

**A Model for Cooperative Advertising and Pricing Decisions in  
manufacturer-Retailer Supply Chain with Discount: A Game Theory Approach**

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**Abstract**

Coordinating the supply chain is among the most important subjects that is extensively addressed in the related literature. If a supply chain is to be coordinated, it is equivalent to say that we must solve a problem related to competition and cooperation. The game theory is obviously one of the most effective methods to solve such problems, in which the players of the supply chain are assumed to engage in cooperative and non-cooperative games. The current study aims to coordinate a two-level supply chain consisting of a manufacturer and a retailer. This will be achieved using cooperative advertisement along with pricing decisions such as the manufacturer offers a price discount to the retailer and the demand is affected by pricing and advertisement. Cooperative advertisement is a coordinated effort made by all the members of the supply chain to increase the customer demand, in which the retailer does the local advertisement and the manufacturer pays for a portion or all the costs of the retailer advertisement. We consider two models for manufacturer-retailer relation using the game theory: the manufacturer-Stackelberg and the retailer-Stackelberg games with asymmetric power distribution.

**Keywords:** Supply Chain; Game Theory; Cooperative Advertisement; Discount; Pricing; Coordination.

## **1. Introduction**

During the last two decades, the subject of supply chain management has grabbed much attention from scientific communities' members and the job owners. Globalization of transactions, increasing the competition and decreasing the difference between quality and the performance of the products, all have persuaded both the scientists and the individuals who are actively engaged in industry section in order to increase the effectiveness and efficiency of their business through revising these activities. They virtually tend to use any possible tool to increase the final profit and their market share. In particular, they seek to create coordination between the supply chain members to achieve the above objectives (Sarmah, et al. 2006). The biggest challenge we face in supply chain management is managing the members, while they are being separate, they depend on each other as well. If we aim to have an effective supply chain, the members of the chain must act as a whole and coherent unit. But what is experienced in practice is somewhat different, because we deal with non-centralized supply chains, in which each member makes decisions to optimize its own objectives. These objectives are frequently conflicting to the objectives of other members. One important subject that is considered in supply chain literature is coordinating and integrating the operations in these independent units to achieve the highest possible profit from the whole chain (Malone and Crowston, 1994). As a result, a key point in supply chain management is to develop the mechanisms that coordinate the objectives of independent components and create balance between their activities and decisions, and thereby optimize the performance of the whole system.

Cooperative advertisement is a mechanism and interaction for coordinating the advertisement activities in the supply chain, in which the manufacturer agrees to pay for a portion or all the costs of local advertisement undertaken by the retailer. The percent of this cost paid by the manufacturer is referred to as "the participation rate" (Bergen and John, 1997). The manufacturer and the retailer employ their advertisement programs to convince the customers to buy their products. But, their efforts are aimed at different purposes that are the manufacturer undertakes the national advertisement to influence the potential customers and enhance the brand name, while the retailer uses local advertisement to encourage the customers to buy the products. It is only a matter of time before these advertisements make the potential customer to believe that the product is desirable and buy it (Huang and Li, 2001). If the investment made in cooperative advertisement is not paid by the manufacturer, the retailer naturally tends to spend less for advertisement than what is considered as desirable by the manufacturer. So, the cooperative advertisement plays an important role in manufacturer-retailer supply chain.

A review on the literature indicates that the studies conducted on this subject are mostly focused on coordinating the pricing and participating in advertisement performed in the supply chain. So, the current study aims at coordinating two-level supply chains by taking a number of variables into consideration. These variables fall into three categories: pricing decisions, advertisement costs and discount. A new model is proposed in this paper that incorporates the discount concept. In addition, a model is proposed for demand function and manufacturer and retailer profit functions. Then, the interaction between the manufacturer and the retailer is considered as a game to determine all their decision variables.

This study investigates a manufacturer-retailer supply chain, in which the members take interactive measures for the interests of themselves and the whole chain. A manufacturer-retailer supply chain involves a manufacturer that wholesales the product to a retailer, that itself sells the product as retail selling to the end customer. The customer demand is affected by the retailing price and advertisement efforts undertaken by the manufacturer and the retailer to promote and introduce the products. Both factors are influential in the market demand. Pricing is a main theme in the literature existed on market research in the distribution channel. It can be suggested that the change of retailing price influences the market demand and the manufacture and the retailer profit, because the price has an effect on the demand. Manufacturer and retailer use advertisement and reduce the products price to attract the customers and increase the sales. The manufacturer can share a portion of the cost of advertisement undertaken by the retailer. This financial aid makes it possible for the retailer to increase the advertisement level, leading to more sales for both the retailer and the manufacturer (Somers, et al., 1990).

This paper is organized as follows: A literature review is presented in section 2. The problem is defined and the parameters and variables are introduced in section 3. The model is solved in section 4. Two scenarios are analyzed and compared subsequently in section 5. Section 6 devotes to numerical simulation for both games. Finally, the paper is summarized and the conclusion and suggestions for future researches are presented in section 7.

## **2. Literature review**

Supply chain coordination has been the focus of many research studies. In the recent years, significant numbers of research has been focused on different factors of supply chain coordination including pricing, order quantity, advertising, etc. Sahin and Robinson (2002) proposed two key factors of supply chain performance including information sharing and coordination. Li and Wang (2007) have reviewed in 2007 the mechanisms of coordinating the supply chain systems within a framework which was based on decision making structure of the supply chain and the nature of the demand. This framework determines the behavioral aspects and information needs in coordinating a supply chain. Recognizing such issues provides good guidelines for future researches in this area.

A main decision for supply-chain members is to determine wholesale prices and retail prices. Pricing is a core theme in the marketing research literature on distribution channels. For example, Jeuland and Shugan (1983), (1988) used two pricing mechanisms; consist of quantity-discount schemes and two-part tariffs to gain channel coordination. In 2008, Xiao and Yang (2008) considered a Stackelberg game for a two-level supply chain consisting of a manufacturer (leader) and a retailer (follower). They showed that the manufacturer can provide a pricing scheme to encourage the retailer to make a decision which maximizes the performance of the chain. Gerstner and Hess (1995) proposed that manufacturers use a price discount. They studied the impact direct discount from manufacturer to the price-sensitive consumers may have on the channel coordination. Dolan (1978) assumed the demand to be constant and considered an estimation of the costs incurred because of provider inventory. Then, he analyzed the discount as a mechanism for encouraging the buyers to choose a purchase quantity which minimizes the total cost of the

system. Corbett and de Groot (2000) considered the seller-buyer model, in which the seller wants from buyer to determine his maintenance cost. Then the seller reports its batch size and quantity discount accordingly. Chen and Xiao (2009) used linear discount mechanism and wholesale price for coordinating a supply chain consisting of a manufacturer, a dominant retailer who determined the price in the market and many secondary retailers. Chen and Wang (2015) examined the impact of power structures on the decision of pricing and channel selection between a free channel and a bundled channel. Their research mainly focused on assessing the impact of supply chain power dynamics on the channel selection problem.

Vertical cooperative advertising is a kind of cooperation between a manufacturer and a retailer and is defined as a financial agreement whereby the manufacturer pays for all the expenditures of local advertisement or part of it to the retailer. This percent of cost paid by the manufacturer is called "participation rate." (Bergen and John, 1997). The first mathematical modeling of cooperative a between the manufacturer and retailer in their advertisement efforts solved by Berger (1972). It is indicated in his study that mathematical modeling can improve the management decisions and the performance of the whole supply chain and that the quantitative analysis can be used for determining the optimization parameters when investing for advertisement. Roslow et al. (1993) studied in 1993 the cooperative advertisement in the supply chain and showed that coordinated advertisement can increase the total profit of the chain. Jorgensen et al. (2001) considered the leadership role in a two-member supply chain; where costumer demand depends on both retail price and advertising goodwill. They obtained both the manufacturer and the retailer's optimal decisions by solving the four game-theoretic models. Huang et al. (2002) used a demand function which depends on both retail price and advertising goodwill. They presented three one-period co-op advertising models and compared the results. Yue et al. (2006) extend the research of Huang et al. (2002). They added price elasticity to their model by noting a price-sensitive demand. They studied the effect of direct discount from manufacturer to the costumer on channel coordination.

A review on the literature indicates that several papers studied about vertical cooperative advertising and pricing decisions in a supply chain, which both factors are considerable determinants of market demand and hence profits gained by channel members. Jorgensen and Zaccour (2003) modeled the customer demand by taking the retailer pricing and goodwill in a dynamic environment into consideration. In this environment, each of the members of the supply chain had access to all information about the product pricing and the advertisement costs of another member. They compared the results obtained from adopting cooperative and non-cooperative strategies with each other in their study. Szmerekovsky and Zhang (2009) studied the collaboration in a supply chain to determine the optimal pricing and co-op advertising decision. This chain involved a manufacturer and a retailer. They considered the manufacturer as the dominant member of the chain. They obtained the desirable decision making of the manufacturer and the retailer through solving the manufacturer-Stackelberg game. Xie and Wei (2009) investigated how pricing and advertising items affect the supply chain coordination in two cooperative and non-cooperative settings. They compared two game models to determine the prices and advertising costs, firstly used Stackelberg non-cooperative model in which the seller acted as leader. Then, they examined the cooperative game model in which both the buyer and the

seller shared the advertising costs and gained higher income compared to non-cooperative case. In this study, Nash bargaining process was employed based on the risk seeking level of the members. In the same year, Xie and Neyret (2009) examined the coordination between the decisions related to pricing and advertising in a two-level supply chain which included a manufacturer and a retailer. They assumed that the demand is affected by advertising costs and retail price. They considered four scenarios in their study including three non-cooperative games in which the manufacturer and the retailer acted independently and a cooperative game. They obtained an optimal solution for coordinated pricing and advertisement in four types of classic relationship.

In 2011, Seyedesfahani et al. (2011) followed a similar approach; they considered the problem of pricing and cooperative advertisement in a single-manufacturer–single-retailer marketing channel; in which the demand was a nonlinear function of pricing. They modified a price demand function which was used by Xie and Neyret (2009) by introducing a new parameter  $\nu$  which can cause either a convex ( $\nu < 1$ ), or a linear ( $\nu = 1$ ) or a concave ( $\nu > 1$ ) curve. Similar approaches with slightly modified demand functions can be found in Aust and Buscher (2012). They modeled a two-level supply chain with one manufacturer and one retailer in which the demand is influenced by both price and advertisement costs. They proposed four relationships among the members of the channel that involved three non-cooperative games and one cooperative game between themselves. Then, they used bargaining model for fair allocation of profit based on the risk taken by the players and their bargaining power. Zhang et al. (2014) examined the effectiveness of such advertising initiative in a leader–follower supply chain with one manufacturer and one retailer. They assumed that both the manufacturer and the retailer can choose to participate in the advertising initiative by reducing their advertising levels. The problem is formulated as a Stackelberg game. They showed that the effectiveness of the advertising initiative critically depends on the leader's participation in the initiative. Jogensen and Zaccour (2014) and Aust and Buscher (2014) provide a good summary of work in cooperative advertising by discussing the studies done. Amrouche and Yan (2015) investigated the benefit of using national brand's advertising (the aggressive strategy) that hurts the private label's demand over using national brand's revenue sharing (the partnership strategy) that fosters collaboration between the retailer and the national brand's manufacturer. They compared each strategy to the benchmark case where none of these strategies is used and where both brands (national and private brands) are offered through a unique retailer. They found that when the national brand's revenue sharing is implemented, the manufacturer and the whole chain are gaining compared to the benchmark case but the retailer is always losing. Chen (2015) evaluated the impact of pricing schemes and cooperative advertising mechanisms on dual-channel supply chain competition. Using a manufacturer Stackelberg game theoretic framework, he examined the problem of determining the optimal local advertising level, the investment in promoting national brand name recognition, the retail channel selling price, and the direct channel selling price for a two-echelon supply chain. His analysis offered structural and quantitative insights into the interplay between upstream and downstream entities in the supply chain, helping managers to understand the interplay between the upstream and downstream entities of a dual channel structure.

A review on the literature indicates that the numerous studies have examined the coordination in the supply chain by taking the factors of pricing, services, advertisement and discount into

consideration separately. There is only a few studies in the literature that have tried to examine the coordination in supply chain by taking the three factors of pricing, discount and participation in advertisement into consideration simultaneously. Also, in the game theory area, there seems to be a gap. So, this research tries to fill the gap existed in the literature and at the same time to model the coordination in the supply chain using the game theory.

### 3. Problem definition

In this section, two different problems related to manufacturer-retailer relations are presented. In the proposed model, the manufacturer offers its products only to the retailer to consumer's dealer and the only act of the retailer is offering the products made by the manufacturer. The manufacturer decides to set the wholesale price ( $w$ ), national advertisement cost ( $A$ ), participation rate ( $t$ ) and discount percent ( $\varepsilon$ ). On the other hand, the retailer decides to set the retail selling price ( $p$ ) and local advertisement cost ( $a$ ). As the next step, we will introduce the demand and the profit functions.

#### 3.1. Definition of demand function

The demand for each product depends on its price offered to the customer and the advertisement done for the product. Provided that the demand for each product depends on its selling price, the pricing offered by the manufacturer to the retailer affects the final demand for the product. Based on the assumptions commonly made in the literature (Jorgensen and Zaccour, 2003); (Szmerekovsky and Zhang, 2009); (Xie and Wei, 2009) and (Xie and Neyret, 2009), the customer demand function  $D(p,a,A)$  can be defined as in Eq. (1):

$$D(p,a,A) = g(p).h(a,A) \quad (1)$$

Where,  $g(p)$  and  $h(a,A)$  reflect the effect of retail selling and the effect of national and local advertisement costs on demand, respectively. In reality, the product price affects the market demand and demand is inversely related to retail selling price. Based on the price elasticity concept, the demand for a product changes, as the price changes. In other words, the demand is decreased when the price increases. The change in the demand as a response to price change is higher for a product with higher elasticity. Such product is called a highly elastic product. There are two types of demand curve. The first and second curves have linear and nonlinear relationships with price, respectively. In the literature review, different linear, convex and concave functions are proposed as the demand function. Piana (2004) suggested three types of society which have different demand curves. The demand curve is linear when the society enjoys a moderate economic level, it is convex when the society is rich and it is concave when the society

is poor.

Price dependence function has been defined as  $\alpha - \beta p$  in many papers such as Xie and Wei (2009) and Xie and Neyret (2009) and it has been extensively shown to be an accurate measure. A linear relationship between demand and price is usually considered as in this paper.  $g(p)$  Is a linear decreasing function with respect to  $p$  which is defined as Eq. (2):

$$g(p) = \alpha - \beta p \quad \alpha \text{ and } \beta \text{ are the positive (2)}$$

constants for this product, such that  $\alpha$  is the maximum possible demand for the product and  $\beta$  is the coefficient of demand changes with respect to the retail price changes. That is, if the product price is increased by one unit, the demand for the product is decreased by  $\beta$  units.

Since both national and local advertisements affect the selling, their effects must be examined separately. To this end, the effect of advertisement ( $h(a, A)$ ) is determined using Eq. (3):

$$h(a, A) = k_1 \sqrt{a} + k_2 \sqrt{A} \quad (3)$$

Where  $k_1$  and  $k_2$  are positive constants which reflects the effects of local and national advertisements on the demand, respectively. As the Eq. (3) indicates,  $h(a, A)$  is an increasing concave function of  $a$  and  $A$ . Because the unnecessary and redundant advertisement continually decreases the efficiency, this property indeed represents “the effect of advertisement saturation”. Saturation occurs when one of the national or local advertisements is more than what is required. According to the proposed definition, we can combine Eqs. (1)- (3) to drive the demand function, as is represented in Eq. (4):

$$D(p, a, A) = (\alpha - \beta p)(k_1 \sqrt{a} + k_2 \sqrt{A}) \quad (4)$$

Eq. (5) must hold to avoid negative demand:

$$D(p, a, A) > 0 \Rightarrow p < \frac{\alpha}{\beta} \quad (5)$$

### **3.2. Definition of profit functions**

In each game, the manufacturer and the retailer consider their profit function as the criterion for their judgment. That is, the players prefer a more profitable game in comparison with other games. The manufacturer and the retailer try to maximize their profit through determining optimal values for their decision variables. The manufacturer profit function is defined as the income driven from

selling the products minus the sum of national advertisement cost and the cost of participation in the retailer advertisement. Also, the retailer profit function is defined as the income driven from the selling minus the cost paid for local advertisement.

It is assumed that the manufacturer will repay  $t$  percent of the cost of local advertisement undertaken by the retailer, such that  $0 \leq t \leq 1$ . That is, if the retailer spends  $a$  units for local advertisement, it will receive  $ta$  from the manufacturer as the incentive for its advertisement efforts.

The manufacturer gives  $\varepsilon$  percent ( $0 \leq \varepsilon < 1$ ) discount for the wholesale price and it causes the moderation of the retail selling price. In other words, the retailer moderates its price based on the discount offered by the manufacturer and this reduction of retail selling price leads to increase in customers demand. As a result, the profit of both members of the chain is increased and this coordinates the chain. Also, when the manufacturer offers more price reduction to the retailer and pays for part of the local advertisement expenditures, the retailer increases the local advertisement.

The profit functions for the manufacturer and the retailer and the whole supply chain are given by Eqs. (6)- (8):

$$\Pi_m = ((1-\varepsilon)w - c)(\alpha - \beta p)(k_1\sqrt{a} + k_2\sqrt{A}) - A - ta \tag{6}$$

$$\Pi_r = (p - (1-\varepsilon)w - d)(\alpha - \beta p)(k_1\sqrt{a} + k_2\sqrt{A}) - (1-t)a \tag{7}$$

$$\Pi_{m+r} = (p - c - d)(\alpha - \beta p)(k_1\sqrt{a} + k_2\sqrt{A}) - A - a \tag{8}$$

Where  $c$  and  $d$  are positive constants and denote the cost of manufacturer for producing a unit of the product and the cost of each retailer unit in addition to the purchase cost, respectively. In this paper,  $m$ ,  $r$  and  $m+r$  represent manufacturer, retailer and the whole system, respectively. We want to prevent negative values of profit function in Eqs. (6)-(8). Then, Eqs. (9)-(11) are obtained:

$$\Pi_m > 0 \Rightarrow (1-\varepsilon)w > c \tag{9}$$

$$\Pi_r > 0 \Rightarrow p > (1-\varepsilon)w + d > (1-\varepsilon)w \tag{10}$$

$$\Pi_{m+r} > 0 \Rightarrow p > c + d \tag{11}$$



In the next step, we combine inequalities (5) and (11) to drive Eq. (12):

$$\alpha - \beta(c + d) > 0 \tag{12}$$

In this paper, we use an appropriate change of variable to simplify the analysis process. So, similar variables as those used in Xie and Neyret (2009) are introduced to obtain Eqs. (13)- (17):

$$\alpha' = \alpha - \beta(c + d) > 0 \tag{13}$$

$$p' = \frac{\beta}{\alpha'}(p - (c + d)) > 0 \tag{14}$$

$$(1 - \varepsilon)w' = \frac{\beta}{\alpha'}((1 - \varepsilon)w - c) > 0 \tag{15}$$

$$k_1' = \frac{\alpha'^2}{\beta}k_1 \tag{16}$$

$$k_2' = \frac{\alpha'^2}{\beta}k_2 \tag{17}$$

Eq. (18) and Eq. (19) are obtained using the above equations:

$$p < \frac{\alpha}{\beta} \Leftrightarrow \beta p - \beta(c + d) < \alpha - \beta(c + d) \Leftrightarrow \frac{\beta p - \beta(c + d)}{\alpha - \beta(c + d)} < 1 \Leftrightarrow p' < 1 \tag{18}$$

$$p > (1 - \varepsilon)w + d \Leftrightarrow p - (c + d) > (1 - \varepsilon)w - c \Leftrightarrow p' > (1 - \varepsilon)w' \tag{19}$$

Also, Eq. (20) is obtained by taking inequalities (18) and (19) into consideration:

$$(1 - \varepsilon)w' < p' < 1 \tag{20}$$

In the next step, we rewrite the profit functions as Eqs. (21)-(23) by applying the changes presented in Eqs. (13)- (17):

$$\Pi_m = (1 - \varepsilon)w'(1 - p')(k_1'\sqrt{a} + k_2'\sqrt{A}) - A - ta \tag{21}$$

$$\Pi_r = (p' - (1 - \varepsilon)w')(1 - p')(k_1'\sqrt{a} + k_2'\sqrt{A}) - (1 - t)a \tag{22}$$

$$\Pi_{m+r} = p'(1 - p')(k_1'\sqrt{a} + k_2'\sqrt{A}) - A - a \tag{23}$$

Note that we have omitted the sign (') in the remaining of the paper.

#### 4. Our proposed model

In this section, the interaction between the manufacturer and the retailer is addressed. This interaction can be modeled using Stackelberg game. When one of the players is able to start the play sooner than the other player, it is able to act as the leader in the game. This is where the concept of Stackelberg equilibrium is used. In a leader-follower scenario, the strategy of each player can be determined through finding the Stackelberg solution. The follower player chooses its best decision by taking the decision of the leader player into consideration. On the other hand, the leader player optimizes its objective function based on the follower response.

##### 4.1. Proposed model: Manufacturer- Stackelberg game

In this section, the relation between the manufacturer and the retailer is modeled as a sequential non-cooperative game, in which the manufacturer is the leader and the retailer is the follower. The Stackelberg game is the solution for this structure. In this game that is played sequentially, the manufacturer has more power and is aware of the response of the retailer in its decision making. So, we firstly solve the retailer decision making problem for identifying its solution function. Next, the manufacturer decision making problem is solved.

Eq. (24) represents the retailer decision making problem in the manufacturer-Stackelberg equilibrium. In addition, the response functions of the retailer are given in Eqs. (25)- (28):

$$\begin{aligned} \max \quad & \Pi_r(p, a) = (p - (1 - \varepsilon)w)(1 - p)(k_1\sqrt{a} + k_2\sqrt{A}) - (1 - t)a \\ \text{s.t.} \quad & \end{aligned} \tag{24}$$

$$w \leq p \leq 1 \quad \text{and} \quad 0 \leq a$$

$$\frac{\partial \Pi_r}{\partial p} = (k_1\sqrt{a} + k_2\sqrt{A})[(1 - p) - (p - (1 - \varepsilon)w)] = 0 \quad \Rightarrow \quad p^* = \frac{(1 - \varepsilon)w + 1}{2} \tag{25}$$

In the next step, we take the second order derivative from the retailer profit function using Eq. (26) to examine whether the optimal values are obtained and whether the second condition for the retailer profit maximization holds:

$$\frac{\partial^2 \Pi_r}{\partial p^2} = -2(k_1 \sqrt{a} + k_2 \sqrt{A}) \tag{26}$$

Since  $k_1 \sqrt{a} + k_2 \sqrt{A} > 0$ , the second order derivative is negative and the sufficient condition for maximization problem is met. This means that the retailing price is optimal. Likewise, Eq. (27) is obtained:

$$p^* = \frac{(1-\varepsilon)w+1}{2} \Rightarrow a^* = \left[ \frac{k_1(1-(1-\varepsilon)w)^2}{8(1-t)} \right]^2 \tag{27}$$

$$\frac{\partial \Pi_r}{\partial a} = \frac{k_1}{2}(p - (1-\varepsilon)w)(1-p)a^{-\frac{1}{2}} - (1-t) = 0$$

We take the second order derivative using Eq. (28) to examine whether the optimal value is obtained:

$$\begin{aligned} \frac{\partial^2 \Pi_r}{\partial a^2} &= -\frac{k_1}{4}(p - (1-\varepsilon)w)(1-p)a^{-\frac{3}{2}} \xrightarrow{p^* = \frac{(1-\varepsilon)w+1}{2}} \frac{\partial^2 \Pi_r}{\partial a^2} = -\frac{k_1}{16}(1-(1-\varepsilon)w)^2 a^{-\frac{3}{2}} \\ \Rightarrow \frac{\partial^2 \Pi_r}{\partial a^2} \Big|_{a=a^*} &= -\frac{8^2}{2k_1^2} \frac{(1-t)^3}{(1-(1-\varepsilon)w)^4} < 0 \Rightarrow \Pi_r = \max \end{aligned} \tag{28}$$

The second order derivative of the retailer objective function is with negative  $a^*$ . So, the sufficient condition for maximization problem is realized. If  $p^*$  and  $a^*$  values obtained from Eq. (25) and Eq. (27) are substituted into Eq. (29) (manufacturer objective function), Eq. (30) is obtained:

$$\max \Pi_m(w, A, t, \varepsilon) = (1-\varepsilon)w(1-p)(k_1 \sqrt{a} + k_2 \sqrt{A}) - A - ta$$

$$s.t. \ 0 \leq w \leq 1, \ 0 \leq A, \ 0 \leq t \leq 1 \quad \text{and} \quad 0 \leq \varepsilon < 1 \tag{29}$$

$$\max \Pi_m(w, A, t, \varepsilon) = \frac{1}{2}(1-\varepsilon)w(1-(1-\varepsilon)w) \left[ k_1 \left[ \frac{(1-(1-\varepsilon)w)^2 k_1}{8(1-t)} \right] + k_2 \sqrt{A} \right] - A - t \frac{(1-(1-\varepsilon)w)^4 k_1^2}{8^2(1-t)^2}$$

$$s.t. \ 0 \leq w \leq 1, \ 0 \leq A, \ 0 \leq t \leq 1 \quad \text{and} \quad 0 \leq \varepsilon < 1 \tag{30}$$

In order for  $\Pi_m$  to be differentiable, Eq. (31) must hold:

$$1-t \neq 0 \Rightarrow t \neq 1$$

$$(1-\varepsilon)w \neq 0 \Rightarrow w \neq 0 \quad \text{and} \quad (1-\varepsilon) \neq 0$$

$$(1-\varepsilon)w \neq 1 \Rightarrow 1-(1-\varepsilon)w \neq 0 \tag{31}$$

Because if  $(1-\varepsilon)w$  or  $1-(1-\varepsilon)w$  is equal to zero, then the manufacturer profit function is equal to  $-A < 0$ . So, the constraints imposed by Eq. (24) are changed as shown in Eq. (32):

$$0 < w \leq 1, \quad 0 \leq A, \quad 0 \leq t < 1, \quad 0 \leq \varepsilon < 1 \quad \text{and} \quad (1-\varepsilon)w \neq 1 \tag{32}$$

to find the decision variables of the manufacturer, we differentiate its profit function with respect to these variables and set the obtained derivative to zero. That is, it can be written based on Eq. (33) that:

$$\frac{\partial \Pi_m}{\partial A} = \frac{k_2(1-\varepsilon)}{2} w(1-(1-\varepsilon)w) A^{-\frac{1}{2}} - 1 = 0 \Rightarrow A^* = \left[ \frac{k_2(1-\varepsilon)w(1-(1-\varepsilon)w)}{4} \right]^2 \tag{33}$$

We take the second order derivative using Eq. (34) to examine the optimal condition:

$$\frac{\partial^2 \Pi_m}{\partial A^2} = -\frac{k_2(1-\varepsilon)}{4} w(1-(1-\varepsilon)w) A^{-\frac{3}{2}} \Rightarrow$$

$$\frac{\partial^2 \Pi_m}{\partial A^2} \Big|_{A=A^*} = -\frac{4^2}{2k_2^2(1-\varepsilon)^2w^2(1-(1-\varepsilon)w)^2} < 0 \Rightarrow \Pi_m = \max \tag{34}$$

The second order derivative of the manufacturer objective function is with negative  $A^*$ . So the sufficient condition for the maximization problem is met. Likewise, Eq. (35) is obtained to calculate  $t^*$ :

$$\frac{\partial \Pi_m}{\partial t} = \frac{k_1^2}{16} (1-\varepsilon)w(1-(1-\varepsilon)w)^3(1-t)^{-2} - \frac{(1-(1-\varepsilon)w)^4 k_1^2}{8^2} \times \frac{1+t}{(1-t)^3} = 0 \Rightarrow$$

$$t^* = \frac{5(1-\varepsilon)w - 1}{3(1-\varepsilon)w + 1} \tag{35}$$

The method for calculating the variable  $t^*$  and examining its optimal condition is provided in appendix 1.

Eq. (35) indicates that the manufacturer has full knowledge about the retailer response and as a result it will participate in the cost of advertisement undertaken by the retailer.

We take derivative from Eq. (29) with respect to  $w$  to get the optimized value of  $w^*$ . Using Eq. (36), it can be written:

$$\frac{\partial \Pi_m}{\partial w} = \frac{k_1^2(1-(1-\varepsilon)w)^2}{8(1-t)} \left[ \frac{1}{2}(1-\varepsilon)(1-(1-\varepsilon)w) - \frac{1}{2}(1-\varepsilon)^2w \right] - \frac{k_1^2}{8(1-t)} \times (1-\varepsilon)^2w(1-(1-\varepsilon)w)^2 +$$

$$\begin{aligned}
 & + \frac{k_2 \sqrt{A}}{2} \left[ (1-\varepsilon)(1-(1-\varepsilon)w) - (1-\varepsilon)^2 w \right] + \frac{tk_1^2}{8^2(1-t)^2} \times 4(1-\varepsilon)(1-(1-\varepsilon)w)^3 = 0 \\
 \Rightarrow w^* & = \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)(1-\varepsilon)} \tag{36}
 \end{aligned}$$

The method for calculating the variable  $w^*$  and examining its optimal condition is provided in appendix 2.

Now, the optimal value of  $\varepsilon^*$  can be obtained using Eq. (37):

$$\begin{aligned}
 \frac{\partial \Pi_m}{\partial \varepsilon} & = \frac{k_1^2(1-(1-\varepsilon)w)^2}{8(1-t)} \left[ -\frac{1}{2}w(1-(1-\varepsilon)w) + \frac{1}{2}(1-\varepsilon)w^2 \right] + \frac{k_1^2}{8(1-t)} \times (1-\varepsilon)w^2(1-(1-\varepsilon)w)^2 + \\
 & + \frac{k_2 \sqrt{A}}{2} \left[ -w(1-(1-\varepsilon)w) - (1-\varepsilon)w^2 \right] - \frac{tk_1^2}{8^2(1-t)^2} \times 4w(1-(1-\varepsilon)w)^3 = 0 \\
 \Rightarrow \Rightarrow 1-\varepsilon & = \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)w} \quad \varepsilon^* = 1 - \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)w} \tag{37}
 \end{aligned}$$

The method for calculating the variable  $\varepsilon^*$  and examining its optimal condition is provided in appendix 3.

Eq. (38) is driven using Eq. (45) and Eq. (46):

$$(1-\varepsilon^*)w^* = \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)} \tag{38}$$

The product  $(1-\varepsilon^*)w^*$  depends on the parameters  $k_1, k_2$  and is constant. So, to optimize  $w^*$  and  $(1-\varepsilon^*)$ , they must be equal. This is because when the product of two variables is constant; these two variables are maximized when they are equal. This is shown in Eq. (39):

$$w^* = 1 - \varepsilon^* \tag{39}$$

Then, Eq. (40) is obtained:

$$w^* = 1 - \varepsilon^* = \left[ \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)} \right]^{\frac{1}{2}} \tag{40}$$

The theorem (1) is obtained using the obtained optimal values:

**Theorem 1:** If the power is distributed asymmetrically between the manufacturer and the retailer and the manufacturer is the leader of the channel, it can be shown using manufacturer-Stackelberg equilibrium that this model has a unique equilibrium, given by Eq. (41):

$$\begin{aligned} \varepsilon^{SM} &= 1 - \left[ \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)} \right]^{\frac{1}{2}}, & w^{SM} &= \left[ \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)} \right]^{\frac{1}{2}} \\ a^{SM} &= \left[ \frac{k_1(1 - (1 - \varepsilon^{SM})w^{SM})(3(1 - \varepsilon^{SM})w^{SM} + 1)}{16} \right]^2, & p^{SM} &= \frac{(1 - \varepsilon^{SM})w^{SM} + 1}{2} \\ A^{SM} &= \left[ \frac{k_2}{4} (1 - \varepsilon^{SM})w^{SM} (1 - (1 - \varepsilon^{SM})w^{SM}) \right]^2, & t^{SM} &= \frac{5(1 - \varepsilon^{SM})w^{SM} - 1}{3(1 - \varepsilon^{SM})w^{SM} + 1} \end{aligned} \quad (41)$$

#### 4.2. Proposed model: retailer-Stackelberg game

Generally, the models proposed in marketing and economics literature for manufacturer-retailer supply chain have focused on a type of relation in which the manufacturer acts as the leader and the retailer follows it. It means that the manufacturer has considerable power and often fully controls the retailer behavior. But, the market structure has shaped during the last two decades in a way that the power has shifted from the manufacturer to the retailer. Indeed, in some cases, the retailers have even gained equal or even higher power than manufacturers. So, in this section, we address the retailer-Stackelberg game by taking this emerging market phenomenon into account.

In this game, the retailer has more influence, power and control as compared to the manufacturer. So, to obtain the equilibrium values, the manufacturer firstly determines the wholesale price, discount percentage, participation rate and national advertisement cost based on the best response function for maximizing its equilibrium profit. That is, the retailer is aware of the manufacturer response in its decision making and the leader decision making problem is solved based on the follower response.

From the manufacturer perspective, it is clear that the optimal value of  $t$  is equal to zero, because it appears with negative coefficient in objective function (29). Similarly, the optimal value for the variable  $\varepsilon$  will be equal to zero, that is the manufacturer doesn't offer any pricing discount and the product is not offered to the retailer with any price reduction. The objective function  $\Pi_m$  has a linear relationship with  $w$ . So, increasing  $w$  will raise the profit function of the manufacturer. Since  $w \leq p \leq 1$ , the optimal value of  $w$  will be equal to  $p$ , but  $w$  cannot be equal to one, since otherwise both parts will not get any profit as is seen in Eq. (42):

$$1 = w \leq p \Rightarrow p = 1 \Rightarrow \Pi_m = \Pi_r = 0 \quad (42)$$

On the other hand, if the manufacturer and the retailer make their optimal decisions simultaneously, their profit margins should be maximized. Based on the profit margin format of the manufacturer and the retailer, if these profit margins are to be maximized, they must be set equal. It means that using Eq. (43), it can be written:

$$\mu_m = w \quad \text{and} \quad \mu_r = p - w$$

$$\mu_m = \mu_r \quad \Rightarrow \quad p - w = w \quad \Rightarrow \quad w^* = p/2 \quad (43)$$

Note that the manufacturer profit increases with  $w$ . But, when the manufacturer is in the follower position, it cannot set the value of  $w$  as large as it wants. In addition, the profit margin of the manufacturer must not be higher than that of the retailer, when the manufacturer is the follower. Indeed, this is a natural constraint that the profit margin of the manufacturer is subjected to. So the value of  $w$  is equal to  $p/2$ . We take derivative from the manufacturer profit function with respect to  $A$  in order to obtain the optimal value of this variable. So, Eq. (44) is obtained:

$$\frac{\partial \Pi_m}{\partial A} = (1 - \varepsilon)w (1 - p) \frac{k_2}{2} A^{-\frac{1}{2}} - 1 = 0 \quad \begin{matrix} w = p/2, \varepsilon = 0 \\ \Rightarrow \end{matrix}$$

$$\frac{p}{2} (1 - p) \frac{k_2}{2} A^{-\frac{1}{2}} - 1 = 0 \Rightarrow A^* = \left( \frac{p}{4} (1 - p) k_2 \right)^2 \quad (44)$$

In the next step, we take the second order derivative from the manufacturer profit function to examine whether the optimal condition and the second condition of the manufacturer profit maximization hold. This is reflected in Eq. (45):

$$\frac{\partial^2 \Pi_m}{\partial A^2} = -\frac{p}{8} (1 - p) k_2 A^{-\frac{3}{2}} \Rightarrow \left. \frac{\partial^2 \Pi_m}{\partial A^2} \right|_{A=A^*} = -\frac{8}{p^2 (1 - p)^2 k_2^2} < 0 \quad \Rightarrow \quad \Pi_m = m \quad a \quad (45)$$

The second order derivative of the manufacturer objective function is with negative  $A^*$ . So, the sufficient condition for maximization problem holds.

We substitute  $w^*$ ,  $t^*$ ,  $A^*$  and  $\varepsilon^*$  into Eq. (24) to obtain Eq. (46):

$$\max \quad \Pi_r(p, a) = \frac{p}{2} (1 - p) \left[ k_1 \sqrt{a} + k_2 \left( \frac{p}{4} (1 - p) k_2 \right) \right] - a$$

(46)

$$s.t. w \leq p \leq 1 \quad \text{and} \quad 0 \leq a$$

Note that, we must have  $p \neq 0$  and  $p \neq 1$ , otherwise the profit function of the retailer is  $-a < 0$ . So, the constraints of Eq. (24) are changed as is represented in Eq. (47):

$$0 < w \leq p < 1 \quad \text{and} \quad 0 \leq a \tag{47}$$

Then, we take derivative from the retailer profit function with respect to these variables and then set the derivative equal to zero to obtain the decision variables of the retailer. That is, it can be written as Eq. (48):

$$p^* = \frac{1}{2} \Rightarrow \tag{48}$$

$$\frac{\partial \Pi_r}{\partial p} = k_1 \sqrt{a} \left( \frac{1}{2}(1-p) - \frac{p}{2} \right) + k_2 \left[ \left( \frac{1}{2}(1-p) - \frac{p}{2} \right) \frac{p}{4}(1-p) + \frac{p}{2}(1-p) \left( \frac{1}{4}(1-p) - \frac{p}{4} \right) \right] = 0$$

Note that the expression  $k_1 \sqrt{a} + k_2 \left( \frac{p}{2}(1-p) \right) > 0$  must be always positive, because Eq. (47)

suggests that  $0 < p < 1$ .

We obtain the second order derivative with respect to this variable using Eq. (49) to examine whether the  $p^*$  value is optimized:

$$\begin{aligned} \frac{\partial^2 \Pi_r}{\partial p^2} &= -k_1 \sqrt{a} - k_2 \frac{p}{2}(1-p) + k_2 \left( \frac{1}{2} - p \right)^2 \\ \Rightarrow \frac{\partial^2 \Pi_r}{\partial p^2} \Big|_{p=p^*} &= -k_1 \sqrt{a} - k_2 \frac{p}{8} < 0 \Rightarrow \Pi_r = \text{max} \end{aligned} \tag{49}$$

The second derivative of the retailer objective function is with negative  $p^*$ . So, the sufficient condition for maximization problem is satisfied.

Eq. (50) can be used to obtain the optimal value of  $a^*$ :

$$\frac{\partial \Pi_r}{\partial a} = \frac{p}{4}(1-p)k_1 a^{-\frac{1}{2}} - 1 = 0 \Rightarrow a^* = \left( \frac{k_1}{16} \right)^2 \tag{50}$$



In order to examine whether the  $a^*$  value is optimal, we take second order derivative with respect to this variable as represented in Eq. (51):

$$\frac{\partial^2 \Pi_r}{\partial a^2} = -\frac{1}{32} k_1 a^{-\frac{3}{2}} \Rightarrow \left. \frac{\partial^2 \Pi_r}{\partial a^2} \right|_{a=a^*} = -\frac{128}{k_1^2} < 0 \Rightarrow \Pi_r = \max \tag{51}$$

The second order derivative of the retailer objective function is with negative  $a^*$ . So, the sufficient Condition for maximization problem holds. The theorem (2) is resulted using the obtained optimal values:

**Theorem 2:** If the power is asymmetrically distributed between the manufacturer and the retailer, and the retailer is the channel leader, it can be shown using the retailer-Stackelberg equilibrium that this model has a unique equilibrium, given by Eq. (52):

$$a^{SR} = \left(\frac{k_1}{16}\right)^2, \quad p^{SR} = 1/2, \quad A^{SR} = \left(\frac{k_2}{16}\right)^2, \quad w^{SR} = 1/4, \quad \varepsilon^{SR} = 0, \quad t^{SR} = 0 \tag{52}$$

**Table 1.** A summary of optimal solutions of two game models

decision variables	manufacturer-Stackelberg game(SM)	retailer-Stackelberg game(SR)
$w$	$\left[ \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)} \right]^{\frac{1}{2}}$	1/4
$p$	$\frac{(1 - \varepsilon^{SM})w^{SM} + 1}{2}$	1/2
$A$	$\left[ \frac{k_2}{4} (1 - \varepsilon^{SM})w^{SM} (1 - (1 - \varepsilon^{SM})w^{SM}) \right]^2$	$(k_2/16)^2$
$a$	$\left[ \frac{k_1(1 - (1 - \varepsilon^{SM})w^{SM})(3(1 - \varepsilon^{SM})w^{SM} + 1)}{16} \right]^2$	$(k_1/16)^2$
$t$	$\frac{5(1 - \varepsilon^{SM})w^{SM} - 1}{3(1 - \varepsilon^{SM})w^{SM} + 1}$	0
$\varepsilon$	$1 - \left[ \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4}}{(9k_1^2 + 16k_2^2)} \right]^{\frac{1}{2}}$	0

SM – Stackelberg manufacturer. SR – Stackelberg retailer.

## 5. Results analysis

In this section, we analyze the optimal solutions of all two game models. Using the optimal solutions summarized in table 1, all decision variables can be considered a function of the parameters  $k_1$  and  $k_2$ . So, all the comparisons are made with respect to various values of  $k_1$  and  $k_2$  which reflect the effect of local and national advertisement on the demand, respectively. The quality of the results is considerably dependent on how well the parameters have been estimated. So, to determine the best policy of both decision makers, we must estimate these parameters as the first step. To this end, a deep research on the market of the product is required in order to determine how the local and national advertisements affect the demand behavior?

### 5.1. Comparison of prices and discount percentage

In the Stackelberg-manufacturer game, the retailing price is higher than an arrangement in which the retailer plays the role of the leader. This is due to the fact that the manufacturer forces a higher wholesale price in this situation and as a result the retailer choose higher retail price to earn significant profit. But, when the retailer is the leader, it sets a low retailing price to earn higher marginal profit, thereby persuades the manufacturer to set a lower wholesale price. It is obvious that the wholesale price in the manufacturer-Stackelberg game is higher than the retailer-Stackelberg game, because the manufacturer is the leader in this setting. As is shown in table 1, the discount percentage of the manufacturer price in the retailer-Stackelberg game is equal to zero, because the manufacturer has not any information about the retailer activity. But, when the manufacturer acts as the channel leader, it is equipped to full knowledge about how the retailer will respond, making the manufacturer to offer price discount to the retailer.

### 5.2. Comparison of advertisement costs and participation rate

The retailer expenditures for local advertisement cost in the the manufacturer-Stackelberg game is more than what it may spend in the retailer-Stackelberg game for the same purpose, because the manufacturer participates in local advertisement ( $t^{SM} > 0$ ).

The results related to national advertisement varies based on the values of the parameters  $k_1$  and  $k_2$ . As is shown in Fig. 1, in the region I, when the ratio of the parameter  $k_1$  to the parameter  $k_2$  is small (approximately 0.1), the cost of national advertisement in the manufacturer-Stackelberg game is equal to this cost in the retailer-Stackelberg game, while in the region II, this cost in the manufacturer-Stackelberg game is lower than the same cost in the retailer.

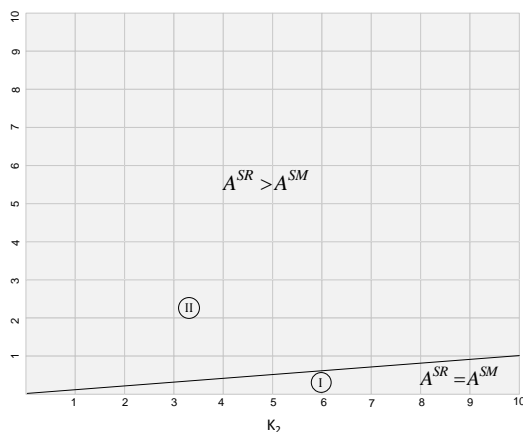


Figure 1. Comparison of national advertisement cost in two game models

Table 1 shows that the participation rate of the manufacturer in the cost of local advertisement undertaken by the retailer is zero in the retailer-Stackelberg game, because the manufacturer has no information about the retailer activity. But, when the manufacturer is the channel leader, it participates in the cost of the retailer advertisement efforts, because it is fully aware of how the retailer will respond.

### 5.3. Comparison of profit

Profit is the most important criterion for measuring the supply chain performance. In this section, we analyze the profit of each member of the chain and the whole system. The desirable profit of the retailer is compared in Fig. 2 for different situations. As it is shown, the retailer prefers to be the channel leader in the region I, while it is interested to be the follower in the region II.

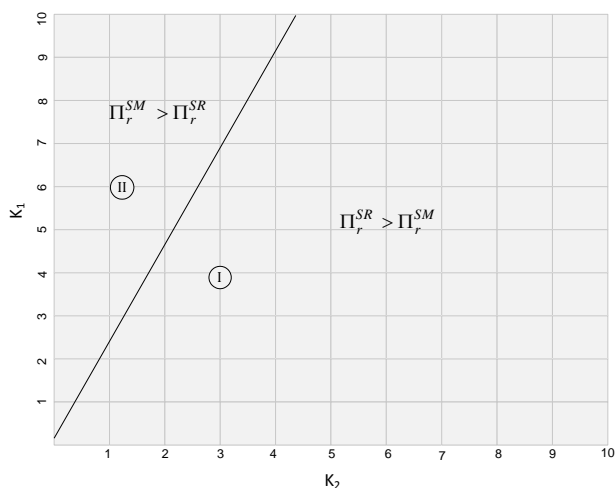


Figure 2. Comparison of retailer profit in two game models

The manufacturer prefers the retailer to be the follower, rather than to be the leader, because the

manufacturer always gains higher profit in this setting. . Generally, Eq. (53) holds:

$$\Pi_m^{SR} > \Pi_m^{SM} \quad \forall(k_1, k_2): \tag{53}$$

The profit of the whole chain in the retailer-Stackelberg game is higher as compared to the manufacturer-Stackelberg game.

### 6. Experimental results

The decision variables are sensitive to the parameters  $k_1$  and  $k_2$ . Thus, in this section, we consider different values for these parameters and calculate the decision variables accordingly. Different numerical values are provided to compare the prices, the advertisement cost and the profit for both proposed scenarios.

The values of  $k_1$  and  $k_2$  are shown in table 2. In each example, the value of decision variables for the manufacturer and the retailer has been calculated for each game. Then, the profit obtained from each game has been specified for both members of the supply chain along with the profit of the whole system. The obtained numerical results are provided in table 3 and table 4.

**Table2.** Values of Parameters  $k_1$  and  $k_2$  in the examples

Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7	Example 8	Example 9	Example 10
$k_1 = 1$	$k_1 = 4$	$k_1 = 7$	$k_1 = 10$	$k_1 = 1$	$k_1 = 2$	$k_1 = 9$	$k_1 = 8$	$k_1 = 9$	$k_1 = 0.5$
$k_2 = 1$	$k_2 = 4$	$k_2 = 2$	$k_2 = 1$	$k_2 = 10$	$k_2 = 5$	$k_2 = 6$	$k_2 = 10$	$k_2 = 4$	$k_2 = 6$

#### 6.1. Numerical examples for manufacturer-Stackelberg game

The numerical results obtained from the manufacturer-Stackelberg game are provided in table 3. The results of 10 examples are shown here.

**Table3.** The results obtained from numerical examples of the manufacturer-Stackelberg game

Example	Example	Example	Example	Example	Example	Example	Example	Example	Example	
1	2	3	4	5	6	7	8	9	10	
0.6451	0.6451	0.5872	0.5786	0.7060	0.6913	0.6187	0.6597	0.5992	0.7063	$w^{SM}$
0.7081	0.7081	0.6724	0.6667	0.7492	0.7390	0.6914	0.7176	0.6795	0.7495	$p^{SM}$
0.6451	0.6451	0.5872	0.5786	0.7060	0.6913	0.6187	0.6597	0.5992	0.7063	$\varepsilon^{SM}$
0.4806	0.4806	0.3559	0.3363	0.5980	0.5710	0.4255	0.5101	0.3828	0.5986	$t^{SM}$
0.0067	0.1077	0.3401	0.6944	0.0061	0.0252	0.5563	0.4239	0.5608	0.0015	$a^{SM}$
0.0037	0.0590	0.0128	0.0031	0.3906	0.0973	0.1256	0.3776	0.0530	0.1406	$A^{SM}$
0.0087	0.1388	0.2433	0.4671	0.3955	0.1171	0.5221	0.6977	0.4407	0.1418	$\Pi_r^{SM}$
0.0104	0.1667	0.3528	0.6975	0.3967	0.1225	0.6819	0.8015	0.6138	0.1422	$\Pi_m^{SM}$
0.0191	0.3055	0.5961	1.1646	0.7922	0.2396	1.2040	1.4992	1.0545	0.2840	$\Pi_{m+r}^{SM}$

manufacturer-Stackelberg game

The following results can be driven from the manufacturer-Stackelberg game:

1. In the manufacturer-Stackelberg game, if the values of the parameters  $k_1$  and  $k_2$  are equal, the wholesale price will be equal to a constant value. The same result holds for the retailing price, the rate of participation in advertisement and the percent of price discount.
2. In the manufacturer-Stackelberg game, the higher the value of  $k_2$  is compared to the value of  $k_1$ , the higher the wholesale price is. The same result holds true for retailing price, the rate of participation in advertisement and the percent of price discount.
3. In the manufacturer-Stackelberg game, the manufacturer profit is higher than the sales profit for every value of the parameters  $k_1$  and  $k_2$ .
4. In the manufacturer-Stackelberg game, the cost of local advertisement is higher than this cost in the retailer-Stackelberg games.

### 6.2. Numerical examples for retailer-Stackelberg game

The numerical results obtained from the retailer-Stackelberg game are provided in table 4. The results of 10 examples are shown in this table.

**Table4.** Results obtained from retailer-Stackelberg game for 10 examples

Example	Example	Example	Example	Example	Example	Example	Example	Example	Example	
1	2	3	4	5	6	7	8	9	10	
0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	$w^{SR}$
0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	$p^{SR}$
0.0039	0.0625	0.1914	0.3906	0.0039	0.0156	0.3164	0.2500	0.3164	0.0010	$a^{SR}$
0.0039	0.0625	0.0156	0.0039	0.3906	0.0977	0.1406	0.3906	0.0625	0.1406	$A^{SR}$
0.0117	0.1875	0.2227	0.3984	0.7852	0.2109	0.5977	1.0313	0.4414	0.2822	$\Pi_r^{SR}$
0.0117	0.1875	0.3984	0.7852	0.3984	0.1289	0.7734	0.8906	0.6953	0.1426	$\Pi_m^{SR}$
0.0234	0.3750	0.6211	1.1836	1.1836	0.3398	1.3711	1.9219	1.1367	0.4248	$\Pi_{m+r}^{SR}$

retailer-Stackelberg game

The following results can be driven from the retailer-Stackelberg game:

1. In the retailer-Stackelberg game, if the values of the parameters  $k_1$  and  $k_2$  are equal, the amount of national and local advertisement will be equal and the manufacturer and the retailer equally gain profit from this game.
2. In the retailer-Stackelberg, the higher the value of  $k_1$  is compared to the value of  $k_2$ , the higher the amount of local advertisement will be compared to national advertisement and vice versa.
3. In the retailer-Stackelberg game, the amount of local advertisement increases as the value of  $k_1$  rises. The same result holds for national advertisement. That is, the amount of national advertisement increases as the value of  $k_2$  rises.
4. In the retailer-Stackelberg game, if the value of  $k_1$  is higher than the value of  $k_2$ , the manufacturer gains higher profit in this game compared to the retailer.

**7. Conclusion and suggestions for future works**

In this study, two classic relations between a manufacturer and a retailer are used based on the game theory to optimize the decisions. The optimized decisions are achieved through using coordination of pricing, cooperative advertisement and price discount. These decisions create cooperation among the supply chain members and increase the profit of the members and the

whole chain. The first and second game models were the manufacturer-Stackelberg and the retailer-Stackelberg games. In both game models, we deal with asymmetric power distribution in the chain. Asymmetric power distribution refers to a situation in which one member has more power compared to other member and one member plays the role of the leader, while other follows it. The demand of the consumer depends on the retailing price and the costs of advertisement undertaken by the chain members. Cooperative advertisement is employed to coordinate two members of the chain, in which the manufacturer pays for a portion of the cost of local advertisement undertaken by the retailer. In addition, the manufacturer offers higher wholesale price discount to create more coordination in the supply chain. This leads to the moderation of retailing price and as a result the moderation of the demand. This increase in demand increases the profit of both members of the supply chain.

We performed sensitivity analysis on these models with respect to the parameters  $k_1$  and  $k_2$ . Then, 10 numerical examples were provided for the supply chain and optimal solutions for each game were obtained. The obtained results indicate that when the manufacturer is the channel leader, the wholesale price is higher than an arrangement in which the manufacturer is the follower of the retailer. As a result, the retailing price in the manufacturer-Stackelberg game is higher than this price in retailer-Stackelberg game. The discount percent of the manufacturer price in the retailer-Stackelberg game is equal to zero, because the manufacturer has no information about the retailer activity. But, when the manufacturer is the channel leader, it is fully aware of the retailer reaction. consequently, the manufacturer offers price discount to the retailer. The retailer spends more in the manufacturer-Stackelberg game compared to the retailer-Stackelberg game, because the manufacturer participates in the cost of local advertisement undertaken by the retailer. But, maximum cost of national advertisement efforts made by the manufacturer is different for two games and depends on the values of the parameters  $k_1$  and  $k_2$ . When the manufacturer follows the retailer, its profit for every value of the parameters  $k_1$  and  $k_2$  is higher than a situation in which it is the channel leader. Also, the maximum profit of the retailer is different in various games and depends on the values of  $k_1$  and  $k_2$ . Additionally, the profit of the whole supply chain in the retailer-Stackelberg game is higher than this profit in manufacturer-Stackelberg game.

The following suggestions may be useful guide for the researchers in their future efforts:

- One possible extension and refinement of the current study is to include three levels or higher levels for supply chain or examine multiple products in the supply chain. Solving and modeling such game is more difficult, yet more applications can be envisioned for it.
- In the demand function used in this model, the dependence of demand on the product price and advertisement is considered as the only factor that affect the demand elasticity. In today economics, other factors beyond the final price of the product such as marketing costs, discounts, special sales, after-sales service etc. affect the product demand. So, more complex

and realistic demand function can be considered in future researches to create collaboration in the supply chain through game theory approach.

- If more retailers as well as decision making in oligopoly setting are included in our model, it may be more realistic.

## References

Amrouche N., Yan R. (2015), Aggressive or partnership strategy: Which choice is better for the national brand?, *International Journal of Production Economics*, Vol. 166, PP. 50–63.

Aust G., Buscher U. (2012), Vertical cooperative advertising and pricing decisions in a manufacturer–retailer supply chain: A game-theoretic approach, *European Journal of Operational Research*, Vol. 223, No. 2, PP. 473–482.

Aust G., Buscher U. (2015), Cooperative advertising models in supply chain management: A review, *European Journal of Operational Research*, Vol. 234, PP. 1–14.

Bergen M., John G. (1997), Understanding cooperative advertising participation rates in conventional channels, *Journal of Marketing Research*, Vol. 34, No. 3, PP. 357–369.

Berger P.D. (1972), Vertical cooperative advertising ventures, *Journal of Marketing Research*, Vol. 9, No., 3, PP. 309–312.

Chen K., Xiao T. (2009), Demand disruption and coordination of the supply chain with a dominant retailer, *European Journal of Operational Research*, Vol. 197, No. 1, PP. 225–234.

Chen T. (2015), Effects of the pricing and cooperative advertising policies in a two-echelon dual-channel supply chain, *Computers & Industrial Engineering*, Vol. 87, PP. 250–259.

Chen X., Wang X. (2015), Free or bundled: Channel selection decisions under different power structures, *Omega*, Vol. 53, PP. 11–20.

Corbett C.J., de Groote X. (2000), A Supplier's Optimal Quantity Discount Policy Under Asymmetric Information, *Management Science*, Vol. 46, No. 3, PP. 444–450.

Dolan R.J. (1978), A normative model of industrial buyer response to quantity discounts, In: Jain, S.C. (Ed.), *Research Frontiers in Marketing: Dialogues and Directions*, American Marketing Association, Chicago, Vol. 23, PP. 121–125.

Gerstner E., Hess J.D. (1995), Pull promotion and channel coordination, *Marketing Science*, Vol.14, No. 1, PP. 43–60.

Huang Z., Li S.X. (2001), Co-op advertising models in manufacturer–retailer supply chains: A game theory approach, *European Journal of Operational Research*, Vol. 135, No. 3, PP. 527–544.

Huang Z., Li S.X., Mahajan V. (2002), An analysis of manufacturer–retailer supply chain coordination in



cooperative advertising, *Decision Sciences*, Vol. 33, No. 3, PP. 469–494.

Jeuland A., Shugan S. (1983), Managing channel profits, *Marketing Science*, Vol. 2, No. 3, PP. 239–272.

Jeuland, A., Shugan, S. (1988), Channel of distribution profits when channel members form conjectures, *Marketing Science*, Vol. 7, No. 2, PP. 202–210.

Jorgensen S., Sigue S.P., Zaccour G. (2001), Stackelberg leadership in a marketing channel, *International Game Theory Review*, Vol. 3, No. 1, PP. 13–26.

Jorgensen S., Zaccour G. (2003), A differential game of retailer promotions, *Automatica*, Vol. 39, No. 7, PP. 1145–1155.

Jorgensen S., Zaccour G. (2014), A survey of game-theoretic models of cooperative advertising, *European Journal of Operational Research*, Vol. 237, No. 1, PP. 1–14.

Li X., Wang Q. (2007), Coordination mechanisms of supply chain systems, *European Journal of Operational Research*, Vol. 179, No. 1, PP. 1-16.

Malone T.W., Crowston K. (1994), The interdisciplinary study of coordination, *Journal ACM Computing Surveys*, Vol. 26, No. 1, PP. 87-119.

Piana V. (2004), Consumer Decision Rules for Agent-Based Models, *Economics Web Institute*.

Roslow S., Laskey H.A., Nicholls J.A.F. (1993), The Enigma of Cooperative Advertising, *Journal of Business & Industrial Marketing*, Vol. 8, No. 2, PP. 70-79.

Sahin F., Robinson E. (2002), Flow coordination and information sharing in supply chains: Review, implications, and directions for future research, *Decision Sciences*, Vol. 33, No. 4, PP. 505–536.

Sarmah S.P., Acharya D., Goyal S.K. (2006), Buyer vendor coordination models in supply chain management, *European Journal of Operational Research*, Vol. 175, No. 1, PP. 1-15.

Seyedesfahani M.M., Biazaran M., Gharakhani M. (2011), A game theoretic approach to coordinate pricing and vertical co-op advertising in manufacturer–retailer supply chains, *European Journal of Operational Research*, Vol. 211, No. 2, PP. 263-273.

Somers T.M., Gupta Y., Harriot S.R. (1990), Analysis of cooperative advertising expenditures: a transfer-function modeling approach, *Journal of Advertising Research*, Vol. 30, No. 5, PP. 35-49.

Szmerekovsky J., Zhang J. (2009), Pricing and two-tier advertising with one manufacturer and one retailer, *European Journal of Operational Research*, Vol. 193, No. 3, PP. 904-917.

Xiao T., Yang D. (2008), Price and service competition of supply chains with risk-averse retailers under demand uncertainty, *International Journal of Production Economics*, Vol. 114, No. 1, PP. 187-200.

Xie J., Neyret A. (2009), Co-op advertising and pricing models in manufacturer-retailer supply chains,

*Computers & Industrial Engineering*, Vol. 56, No. 4, PP. 1375-1385.

Xie J., Wei J. (2009), Coordinating advertising and pricing in a manufacturer–retailer channel, *European Journal of Operational Research*, Vol.192, No. 2, PP. 785-791.

Yue J., Austin J., Wang M., Huang Z. (2006), Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount, *European Journal of Operational Research*, Vol. 168, No. 1, PP. 65–85.

Zhang W., Li Sh., Zhang D., Hou W. (2014), On the impact of advertising initiatives in supply chains, *European Journal of Operational Research*, Vol. 234, No. 1, PP. 99–107.

## Appendix

### Appendix 1: Calculating optimal value of $t$ in manufacturer-Stackelberg game and examining whether $t^*$ is optimal

We take derivative from Eq. (30) with respect to  $t$  that we could obtain the optimal value of  $t$  in the manufacturer-Stackelberg game. Eq. (A1) gives us:

$$\frac{\partial \Pi_m}{\partial t} = \frac{k_1^2}{16} (1 - (1 - \varepsilon)w)^3 (1 - t)^{-2} \left[ (1 - \varepsilon)w - \frac{1 + t}{4(1 - t)} (1 - (1 - \varepsilon)w) \right] = 0 \quad (A1)$$

Using Eq. (31), it can be concluded that the conditions  $1 - t \neq 0$  and  $1 - (1 - \varepsilon)w \neq 0$  must be satisfied for differentiability of  $\Pi_m$ . As a result, using Eq. (A2), we have:

$$\left[ (1 - \varepsilon)w - \frac{1 + t}{4(1 - t)} (1 - (1 - \varepsilon)w) \right] = 0 \Rightarrow t^* = \frac{5(1 - \varepsilon)w - 1}{3(1 - \varepsilon)w + 1} \quad (A2)$$

Now, we take the second order derivative from  $\Pi_m$  with respect to  $t$  using Eq. (A3) to examine whether  $t^*$  is optimal:

$$\frac{\partial^2 \Pi_m}{\partial t^2} = -\frac{k_1^2}{8} (1 - \varepsilon)w (1 - (1 - \varepsilon)w)^3 (1 - t)^{-3} - \frac{(1 - (1 - \varepsilon)w)^4 k_1^2}{8^2} \times \frac{2(2 + t)}{(1 - t)^4} \quad (A3)$$

In the next step, we simplify Eq. (A3) to drive Eq. (A4):

$$\frac{\partial^2 \Pi_m}{\partial t^2} \Big|_{t=t^*} = -\frac{k_1^2}{8} \left( \frac{1}{3(1 - \varepsilon)w + 1} \right)^3 \left[ (1 - \varepsilon)w + \frac{11(1 - \varepsilon)w + 1}{8} \right] < 0 \Rightarrow \Pi_m = \max \quad (A4)$$

The second order derivative of the manufacturer objective function is with negative  $t^*$ . So, the sufficient condition for maximization problem is met.

**Appendix 2: Calculating optimal value of  $w$  in manufacturer-Stackelberg game and examining whether  $w^*$  is optimal**

We take derivative from Eq. (30) with respect to  $w$  to obtain the optimal value of  $w$  in the manufacturer-Stackelberg game. Eq. (A5) gives us:

$$\frac{\partial \Pi_m}{\partial w} = \frac{k_1^2(1-(1-\varepsilon)w)^2}{8(1-t)} \left[ \frac{1}{2}(1-\varepsilon)(1-(1-\varepsilon)w) - \frac{1}{2}(1-\varepsilon)^2w \right] - \frac{k_1^2}{8(1-t)} \times (1-\varepsilon)^2w(1-(1-\varepsilon)w)^2 + \frac{k_2\sqrt{A}}{2} \left[ (1-\varepsilon)(1-(1-\varepsilon)w) - (1-\varepsilon)^2w \right] + \frac{tk_1^2}{8^2(1-t)^2} \times 4(1-\varepsilon)(1-(1-\varepsilon)w)^3 = 0 \tag{A5}$$

Substituting  $A^*$  and  $t^*$  from Eq. (33) and Eq. (35) into Eq. (A5) gives us Eq. (A6):

$$\frac{\partial \Pi_m}{\partial w} = \frac{(1-\varepsilon)}{64} (1-(1-\varepsilon)w) \left[ 2k_1^2(3(1-\varepsilon)w+1)(1-4(1-\varepsilon)w) + 8k_2^2(1-\varepsilon)w(1-2(1-\varepsilon)w) + k_1^2(5(1-\varepsilon)w-1)(3(1-\varepsilon)w+1) \right] = 0 \tag{A6}$$

Using Eq. (31), it can be concluded that the conditions  $1-\varepsilon \neq 0$  and  $1-(1-\varepsilon)w \neq 0$  must be satisfied for differentiability of  $\Pi_m$ . As a result, using Eq. (A7), we have:

$$\left[ 2k_1^2(3(1-\varepsilon)w+1)(1-4(1-\varepsilon)w) + 8k_2^2(1-\varepsilon)w(1-2(1-\varepsilon)w) + k_1^2(5(1-\varepsilon)w-1)(3(1-\varepsilon)w+1) \right] = 0$$

$$\Rightarrow w^* = \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2+k_1^2) + 9k_1^2}}{(9k_1^2 + 16k_2^2)(1-\varepsilon)} \tag{A7}$$

Now, we take the second order derivative from  $\Pi_m$  with respect to  $w$  using Eq. (A8) to examine whether  $w^*$  is optimal:

$$\frac{\partial^2 \Pi_m}{\partial w^2} = 2 \left[ -9k_1^2(1-\varepsilon)^2 - 16k_2^2(1-\varepsilon)^2 \right] w + 8k_2^2(1-\varepsilon)$$

$$\Pi_m = \max \tag{A8}$$

$$\frac{\partial^2 \Pi_m}{\partial w^2} \Big|_{w=w^*} = -2(1-\varepsilon)\sqrt{16k_2^2(k_2^2+k_1^2)+9k_1^4} < 0 \Rightarrow$$

The second order derivative of the manufacturer objective function is with negative  $w^*$ . So, the sufficient condition for maximization problem is met.

**Appendix 3: Calculating optimal value of  $\varepsilon$  in manufacturer-Stackelberg game and examining whether  $\varepsilon^*$  is optimal**

We take derivative from Eq. (30) with respect to  $\varepsilon$  to obtain the optimal value of  $\varepsilon$  in the manufacturer- Stackelberg game. Eq. (A9) gives us:

$$\frac{\partial \Pi_m}{\partial \varepsilon} = \frac{k_1^2(1-(1-\varepsilon)w)^2}{16(1-t)} [-w(1-4(1-\varepsilon)w)] + \frac{k_2\sqrt{A}}{2} [-w(1-2(1-\varepsilon)w)] - \frac{tk_1^2}{8^2(1-t)^2} \times 4w(1-(1-\varepsilon)w)^3 = 0 \tag{A9}$$

Substituting  $A^*$  and  $t^*$  from Eq. (33) and Eq. (35) into Eq. (A9) gives us Eq. (A10):

$$\frac{\partial \Pi_m}{\partial \varepsilon} = \frac{w}{64} (1-(1-\varepsilon)w) [-2k_1^2(3(1-\varepsilon)w+1)(1-4(1-\varepsilon)w) - 4k_2^2(1-\varepsilon)w(1-2(1-\varepsilon)w) - k_1^2(3(1-\varepsilon)w+1)(5(1-\varepsilon)w-1)] = 0 \tag{A10}$$

Using Eq. (31), it can be concluded that the conditions  $1-(1-\varepsilon)w \neq 0$  must be satisfied for differentiability of  $\Pi_m$ . As a result, using Eq. (A11), we have:

$$[-2k_1^2(3(1-\varepsilon)w+1)(1-4(1-\varepsilon)w) - 4k_2^2(1-\varepsilon)w(1-2(1-\varepsilon)w) - k_1^2(3(1-\varepsilon)w+1)(5(1-\varepsilon)w-1)] = 0$$

$$\Rightarrow \varepsilon^* = 1 - \frac{4k_2^2 + \sqrt{16k_2^2(k_2^2+k_1^2)+9k_1^4}}{(9k_1^2+16k_2^2)w} \tag{A11}$$

Now, we take the second order derivative from  $\Pi_m$  with respect to  $\varepsilon$  using Eq. (A12) to examine whether  $\varepsilon^*$  is optimal:

$$\frac{\partial^2 \Pi_m}{\partial \varepsilon^2} = -2[9k_1^2 w^2 + 16k_2^2 w^2](1 - \varepsilon) + 8k_2^2 w$$

$$\frac{\partial^2 \Pi_m}{\partial w^2} \Big|_{w=w^*} = -2w \sqrt{16k_2^2(k_2^2 + k_1^2) + 9k_1^4} < 0 \Rightarrow \Pi_m = \text{max} \quad (\text{A12})$$

The second order derivative of the manufacturer objective function is with negative  $\varepsilon^*$ . So, the sufficient condition for maximization problem is met.