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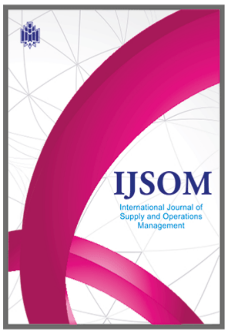

Inventory Model for Non – Instantaneous Deteriorating Items, stock dependent demand, partial backlogging, and inflation over a finite time horizon

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Abstract

In the present study, the Economic Order Quantity (EOQ) model of two-warehouse deals with non-instantaneous deteriorating items, the demand rate is considered as stock dependent and model is affected by inflation under the pattern of time value of money over a finite planning horizon. Shortages are allowed and partially backordered depending on the waiting time for the next replenishment. The main objective of this work is to minimize the total inventory cost and finding the optimal interval and the optimal order quantity. An algorithm is designed to find the optimum solution of the proposed model. Numerical examples are given to demonstrate the results. Also, the effect of changes in the different parameters on the optimal total cost is graphically presented.

Keywords: Two-warehouse; partial backlogging; stock-dependent demand; Inflation; Deterioration; shortages.

1. Introduction

In some pragmatic situations, while suppliers recommend price discounts for mass purchases or if the goods are seasonal, the retailers will possibly buy the superfluous goods that can be stored in own warehouse (OW). And rented warehouse (RW) is used as a store over the certain capacity W_1 of the own warehouse. Generally, the rented warehouse may have a costly superior unit holding cost than the own warehouse due to surplus cost of maintenance, material handling, etc. Hartely (1976) was the original instigator to consider the impact of a two-warehouse model in inventory

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research and developed an inventory model with a RW storage principle. Sarma (1983) developed a two-warehouse model for deteriorating items with an infinite replenishment rate and shortages. Sarma and Sastry (1988) introduced a deterministic inventory model with an infinite production rate, permissible shortage and two levels of storage. Pakkala and Achary (1992) considered a two-warehouse model for deteriorating items with finite replenishment rate and shortages. Lee and Ma (2000) compared an optimal inventory policy for deteriorating items with two-warehouse and time-dependent demand. Yang (2004) produced a two-warehouse inventory model with constant deteriorating items, constant demand rate and shortages under inflation. Yang (2006) investigated the two-warehouse partial backlogging inventory models for deteriorating items under inflation. Kumar et.al (2008) developed a Two-Warehouse inventory model without shortage for exponential demand rate and an optimum release rule. Kumar et al. (2011) produced a Deterministic Two-warehouse Inventory Model for Deteriorating Items with Stock-dependent Demand and Shortages under the condition of permissible delay. Yang (2012) analyzed two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. Sett et al. (2012) introduced a two warehouse inventory model with increasing demand and time varying deterioration. Kumar et al. (2013) developed Learning effect on an inventory model with two-level storage and partial backlogging under inflation. Yang and Chang (2013) perused a two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. Guchhaita et al. (2013) investigated a two storage inventory model of a deteriorating item with variable demand under partial credit period. Considering the deterioration was the most important contribution in lots of inventory systems. Normally, an inventory model is understood with non-deteriorating items and instantaneous deteriorating items. Major part of goods undergo waste or deterioration over time, examples are medicines, volatile liquids, blood banks, and so on. Therefore, waste or deterioration of physical goods in stock is a more realistic assumption that is necessary to be considered in inventory modeling. The primary effort to describe the optimal ordering policies for such items was made by Ghare and Schrader (1963). They presented an EOQ model for an exponentially decaying inventory. Philip (1974) developed an inventory model with three parameter Weibull distribution rate without considering shortages. Deb and Chaudhari (1986) derived inventory model with time-dependent deterioration rate. A meticulous assessment of deteriorating inventory literature is given by Goyal and Giri (2001). Numerous researchers imagine that the deterioration of the items in inventory starts from the instant of their arrival in stock. If truth be told most goods would have a duration of maintaining the quality or original condition (e.g. Vegetables, fruit, fish, meat and so on), namely, during that period, there is no deterioration occurring. Wu et al. (2006) defined the phenomenon as ‘‘non-instantaneous deterioration’’. The examples of this type of phenomenon in the real world are firsthand vegetables and fruits that have a short lifespan with respect to maintenance of freshness and quality. Obviously there are many factors such as temperature, humidity, etc. that prevent vegetables and fruits to spoil immediately. However on a later part, the items like above-said will start to decompose naturally. For these kinds of items, the assumption that the deterioration starts from the instant of arrival in stock may retailers to make inappropriate replenishment policies due to overvaluing the total annual relevant inventory cost. In this way Ouyang et al. (2006) developed an inventory model for non-instantaneous deteriorating items with permissible delay in payments. Liao (2008) studied an EOQ model with non- instantaneous receipt and exponential deteriorating item under two level trade credits. Geetha and Uthayakumar (2009) formulated a replenishment policy for non-instantaneous deteriorating inventory system with partial backlogging. Chung (2009) provided a complete proof on the solution procedure for non-instantaneous deteriorating items with permissible delay in payment. Chang et al. (2010) framed optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. Joint control of inventory and its pricing for non-instantaneous deteriorating items under permissible delay in payments and partial backlogging is developed by Maihami and Abadi (2012a). Maihami and Kamalabadi (2012b) considered an inventory control model for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. Dye (2013) investigated the effect of preservation technology investment in a non-instantaneous deteriorating inventory model.

Inflation is critically responsible for determining the optimal order policy and influences the demand of certain products. As inflation increases, the value of money goes down and erodes the future worth of savings and forces one for more current spendings. Usually, these spendings are on peripherals and luxury items that give rise to the demand of these items. As a result, the effect of inflation and the time value of the money cannot be ignored in determining the optimal inventory policy. As mentioned above, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. The concept of the inflation should be considered especially for long-term investment and forecasting. Buzacott (1975) first developed the EOQ model taking inflation into account. Misra (1979) then extended the EOQ model with different inflation rates for various associated costs. Datta and Pal (1991) considered the effects of inflation and time value of money of an inventory model with a linear time-dependent demand rate and shortages. Sarker and Pan (1994) considered a finite replenishment model when the shortage is allowed. Chung (1996) developed an algorithm with finite replenishment and infinite planning horizon. Ray and Chaudhari (1997) provided an EOQ model under inflation and time discounting. Thangam and Uthayakumar (2010) studied an inventory model for deteriorating items with inflation induced demand and exponential partial backorders–a discounted cash flow approach. Valliathal and Uthayakumar (2010) discussed the Production - inventory problem for ameliorating/deteriorating items with a non-linear shortage cost under inflation and time discounting. Tolgari et al. (2012) studied an inventory model for imperfect items under inflationary conditions by considering inspection errors. Guria et al. (2013) formulated an inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. In the case of perishable product, the retailer may need to backlog demand to avoid costs due of deterioration. When the shortage occurs, some customers are willing to wait for back order and others would turn to buy from other sellers. Inventory model of deteriorating items with time proportional backlogging rate has been developed by Dye et al. (2007). Wang (2002) studied shortages and partial backlogging of items. Min and Zhou (2009) derived a perishable inventory model under stock-dependent selling rate and shortage-dependent partial backlogging with capacity constraint. An Economic Order Quantity model for Weibull deteriorating items with power demand and partial backlogging have been considered by Tripathy and Pardhan (2010). Yang (2011) discussed partial backlogging production-inventory lot-size model for deteriorating items with time-varying production and demand rate over a finite time horizon. Roy et al. (2011) gave an economic order quantity model of imperfect quality items with partial backlogging. Cheng et al. (2011) developed an optimal policy for deteriorating items with trapezoidal type demand and partial backlogging. Sarkar et al. (2012) studied an optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles. Sarkar and Sarkar (2013) developed an improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. Taleizadeh and Pentico (2013) provided an economic order quantity model with a known price increase and partial backordering. Taleizadeh et al. (2013) discussed an EOQ model with partial delayed payment and partial backordering. Tan and Weng (2013) developed the discrete-in-time deteriorating inventory model with time-varying demand, variable deterioration rate and waiting-time-dependent partial backlogging. Kumar et al. (2014) discussed Effect of Salvage Value on a Two-Warehouse Inventory Model for Deteriorating Items with Stock-Dependent Demand Rate and Partial Backlogging.

Tayal et al. (2015) developed an inventory model for deteriorating items with seasonal products and an option of an alternative market .Kumar and Kumar (2016) developed an inventory model with stock dependent demand rate for deterioration items.

In this paper, we have estimated a two-warehouse inventory model for non-instantaneous deteriorating things underneath the impact of inflation and demand is stock-dependent. Shortages are allowed and partially backordered depending on the waiting time for the next replenishment. The main objective of this work is minimizing the total inventory cost and finding the new optimal interval and the optimal order quantity. The model shows the effects of changes in various parameters by taking suitable numerical examples and sensitivity analysis. So, our model has a new managerial insight that helps a retailer/supplier to optimize the total cost of the system.

2. Notations and Assumptions

2.1 Notations

The following notations are used throughout this paper:

A	The ordering cost per order
C_{hr}	The holding cost per item in RW
C_{ho}	The holding cost per item in OW, $C_{hr} > C_{ho}$
C_2	The deterioration cost per unit per cycle
C_3	The shortage cost for backlogged items per unit per cycle
C_4	The unit cost of lost sales per cycle
p	The purchasing cost per unit
s	The selling price per unit, $s > p$
μ_1	The life time of the items in OW
μ_2	The life time of the items in RW, $\mu_1 < \mu_2$

α	The deterioration rate in OW, $0 \le \alpha < 1$
β	The deterioration rate in RW, $0 \le \beta < 1, \alpha > \beta$
T	The length of the order cycle (decision variable)
H	The planning horizon
m	The number of replenishments during the planning horizon, m = H/T (decision variable)
W_1	The capacity of OW
W_2	The maximum inventory level in RW (decision variable)
S	The maximum inventory level per cycle (decision variable)
BI	The maximum amount of shortage demand to be backlogged (decision variable)
Q	The $2^{nd}, 3^{rd}, ..., m^{th}$ order size (Decision variable)
r	The discount rate representing the time value of money.
f	The inflation rate
R	The net discount rate of inflation i.e. R = r – f
$q_r(t)$	The inventory level in RW at time t
$q_o(t)$	The inventory level in OW at time t
$q_s(t)$	The negative inventory level at time t
T_j	The total elapsed time including the jth replenishment cycle (j = 1, 2, 3….)
t_r	Length of period during which inventory level reaches to zero in RW
t_j	The time at which the inventory level in OW in the j^{th} replenishment cycle drops to zero (j = 1, 2,…., m).
$T_j - t_j$	The time period when shortage occurs ($j = 1, 2, ..., m$)
TC_f	The total cost for the first replenishment cycle
TC	The total cost of the system over a finite planning horizon

2.2 Assumptions

To develop the mathematical model, the following assumptions are being made:

1. A single item is considered over the prescribed period of planning horizon.

2. There are no replacement or repair of deteriorated items in a given cycle.

3. The lead time is zero.

4. Deterioration takes place after the life time of items. That is, during the fixed period, the Product has no deterioration. After that, it will deteriorate with constant rate.

5. The replenishment takes place at an infinite rate.

6. The effects of inflation and time value of money are considered.

7. The demand rate $(a + bq_r(t))$ is stock dependent.

8. Shortages are allowed and partially backlogged. During the stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. So the Backlogging rate of negative inventory is, $1/(1 + \delta(T - t))$, where δ is backlogging parameter $0 \le \delta \le 1$ and $(T - t)$ is waiting time $(t_j \le t \le T)$, $(j = 1, 2, \ldots, m)$. The remaining fraction $(1 - B(t))$ is lost.

9. The OW has limited capacity of W_1 units and the RW has unlimited capacity. For economic reasons, the items of RW are consumed first and next the items of OW.

3. Formulation and solution of the model

Suppose that the planning horizon H is divided into m equal parts of length T = H/m. Hence the reorder times over the planning horizon H are $T_j = jT$ (j=0, 1, 2, …, m). When the inventory is positive, demand rate is stock dependent, whereas for negative inventory, the demand is partially backlogged. The period for which there is no-shortage in each interval [jT, (j+1)T] is a fraction of the scheduling period T and is equal to kT (0 < k < 1). Shortages occur at time $t_j = (k+j-1)T$, (j = 1, 2, …, m) and accumulate until time t = jT (j = 1, 2, …, m) before they are backordered. This model is demonstrated in Figure-1. The first replenishment lot size of S is replenished at $T_0 = 0$. W_1 units are kept in OW and the rest is stored in RW. The items of OW are consumed only after consuming the goods kept in RW. In the RW, during the time interval $[0, \mu_2]$, the inventory level is decreasing only due to demand rate and the inventory level is dropping to zero owing to demand and deterioration during the time interval $[\mu_2, t_r]$. In OW, during the time interval $[0, \mu_1]$, there is no change in the inventory level. However, the inventory W_1 decreases during $[\mu_1, t_r]$ due to deterioration only, but during $[t_r, t_1]$, the inventory is depleted due to both demand and deterioration. By the time t_1, both warehouses are empty. Finally, during the interval $[t_1, T]$, shortages occur and accumulate until $t = T_1$ before they are partially backlogged.

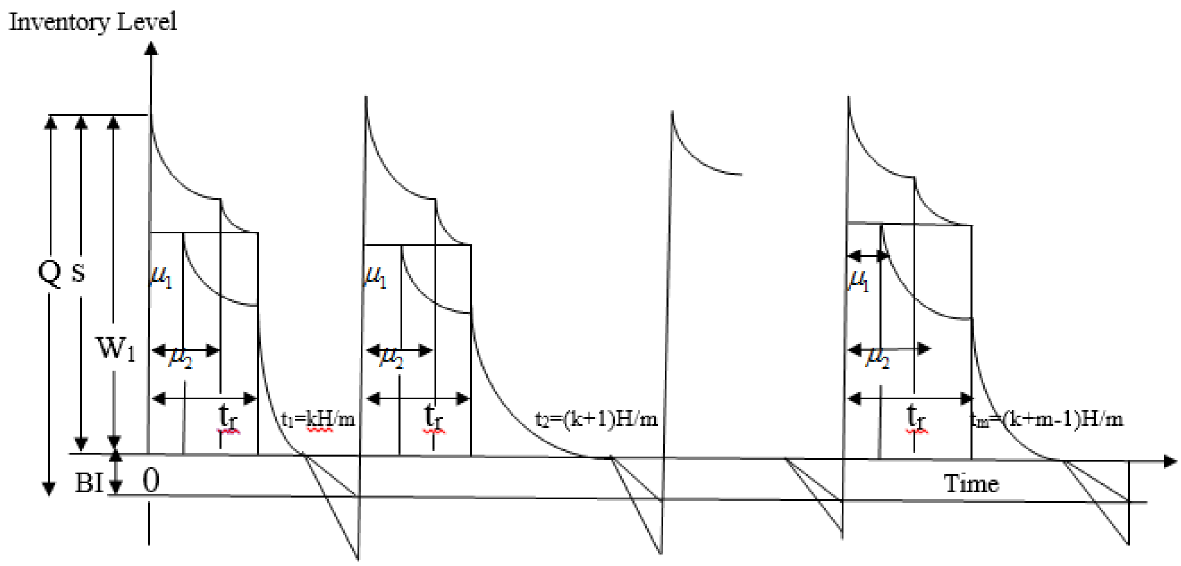

Figure 1. Graphical representation of the two warehouse inventory system

Based on the above description, during the time interval [0, μ2], the inventory level in RW is decreasing only due to demand rate and the differential equation representing the inventory status is given by

$$\frac{dq_r(t)}{dt} = -\left(a + bq_r(t)\right) \qquad 0 \le t \le \mu_2 \qquad (1)$$

Under the condition $q_r(0) = W_2$, the solution of equation (1) is

$$q_r(t) = \frac{-a}{b} + \left(\frac{a}{b} + W_2\right)e^{-bt} \qquad (2)$$

In the second interval [μ_2, t_r] in RW, the inventory level decreases due to demand and deterioration. Thus, the differential equation below represents the inventory status:-

$$\frac{dq_r(t)}{dt} + \beta q_r(t) = -\left(a + bq_r(t)\right) \qquad \mu_2 \le t \le t_r \qquad (3)$$

$$\frac{dq_r(t)}{dt} + \beta q_r(t) + bq_r(t) = -a \qquad \mu_2 \le t \le t_r \qquad (4)$$

Under the condition $q_r(t_r) = 0$, the solution of equation (4) is:-

$$q_r(t) = \frac{-a}{\beta + b} + \frac{a}{\beta + b}e^{(\beta + b)(t_r - t)} \qquad \mu_2 \le t \le t_r \qquad (5)$$

Putting t = μ_2 in equations (2) &(5) we get the value of W_2 as:-

$$W_2 = \frac{a\beta}{b(\beta+b)}\left\{e^{b\mu_2} - 1 - \frac{b}{\beta}\left(1 - e^{(\beta+b)(t_r-\mu_2)+b\mu_2}\right)\right\} \tag{6}$$

Putting the value of W_2 in equation (2) we get:-

$$q_r(t) = \frac{a}{b}\left[-1 + e^{-bt}\right] + \frac{a\beta}{b(\beta+b)}\left\{e^{b\mu_2} - 1 - \frac{b}{\beta}\left(1 - e^{(\beta+b)(t_r-\mu_2)+b\mu_2}\right)\right\}, \qquad 0 \le t \le \mu_2 \tag{7}$$

In OW, during the interval $[0, \mu_1]$, there is no change in the inventory level and during $[\mu_1, t_r]$ the inventory W_1 decreases only due to deterioration.
Therefore the rate of change in the inventory level is given by:-

$$\frac{dq_o(t)}{dt} = 0 \qquad 0 \le t \le \mu_1 \tag{8}$$

$$\frac{dq_o(t)}{dt} + \alpha q_o(t) = 0 \qquad \mu_1 \le t \le t_r \tag{9}$$

Under the conditions $q_0(0) = W_1$ and $q_0(\mu_1) = W_1$, the solutions of equations (8) & (9) are

$$q_o(t) = W_1 \qquad 0 \le t \le \mu_1 \tag{10}$$

$$q_o(t) = W_1 e^{\alpha(\mu_1 - t)} \qquad \mu_1 \le t \le t_r \tag{11}$$

During the time interval $[t_r, t_1]$ in own warehouse, the inventory level decreases due to demand and deterioration. Thus, the differential equation is:-

$$\frac{dq_o(t)}{dt} + \alpha q_o(t) = -\left(a + bq_r(t)\right) \qquad t_r \le t \le t_1 \tag{12}$$

Under the condition $q_0(t_1) = 0$, the solution of equation (12) is:-

$$q_o(t) = \frac{a}{\alpha+b}\left(e^{(\alpha+b)(t_1-t)} - 1\right) \qquad t_r \le t \le t_1 \tag{13}$$

By putting $t = t_r$ in equations (11) and (13) we get,

$$t_r = \frac{\left(W_1\alpha e^{\alpha\mu_1} + ae^{(\alpha+b)t_1}\right) \pm \sqrt{\left(W_1\alpha e^{\alpha\mu_1} + ae^{(\alpha+b)t_1}\right)^2 + \frac{4a\alpha b}{\alpha+b}\left(W_1 e^{\alpha\mu_1} - \frac{a}{\alpha+b}e^{(\alpha+b)t_1} + \frac{a}{\alpha+b}\right)}}{2\left(\frac{-a\alpha b}{\alpha+b}\right)} \tag{14}$$

During the interval $[t_1, T]$, shortages occur and the demand is partially backlogged. That is, the inventory level at time t is governed by the following differential equation:-

$$\frac{dq_s(t)}{dt} = \frac{-a}{1+\delta(T-t)} \qquad t_1 \le t \le T \tag{15}$$

Under the condition $q_s(t_1) = 0$ the solution of equation (15) is:-

$$q_s(t) = a(t_1 - t)\left[1 - \delta T + \frac{\delta}{2}(t_1 + t)\right] \qquad t_1 \le t \le T \tag{16}$$

Therefore the maximum inventory level and maximum amount of shortage demand to be backlogged during the first replenishment cycle are:-

$$S = W_1 + \frac{a\beta}{b(\beta + b)}\left\{e^{b\mu_2} - 1 - \frac{b}{\beta}\left(1 - e^{(\beta+b)(t_r - \mu_2) + b\mu_2}\right)\right\} \tag{17}$$

$$BI = \frac{aH(1-k)}{2m^2}\left[2m - \delta H(1-k)\right] \tag{18}$$

There are m cycles during the planning horizon. Since, inventory is assumed to start and end at zero, an extra replenishment at T_m = H is required to satisfy the backorders of the last cycle in the planning horizon. Therefore, there are m + 1 replenishments over the planning horizon H.
The first replenishment lot size is S.

The 2nd, 3rd,..., m^{th} replenishment order size is: -Q = S + BI (19)

The last or $(m + 1)^{th}$ replenishment lot size is BI.

Since replenishment is done at the beginning of each cycle, the present value of ordering cost during the first cycle is: -OC=A (20)

The holding cost for the RW during the first replenishment cycle is:-

$$HC_r = C_{hr}\left[\int_0^{\mu_2} q_r(t)e^{-Rt}dt + \int_{\mu_2}^{t_r} q_r(t)e^{-Rt}dt\right] \tag{21}$$

$$= C_{hr}\left[-\frac{a}{b}\left\{\frac{1 - e^{-R\mu_2}}{R}\right\} - \left(\frac{a}{b} + W_2\right)\left\{\frac{e^{-(R+b)\mu_2} - 1}{R + b}\right\}\right] + C_{hr}\left[\frac{a}{\beta + b}\left\{e^{-Rt_r}\left(\frac{\beta + b}{R(\beta + b + R)}\right) + e^{-R\mu_2}\left(\frac{e^{(t_r - \mu_2)(\beta + b)}}{\beta + b + R} - \frac{1}{R}\right)\right\}\right] \tag{22}$$

The holding cost for the OW during the first replenishment is:-

$$HC_o = C_{ho}\left[\int_0^{\mu_1} q_o(t)e^{-Rt}dt + \int_{\mu_1}^{t_r} q_o(t)e^{-Rt}dt + \int_{t_r}^{t_1} q_o(t)e^{-Rt}\right]$$

$$= C_{ho}\left[\frac{W_1}{R}\left(1 - e^{-R\mu_1}\right) + \frac{W_1 e^{\alpha\mu_1}}{\alpha + R}\left(e^{-(\alpha+R)\mu_1} - e^{-(\alpha+R)t_r}\right) + \frac{a}{\alpha + b}\left\{e^{-Rt_1}\left(\frac{\alpha + b}{R(\alpha + b + R)}\right) + e^{-Rt_r}\left(\frac{e^{(\alpha+b)(t_1 - t_r)}}{\alpha + b + R} - \frac{1}{R}\right)\right\}\right] \tag{23}$$

The deteriorating cost for RW during the first replenishment cycle is:-

$$DC_r = C_2 \int_{\mu_2}^{t_r} \beta q_r(t) e^{-Rt} dt$$

$$= C_2 \frac{a\beta}{\beta+b} \left\{ e^{-Rt_r} \left(\frac{\beta+b}{R(\beta+b+R)} \right) + e^{-R\mu_2} \left(\frac{e^{(t_r-\mu_2)(\beta+b)}}{\beta+b+R} - \frac{1}{R} \right) \right\} \quad (24)$$

The deteriorating cost for OW during the first replenishment cycle is:-

$$DC_o = C_2\alpha \left[\int_{\mu_1}^{t_r} q_o(t) e^{-Rt} dt + \int_{t_r}^{t_1} q_o(t) e^{-Rt} dt \right]$$

$$= C_2\alpha \left[\frac{W_1 e^{\alpha\mu_1}}{\alpha+R} \left(e^{-\mu_1(\alpha+R)} - e^{-(\alpha+R)t_r} \right) + \frac{a}{\alpha+b} e^{-Rt_1} \left(\frac{\alpha+b}{R(\alpha+b+R)} \right) + \frac{a}{\alpha+b} e^{-Rt_r} \left(\frac{e^{(t_1-t_r)(\alpha+b)}}{\alpha+b+R} - \frac{1}{R} \right) \right] \quad (25)$$

Total shortage cost during the first replenishment cycle is:-

$$SC = -C_3 \int_{t_1}^{T} q_s(t) e^{-Rt} dt$$

$$= \frac{-C_3 a}{R} \left\{ e^{\frac{-RH}{m}} \left[\frac{KH^2\delta}{m^2} \left(1 - \frac{K}{2} - \frac{m}{\delta H} \right) + \frac{H}{m} \left(1 - \frac{\delta H}{2m} \right) + \frac{1}{R} \left(1 + \frac{\delta}{R} \right) \right] + \frac{e^{\frac{-RKH}{m}}}{R} \left[\frac{\delta H}{m} \left(1 - K - \frac{m}{HR} \right) - 1 \right] \right\}$$

(26)

The lost sale cost during the first replenishment cycle is:-

$$LC = C_4 \int_{t_1}^{T} \left(1 - \frac{1}{1+\delta(T-t)} \right) a e^{-Rt} dt$$

$$= \frac{-C_4 a\delta}{R^2} \left[e^{\frac{-RKH}{m}} + e^{\frac{-RKH}{m}} \left(\frac{RH}{m}(1-K) - 1 \right) \right] \quad (27)$$

Replenishment is done at t = 0 and T. The present value of purchasing cost PC during the first replenishment cycle is:- $PC = pS + pe^{-RT}(BI)$

$$= p\left\{ W_1 + \frac{a\beta}{b(\beta+b)} \left\{ e^{b\mu_2} - 1 - \frac{b}{\beta} \left(1 - e^{(\beta+b)(t_r-\mu_2)+b\mu_2} \right) \right\} + ae^{\frac{-RH}{m}} \frac{H}{M}(1-k) \left[1 + \frac{\delta H}{2m}(K-1) \right] \right\} \quad (28)$$

So, the total cost = Ordering cost + inventory holding cost in RW + inventory holding cost in OW + deterioration cost in RW + deterioration cost in OW + shortage cost + lost sales cost + purchasing cost. $TC_F = OC + HC_r + HC_0 + DC_r + DC_0 + SC + LC + PC$

So, the present value of the total cost of the system over a finite planning horizon H is:-

$$TC(m,k)=\sum_{j=0}^{m-1}TC_f e^{-RjT}+Ae^{-RH}=TC_F\left(\frac{1-e^{-RH}}{1-e^{\frac{-RH}{m}}}\right)+Ae^{-RH} \tag{29}$$

Where T = H/m and TC_F derived by substituting equations (21) to (28) in equation (29).
On simplification we get:-

$$TC(m,k)=Ae^{-RH}+G\left[A+C_{hr}\left\{\frac{a}{b}\left(\frac{e^{-R\mu_2}-1}{R}\right)+\left(\frac{a}{b}+W_2\right)\left(\frac{e^{-(R+b)\mu_2}-1}{R+b}\right)\right\}\right.$$

$$+\left(\frac{C_{hr}a}{\beta+b}+\frac{C_2a\beta}{\beta+b}\right)\left\{\frac{e^{-Rt_r}(\beta+b)}{R(\beta+b+R)}+e^{-R\mu_2}\left(\frac{e^{(t_r-\mu_2)(\beta+b)}}{\beta+b+R}-\frac{1}{R}\right)\right\}$$

$$+C_{ho}\left[\frac{W_1}{R}\left(1-e^{-R\mu_1}\right)+\frac{W_1e^{\alpha\mu_2}}{\alpha+R}\left(e^{-(\alpha+R)\mu_1}-e^{-(\alpha+R)t_r}\right)\right]+\left(C_{ho}+C_2\alpha\right)\left[\frac{W_1e^{\alpha\mu_2}}{\alpha+R}\left(e^{-\mu_1(\alpha+R)}-e^{-(\alpha+R)t_r}\right)\right]$$

$$+C_2\alpha\left[\frac{a}{R(\alpha+b+R)}e^{\frac{-RKH}{m}}+\frac{a}{\alpha+b}\left(\frac{e^{\left(\frac{KH}{m}-t_r\right)(\alpha+b)}}{\alpha+b+R}-\frac{1}{R}\right)e^{-Rt_r}\right]$$

$$-\frac{C_3a}{R}\left[e^{\frac{-RH}{m}}\left\{\frac{KH^2\delta}{m^2}\left(1-\frac{K}{2}-\frac{m}{\delta H}\right)+\frac{H}{m}\left(1-\frac{\delta H}{2m}\right)+\frac{1}{R}\left(1+\frac{\delta}{R}\right)\right\}+\frac{e^{\frac{-RKH}{m}}}{R}\left[\frac{\delta H}{m}\left(1-K-\frac{m}{HR}\right)-1\right.\right.$$

$$-\frac{C_4a\delta}{R^2}\left[e^{\frac{-RKH}{m}}-e^{\frac{-RKH}{m}}\left(\frac{RH}{m}(1-K)-1\right)\right]$$

$$+p\left[W_1+\frac{a\beta}{b(\beta+b)}\left\{e^{b\mu_2}-1-\frac{b}{\beta}\left(1-e^{(\beta+b)(t_r-\mu_2)+b\mu_2}\right)\right\}\right]+ae^{\frac{-RH}{m}}\frac{H}{m}(1-K)\left(1-\frac{\delta H}{2m}(K-1)\right)\Bigg]$$

Where $G=\left(\frac{1-e^{RH}}{1-e^{-RH/m}}\right)$

(30)

4. Solution Procedure

The present value of total cost TC (m, k) is a function of two variables m and k where m is a discrete variable and k is a continuous variable. For a given value of m, the necessary condition for TC (m, k) to be minimized is dTC (m, k) / dk = 0 which gives

$$\frac{dTC(\mathrm{m,k})}{dk}=C_2\alpha\left[\frac{aH\ e^{\frac{kH}{m}(\alpha+b)}}{m(\alpha+b+R)}-\frac{aH\,e^{\frac{-RHk}{m}}}{m(\alpha+b+R)}\right]+C_3\left(\frac{aH}{Rm}e^{\frac{-RH}{m}}\right)+\frac{H^2\delta}{m^2}e^{\frac{-RHk}{m}}(k-1)$$

$$+\frac{H\delta}{mR}e^{\frac{-RHk}{m}}+\frac{H}{m}e^{\frac{-RHk}{m}}-C_4\frac{\delta a}{R^2}\left[\frac{R^2H^2}{m^2}e^{\frac{-RHk}{m}}(1-k)-\frac{RH}{m}e^{\frac{-RHk}{m}}\right]+P\left[(1-k)\frac{a\,H}{m}e^{\frac{-RH}{m}}\left\{1-\frac{H\delta}{m}(1+k)\right\}\right]=0$$

(31)

$$\frac{d^2TC(\mathrm{m},\mathrm{k})}{dk^2}=C_2\alpha\left[\frac{aH^2}{m^2}\left(\frac{R\,e^{\frac{-RHk}{m}}}{(\alpha+b+R)}+(\alpha+b)e^{\frac{kH}{m}(\alpha+b)}\right)\right]+\frac{H^3\delta R}{m^3}e^{\frac{-RHk}{m}}(1-k)-C_4\frac{\delta a}{R^2}\left[\frac{R^3H^3}{m^3}e^{\frac{-RHk}{m}}(k-1)\right]$$

$$+P\left[\frac{a\,\delta H^2}{2m^2}\left(e^{\frac{-RH}{m}}-1\right)+\frac{a\,H}{m}e^{-\frac{RH}{m}}\left(\frac{\delta kH}{m}-1\right)\right]\geq 0 \qquad (32)$$

Furthermore, the equation (32) shows that TC (m, k) is convex with respect to k. So, for a given positive integer m, the optimal value of k can be obtained from (31).

Algorithm

Step 1: Start with m = 1.

Step 2: solve (31) for k. Then substitute the obtained solution into (30) to compute the total inventory cost.)

Step 3: Increase m by one and repeat step 2.

Step 4: Repeat step 2 and step 3 until TC (m, k) increases. The value of m which corresponds to the increase of TC for the first time is taken as the optimal value of m (denoted by m*) and the corresponding k (denoted by k*) is the optimal value of k.

Using the optimal solution procedure described above, we can find the optimal order quantity and maximum inventory levels as:

$$W_2=\frac{a\beta}{b(\beta+b)}\left\{e^{b\mu_2}-1-\frac{b}{\beta}\left(1-e^{(\beta+b)(t_r-\mu_2)+b\mu_2}\right)\right\}$$

$$S^* = W_1+\frac{a\beta}{b(\beta+b)}\left\{e^{b\mu_2}-1-\frac{b}{\beta}\left(1-e^{(\beta+b)(t_r-\mu_2)+b\mu_2}\right)\right\}$$

$$Q^* = W_1+\frac{a\beta}{b(\beta+b)}\left\{e^{b\mu_2}-1-\frac{b}{\beta}\left(1-e^{(\beta+b)(t_r-\mu_2)+b\mu_2}\right)\right\}+\frac{aH(1-k)}{2m^2}\left[2m-\delta H(1-k)\right]$$

Where t_r = $$\frac{\left(W_1\alpha e^{\alpha\mu_1}+ae^{(\alpha+b)t_1}\right)\pm\sqrt{\left(W_1\alpha e^{\alpha\mu_1}+ae^{(\alpha+b)t_1}\right)^2+\frac{4a\alpha b}{\alpha+b}\left(W_1e^{\alpha\mu_1}-\frac{a}{\alpha+b}e^{(\alpha+b)t_1}+\frac{a}{\alpha+b}\right)}}{2\left(\frac{-a\alpha b}{\alpha+b}\right)}$$

5. Numerical Examples

Example 1

Consider an inventory system with the following data: D = 100 units; W_1 = 50 units; p = \$4; s = \$15; A = \$150; C_{hr} = \$2; C_{ho} = \$1.2; C_2 = \$1.5; C_3 = \$5; C_4 = \$10; α = 0.8; β = 0.2; δ = 0.008; μ_1 = 5/12 year; μ_2 = 8/12 year; R = 0.2; H = 20 years.

Using the solution procedure described above, the results are presented in Table 1. From this table we see that when the number of replenishments m = 7, the total cost TC becomes minimum. Hence, the optimal values of m and k are m^* = 7, k^* = **0.2995** respectively, and the minimum total cost TC (m^*, k^*) = \$**5594**. We then have, $T^* = H/m^*$ = 20/7 = 2.8571 year, t_r^* = 0.8767 year, $t_1^* = k^*H/m^*$ =1.6349 year, W_2 = 174 units, Q^* = 409 units.

Table 1. Optimal total cost with respect to *m*

M	k(m)	t_r	t_1	T	Q	TC (m, k)
1	0.4485	8.9519	8.8552	20	3922	26496
2	0.4993	4.8536	4.6241	10	1482	11060
3	0.4875	3.1815	3.9620	6.2	925	7641
4	0.4599	2.2128	3.0957	4.97	659	6513
5	0.4306	1.5241	2.3648	3.812	498	5897
6	0.3582	1.9312	1.6842	2.76	452	5658
7	**0.2995**	**0.8767**	**1.6349**	**2.8571**	**409**	**5594**
8	0.2365	0.5145	1.321	2.40	349	5604
9	0.1619	0.3610	1.0822	2.498	325	5758
10	0.1062	0.2138	0.9654	2.12	319	5810

From the table 1, we observed that the total cost decreases with the number of replenishment m and it attains the minimum value \$5594 at m = 7.if the number of replenishment crosses 7, the total cost then increases. Therefore, if the retailer replenishes the quantities 7times during the finite horizon, it will attain the minimum cost.

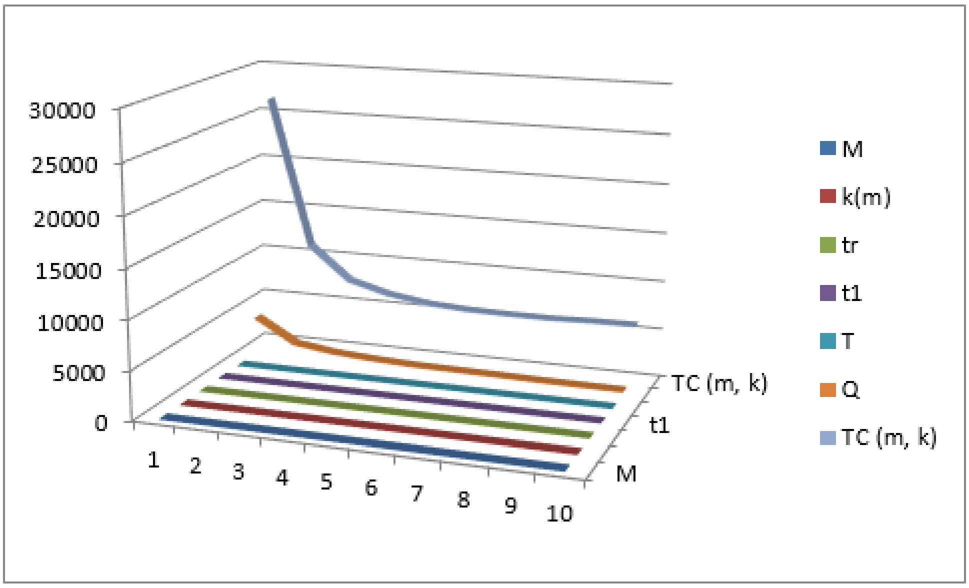

Figure 2. Total cost with respect to number of replenishment

Moreover, if $\mu_1 = 0$ and $\mu_2 = 0$, this model becomes the instantaneous deteriorating item case, and the optimal values of m and k are $m^* = 7$, $k^* = 0.2098$ respectively. Furthermore, $T^* = 1.953$ year, $t_r^* = 0.5167$, $t_1 = 1.7737$ year, $W_2 = 213$ units, $Q = 456$ units and the minimum total cost $TC(m^*, k^*) = \$6690$. The results are illustrated in table 2.

Table 2: Optimal total cost with respect to *m* when $\mu_1=0$ and $\mu_2=0$

M	k(m)	t_r	t_1	T	Q	TC (m, k)
1	0.4170	8.4331	8.6052	20	4022	27496
2	0.4593	4.4536	4.6281	10	1532	11865
3	0.4395	2.7815	3.9925	5.6	1020	7649
4	0.3999	2.0120	3.0025	4.92	735	7553
5	0.3426	1.2041	2.3410	2.312	598	6895
6	0.2782	0.8312	2.0042	1.763	612	6758
7	**0.2098**	**0.5167**	**1.7737**	**1.953**	**456**	**6690**
8	0.1365	0.3145	2.321	2.10	399	6894
9	0.0618	0.1210	1.9872	1.998	385	7058

From figure 3, it is observed that the total cost decreases with the number of replenishment m and it attains the minimum value $\$ = 5594$ at m=7. If the number of replenishment crosses 7, the total cost then increases. Therefore, if the retailer replenishes the quantities 7 times during the finite horizon, it will attain the minimum cost.

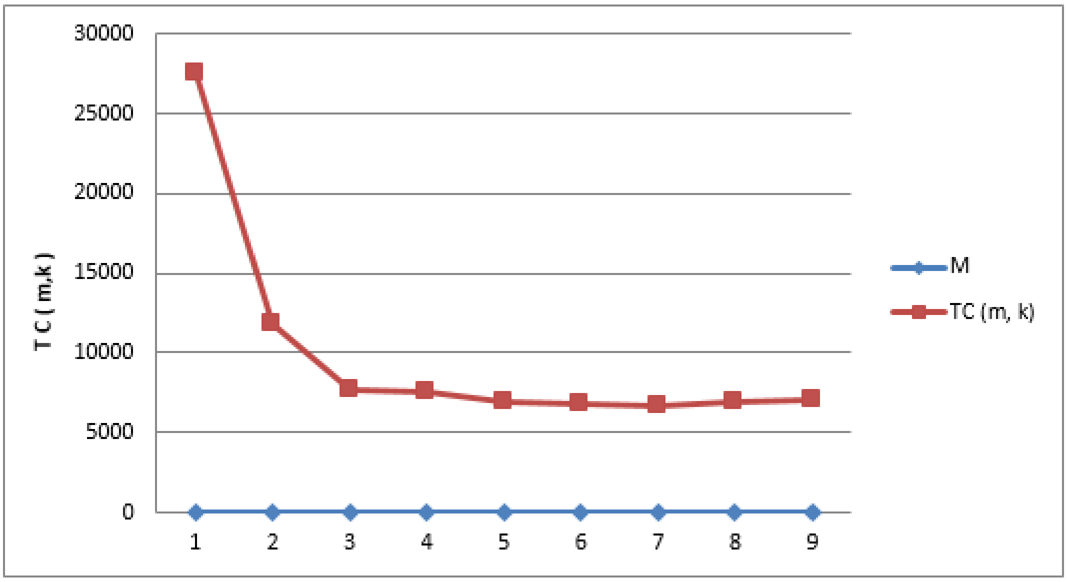

Figure 3. Total cost with respect t(o) number of replenishment when $\mu_1 = 0$

Example 2

Consider an inventory system with the following data: D = 400 units; W_1 = 300 units; p = \$4; s = \$20; A = \$100; C_{hr} = \$3; C_{ho} = \$2; C_2 = \$1; C_3 = \$6; C_4 = \$8; α = 0.9; β = 0.2; δ = 0.005; μ_1 = 2/12 year; μ_2 = 4/12 year; R = 0.1; H = 15 years.

Using the solution procedure described above, the results are presented in Table 3. From this table we see that when the number of replenishments m = 6, the total cost TC becomes minimum. Hence, the optimal values of m and k are m^* = 6, k^* = **0.5455** respectively, and the minimum total cost TC (m^*, k^*) = \$**31560**. We then have, $T^* = H/m^*$ = 15/6 = 2.5 year, t_r^* = 0.6345 year, $t_1^* = k^*H/m^*$ =0.8856 year, W_2 = 401 units, Q^* = 1421 units.

Table 3. Optimal total cost with respect to *m*

M	k(m)	t_r	t_1	T	Q	TC (m, k)
1	0.5455	9.9812	9.7562	15	11580	125635
2	0.5493	4.2316	4.6344	7.5	4126	54469
3	0.5162	2.4815	2.8620	5	2485	38546
4	0.4395	1.2128	1.0957	3.75	2054	34356
5	0.3356	1.0241	1.1642	3	1548	31462
6	**0.2582**	**0.6345**	**0.8856**	**2.5**	**1352**	**31560**
7	0.1795	0.3967	.6349	2.056	1240	32559
8	0.0865	0.1845	0.3218	1.472	1129	36504

From the figure 4, we observed that the total cost decreases with the number of replenishment m and it attains the minimum value $31560 at m = 6.if the number of replenishment crosses 6, the total cost then increases. Therefore, if the retailer replenishes the quantities 6times during the finite horizon, it will attain the minimum cost.

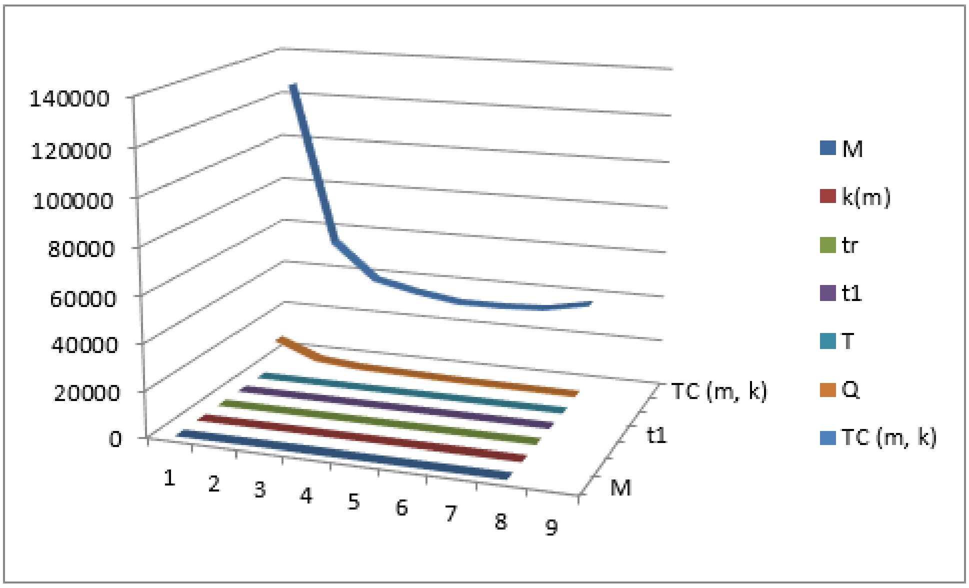

Figure 4. Total cost with respect to number of replenishment

Moreover, if $\mu_1 = 0$ and $\mu_2 = 0$, this model becomes the instantaneous deteriorating item case, and the optimal values of m and k are $m^* = 6$, $k^* = 0.2582$ respectively. Furthermore,

$T^* = 1.953$ year, $t_r^* = 0.6345$, $t_1 = .8856$ year, $W_2 = 401$ units, $Q = 1421$ units and the minimum total cost $TC(m^*, k^*) = \$31560$. The results are illustrated in table 3. The results are illustrated in Table 4 and figure 5shows the convexity of the total cost function with respect to m when $\mu_1 = 0$ and $\mu_2 = 0$.

Table 4: Optimal total cost with respect to *m* when $\mu_{1=0}$ and $\mu_{2=0}$

M	k(m)	t_r	t_1	T	Q	TC (m, k)
1	0.5370	8.0332	8.3154	15	11865	120653
2	0.5246	3.6536	3.9282	7.5	4465	56862
3	0.4595	2.3812	2.9322	5	2798	41764
4	0.3192	1.5122	1.0625	3.75	2035	37553
5	0.3126	.9647	1.3415	3	1690	36595
6	**0.2350**	**0.5862**	**1.0149**	**2.5**	**1412**	**35750**
7	0.1498	0.3167	.7237	1.958	1248	36458
8	0.1365	0.1045	.5321	1.653	1152	37694

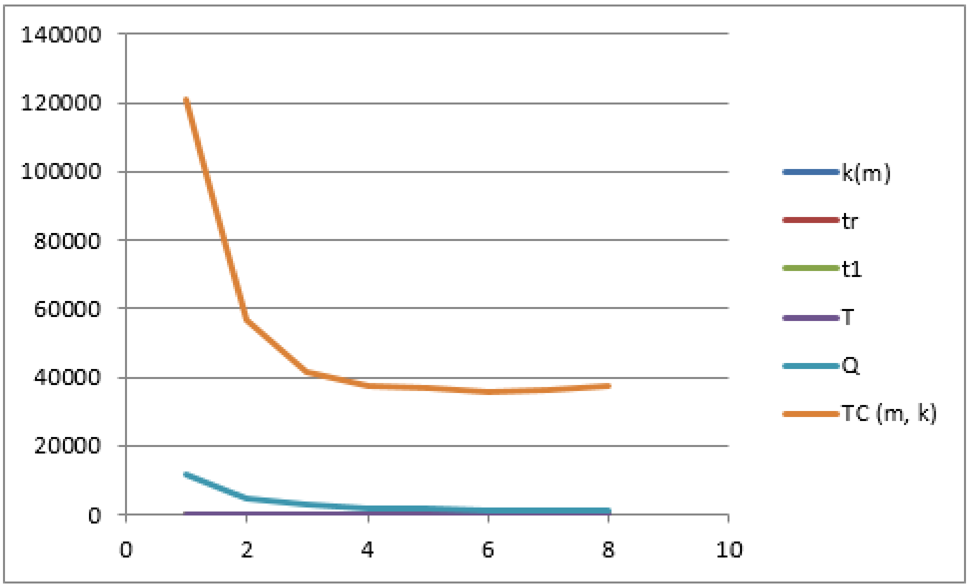

Figure 5. Total cost with respect to number of replenishment when $\mu_1 = 0$

It can be seen that there is a decrease in total cost of the non-instantaneous deteriorating item model. This implies that if the retailer can convert the instantaneously deteriorating items to non-instantaneous deteriorating items by improving stock control, then the total cost per unit time will decrease.

6. Sensitivity Analysis

We now learn the effects of changes in the values of the system parameters H, β, W, p, A, δ and R on the optimal replenishment policy of the example 1. We change one parameter while keeping the others unchanged. The results are summarized in Table 1.

Table 5. Sensitivity Analysis for various inventory parameters

Parameter	Parameter value	M	K	t_1	Q	TC(m,k)
H	14	5	0.3364	1.6812	430.6838	5449.7840
	19	6	0.3061	1.5249	415.2560	5598.7815
	23	8	0.3026	1.5348	406.5648	5500.6245
β	0.1	7	0.3185	1.6024	410.6583	5495.6824
	0.2	7	0.3142	1.6113	415.2546	5562.7845
	0.3	7	0.2996	1.4823	419.8324	5687.8823
δ	0.004	7	0.3124	1.5846	416.1875	5596.7615
	0.008	7	0.3054	1.5840	415.2596	5546.4526
	0.012	7	0.3087	1.5832	414.8542	5548.3256
R	0.1	6	0.3212	1.8256	468.4356	9856.7462
	0.2	7	0.3126	1.6247	416.2056	5518.7503
	0.3	8	0.3022	1.4826	376.6023	3865.2387
W_1	25	6	0.2465	2.3682	548.3024	7215.1245
	50	7	0.3044	1.2536	415.6526	5459.8254
	75	7	0.3654	1.1455	354.4562	4856.6522
P	4	7	0.3056	1.2654	416.9658	5418.9654
	6	6	0.2458	1.3236	452.3562	6859.2546
	8	5	0.2368	1.6542	526.1254	8396.6954
A	100	7	0.3156	1.6255	416.1564	5402.3652
	150	7	0.3049	1.6245	416.0654	5546.1243
	200	7	0.3045	1.6234	416.0212	5648.9564

Based on our numerical results, we obtain the following managerial phenomena:

1. The increment of time horizon H decreases the order quantity and total cost. On the other hand when the time horizon increases, the number of replenishments and consequently the ordering cost increase. In order to minimize the total cost, the retailer should decrease the time horizon.

2. When the deterioration rate β in RW increases, then the total cost of the retailer and the order quantity also increase. Then, if the retailer minimizes the deterioration rate of the item, then he/she

will reduce the total cost. Moreover the minimum deterioration rate of the products will minimize the deterioration cost of the items for the retailer.

3. If backlogging parameter δ is increased then the total cost and the order quantity will be decreased. But the number of replenishment will remain steady. Hence, in order to minimize the cost, the retailers should increase the backlogging parameter.

4. The increase of the net discount rate of inflation R, reduces the optimal cost and order quantity. When the inflation rate increases, the number of replenishment also increases. It will decrease the total cost of the retailer in turn. Then, for higher values of the net discount rate of inflation, the total cost of the retailer will be minimized.

5. When the purchasing price p increases, the total optimal cost and the order quantity increase significantly. Whereas the increment of the purchase price, boosts the number of replenishments and also the total cost of the retailer.

6. When the setup cost A increases, the total cost also increases. We can see that the minimum setup cost will minimize the total cost of the retailer.

7. Conclusion

In the present article, we have considered two warehouses, shortages permitted and partially backordered depending on the waiting time for the next replenishment. In the given model, demand rate have been considered as stock dependent and model is affected by inflation. An algorithm is designed to find the optimum solution of the proposed model. The performance of the proposed model was assessed by some numerical examples and the results of the planned model show that there is a decrease in total cost of the non-instantaneously deteriorating items compared with instantaneously deteriorating items. The optimal total cost will be decreased when the net discount rate of inflation and the backlogging rate are increased. Furthermore, sensitivity analysis was carried out with respect to the key parameters and useful managerial insights were obtained. The proposed model incorporates some realistic features that are likely to be associated with some kinds of inventories. Furthermore, this model can be adopted in the inventory control of retail businesses such as food industries, seasonable cloths, domestic goods, automobile, electronic components etc.

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References

Buzacott, JA. , (1975). Economic order quantities with inflation. *Operations Research Quarterly*, 26(3), 553–558.

Chang, CT., Teng, JT., & Goyal, SK., (2010). Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. *International Journal of Production Economics*, 123(1), 62–68.

Cheng, M., Zhang, B., & Wang, G., (2011). Optimal policy for deteriorating items with trapezoidal type demand and partial backlogging. *Applied Mathematical Modelling,* 35(7), 3552–3560.
Chung, K.J., (1996). Optimal ordering time interval taking account of time value. *Production Planning and Control*, 7(3), 264–267.

Chung, K.J., (2009). A complete proof on the solution procedure for non-instantaneous deteriorating items with permissible delay in payment. *Computers & Industrial Engineering*, 56(1), 267–273.

Data, T.K., Pal, A.K., (1991). Effects of inflation and time value of money on an inventory model with linear time dependent demand rate and shortages. *European Journal of Operational Research,* 52(3), 326–333.

Deb, M., Chaudhuri, K.S., (1986). An EOQ model for items with finite rate of production and variable rate of deterioration. *Opsearch,* 23(1), 175-181.

Dye, C.Y., (2013). The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega,* 41(5), 872–880.

Dye, C.Y., Ouyang, L.Y., & Hsieh, T.P., (2007). Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. *European Journal of Operational research*, 178(3), 789-807.

Geetha, K.V., Uthayakumar, R., (2009). Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments. *Journal of Computational and Applied Mathematics*, 233(10), 2492-2505.

Ghare, P.M., Schrader, G.H., (1963). A model for exponentially decaying inventory system. *International Journal of Production Research*, 21(), 449-460.

Goyal, S.K., Giri, B.C., (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research*, 134(1), 1-16.

Guchhaita, P., Maiti , M. K., & Maiti, M., (2013). Two storage inventory model of a deteriorating item with variable demand under partial credit period. *Applied Soft Computing*, 13(1), 428–448.

Guria, A., Das, B., Mondal, S., & Maiti, M., (2013). Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. *Applied Mathematical Modelling*, 37(1), 240–257.

Hartley R.V. (1976). Operations Research—A Managerial Emphasis. Good Year Publishing Company, California, 315-317.

Hou KL, Lin LC (2006) An EOQ model for deteriorating items with price- and stock-dependent selling rates under inflation and time value of money. *International Journal of Systems Science* 37:1131-1139.

Hsieh TP, Dye CY, Ouyang LY (2008) determining optimal lot size for a two-warehouse system with deterioration and shortages using net present value. *European Journal of Operational Research* 191:182–192.

Kumar N, Singh S R, Kumari R (2008) a Two-Warehouse inventory model without shortage for exponential demand rate and an optimum release rule. *Ultra Scientist of Physical Sciences* 20:395-402.

Kumar N and Kumar S (2016) Effect of learning and salvage worth on an inventory model for deteriorating items with inventory-dependent demand rate and partial backlogging with capability constraints. *Uncertain Supply Chain Management* 4: 123-136.

Kumar N, Singh S R, Kumari R (2011) a deterministic Two-warehouse Inventory Model for Deteriorating Items with Sock-dependent Demand and Shortages under the Conditions of Permissible Delay. *International Journal of Mathematical Modeling and Numerical Optimizations* 11:357-375.

Kumar N, Singh S R, Kumari R (2013) Learning effect on an inventory model with two-level storage and partial backlogging under inflation. *International Journal of Services and Operations Management* 16:105 – 122.

Kumar N, Singh S R, Kumari R (2014) Effect of Salvage Value on a Two-Warehouse Inventory Model for Deteriorating Items with Stock-Dependent Demand Rate and Partial Backlogging .*International Journal of Operational Research* 19: 479 – 496.

Lee C, Ma C (2000) optimal inventory policy for deteriorating items with two-warehouse and timedependent demands. *Production Planning and Control* 11:689–696.

Liao J (2008) An EOQ model with non-instantaneous receipt and exponential deteriorating item under two-level trade credit. *International Journal of Production Economics* 113:852–861

Maihami R, Abadi INK (2012a) Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. *Mathematical and Computer Modelling* 55:1722–1733.

Maihami R, Kamalabadi IN (2012b) Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand. *International. Journal of Production Economics* 136:116–122.

Min J, Zhou YW (2009) a perishable inventory model under stock-dependent selling rate and shortage dependent partial backlogging with capacity constraint. *International Journal of Systems Science* 40:33-44.

Misra RB (1979) A note on optical inventory management under inflation. *Naval Research Logistics Quarterly* 26:161–165.

Ouyang LY, Wu KS, Yang CT (2006) A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. *Computers & Industrial Engineering* 51:637–651.

Pakkala, T.P.M., Achary, K.K., (1992). A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate. *European Journal of Operational Research*, 57(), 71-76.

Philip GC (1974) A generalized EOQ model for items with weibull distribution. *AIIE Transactions* 6:159-162.

Ray J, Chaudhuri KS (1997) An EOQ model with stock-dependent demand, shortage, inflation and time discounting. *International Journal of Production Economics* 53:171-180.

Roy MD, Sana SS, and Chaudhuri KS (2011) An economic order quantity model of imperfect quality items with partial backlogging. *International Journal of Systems Science* 42:1409-1419.

Sarkar B, Sarkar S (2013) An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand. *Economic Modelling* 30:924–932.

Sarkar T, Ghosh SK, Chaudhuri KS (2012) An optimal inventory replenishment policy for a deteriorating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles. *Applied Mathematics and Computation* 218:9147–9155.

Sarker BR, Pan H (1994) Effects of inflation and time value of money on order quantity and allowable shortage. *International Journal of Production Economics* 34:65–72.

Sarma KVS (1983) A deterministic inventory model with two levels of storage and an optimum release rule. *Opsearch* 20:175-180.

Sarma KVS, Sastry MP (1988) Optimum inventory for systems with two levels of storage. *Industrial Engineering Journal* 8:12-19.

Sett BK, Sarkar B, Goswami A (2012) A two-warehouse inventory model with increasing demand and time varying deterioration. *Scientia Iranica* E 19:1969–1977.

Taleizadeh AA, Pentico DW (2013a) An economic order quantity model with a known price increase and partial backordering. *European Journal of Operational Research* 228:516–525.

Taleizadeh AA, Pentico DW, Jabalameli MS Aryanezhad M (2013b) An EOQ model with partial delayed payment and partial backordering. *Omega* 41:354–368.

Tan Y, Weng MX (2013) A discrete-in-time deteriorating inventory model with time-varying demand, variable deterioration rate and waiting-time-dependent partial backlogging. *International Journal of Systems Science* 44:1483-1493.

Tayal S, Singh S R, Sharma S (2015) An inventory model for deteriorating items with seasonal products and an option of an alternative market. *Uncertain Supply Chain Management* 3: 69-86.

Thangam A, Uthayakumar R (2010) An inventory model for deteriorating items with inflation induced demand and exponential partial backorders–a discounted cash flow approach. *International Journal of Management Science and Engineering Management* 5:170-174.

Tolgari JT, Mirzazadeh A, Jolai A (2012) An inventory model for imperfect items under inflationary conditions with considering inspection errors. *Computers and Mathematics with Applications* 63:1007– 1019.

Tripathy CK, Pradhan LM (2010) An EOQ model for Weibull deteriorating items with power demand and partial backlogging. *International Journal of Contemporary Mathematical Sciences* 5:1895-1904.

Valliathal M, Uthayakumar R (2010) The Production - Inventory Problem for Ameliorating/ Deteriorating Items with Non-Linear Shortage Cost under Inflation and Time Discounting. *Applied Mathematical Sciences* 4:289 – 304.

Wu KS, Ouyang LY, Yang CT (2006) An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *International Journal of Production Economics* 101:369-384.

Yang HL (2004) Two-warehouse inventory models for deteriorating items with shortages under inflation. *European Journal of Operational Research* 157:344–356.

Yang HL (2006) Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *International Journal of Production Economics* 103:362–370.

Yang HL (2011) A partial backlogging production-inventory lot-size model for deteriorating items with time-varying production and demand rate over a finite time horizon. *International Journal of Systems Science* 42:1397-1407.

Yang HL (2012) Two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. *International Journal of Production Economics* 138:107–116.

Yang HL, Chang CT (2013) A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. *Applied Mathematical Modelling* 37:2717–2726.