss\begin{pmatrix} Z^{(1)}\left(x^\wedge,\tilde{C}^{(1)},\tilde{Q}^{(1)}\right)\le Z^{(1)}\left(x^*,\tilde{C}^{(1)},\tilde{Q}^{(1)}\right), Z^{(2)}\left(x^\wedge,\tilde{C}^{(2)},\tilde{Q}^{(2)}\right)\le Z^{(2)}\left(x^*,\tilde{C}^{(2)},\tilde{Q}^{(2)}\right),\ldots, \\ Z^{(k-1)}\left(x^\wedge,C^{(k-1)},\tilde{Q}^{(k-1)}\right)\le Z^{(k-1)}\left(x^*,\tilde{C}^{(k-1)},\tilde{Q}^{(k-1)}\right), Z^{(k)}\left(x^\wedge,\tilde{C}^{(k)},\tilde{Q}^{(k)}\right)\le Z^{(k)}\left(x^*,\tilde{C}^{(k)},\tilde{Q}^{(k)}\right), \\ Z^{(k+1)}\left(x^\wedge,\tilde{C}^{(k+1)},\tilde{Q}^{(k+1)}\right)\le Z^{(k+1)}\left(x^*,\tilde{C}^{(k+1)},\tilde{Q}^{(k+1)}\right),\ldots,Z^{(K)}\left(x^\wedge,\tilde{C}^{(K)},\tilde{Q}^{(K)}\right)\le Z^{(K)}\left(x^*,\tilde{C}^{(K)},\tilde{Q}^{(K)}\right) \end{pmatrix}\ge\beta$$

i.e.,

$$\underset{D,E}{Sup}\ \min\left(\varphi_{\tilde{C}^{(1)}}\left(C^{(1)}\right),\ \varphi_{\tilde{C}^{(2)}}\left(C^{(2)}\right),\ldots,\varphi_{\tilde{C}^{(K)}}\left(C^{(K)}\right);\ \varphi_{\tilde{Q}^{(1)}}\left(Q^{(1)}\right),\varphi_{\tilde{Q}^{(2)}}\left(Q^{(2)}\right),\ldots,\varphi_{\tilde{Q}^{(K)}}\left(Q^{(K)}\right)\right)\ge\beta. \tag{5}$$

Where,

$$(D,E)=\left\{\begin{matrix} \left(\left(C^{(1)},Q^{(1)}\right),\left(C^{(2)},Q^{(2)}\right),\ldots,\left(C^{(K)},Q^{(K)}\right)\right)\in R^{n\times1}\times R^{n\times n}:\ Z^{(1)}\left(x^\wedge,\tilde{C}^{(1)},\tilde{Q}^{(1)}\right)\le Z^{(1)}\left(x^*,\tilde{C}^{(1)},\tilde{Q}^{(1)}\right), \\ Z^{(2)}\left(x^\wedge,\tilde{C}^{(2)},\tilde{Q}^{(2)}\right)\le Z^{(2)}\left(x^*,\tilde{C}^{(2)},\tilde{Q}^{(2)}\right),\ldots,\ Z^{(k-1)}\left(x^\wedge,\tilde{C}^{(k-1)},\ \tilde{Q}^{(k-1)}\right)\le Z^{(k-1)}\left(x^*,\tilde{C}^{(k-1)},\tilde{Q}^{(k-1)}\right), \\ Z^{(k)}\left(x^\wedge,\tilde{C}^{(k)},\tilde{Q}^{(k)}\right)\le Z^{(k)}\left(x^*,\tilde{C}^{(k)},\tilde{Q}^{(k)}\right), Z^{(k+1)}\left(x^\wedge,\tilde{C}^{(k+1)},\tilde{Q}^{(k+1)}\right)\le Z^{(k+1)}\left(x^*,\tilde{C}^{(k+1)},\tilde{Q}^{(k+1)}\right),\ldots, \\ Z^{(K)}\left(x^\wedge,\tilde{C}^{(K)},\tilde{Q}^{(K)}\right)\le Z^{(K)}\left(x^*,\tilde{C}^{(K)},\tilde{Q}^{(K)}\right) \end{matrix}\right\}$$

For the supremum to exist, there is $\left(V^{(1)},V^{(2)},\ldots,V^{(K)}\right)\in D$, and $\left(W^{(1)},W^{(2)},\ldots,W^{(K)}\right)\in E$ with $\min\left(\varphi_{\tilde{C}^{(1)}}\left(V^{(1)}\right),\ \varphi_{\tilde{C}^{(2)}}\left(V^{(2)}\right),\ldots,\varphi_{\tilde{C}^{(K)}}\left(V^{(K)}\right)\right)<\beta$, and $\min\left(\varphi_{\tilde{Q}^{(1)}}\left(W^{(1)}\right),\ \varphi_{\tilde{Q}^{(2)}}\left(W^{(2)}\right),\ldots,\varphi_{\tilde{Q}^{(K)}}\left(W^{(K)}\right)\right)<\beta$

then

$$\underset{D,E}{\sup}\ \min\left(\varphi_{\tilde{C}^{(1)}}\left(V^{(1)}\right),\ \varphi_{\tilde{C}^{(2)}}\left(V^{(2)}\right),\ldots,\varphi_{\tilde{C}^{(K)}}\left(V^{(K)}\right);\ \varphi_{\tilde{Q}^{(1)}}\left(W^{(1)}\right),\ \varphi_{\tilde{Q}^{(2)}}\left(W^{(2)}\right),\ldots,\varphi_{\tilde{Q}^{(K)}}\left(W^{(K)}\right)\right)<\beta.$$

This contradicts (5). Then there is $\left(V^{(1)},V^{(2)},\ldots,V^{(K)}\right)\in D$, and $\left(W^{(1)},W^{(2)},\ldots,W^{(K)}\right)\in E$ satisfying

$$\min\left(\varphi_{\tilde{C}^{(1)}}\left(V^{(1)}\right),\ \varphi_{\tilde{C}^{(2)}}\left(V^{(2)}\right),\ldots,\varphi_{\tilde{C}^{(K)}}\left(V^{(K)}\right);\ \varphi_{\tilde{Q}^{(1)}}\left(W^{(1)}\right),\ \varphi_{\tilde{Q}^{(2)}}\left(W^{(2)}\right),\ldots,\varphi_{\tilde{Q}^{(K)}}\left(W^{(K)}\right)\right)\ge\beta, \tag{6}$$

i.e.,

$$V^{(k)}\in\pi_\beta^{(k)},W^{(k)}\in\psi_\beta^{(k)},k=1,2,\ldots,K \tag{7}$$

Equations (5) and (7) contradict the efficiency of x^* for $\beta-$PMOQP problem, and the efficiency part is established.

The $\beta-$ PMOQP problem can be treated using the weighting approach (Miettinen, 1999) that is by defining the following problem:

$$(\beta\text{P})\qquad F_\beta(\lambda,C,Q)=\min\sum_{k=1}^{K}\lambda^k Z^{(k)}\left(x,C^{(k)},Q^{(k)}\right)=\sum_{k=1}^{K}\lambda^k\left(\left(C^{(k)}\right)^T x+\frac{1}{2}x^T Q^{(k)}x\right)$$

Subject to

$$x\in M,C^{(k)}\in\left(\tilde{C}^{(k)}\right)_\beta,Q^{(k)}\in\left(\tilde{Q}^{(k)}\right)_\beta,\text{ and }\lambda\in\Omega=\left\{\lambda\in R:\sum_{k=1}^{K}\lambda^k=1,\ \lambda^k\ge0\right\}.$$

We see that x^* is a $\beta-$parametric efficient solution of $\beta-$ PMOQP problem if there exists $\lambda^*\ge0$ such that x^* is the unique optimal solution of (βP) corresponding to the $\beta-$ level.

It is clear that (βP) problem can be written in the following form (see, Bazaraa et al., 1990; and Steuer, 1986)

$$(\beta\text{P})_0\qquad \min\sum_{k=1}^{K}\lambda^k\left(\left(\left(C^{(k)}(0)\right)^L+\omega\left(C^{(k)}(0)\right)^U\right)^T+\frac{1}{2}x^T\left(\left(Q^{(k)}(0)\right)^L+\omega\left(Q^{(k)}(0)\right)^U x\right)\right)$$

Subject to

$x \in M, \ \lambda \in \Omega.$, and $\omega \in [0, 1]$.

5. Numerical example

Consider a multi-objective quadratic programming problem in this form:

$$\text{(Poss- MOQP)} \quad \min \left((x_1 \quad x_2) \begin{pmatrix} \tilde{q}_{11}^{(1)} & \tilde{q}_{12}^{(1)} \\ \tilde{q}_{21}^{(1)} & \tilde{q}_{22}^{(1)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad (x_1 \quad x_2) \begin{pmatrix} \tilde{q}_{11}^{(2)} & \tilde{q}_{12}^{(2)} \\ \tilde{q}_{21}^{(2)} & \tilde{q}_{22}^{(2)} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right)$$

Subject to

$$\begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \ x_1, x_2 \ge 0.$$

In this example, two objectives are considered simultaneously. Also, the possibilistic variables $\tilde{q}_{11}^{(1)}$, and $\tilde{q}_{11}^{(2)}$ are characterized by possibility distributions ($\varphi_{\tilde{q}_{11}^{(1)}}(\cdot)$, and $\varphi_{\tilde{q}_{11}^{(2)}}(\cdot)$). The supports of the possibilistic variables ($\tilde{q}_{11}^{(1)}$, and $\tilde{q}_{11}^{(2)}$) are $[2, 4]$, and $[3, 5]$. It is appropriate to characterize this supports by parametric functions beginning by the points of maximum possibility. Then, the parametric functions of ω to the supports, for $0 \le \omega \le 1$, are:

Supp ($\tilde{q}_{11}^{(1)}$) $= 2 + 2\omega$ $\quad \varphi_{\tilde{q}_{11}^{(1)}} = 0$,

Supp ($\tilde{q}_{12}^{(1)}$) $= 2 - 2\omega$ $\quad \varphi_{\tilde{q}_{12}^{(1)}} = 0$,

Supp ($\tilde{q}_{21}^{(1)}$) $= 2 - 2\omega$ $\quad \varphi_{\tilde{q}_{21}^{(1)}} = 0$,

Supp ($\tilde{q}_{22}^{(1)}$) $= 1 + 2\omega$ $\quad \varphi_{\tilde{q}_{22}^{(1)}} = 0$,

Supp ($\tilde{q}_{11}^{(2)}$) $= 1 + 2\omega$ $\quad \varphi_{\tilde{q}_{11}^{(2)}} = 0$,

Supp ($\tilde{q}_{12}^{(2)}$) $= 3 - 2\omega$ $\quad \varphi_{\tilde{q}_{12}^{(2)}} = 0$,

Supp ($\tilde{q}_{21}^{(2)}$) $= 1 + 2\omega$ $\quad \varphi_{\tilde{q}_{21}^{(2)}} = 0$,

Supp ($\tilde{q}_{22}^{(2)}$) $= 1 + 2\omega$ $\quad \varphi_{\tilde{q}_{22}^{(2)}} = 0$.

Then $(\beta \text{P})_0$ problem becomes

$$\min \ \left(\lambda^{(1)}\left(((2+2\omega)x_1^2 + 2(2-2\omega)x_1x_2 + (1+2\omega)x_2^2\right) + \lambda^{(2)}\left(((1+2\omega)x_1^2 + 4x_1x_2 + (1+2\omega)x_2^2\right)\right)$$

Subject to

$2x_1 + 3x_2 \le 6,$

$2x_1 + x_2 \le 4, \omega \in [0, 1].$

At $\lambda^{(1)} = \lambda^{(2)} = 0.5,$ and $\omega = 0.3$, the $(\beta \text{P})_0$ problem becomes

$$\min \ \left(2.1x_1^2 + 3.7x_1x_2 + 1.6x_2^2\right)$$

Subject to

$2x_1 + 3x_2 \le 6,$

$2x_1 + x_2 \le 4.$

The solution of the problem is: $(x_1, \ x_2) = (1.5, 1)$, with $Z^{(1)} = 11.65$, $Z^{(2)} = 11.2$

6. Concluding remarks

In this paper, multi- objective quadratic programming (Poss- MOQP) problem with possibilistic variables coefficients matrix in the objective functions has been introduced. The (Poss- MOQP) is considered by incorporating possibilistic data in the objective functions coefficients. Through the use of the α - level sets, the problem has been converted into the corresponding deterministic quadratic problem and hence the concept of α - efficient solution has been introduced instead of efficient solution. A necessary and sufficient condition for such solution has been established and hence the relationship between solutions of possibilistic levels has been constructed. Also, numerical example has been given for illustration.

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