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Mathematical modeling for EOQ inventory system with advance payment and fuzzy Parameters

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Abstract
This study considers an EOQ inventory model with advance payment policy in a fuzzy situation by employing two types of fuzzy numbers that are trapezoidal and triangular. Two fuzzy models are developed here. In the first model the cost parameters are fuzzified, but the demand rate is treated as crisp constant. In the second model, the demand rate is fuzzified but the cost parameters are treated as crisp constants. For each fuzzy model, we use signed distance method to defuzzify the fuzzy total cost and obtain an estimate of the total cost in the fuzzy sense. Numerical example is provided to ascertain the sensitiveness in the decision variables about fuzziness in the components. In practical situations, costs may be dependent on some foreign monetary unit. In such a case, due to a change in the exchange rates, the costs are often not known precisely. The first model can be used in this situation. In actual applications, demand is uncertain and must be predicted. Accordingly, the decision maker faces a fuzzy environment rather than a stochastic one in these cases. The second model can be used in this situation. Moreover, the proposed models can be expended for imperfect production process.

Keywords: Inventory costs; Advance payment; Fuzzy numbers; Signed distance method.

1. Introduction

An advance payment is a sum of money that is provided in advance of the date that it is contractually due, or in advance of delivery of the products or services that the payment is intended to pay for. Such payments may be structured into a contract or offered to address a specific adversity situation. In some cases, the full amount due will be paid in advance, while in

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others, part of the money will be offered in advance and the other part will be paid later. A common reason to offer an advance payment is because someone is in immediate need of funds that will be due even though the person has not yet earned them. Many authors are offered advances on their work to provide them with money to pay off debts and to live on while they finish their books.
Likewise, it is not uncommon for an insurance company to provide part of a settlement in advance to help the customer recover more quickly. People can also arrange advances on paychecks and salary through their employers to address troubled economic circumstances. Some everyday examples of advance payments are prepaid cell phones, or simply prepaying your rent or utilities as many people do now. In Chinese automobile industry, advance payment to the manufacturer is a common practice for the 4S (sale, spare-part, service, and survey) stores who are required to pay all of the payment before delivery. In Chinese steel industry, huge steel plants also require advance payment, especially for small buyers. Sometimes, the buyer is offered a price discount if he/she gives advance payment even not required by the seller, such cases can be found in bricks and tile factories in India (Zhang et al. (2014)).

Economic order quantity-based inventory models minimize the sum of mainly two costs, that are the holding and the ordering costs. These models assume that the input parameters as cost and demand are described as crisp values or having crisp statistical distributions where their total inventory cost functions are minimized without ambiguity in the results. But exact values of the cost characteristics are rarely attained as they may be vague and imprecise to certain extent in practical situations. For instance, the shortage in an inventory system may occur due to various causes, viz. sudden increase of demand, transportation problems, hike in wages, delayed production, etc. Shortage brings loss of goodwill and it is difficult to measure the exact amount of shortage cost. The same problem is also experienced in the case of the ordering and holding costs. Similarly, the demand becomes extremely variable because of shorter product life cycles in the highly competitive market. Thus in inventory system, the decision maker may allow some flexibility in the cost parameter values to tackle the uncertainties that always fit the real situations. Statistical treatment of the cost characteristics is inefficient for these models, because of the lack of random observations. Therefore, the practitioner should be more careful in accounting flexibility in the cost components and demand.

In this paper, an EOQ inventory model with advance payment policy is described by establishing fuzziness in the cost and demand parameters. The main contribution of this paper is to establish the mathematical model and propose a solving approach for the EOQ inventory problem with advance payment policy in fuzzy random environment. Our main objective is to study the impact and sensitiveness of the impreciseness of cost components in the decision variables and the total cost. This research may be very useful for Chinese automobile and steel industries. The next section reviews the relevant literature on EOQ inventory models.

2. Literature review

Some vendors offer their buyers a delay period in purchasing payment. Therefore, before the end of delay period, buyers can sell the goods and accumulate revenue and earn interest. It is a very common policy used by vendors to promote their commodities. The buyer's inventory policies under delayed payment have been widely addressed in recent decades. In literature, Goyal (1985) was the first to consider the vendor's delayed payment in analyzing the buyer's EOQ. Later, lots of papers extended the work of Goyal (1985) under various practical situations. Reviews of literature on inventory systems with trade credit are available in Seifert et al. (2013).

Although the influences of delayed payment on inventory policies have attracted great attention, the advance payment and its influences on inventory decisions are rarely discussed in the literature. Maiti et al. (2009) considered the advance payment and developed an inventory model for one item in stochastic environment with price-dependent demand over a finite time horizon. They assumed that the buyer's procurement price dependent on the fraction of the advance payment. Gupta et al. (2009) further extended the model by considering the imprecise information of the cost. Thangam (2012) incorporated the advance payment scheme and two-echelon trade credits into an EOQ model for perishable items. Recently, Zhang et al. (2014) developed an EOQ model with two types of advance payment, i.e., all payment paid in advance and partial advance payment.

Fuzzy set theory, introduced by Zadeh (1965), has been receiving considerable attention amongst researchers in production and inventory management. Several authors have applied the fuzzy set concepts to deal with the inventory control problems. Ishii and Konno (1998) fuzzified the shortage cost to an L-fuzzy number in the classical newsboy problem. A backorder inventory model which fuzzifies the order quantity as triangular and trapezoidal fuzzy numbers and keeps the shortage cost as a crisp parameter was developed by Yao and Lee (1999). Yao and Chiang (2003) developed an EOQ model with the total demand and the unit carrying cost being triangular fuzzy numbers. They used the signed distance and the centroid as defuzzification methods. Vijayan and Kumaran (2008) considered inventory models with partial backorders and lost sales and fuzzy stock-out periods and used signed distance method to defuzzify. Bjrk (2009) addressed an EOQ inventory model with backorders, where the demand and the lead times are kept fuzzy (general triangular fuzzy numbers). Sadjadi et al. (2010) propose a pricing and marketing model with fuzzy parameters. In their model, there are three elasticity, namely, selling price, marketing expenditure, and lot size are assumed to be fuzzy numbers. Liu (2012) developed a solution for fuzzy integrated production and marketing planning based on extension principle method. Recently, Mahata and Goswami (2013) developed inventory models for items with imperfect quality and shortage backordering in fuzzy environments by employing two types of fuzzy numbers such as trapezoidal and triangular.

From the authors' literature, none of the authors developed EOQ inventory model with advance payment policy and fuzziness in the cost and demand parameters. Therefore, this paper intends to fill this remarkable gap in the literature. In this study, we represent the cost parameters such as ordering cost A , holding cost h and purchasing cost p, and demand rate D by two types of fuzzy numbers which are trapezoidal and triangular. Two fuzzy models are developed. In the first model the cost parameters are fuzzified while the demand rate is treated as crisp constant. In the second model the demand rate is fuzzified but the cost parameters are treated as crisp constants. For each fuzzy model, a method of defuzzification, namely the signed distance, is employed to find the estimate of total cost in the fuzzy sense, and then the corresponding optimal cycle time is derived to minimize the total cost. Numerical example is carried out to investigate the behavior of our proposed models, and the results are compared with those obtained from the crisp model.

The article is designed as follows: Section 3 describes the research methodology to demonstrate the nature of the research. Section 4 outlines preliminary concepts that have been used for model building purposes. In Section 5, notations and assumptions are given. The formulations of both models are discussed in Section 6 and 7, respectively. Then we use an example to illustrate the idea of this paper in Section 8. Finally, some conclusions are drawn from the discussion in Section 9.

3. Research methodology

The methodology used in this paper is the quantitative method, which is based on the principles of Operational Research and Management Science. The models are built on mathematically oriented inventory theory. Solving models cannot be meaningfully discussed until the formulation process is completed. In this study, we shall concentrate on showing how mathematical analysis can be used to help operating rules for controlling EOQ inventory system. It is necessary to describe mathematically the system to be studied when mathematics is applied to the solution of inventory problems. Such a description is often related to as a mathematical model. Mathematical models are idealized representations, but that are expressed in terms of mathematical symbols and notations. A mathematical model of a real life problem includes the system of equations that describe the essence of the problem. This study is based upon mathematical modeling.

Mathematical modeling is to describe a real world problem with tractable mathematical formulations. These mathematical formulations should not only produce values of performance measures, but also provide insights, and a deeper understanding for the phenomenon studied. The purpose of modeling inventory situations is to derive an operating doctrine. For this, the following four simple steps are involved.

- Examining the inventory situation, listing characteristics and assumptions concerning the situation.
- Finding the total annual relevant cost equation in narrative.
- Transforming the total annual cost equation from narrative into the shorthand logic of mathematics.
- Optimizing the cost equation, finding the optimum for when to order (*cycle time*).

In this paper, the methodology followed to find the optimal order quantity is listed below:

1. Mathematical model formulation
2. Solution procedure
3. Numerical example

4. Preliminaries

The fuzzy set theory was introduced to deal with problems in which fuzzy phenomena exist. In a universe of discourse X , a fuzzy subset $\tilde{a}$ of X is defined by the membership function $\mu_{\tilde{a}}(x)$ which maps each element x in X to a real number in the interval [0, 1]. The function value of $\mu_{\tilde{a}}(x)$ denotes the grade of membership.

Definition 1:
Fuzzy convex (Vijayan and Kumaran (2008)): A fuzzy set $\tilde{a}$ on X is convex if $\mu_{\tilde{a}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{a}}(x_1), \mu_{\tilde{a}}(x_2))$ for all x_1, x_2 in X and for $\lambda \in [0,1]$, where *min* denotes the minimum operator.

Definition 2:

Fuzzy point (Pu and Liu (1980)): Let $\tilde{a}$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, x = a \\ 0, x \neq a \end{cases} \tag{1}$$

Definition 3:
Fuzzy number (Vijayan and Kumaran (2008)): A fuzzy number is a fuzzy subset of the real line which is both normal and convex. For a fuzzy number $\tilde{A}$, its membership function can be denoted by

$$\mu_{\tilde{A}}(x) = \begin{cases} l(x), x < m \\ 1, \ m \leq x \leq n \\ u(x), \ x > n \end{cases} \tag{2}$$

where $l(x)$ is upper semi continuous, strictly increasing for $x < m$ and there exists $m_1 < m$ such that $l(x) = 0$ for $x \leq m_1$, $u(x)$ is continuous, strictly decreasing function for $x > n$ and there exists $n_1 \geq n$ such that $u(x) = 0$ for $x > n_1$, $l(x)$ and $u(x)$ are called the left and right reference functions, respectively.

Definition 4:

Trapezoidal fuzzy number (Zimmerman (1991)): The fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that

$a_1 < a_2 < a_3 < a_4$, whose membership function, representing a trapezoid, can be denoted by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ 1, \qquad a_2 \le x \le a_3 \\ \frac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4 \\ 0, \qquad otherwise \end{cases} \tag{3}$$

where a_1, a_2, a_3, a_4 are the lower limit, lower mode, upper mode and upper limit, respectively of the fuzzy number $\tilde{A}$. The interval $[a_1, a_4]$ is called the support of the fuzzy number and it gives the range of all possible values of $\tilde{A}$ that are at least marginally possible or plausible. The interval $[a_2, a_3]$ corresponds to the core of fuzzy number and gives the range of most plausible values. The intervals $[a_1, a_2]$ and $[a_3, a_4]$ are called penumbra of the fuzzy number $\tilde{A}$.

Let $\tilde{A}_1 = (a_{11}, a_{12}, a_{13}, a_{14})$, $\tilde{A}_2 = (a_{21}, a_{22}, a_{23}, a_{24})$ be two trapezoidal fuzzy numbers, then $\tilde{A}_1 + \tilde{A}_2 = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24})$ and, for all $b \ge 0$, $b\tilde{A}_1 = (ba_{11}, ba_{12}, ba_{13}, ba_{14})$.

Definition 5:

Triangular Fuzzy Numbers (Chiang et al. (2005)): Let $\tilde{A} = (a_1, a_2, a_3), a_1 < a_2 < a_3$, be a fuzzy set on $R = (-\infty, \infty)$. It is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, \qquad otherwise \end{cases} \tag{4}$$

The set $\tilde{A}(\alpha) = \{x : \mu_{\tilde{A}}(x) \ge \alpha\}$, where $\alpha \in [0,1]$ is called the α cut of $\tilde{A}$. $\tilde{A}(\alpha)$ is a nonempty bounded closed interval contained in the set of real numbers and it can be denoted by $\tilde{A}(\alpha) = [\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)]$. $\tilde{A}_L(\alpha)$ and $\tilde{A}_R(\alpha)$ are respectively the left and right limits of $\tilde{A}(\alpha)$

and are usually known as the left and right α cuts of $\tilde{A}$. $\tilde{A}_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $\tilde{A}_R(\alpha) = a_4 + (a_4 - a_3)\alpha$ for a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{A}_L(\alpha) = a_1 + (a_2 - a_1)\alpha$ and $\tilde{A}_R(\alpha) = a_3 + (a_3 - a_2)\alpha$ for triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$.

Definition 6:

Level α Fuzzy Interval (Chiang et al. (2005)): Let $[a, b, \alpha]$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a level α fuzzy interval, $0 \le \alpha \le 1$, $a < b$, if its membership function is

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & a \le x \le b \\ 0, & otherwise \end{cases} \tag{5}$$

Next, as in Yao and Wu (2000), we introduce the concept of the signed distance which will be needed later. We first consider the signed distance on R.

4.1 Signed distance method

The signed distance between the real numbers a and 0, denoted by $d_0(a,0)$ is given by $d_0(a,0) = a$. Hence the signed distance of $\tilde{A}_L(\alpha)$ and $\tilde{A}_R(\alpha)$ measured from 0 are $d_0(\tilde{A}_L(\alpha),0) = \tilde{A}_L(\alpha)$ and $d_0(\tilde{A}_R(\alpha),0) = \tilde{A}_R(\alpha)$, respectively.

The signed distance of the interval $(\tilde{A}_L(\alpha), \tilde{A}_R(\alpha))$ measured from the origin 0 by

$$d_0((\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)),0) = \frac{1}{2}\left[d_0(\tilde{A}_L(\alpha),0) + d_0(\tilde{A}_R(\alpha),0)\right] = \frac{1}{2}\left(\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)\right) \tag{6}$$

where $\tilde{A}_L(\alpha)$ and $\tilde{A}_R(\alpha)$ exist and are integrable for $\alpha \in [0,1]$.

For each $\alpha \in [0,1]$, the crisp interval $\left[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)\right]$ and the level α fuzzy interval $\left[[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)];\alpha\right]$ are in one to one correspondence. The signed distance from $\left[[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)];\alpha\right]$ to $\tilde{0}$ (where $\tilde{0}$ is the 1 level fuzzy point which maps to the origin) is

$$\left(\left[[\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)];\alpha\right], \tilde{0}\right) = d_0((\tilde{A}_L(\alpha), \tilde{A}_R(\alpha)),0) = \frac{1}{2}\left(\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)\right) \tag{7}$$

The signed distance of $\tilde{A}$ measured from $\tilde{0}$ defined as

$$d(\tilde{A},\tilde{0}) = \frac{1}{2}\int_0^1 \left(\tilde{A}_L(\alpha) + \tilde{A}_R(\alpha)\right) d\alpha \tag{8}$$

Lemma 1:

Let $\tilde{A}_i$, $i = 1,2,3 \ldots N$ be N fuzzy numbers and b_i, $i = 1,2,3 \ldots N$ are real crisp constants. Then

$$d\left(\sum_{i=1}^{N} b_i \tilde{A}_i, \tilde{0}\right) = \sum_{i=1}^{N} b_i d(\tilde{A}\ ,\tilde{0}) \tag{9}$$

According to Yao and Wu (2000), the signed distance formula in Eq. (8) is considered when ranking fuzzy numbers. Then the fuzzy numbers $\tilde{A}_1$ and $\tilde{A}_2$ ranked as

$$\tilde{A}_1 < \tilde{A}_2 \quad \text{if} \quad d(\tilde{A}_1,\tilde{0}) < d(\tilde{A}_2,\tilde{0}) \tag{10}$$

5. Notations and Assumptions

We adopt the following notations and assumptions in order to develop the proposed model.

5.1. Notations

T	time interval between successive orders (*decision variable*)
D	demand rate
A	ordering cost per order
h	unit stock-holding cost per item per unit time excluding interest charges
p	unit purchase cost in $
β	price discount factor for advance payment
t	length of advance payment
I_c	interest charges per $ investment in stocks per year

5.2. Assumptions

1. The vendor offers price discount for the buyer if all payment is paid in advance.
2. Replenishments are instantaneous and the shortages are not allowed.
3. Time horizon is infinite.

6. Crisp mathematical modeling of the EOQ problem with advance payment

In this study the vendor gives all the payment paid in advance payment term similar to Zhang et al. (2014). In the proposed scenario, the buyer pays a purchase cost $DTp\beta$ and incurs an ordering cost A at time zero. The inventory level before arrival of a procurement is zero. The purchase cost has to be financed at interest rate I_c , and the loan interest cost equals $\frac{DTp\beta I_c t}{T} = Dp\beta I_c t$ during this period. During the stock period, that is, from time t to time $t+T$, the buyer makes payment to the interest-bearing account immediately after the selling of the goods. As the loan is being paid back, the interest payable is decreasing. On the last day of stock period, the buyer pays the remaining balance. Hence, the average outstanding amount of the loan is $DTp\beta$, and the interest cost is $\frac{DT^2 p\beta I_c}{2}$ from time t to time $t+T$ in one cycle. The physical holding cost is the same as that of the traditional economic order quantity inventory model and is not influenced by the payment terms. The behavior of inventory for this model is depicted in Fig. 1.

Based on the assumptions described above with Fig. 1, the buyer's total cost per unit time can be obtained as Zhang et al. (2014) is

$$\Pi(T) = \frac{A}{T} + \frac{DTh}{2} + Dp\beta I_c t + \frac{Dp\beta T I_c}{2} \tag{11}$$

where the first term is ordering cost, the second term is holding cost ((excluding interest charge), the third term is cost of interest charges at the time of advance account payment and the last term is cost of interest charges when the goods are kept in stock.

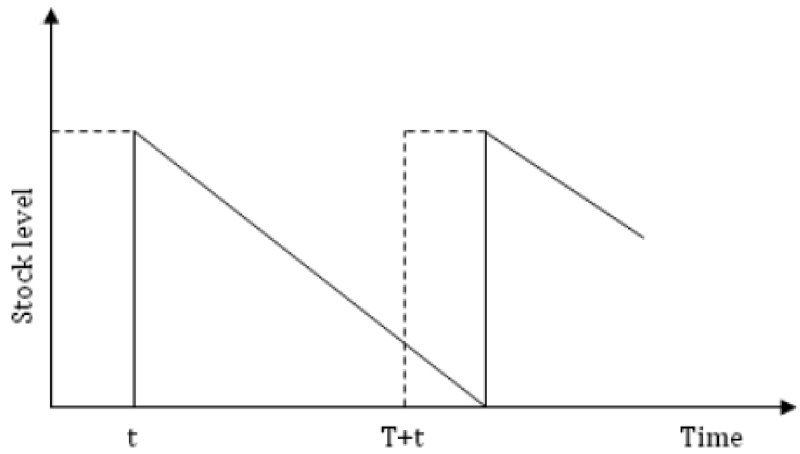

Figure 1. Time-weighted inventory when all the payment paid in advance

6.1. Solution procedure

We proved the convexity of the total cost $\Pi(T)$ in Lemma 2 based on classical differential calculus optimization technique.

Lemma 2. *The total cost* $\Pi(T)$ *is a convex function of cycle time* T.

Proof. Taking the first and second partial derivatives of $\Pi(T)$ with respect to T, we have

$$\frac{\partial \Pi(T)}{\partial T} = \frac{-A}{T^2} + \frac{Dh}{2} + \frac{Dp\beta\, I_c}{2} \tag{12}$$

and

$$\frac{\partial^2 \Pi(T)}{\partial T^2} = \frac{2A}{T^3} > 0 .$$

Therefore $\Pi(T)$ is a convex function T.

This completes the proof of Lemma 2.
Hence, the unique optimal value of the buyer's optimal replenishment cycle T can be obtain by setting Eq. (12) to zero as

$$T_0^* = \sqrt{\frac{2A}{D(h + p\beta\, I_c)}} \tag{13}$$

The buyer's optimal replenishment cycle is not related to the length of the advance payment. However, the price discount associated with advance payment makes the capital cost of inventory smaller which leads to a larger replenishment cycle. Hence, the optimal order quantity is larger than that of the traditional EOQ model. When the supplier is huge and the buyer is small, such as the automobile industry in China, the supplier may ask the buyer to pay in advance without any discount, that is $\beta = 1$. In this situation, the buyer's optimal replenishment cycle is the same as the outcome of the EOQ model Zhang et al. (2014).

7. Fuzzy mathematical modeling of the EOQ problem with advance payment

There are two types of fuzzy models that developed in this section. In the first, the cost parameters are fuzzified while the demand rate is treated as crisp constant. In the second model the demand rate is fuzzified but the cost parameters are treated as crisp constants.

7.1 The EOQ advance payment model with all cost components fuzzy

Considering that the cost components, ordering cost A, holding cost h and purchase cost p are all fuzzy but the demand rate D is crisp constant. We represent cost components A, h and p by trapezoidal fuzzy numbers as given below

$$\begin{aligned}
\tilde{A} &= (A-\delta_1, A-\delta_2, A+\delta_3, A+\delta_4) \\
\tilde{h} &= (h-\delta_5, h-\delta_6, h+\delta_7, h+\delta_8) \\
\tilde{p} &= (p-\delta_9, p-\delta_{10}, p+\delta_{11}, p+\delta_{12})
\end{aligned} \tag{14}$$

where $\delta_l, l = 1,2,3\ldots,12$, are arbitrary positive numbers under the following restrictions $S > \delta_1 > \delta_2, \quad \delta_3 < \delta_4; \quad A > \delta_5 > \delta_6, \quad \delta_7 < \delta_8; \quad C_b > \delta_9 > \delta_{10}, \quad \delta_{11} < \delta_{12}.$

Then the left and right limits of α cuts of $\tilde{A}, \tilde{h}$ and $\tilde{p}$ are given below

$$\begin{aligned} &\tilde{A}_L(\alpha) = A - \delta_1 + (\delta_1 - \delta_2)\alpha > 0, && \tilde{A}_R(\alpha) = A + \delta_4 - (\delta_4 - \delta_3)\alpha > 0 \\ &\tilde{h}_L(\alpha) = h - \delta_5 + (\delta_5 - \delta_6)\alpha > 0, && \tilde{h}_R(\alpha) = h + \delta_8 - (\delta_8 - \delta_7)\alpha > 0 \\ &\tilde{p}_L(\alpha) = p - \delta_9 + (\delta_9 - \delta_{10})\alpha > 0, && \tilde{p}_R(\alpha) = p + \delta_{12} - (\delta_{12} - \delta_{11})\alpha > 0 \end{aligned} \tag{15}$$

Accordingly, when the costs A, h and p in Eq. (11) are fuzzified to be $\tilde{A}, \tilde{h}$ and $\tilde{p}$ as expressed in Eq. (14), we can obtain the total cost function in the fuzzy sense is given by

$$\tilde{\Pi}(T) = \frac{\tilde{A}}{T} + \frac{DT\tilde{h}}{2} + D\tilde{p}\beta I_c t + \frac{D\tilde{p}\beta T I_c}{2} \tag{16}$$

From Eq. (15), we can obtain the left and right hand side of the α - cut, $(0 \le \alpha \le 1)$, of $\tilde{\Pi}(T)$ by the following form

$$\tilde{\Pi}(T)_L(\alpha) = \frac{\tilde{A}_L(\alpha)}{T} + \frac{DT\tilde{h}_L(\alpha)}{2} + D\tilde{p}_L(\alpha)\beta I_c t + \frac{D\tilde{p}_L(\alpha)\beta T I_c}{2} \tag{17}$$

$$\tilde{\Pi}(T)_R(\alpha) = \frac{\tilde{A}_R(\alpha)}{T} + \frac{DT\tilde{h}_R(\alpha)}{2} + D\tilde{p}_R(\alpha)\beta I_c t + \frac{D\tilde{p}_R(\alpha)\beta T I_c}{2} \tag{18}$$

respectively.

Hence, when the costs describe with trapezoidal number, using Eqs. (8), (9), (17) and (18), the defuzzified value of $\tilde{\Pi}(T)$ (as expressed in Eq. (16)) is given by

$$d(\tilde{\Pi}(T), \tilde{0}) = \frac{H_1}{T} + \frac{DTH_2}{2} + DH_3\beta I_c t + \frac{DH_3\beta T I_c}{2} \tag{19}$$

where

$$H_1 = A + \frac{1}{4}(\delta_4 + \delta_3 - \delta_2 - \delta_1) > 0,$$

$$H_2 = h + \frac{1}{4}(\delta_8 + \delta_7 - \delta_6 - \delta_5) > 0,$$

$$H_3 = p + \frac{1}{4}(\delta_{12} + \delta_{11} - \delta_{10} - \delta_9) > 0.$$

The defuzzified value $d(\tilde{\Pi}(T), \tilde{0})$ (as expressed in Eq. (19)) is taken as the estimate of fuzzy cost function in Eq. (15), denoted by $E^{\Theta}(\tilde{\Pi}(T))$, when the costs represented by trapezoidal number. The estimate in Eq. (19) is a convex function of T similar to Eq. (11). Then a unique minimum of $E^{\Theta}(\tilde{\Pi}(T))$ is obtained by equating to zero the first order partial derivatives of $E^{\Theta}(\tilde{\Pi}(T))$ with respect to T. That is

$$\frac{\partial E^{\Theta}(\tilde{\Pi}(T))}{\partial T} = 0 \tag{20}$$

Solving Eq. (20), we obtain

$$T_{\Theta}^{*} = \sqrt{\frac{2H_1}{D(H_2 + H_3 \beta I_c)}} \tag{21}$$

On the other hand, if we treat the cost components A, h and p are the triangular fuzzy numbers as given below.

$$\begin{aligned} \tilde{A} &= (A-\Delta_1, A, A+\Delta_2), \quad 0<\Delta_1< A, 0<\Delta_2 \\ \tilde{h} &= (h-\Delta_3, h, h+\Delta_4), \quad 0<\Delta_3< h, 0<\Delta_4 \\ \tilde{p} &= (p-\Delta_5, p, p+\Delta_6), \quad 0<\Delta_5< p, 0<\Delta_6 \end{aligned} \tag{22}$$

Then we obtain the left and right limits of α cuts of $\tilde{A}, \tilde{h}$ and $\tilde{p}$ as follows.

$$\begin{aligned} \tilde{A}_L(\alpha) &= A-\Delta_1+\alpha\Delta_1 > 0, \qquad \tilde{A}_R(\alpha) = A+\Delta_2-\alpha\Delta_2 > 0 \\ \tilde{h}_L(\alpha) &= h-\Delta_3+\alpha\Delta_3 > 0, \qquad \tilde{h}_R(\alpha) = h+\Delta_4-\alpha\Delta_4 > 0 \\ \tilde{p}_L(\alpha) &= p-\Delta_5+\alpha\Delta_5 > 0, \qquad \tilde{p}_R(\alpha) = p+\Delta_6-\alpha\Delta_6 > 0 \end{aligned} \tag{23}$$

Hence, when the costs A, h and p in Eq. (11) are described with triangular numbers $\tilde{A}$, $\tilde{h}$ and $\tilde{p}$ as expressed in Eq. (22), following similar steps as in the trapezoidal case and using Eq. (23), the signed distance value (defuzzified value) of total cost is established as

$$d(\tilde{\Pi}(Q,L,n),\tilde{0}) = \frac{G_1}{T} + \frac{DTG_2}{2} + DG_3 \beta I_c t + \frac{DG_3 \beta T I_c}{2} \tag{24}$$

Where

$$G_1 = A + \frac{1}{4}(\Delta_2 - \Delta_1) > 0,$$

$$G_2 = h + \frac{1}{4}(\Delta_4 - \Delta_3) > 0,$$

$$G_3 = p + \frac{1}{4}(\Delta_6 - \Delta_5) > 0.$$

Similarly, the defuzzified value $d(\tilde{\Pi}(T),\tilde{0})$ (as given in Eq. (24)) is taken as the estimate of fuzzy cost function in Eq. (11), denoted by $E^{\Delta}(\tilde{\Pi}(T))$, when the costs are represented by triangular number. The estimate in Eq. (24) is a convex function of T similar to Eq. (19). Then a unique minimum of $E^{\Delta}(\tilde{\Pi}(T))$ is obtained by equating to zero the first order partial derivatives of $E^{\Delta}(\tilde{\Pi}(T))$ with respect to Q. That is

$$\frac{\partial E^{\Delta}(\tilde{\Pi}(T))}{\partial T} = 0 \tag{25}$$

Solving Eq. (25), we obtain

$$T_{\Delta}^{*} = \sqrt{\frac{2G_1}{D(G_2 + G_3 \beta I_c)}} \tag{26}$$

7.2 The EOQ advance payment model with fuzzy demand

In this case we consider that the demand rate D is fuzzy in Eq. (11) while the cost components are crisp constants. We represent demand rate D by trapezoidal and triangular fuzzy numbers as given below

$\tilde{D} = (D-\delta_{13}, D-\delta_{14}, D+\delta_{15}, D+\delta_{16})$ where $D > \delta_{13} > \delta_{14}, \delta_{15} < \delta_{16}$

And

$\tilde{D} = (D-\Delta_{13}, D, D+\Delta_{14})$ where $0 < \Delta_7 < A, 0 < \Delta_8$

respectively.

Then the left and right limits of α cuts of $\tilde{D}$ for trapezoidal and triangular fuzzy numbers are given by

$$\tilde{D}_L(\alpha) = D - \delta_{13} + (\delta_{13} - \delta_{14})\alpha > 0, \qquad \tilde{D}_R(\alpha) = D + \delta_{15} - (\delta_{15} - \delta_{16})\alpha > 0$$

And

$$\tilde{D}_L(\alpha) = D - \Delta_7 + \alpha\Delta_7 > 0, \qquad \tilde{D}_R(\alpha) = D + \Delta_8 - \alpha\Delta_8 > 0$$

respectively.

Hence, when the demand D in Eq. (11) are fuzzified to be $\tilde{D}$, we can obtain the total cost function in the fuzzy sense is given by

$$\tilde{\Pi}(T) = \frac{A}{T} + \frac{\tilde{D}Th}{2} + \tilde{D}p\beta I_c t + \frac{\tilde{D}p\beta T I_c}{2} \tag{27}$$

Now, the left and right α cuts of the function in Eq. (27) for trapezoidal and triangular fuzzy number cases are given by

$$\tilde{\Pi}(T)_L(\alpha) = \frac{A}{T} + \frac{\tilde{D}_L(\alpha)Th}{2} + \tilde{D}_L(\alpha)p\beta I_c t + \frac{\tilde{D}_L(\alpha)p\beta T I_c}{2} \tag{28}$$

and

$$\tilde{\Pi}(T)_R(\alpha) = \frac{A}{T} + \frac{\tilde{D}_R(\alpha)Th}{2} + \tilde{D}_R(\alpha)p\beta I_c t + \frac{\tilde{D}_R(\alpha)p\beta T I_c}{2} \tag{29}$$

respectively.

Hence, using Eqs. (19), (20), (28) and (29), the estimate of total cost function for the trapezoidal and triangular number cases under fuzzy demand are expressed as

$$d(\tilde{\Pi}(T),\tilde{0}) = \frac{A}{T} + \frac{H_4 Th}{2} + H_4 p\beta I_c t + \frac{H_4 p\beta T I_c}{2} \tag{30}$$

where $H_4 = D + \frac{1}{4}(\delta_{16}+\delta_{15}-\delta_{14}-\delta_{13}) > 0$

and

$$d(\tilde{\Pi}(T),\tilde{0}) = \frac{A}{T} + \frac{G_4 Th}{2} + G_4 p\beta I_c t + \frac{G_4 p\beta T I_c}{2} \tag{31}$$

where $G_4 = D + \frac{1}{4}(\Delta_8 - \Delta_7) > 0$

respectively.

Similarly, the defuzzified values as given in Eqs. (30) and (31) are taken as the estimate of fuzzy cost functions for the trapezoidal and triangular number cases in Eq. (11) denoted by $E_D^{\Theta}(\tilde{\Pi}(T))$ and $E_D^{\Delta}(\tilde{\Pi}(T))$, respectively. Again, similar to Eqs. (19) and (24), the estimates in Eqs. (30) and (31) are convex functions of T. Then the unique minimum of $E_D^{\Theta}(\tilde{\Pi}(T))$ and $E_D^{\Delta}(\tilde{\Pi}(T))$ are obtained by equating to zero the first order partial derivatives of $E_D^{\Theta}(\tilde{\Pi}(T))$ and $E_D^{\Delta}(\tilde{\Pi}(T))$ with respect to T.

Hence the optimum value of T is obtained for the trapezoidal and triangular number cases from the following expressions

$$T_{\Theta}^{*} = \sqrt{\frac{2A}{H_1(h + p\beta I_c)}} \tag{32}$$

and

$$T_{\Delta}^{*} = \sqrt{\frac{2A}{G_1(h + p\beta I_c)}}$$

(33)

respectively.

8. Numerical analysis

Extensive numerical analysis has been done to assess the impact of the level of fuzziness in the input parameters over the decision variable. Let us consider an EOQ inventory model with the following data: $\beta = 0.9,\ t = 0.1$ and $I_c = 0.2$. Besides, the crisp costs $A = 30, p = 25$ and $h = 20$ as well as the crisp demand $D = 400$. Zhang et al. (2014)'s statistical data is more appropriate for our proposed model. Therefore, we use the same numerical data as in Zhang et al. (2014) to verify the results obtained by this paper.

We set some trapezoidal and triangular fuzzy numbers of the input parameters $(p, D, h$ and $A)$ in Tables 1(a, b) and 2(a, b), respectively, to represent the components of fuzzy models developed in Section 4. For each of these parameters, the variations in the values are arranged arbitrary and their defuzzified values are determined by applying the signed distance method. The signed distance value (defuzzified value) and the corresponding percentage difference under fuzzy case (based on the defuzzified value) from the crisp values denoted by $\hat{p}, \hat{D}, \hat{h}, \hat{A}, \hat{E}^{\Theta}(\tilde{\Pi}(T))$ and $\hat{E}^{\Delta}(\tilde{\Pi}(T))$ for the components $p, D, h, A, E^{\Theta}(\tilde{\Pi}(T))$ and $E^{\Delta}(\tilde{\Pi}(T))$, respectively, are also shown along with the fuzzy numbers in Tables 1(a, b) and 2(a, b). Note that fuzzy cost parameters chosen are in the ranking order indicated in Eq. (10).

Based on these values, the optimal replenishment cycle T^* with the minimum total cost for the fuzzy model developed in Section 5 are computed for each set of trapezoidal and triangular fuzzy numbers. Moreover, we use Section 4 for the crisp EOQ inventory model. The results are summarized in Tables 3 and 4.

Table 1(a). Fuzzy trapezoidal values for the input parameters for *p* and *D*

$\tilde{p}$	$d(\tilde{p},\tilde{0})$	$\hat{p}$	$\tilde{D}$	$d(\tilde{D},\tilde{0})$	$\hat{D}$
(2,4,28,30)	16	-36	(100,150,415,455)	280	-30
(5,7,32,40)	21	-16	(120,200,455,505)	320	-20
(11,20,30,55)	29	+16	(220,290,630,780)	480	+20
(10,17,30,75)	34	+36	(280,320,680,800)	520	+30
(15,20,42,103)	45	+80	(320,360,790,930)	600	+50

Table 1(b). Fuzzy trapezoidal values for the input parameters for *h* and *A*

$\tilde{h}$	$d(\tilde{h},\tilde{0})$	$\hat{h}$	$\tilde{A}$	$d(\tilde{A},\tilde{0})$	$\hat{A}$
(1,3,21,23)	12	-40	(2,5,32,33)	18	-40
(3,6,25,30)	16	-20	(5,9,37,45)	24	-20
(6,15,25,50)	24	+20	(10,24,50,60)	36	+20
(5,12,25,70)	28	+40	(12,20,37,99)	42	+40
(10,15,37,98)	40	+100	(23,27,90,100)	60	+100

Table 2(a). Fuzzy triangular values for the input parameters for *p* and *D*

$\tilde{p}$	$d(\tilde{p},\tilde{0})$	$\hat{p}$	$\tilde{D}$	$d(\tilde{D},\tilde{0})$	$\hat{D}$
(3,25,27)	20	-20	(70,400,410)	320	-20
(7,25,31)	22	-12	(140,400,420)	340	-15
(19,25,43)	28	+12	(390,400,650)	460	+15
(22,25,48)	30	+20	(380,400,740)	480	+20
(20,25,70)	35	+40	(380,400,900)	520	+30

Table 2(b). Fuzzy triangular values for the input parameters for *h* and *A*

$\tilde{h}$	$d(\tilde{h},\tilde{0})$	$\hat{h}$	$\tilde{A}$	$d(\tilde{A},\tilde{0})$	$\hat{A}$
(2,20,22)	16	-20	(3,30,33)	24	-20
(4,20,28)	18	-10	(10,30,38)	27	-10
(12,20,36)	22	+10	(18,30,54)	33	+10
(10,20,46)	24	+20	(26,30,68)	36	+20
(15,20,65)	30	+30	(25,30,83)	42	+30

Table 3. Optimal inventory policy with fuzzy costs for trapezoidal and triangular fuzzy numbers

Trapezoidal number case						Triangular number case					
$\hat{p}$	$\hat{h}$	$\hat{A}$	T^*	$E^{\Theta}(\tilde{\Pi}(T))$	$\hat{E}^{\Theta}(\tilde{\Pi}(T))$	$\hat{p}$	$\hat{h}$	$\hat{A}$	T^*	$E^{\Delta}(\tilde{\Pi}(T))$	$\hat{E}^{\Delta}(\tilde{\Pi}(T))$
-36	-40	-40	0.0778	578.1	39	-20	+20	-20	0.0782	757.4	20.0
-16	-20	-20	0.0779	767.5	19	-12	+10	-10	0.0784	847.1	10.5
0	0	0	0.0782	946.8	0	0	0	0	0.0782	946.8	0
+16	+20	+20	0.0785	1126.2	19	+12	+10	+10	0.0781	1046.5	10.5
+36	+20	+40	0.0785	1315.5	39	+20	+20	+20	0.0782	1136.2	20.0
+80	+100	+100	0.0790	1843.5	94.5	+40	+30	+40	0.0761	1356.4	43.3

Table 4. Optimal inventory policy with fuzzy demand for trapezoidal and triangular fuzzy numbers

Trapezoidal number case				Triangular number case			
$\hat{D}$	T^*	$E^{\Theta}(\tilde{\Pi}(T))$	$\hat{E}^{\Theta}(\tilde{\Pi}(T))$	$\hat{D}$	T^*	$E^{\Delta}(\tilde{\Pi}(T))$	$\hat{E}^{\Delta}(\tilde{\Pi}(T))$
-30	0.0935	767.6	19.0	-20	0.0875	829.9	12.3
-20	0.0875	829.9	12.3	-15	0.0849	860.0	9.2
0	0.0782	946.8	0	0	0.0782	946.8	0
+20	0.0714	1056.0	11.5	+15	0.0730	1029.3	8.7
+30	0.0686	1108.3	17.1	+20	0.0714	1056.0	11.5
+50	0.0639	1209.1	27.7	+30	0.0686	1108.3	17.1

9. Conclusion

Huge steel plants require advance payment in Chinese steel industry especially for small buyers. In addition, in real world applications, the input cost and other parameters in the EOQ inventory problem may not be known precisely or it may be uncertain due to some uncontrollable factors. Hence, approximate solution methodologies have been illustrated for the solution of a class of realistic inventory problems. Fuzzy methodologies provide a useful way to model vagueness in human recognition and judgment. Uncertainties characterized by imprecise expression can be represented by fuzzy sets. Moreover, fuzzy numbers are largely applied on data analysis, artificial intelligence, and decision making. Fuzzy numbers are frequently used in applications and they also make handling the realistic problem easier.

This paper proposed two fuzzy models for an EOQ inventory model with advance payment option under fuzzy costs and demand environment. The fuzziness in the cost components and demand are represented by the trapezoidal and triangular fuzzy numbers. Numerous methods for total ordering of fuzzy numbers have been suggested in the literature. Each method appears to have some advantages as well as disadvantages. In the context of each application, some methods seem more appropriate than others. However, the issue of choosing a proper ordering method in a given context is still a subject of active research. But some existing studies show that the signed distance method is better for defuzzification than other methods. Therefore, we use signed distance method to defuzzify the fuzzy total cost and obtain an estimate of the total cost in the fuzzy sense. Classical differential calculus optimization technique is used to find the optimal solution of the models. Numerical example is provided to ascertain the sensitiveness in the decision variables about fuzziness in the components.

Our results indicate that the optimal solutions of the fuzzy model slightly fluctuate from the solutions of the crisp model (see Tables 3 and 4). Hence, the research reveals that in all the models, the decision variables and the total expected cost are sensitive to the level of fuzziness in the cost components and demand. Hence the practitioner is advised to be more careful in accounting flexibility in the ordering cost, holding cost, purchasing cost and demand rate.

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