International Journal of Supply and Operations Management

IJSOM

August 2014, Volume 1, Issue 2, pp. 245-259

ISSN-Print: 2383-1359

ISSN-Online: 2383-2525

www.ijsom.com

A novel hierarchical model to locate health care facilities with fuzzy demand solved by harmony search algorithm

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Abstract

In the field of health losses resulting from failure to establish the facilities in a suitable location and the required number, beyond the cost and quality of service will result in an increase in mortality and the spread of diseases. So the facility location models have special importance in this area. In this paper, a successively inclusive hierarchical model for location of health centers in term of the transfer of patients from a lower level to a higher level of health centers has been developed. Since determination the exact number of demand for health care in the future is difficult and in order to make the model close to the real conditions of demand uncertainty, a fuzzy programming model based on credibility theory is considered. To evaluate the proposed model, several numerical examples are solved in small size. In order to solve large scale problems, a meta-heuristic algorithm based on harmony search algorithm was developed in conjunction with the GAMS software which indicants the performance of the proposed algorithm.

Keywords: Hierarchical Facility Location, Successively Inclusive, Fuzzy Credibility Theory, Harmony Search Algorithm, health care facilities.

1. Introduction

Proper location of health facilities and medical centers has a significant role in increasing access to health services and the satisfaction of the sector. Growing population and inclination to urbanity have given rise to unbalanced development of cities, so that most of the immigrant population settled in the cities border, and this has resulted in declining living standards, lack of medical facilities and, ultimately, unequal distribution of facilities. It is predicted that by 2025, more than 5 billion of people in the world will be living in urban areas, so that 80 percent of the population will be living in cities of less developed, and this will be a challenge for urban planners

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and managers (2003). The main task of urban planners and decision makers is to determine the optimal location of public services such as health centers, so that all urban residents have access to them easily. In addition, planners are trying to optimize distribution of services in urban environments, and this distribution should adjust to population and their demand in different regions of the city. This paper seeks to locate and organize optimal health care centers according to the demand of the population in different parts of the city. In hierarchical location, facilities, based on the type and quantity of product / service offered, are to be classified at different levels. The HFLP[1] models have been applied in different fields such as education systems, health care centers, production–distribution networks, and telecommunication systems (2014). In HFLP models, service availability refers to the customer accessibility to a service at different levels of hierarchy. Nested and non-nested networks are defined as successively inclusive and successively exclusive networks (1984). In a successively inclusive hierarchy, a facility at level q provides all types of services to facilities at lower levels. On the contrary, in a successively exclusive hierarchy, a facility at level k offers only service types to facilities at lower levels.

This study focuses only on health care centers from among the HFLP models. Calvo and Marks model (1973) is of early HFLP models in health care centers. In this multi-flow pattern, k level of the facilities with limited capacity is considered and the number of facilities is determined by the model itself[2]. It is suggested that alternative optimizations can be according to the utility function specified for the user, operator and community sectors, respectively. Narula and Ogbu (1979) have presented a two-level model by considering facilities with capacity constraints, so that at the level 1, health centers and at level 2, hospitals are located. They have also offered five heuristic methods to solve the proposed model, and reported their computation results. Okabe et all (1997) offered a computational approach to optimize a system with k-level hierarchical and non-exclusive facilities. Numerical results indicate the achieving near-optimal solution in a relatively small number of iterations. Tien and El-Tell (1984) offered a two-level model with a single flow pattern that is one of the few single flow hierarchy models to locate treatment centers. Gerrard and Church (1994) offered a two-level model and Boffey (2003) presented a three-level model with regard to the transfer of patients from treatment centers at a lower level to a higher level. Galvão et all (2006) also offered a three-level model. Suppose that in addition to referring the patient to a higher level facility, capacity constraints are added at the high level and the problem was solved by lagrangian Relaxation. Yasenovskiy and Hodgson (2007) presented a three-level model which was based on the assumption that, in reality, due to some reasons, people probably would prefer health center with facilities at higher service levels at farther distance in comparison to nearer ones which offer facilities at lower service level. Hodgson and Jacobs (2009) developed previous models based on patient behavior and considering the various possibilities for referring the patient to a higher level of service required by them.

Innovation of this paper can provide a new mathematical hierarchical model for locating treatment centers for patients to access appropriate medical care, considering the distance minimization objective function and the penalty for demand exceeds the capacity of the facility. According to the authors' knowledge, in this paper, the possibility of transferring patients to different levels, after an initial review of the medical centers with a higher level of service is considered. Another innovation of the paper, taking into account the uncertainty of demand with credibility theory and finally a hybrid Meta heuristic method based on harmony search algorithm and GAMS software is presented. In the second part, the literature on hierarchical location of health care facilities is denoted. In third section, fuzzy credibility theory is discussed. In fourth section, mathematical model of the problem is presented. In fifth Section, Harmony search algorithm is expressed, and finally, conclusion is discussed.

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[1]Hierarchical Facility Location Problem
[2]Endogenous

2. Fuzzy theory and the concept of credibility theory

In a classic series, limits are set exactly defined. In these collections, an element is or is not definitely a member of the set. But in practice there are many sets that their boundaries cannot be defined exactly. Such sets are called fuzzy. Fuzzy set theory was introduced first by LotfiZadeh (1965) and then developed by many other researchers for various problems. Liu's credibility theory was introduced as a way to solve fuzzy model (2004). In this section, we briefly discuss the basic concepts of fuzzy sets and fuzzy measures. Ghaffari-Nasab(2013) has shown axioms of the possibility measure theory are introduced, which form the basis of the credibility measure theory. Let Θ be a nonempty set, and let P (Θ) be the power set of Θ. Each element in P (Θ) is called an event, and let ∅ be an empty set. For each event A∈P(Θ), there is a nonnegative number Pos{A} which is in compliance with the following four axioms:

Axiom 1: Pos{∅}=0

Axiom 2: Pos{Θ}=1

Axiom 3: $Pos\{\cup_k A_k\} = \sup_k (A_k)$ For any arbitrary collection $\{A_K\} \in P(\Theta)$

(Θ, P (Θ ,Pos)) is called a possibility space, and the function Pos{} is referred to as a possibility measure.

Axiom4: If Θi is a non-empty set, $Pos_i\{\}$ $i = 1, 2, ..., n$ satisfies the conditions stated in the above three axioms, and $\Theta = \Theta_1 \times \Theta_2 \times ... \times \Theta_n$ Then, for each

A ϵ P (Θ), $Pos_1\{\Theta_1\} \wedge Pos_2\{\Theta_2\} \wedge ... \wedge Pos_n\{\Theta_n\}$ (1)

The following definitions are provided by Ghaffari-Nasab(2013):

Definition 1 Liu(2004). Let (Θ, P (Θ), Pos) be a possibility space, and A be a set in P (Θ). Then the necessity measure of A is defined by:

$$Nec\{A\} = 1 - Pos\{A^C\} \quad (2)$$

Definition 2 Liu(2004). Let (Θ, P (Θ), Pos) be a possibility space, and A be a set in P (Θ). Then the credibility measure of A is defined by:

$$Cr\{A\} = \frac{1}{2}(Nec\{A\} + Pos\{A\}) \quad (3)$$

If $\mu_{\tilde{D}}(x)$ is the membership function of $\tilde{D}$ (the fuzzy variable) then the possibility, necessity, and credibility of fuzzy event can be represented respectively by:

$$Pos\{\tilde{D} \geq r\} = \sup_{x \geq r} \mu_{\tilde{D}}(x) \quad (4)$$

$$Nec\{\tilde{D} \geq r\} = 1 - \sup_{x < r} \mu_{\tilde{D}}(x) \quad (5)$$

$$Cr\{\tilde{D} \geq r\} = \frac{1}{2}(Pos\{\tilde{D} \geq r\} + Nec\{\tilde{D} \geq r\}) \quad (6)$$

Here, the credibility of a fuzzy event is defined as the average of its possibility and necessity. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. For this reason, Ghaffari-Nasab(2013) and Cao(2010) have shown that the credibility measure uses combining these two functions and essentially plays the role of probability in fuzzy terms.

A fuzzy triangle variable is measured as follow:

$$\mu_{\tilde{D}}(x)\begin{cases}\dfrac{(x-d_1)}{(d_2-d_1)} & d_1 \le x \le d_2 \\ 1 & x = d_2 \\ \dfrac{(d_3-x)}{(d_3-d_2)} & d_2 \le x \le d_3 \\ 0 & otherwise\end{cases} \tag{7}$$

Based on these definitions, the possibility, necessity and credibility can be rewritten as follows:

$$Pos\{\tilde{D} \ge r\}\begin{cases}1 & if\ r \le d_2 \\ \dfrac{d_3-r}{d_3-d_2} & if\ \ d_2 \le r \le d_3 \\ 0 & if\ r \ge d_3\end{cases} \tag{8}$$

$$Nec\{\tilde{D} \ge r\}\begin{cases}1 & if\ r \le d_1 \\ \dfrac{d_2-r}{d_2-d_1} & if\ \ d_1 \le r \le d_2 \\ 0 & if\ r \ge d_2\end{cases} \tag{9}$$

$$Cr\{\tilde{D} \ge r\}\ \begin{cases}1 & if\ \ r \le d_1 \\ \dfrac{2d_2-d_1-r}{2(d_2-d_1)} & if\ \ d_1 \le r \le d_2 \\ \dfrac{d_3-r}{2(d_3-d_2)} & if\ \ d_2 \le r \le d_3 \\ 0 & if\ \ \ \mathrm{r} \ge \mathrm{d3}\end{cases} \tag{10}$$

3. Problem definition

The proposed model aims to establish health care centers in a way that accessibility to facilities in the shortest time, is possible. Each candidate demand node can be considered for the establishment of facility, too. There are K different types of health care centers and C = K different service level is required. The health centers of type k offer the service levels of c = 1,…, k and service levels of c=k+1,…, C in these centers are not proposed. In this problem, patients to be transferred according to the requested service as well as the distance to an established center that offers the level of service required. After transferring the proportion of patients may be move to a higher level service due to complications detected. In this situation, if the new requested service is not provided at that medical center, the patient will be referred to a medical facility with a higher level of service.

The model assumes:

- In this model, k types of facility and C types of service are considered.
- A facility at level k provides all types of services to facilities at lower levels (successively inclusive).
- At each node, you can establish at the most one facility, and facility capacity in each level is limited to any particular service.
- Shortage cost is considered for demand beyond facility capacity at each level of service.
- In this model, the hypothesis of a possible transfer of patients from each level of services to the higher levels is considered.

- Because the determination of demand is based on the population of each node, and also because in practice it is difficult to determine the exact population in the future, so in this paper, demand is considered a fuzzy number with membership function of the triangular such as $\tilde{w}_i = (w_{1i}, w_{2i}, w_{3i})$.

3.1. Sets

CU : Number of nodes
i, j :Set of points, including the demand nodes and the candidates for the establishment of health centers i , $j = 1, 2, ..., n$
k : set of hierarchical levels $k = 1, 2, ..., K$
c :set of level of services provided $c = 1, 2, 3, ..., C$

3.2. Parameters

$\tilde{w}_i$: Fuzzy demand at node i
Ra_{cc+l} :The rate of the transition from service level c to the service level c + l
Q_{kc} : The capacity of a facility type k at service level c
u_{ic} : Percentage of requested service type c, at demand point i
$Budget$: The total available budget (money unit)
d_{ij} : Distance between facility i to facility j
d_{hj} : Distance between facility h to facility j
M : A very large number

3.3. Decision variables

x_{ijkc}	Number of patients at point i with the requested service level c from the established facility type k at point j
r_{jkhc}	The crowed referred from facility j to facility h of type k in order to service level c
y_{jk}	A binary variable which is 1 if facility type k is established and otherwise is 0
sh_{jc}	Shortage cost related to the lack of capacity of facility j at service level c

3.4. Mathematical model

In this section, the mathematical model is discussed.

$$Minimize \quad Z = \sum_{i}^{n}\sum_{j}^{n}\sum_{k}^{K}\sum_{c}^{C}\theta d_{ij} \times x_{ijkc} + \sum_{j}^{n}\sum_{h}^{n}\sum_{k}^{K}\sum_{c}^{C}\theta d_{jh} \times r_{jhkc} + \sum_{j=1}^{n}\sum_{c}^{C}\alpha \times Sh_{jc} + \sum_{j}^{n}\sum_{k}^{K} CO_k \times y_{jk} \tag{11}$$

Subject to

$$\sum_{i}^{n}\sum_{j}^{n}\sum_{k}^{K-1}\sum_{c=k+1}^{C} x_{ijkc} = 0 \tag{12}$$

$$\sum_{j}^{n}\sum_{h}^{n}\sum_{k}^{K-1}\sum_{c=k+1}^{C} r_{jhkc} = 0 \tag{13}$$

$$\sum_{k=c}^{K}\sum_{j=1}^{n} x_{ijkc} = u_{ic} \times \tilde{w}_{i} \quad c = 1,2,3,\ldots,C \quad i = 1,2,3,\ldots,n \tag{14}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} y_{jk} \le CU \tag{15}$$

$$\sum_{k=1}^{K} y_{jk} \le 1 \quad j = 1,2,\ldots,\mathrm{n} \tag{16}$$

$$x_{ijkc} \le M \times \mathrm{y}_{jk} \quad i,j = 1,2,\ldots,\mathrm{n} \quad \mathrm{k}=1,2,3,\ldots,\mathrm{K} \quad \mathrm{c}=1,2,3,\ldots,\mathrm{C} \quad ,\mathrm{k} \ge \mathrm{c} \tag{17}$$

$$\sum_{h=1}^{n}\sum_{k \ge c+l}^{K} r_{jh(k)(\mathrm{c}+l)} = \sum_{i=1}^{n}\sum_{k=c}^{k<c+l} Ra_{(\mathrm{c})(c+l)} x_{ijkc} \quad j = 1,2,\ldots,n, \quad j \ne h, \quad c = 1,2,3,\ldots,C-l, \quad l = 1,2,3,\ldots \mathrm{C}-1 \tag{18}$$

$$\sum_{j=1}^{n} r_{jhkc} \le M \times y_{hk} \quad j \ne h \quad h = 1,2,\ldots,\mathrm{n} \quad k = 1,2,3,\ldots,\mathrm{K}, \quad c = 1,2,3,\ldots,C \tag{19}$$

$$\sum_{i=1}^{n}\sum_{k=1}^{K} x_{ijkc} + \sum_{i=1}^{n}\sum_{k=1}^{K} Ra_{(c-l)(\mathrm{c})} x_{ijk(c-l)} + \sum_{h=1}^{n}\sum_{k=1}^{K} r_{hjkc} \le Q_{kc} \times y_{jk} + Sh_{\mathrm{jc}} \quad j = 1,2,\ldots,\mathrm{n} \quad c = 1,2,3,\ldots,C \tag{20}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{3} CO_{k} \times y_{jk} \le Budget \tag{21}$$

Equation (11) expresses the objective function that the first term computes the expected cost of transferred patients from demand node to the appropriate facility. The second term computes the expected transportation cost of transferred patients to the higher level facilities. It should be noted that in this paper, the transportation cost is considered based on multiple distances. The third term computes the expected shortage cost for lack of capacity of facilities at different services and finally the fourth term computes the fixed cost of locating facilities.

Constraints (12) and (13) guarantee that demand of service type c can be provided by no facility of lower levels. Constraint (14) ensures that all demands at each node i for each level of service c, has been answered. Constraint (15) states that the maximum total number of established facilities cannot be more than candidate nodes. Constraint (16) specifies that there can be maximum one type of facility at each node. By Constraint (17), service may only be obtained at points where facilities of appropriate level are located (level c service may be obtained at a level k facility only if k of an equal or higher level than of c) successively inclusive is mandated (18) specifies proportion of patients who were referred to the higher level facilities based on transition rate. In fact, for this group of patients, current facility, not able to respond to their needed services and must be transferred to the facility at higher levels. Constraint (19) indicates that the patient should be referred to a facility that was established. Constraint (20) is capacity limitation based on the type of service provided in each facility. For an appropriate facility j the number of people who are looking for the specified service c of that facility (first term) and proportion of patients who are referred from lower level services of the facility to its higher level service c (the second term) and the number of patients who are transferred from the other lower level facilities to the specified facility for the service c (the third term) all should be less than or equal to capacity limitation service c applied to the facility based on its type. If the number of patients (demand) for the specified service c, at facility j is more than its capacity, shortage is considered for service level c of the facility j. Constraint (21) is the budget constraint. It states that the construction costs of all type of facilities should be less than or equal to the available budget.
In order to solve the above model due to fuzzy demand, Cr factor will change in the range [0, 1] then according to the credibility function demand at each node to be specified. Using harmony search algorithm and CPLEX solver best possible answers were determined according to the parameter Cr.

4. Solution procedure

Solving the proposed model in the simplest condition is identical to solving the incapacitated location-allocation problem which Megiddo and Supowit (1984) have shown that it is NP-hard. This reveals that solving the study's model in a reasonable time is extremely hard. Therefore, the use of meta-heuristic to find approximate solution to large instances becomes a practical consideration. In order to solve this model, an efficient meta-heuristic is applied based on HS then a local search is applied on the best achieved solution.

4.1. Harmony search algorithm

In this section, harmony search algorithm is described. For the first time, HS was presented by Geem et all (2001), inspired by a piece of music. Despite the fact that not much time has passed since the introduction and utilization of these algorithms, it is used for solving optimization problems (discrete and continuous) a lot. The possibility of combining this algorithm with other heuristic and Meta heuristic algorithms have also been studied.

4.2. HS steps

First, the parameters in harmony search algorithm are expressed, then Harmony search algorithm is reviewed step by step. (Note that our problem space is discrete and thus the algorithm described in the discrete space).
Harmony search algorithm parameters:
HMS: The number of vectors in the harmony memory matrix

HMCR: Harmony memory consideration rate
PAR: Pitch adjustment rate
BW: Band width (this parameter is used in continuous space)
MaxIt: the maximum number of iterations
The general shape memory matrix is shown below:

$$\left[\begin{array}{cccc|c} A_1^1 & A_2^1 & \cdots & A_N^1 & f(A^1) \\ A_1^2 & A_2^2 & \cdots & A_N^2 & f(A^2) \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ A_1^1 & A_2^1 & \cdots & A_N^{HMS} & f(A^{HMS}) \end{array}\right] \tag{22}$$

The number of rows in the matrix is HMS; each row of HMS is a solution or answer to the problem. N is the number of variables in each solution vector.
The algorithm has four phases:

1. Create an initial generation (initialization)
2. Producing new Harmony
3. Harmony memory updates
4. Stop condition

In the first phase, early-generation Harmonies randomly generated and stored in the harmony memory (HM).In the second phase, in order to create a new harmony for making the variable i, first, a number is produced between [0, 1] then this number is compared to HMCR. If it is less, the value of variable is chosen by considering the HM, otherwise it is created randomly. Lee and Geem(2005) suggest that if the value of variable i in New Harmony is chosen from the HM, it can be adjusted by using PAR. In the third phase, the value of newly produced harmony is compared to the worst harmony in the matrix, if it is better, the New Harmony is replaced by it. In the fourth phase, the second and third phases are repeated until a stop condition is ensured.

4.3. Adjustment of parameters in harmony search algorithm

To use the harmony search algorithm, delete by the method of trial and error and solving some typical problems, parameters are regulated as follows. The PAR is considered constant and equal to 0.1, HMS = 300, MaxIt = 3000 and subsequently, the HMCR has been set to increase linearly with the number of iterations, the amount HMCR in each iteration is obtained as following:

$$HMCR(t) = \frac{(HMCR_F - HMCR_I)(t-1)}{NI-1} \tag{23}$$

In the above equation, $HMCR_I$ shows HMCR value in the first iteration and $HMCR_f$ shows its value in the final iteration. The initial and final HMCR value is considered as following Landa-Torres et all(2012).

$$\{HMCR_I, HMCR_f\} = \{0.1,\ 0.03\}$$

4.4. Solution representation

The solution representation used in this paper, is a string length of the candidate node, such that each node is between 0 – k. For example, for k=3, the value of one node is 0 if there is not any facility in that point, 1 if the facility of type low is established, 2 if a facility of type mid is established and finally 3 is allocated if the facility type high is established there.
For example, for a problem with 10 candidate nodes and three types of facility, the solution string is the same fig 1 as follows:

0	1	3	3	2	0	0	2	1	3

Figure 1- solution presentation for problem with 10 candidate nodes and 3 kind of facilities

At the above string in the locations 1, 6 and 7, there is no facility. At places 2 and 9 there are two low facilities. In the locations 5 and 8, there are two mid facilities and finally, there are three high facilities in locations 3, 4 and 10.

So, for initial solution generation, random procedure is used. Note that in order to create the initial solutions for harmony vectors and along the harmony search algorithm, the cost of established facility should be considered. For this purpose, a modified function is defined. In such a way that whenever the number of facilities and their establishment cost is more than the available budget, zero is randomly assigned to one k type facility, if all except one of them (note that there must be at least one facility of type k), then facilities type k-1 are going to get zero value and making them one by one zero. This procedure is repeated until cost is within budget restrictions.

4.5. Local search algorithm

In order to improve the solution of the proposed harmony algorithm, a local search is applied on the best achieved solution. For this purpose, the output of the harmony search algorithm can be used as a starting solution. Improvements may be accomplished by the exchange node of a facility (a randomly selected) with node where no facility has been established if better solution is generated, then we update the string and this will be continued until further improvement is not possible.

4.6. The impact of the Cr on the quality of the solution

It is used in order to estimate patient demand by credibility theory method in this paper. For this purpose, based on a specified Cr level, demand of patients is estimated and then with regard to estimated demands, location is done. Finding the optimum Cr has decisive role in the real objective function value of the problem. Determination of patient's primary demand is an estimation based on Cr and there is obviously a possibility of different amount of demand than predetermined. If the chosen values for Cr is lower, then the intended values for patients demand at every node is estimated near to its max value. This increases the cost of construction but decreases the possibility of deficiencies and penalties. In this situation, the overestimation of demand causes a waste of resources and inappropriate use of resources, because the estimated demand may be greater than its actual value. Also, picking up large amounts of CR causes low value to be selected for each demand node. This reduces the cost of construction, but lack of capacity may occur. In order to find the best amount of the CR based on which is the amount of customer demand, it is estimated that Monte Carlo simulation tools need to be used.

4.7. Determination of the optimal CR

Now, Ghaffari-Nasab(2013) and Cao(2010) suggest the following algorithm in order to find the optimal value of the CR parameter.

CR Optimization Algorithm
Step 1: Cr value in the distance between [0,1] 0.1 to 0.1 is changed, and second to fifth steps are repeated.
Step 2: in specified Cr patient demand and the optimal solutions of the problem are identified.
Step 3: the subprogram for generating a random number is run to determine the level of demand at each node.

The subprogram generating a random number
Sub step 1: i=1
Sub step 2: a random number m_i between the lower and upper bound of fuzzy demand is produced at node i and the amount of membership degree is determined based on the membership function of above-mentioned demand node.
Sub step 3: random number r at the interval [0, 1] is generated.
Sub step 4: If $r \leq f$ m_i is considered as real amount of node i demand and go to step 4, otherwise go to step 2
Sub step 5: i increase by one unit. If $i \leq n$ go to sub step 2, otherwise go to sub step 6
Sub step 6: end
Step 4: According to the Simulated demand point and the location and type of facility in each identified level of Cr, cost to get
Step 5: The third and fourth steps performed 1,000 times and the average cost is calculated
Step 6: The amount of Cr with the lowest mean value of the objective function to be reported as optimal Cr.

5. Numerical results

In this section, at first, optimal Cr is evaluated, then some sample problems are solved and suggested algorithm is examined. Producing sample problems is done as follows:

5.1. Producing random samples

Fuzzy demand numbers of the patient is produced randomly. The number of demand nodes and candidate points are equal (Network). In all problems k=c=3 and α is considered twice as the maximum distance between demand node and established facility. It is assumed that all facilities with one type have equal capacity and the fixed cost for establishing one facility is 1.5 times its capacity based on the type of that facility. In order to produce the problems, it is started with small size problems and with increasing demand nodes and budget extends to large scale problems.

5.2. Numerical results of optimal Cr determination

In order to find optimal Cr, the algorithm in section 5.4 is used. At first, 9 sample problems with respect to above descriptions are produced. Generated problems are shown in Table 1. At the first column, the number of sample problem is shown. At the second column, n is the number of all demand nodes (candidate nodes) and finally the available budget is presented at the third column.

Table 1-generated random problems

#	n	Budget(money unit)
P1	5	800
P2	7	800
P3	7	300
P4	9	800
P5	10	400
P6	10	800
P7	12	800
P8	8	800
P9	11	1600

In this stage, demand of each node is estimated for every problem with respect to each Cr and the optimal value of objective is obtained by CPLEX solver for small size problems. Then for each level of Cr and based on optimal solution obtained, location of facility is considered for each problem and by use of subsection 5.4, demand is simulated for each node. Objective function of the problem is calculated for simulated demands. Producing demand and cost calculation is repeated 1000 times and the average of 1000 times simulation is considered as the cost of the problem at the specified Cr. based on the average cost of considered problems, optimal Cr is determined. Table 2 shows the results of the optimal solution and simulation.

Table 2- planned objective function values and the average objective function for small size problems

Problem	Cr	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
P1	Z1	213.3	232.1	241.5	241.6	241.7	241.8	254.4	264.1	278.9	279.5	279.7
	M1	372.8	385.3	275.7	275.9	275.9	275.9	276.19	267.8	267	279.1	279.1
P2	Z1	280.3	280.5	280.6	280.8	280.9	282.3	318.2	318.6	335.8	359.3	392.5
	M1	353.3	352.3	351.2	351.6	351.7	350.9	321.3	321.2	321.4	321.1	392.1
P3	Z1	280.2	280.4	280.6	280.7	280.9	282.3	320.8	359.4	398.1	443.1	494.4
	M1	352.6	352.5	352.3	351.04	351.5	349.1	349.9	351	350.3	349.9	351.6
P4	Z1	280.6	280.8	281	301	318.3	318.5	336.9	368.7	392.7	393.2	393.6
	M1	378.8	378.1	378.4	377.6	327.6	327.1	327.7	327.5	391.8	391.8	391.5
P5	Z1	292.7	317.7	318	328.8	345.3	369.1	392.4	392.7	402.4	435.3	444.5
	M1	611.8	492.1	491.1	491.9	490.5	488	393.5	393.6	393.5	393.6	444.6
P6	Z1	323.5	342.8	369.9	392.4	392.6	392.8	405.9	452.9	510.5	568.12	625.7
	M1	695.7	693.3	692.4	443.6	446.7	443.5	442.9	443.5	443.1	442.9	444.3
P7	Z1	323.5	342.8	369.9	392.5	392.7	392.8	405.9	445.4	458.5	483.1	483.6
	M1	689.1	691.2	692	444.9	445.7	445.1	445	446.3	446.2	482.6	482.6
P8	Z1	363.9	392.7	392.9	393.1	399.9	416	445.8	475.4	483.5	507.9	558
	M1	715.1	550.6	451.5	453.5	453.8	532.6	480.1	477.6	483.2	483.2	556.1
P9	Z1	392.9	393.1	401.5	423.9	445.7	453.7	483.5	492.6	543.3	558.4	587.2
	M1	546.2	542.7	545.7	545.4	484	484.3	483.3	483.3	483.3	557.3	557.4
Average		414.8	405.6	398.1	376.1	373.6	380.3	382.4	393.3	410.2	429.4	453.2

In the table above, Z1 represents the objective function of the problem with the identified demand at any given level of credibility and M1 is the average of objective function for demand simulation. At the first row of table 2, Cr values that are changed in the distance between [0, 1] 0.1 to 0.1) are shown. At each column, Z1, and M1 are represented for specified Cr in each problem. As indicated in Figure 1, the objective function is reduced by increasing the credibility, and this trend extends up to 0.6 credits, then the average of the objective function increases. So according to the obtained values, 0.6 is considered the optimal level of credit.

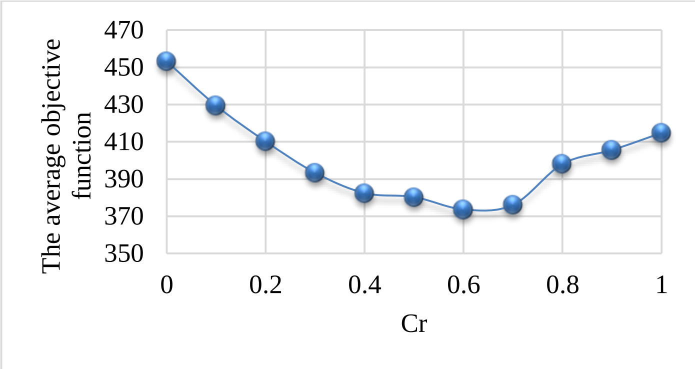

Figure 2- the average objective function for small size problems

5.3. Performance evaluation of HS algorithm in small size problems

In order to evaluate the performance of the algorithm, the results of the algorithm in solving the small size problems was compared with the results of exact solution by CPLEX solver. Table 3 shows the results of this comparison.

Table 3- compares the output of the harmony search algorithm and CPLEX solver

Prob	CPLEX		HS algorithm		GAP (%)
	Object value	T(s)	Object value	T(s)	
P1	241.7	2.8	241.7	195.5	0.00
P2	280.9	2.9	281.05	199.2	0.0005
P3	280.9	4.2	281.07	191.8	0.0005
P4	318.3	30.6	318.3	198.3	0.00
P5	345.3	52.9	345.3	200.5	0.00
P6	392.6	99.8	392.6	204.6	0.00
P7	392.7	97.7	393.01	205.7	0.0007
P8	399.9	310.9	400.3	218.1	0.001
P9	445.7	1525.5	446.46	240.2	0.001
AVG	344.2	236.4	355.09	205.98	

In the above table, the first column indicates the number of sample problems of small size and at the next columns objective function and T, solution time (second), obtained by CPLEX solver and harmony search (HS) are presented, and GAP% is the difference between the optimal value of objective function obtained by CPLEX solver and the best objective function value is obtained by harmony search algorithm divide by optimal value of objective function multiplied by 100.
According to the above table, the average time needed to solve the small size problem by suggested algorithm is equal to 344 seconds .On issues such as problems with the network of very small dimensions of 5 * 5 or 7 * 7 solution time by CPLEX solver to lower the resolution to be obtained from the harmony algorithm and it is because of link between GAMS & MATLAB. But the problems with dimensions of 10 * 10 or 12 *12 solution time by CPLEX solver is much less than the HS time resolution.

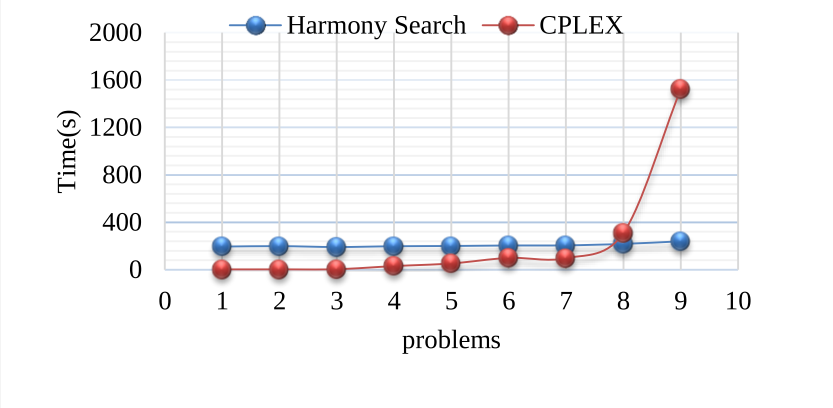

Figure 3- solution times by CPLEX and HS algorithm for small size problems

The average GAP in the harmony search method in comparison with the exact method is equal to 1%.

5.4. Performance of the HS Algorithm in large size problems

In order to evaluate the performance of HS algorithm and the algorithm combined with a heuristic algorithm with large size problems, six randomly generated problems are given in Table4. In this table, the first column is the number of large size problems. At the second column, n is the number of all demand nodes (candidate nodes) and finally the available Budget budget is presented at the third column.

Table 4- The specification of 6 problems in large size problems

#	n	Budget(money unit)
P10	15	600
P11	17	800
P12	20	800
P13	22	1000
P14	25	1000
P15	30	1000

The results of problem solving with HS algorithm and harmony combination heuristic algorithm are presented in Table 5. In this table, objective function and T, solution time (second), obtained by harmony search (HS) and harmony combination heuristic algorithm are presented, and GAP% is calculated.

Table 5-Comparing the performance of the HS algorithm and HS combination heuristic algorithm

#	HS algorithm		GAP (%)	HS combination heuristic algorithms		GAP (%)
	Object value	T(s)		Object value	T(s)	
P10	502.46	439.13	0.001	501.67	471.86	0.00
P11	560.95	467.67	0.0025	560	511.17	0.00
P12	600.004	523.1	0.001	598.98	554.2	0.00
P13	668.7	566.02	0.0024	667.04	702.92	0.00
P14	765.72	650.5	0.0016	764.5	1004.3	0.00
P15	913.33	792.34	0.04	877.3	1320.74	0.00
AVG	668.53	573.13	0.008	661.58	760.86	0.00

Due to the use of the HS Output for local search algorithm, the HS average time is less than the combined algorithms. Average time solution for harmony algorithm is 573.13, while it is 760.86 for the hybrid algorithm. In terms of solution quality on average 0.8%, local search is able to improve the solutions. The greatest improvement occurred in response to the combination of the two algorithms in problem 15.

6. Result

In this study, a linear hierarchical facility location model is presented for health care centers with the objective of minimizing the time to reach relief centers and the cost of providing facilities. In this model, the demand was considered as fuzzy and to optimize the conditions of uncertainty, fuzzy credibility theory was used. Harmony search algorithm for solving the model was developed. The results show that the proposed harmony search algorithm offers near-optimal solution in problems with small sizes. For large scale problems, combined harmony search algorithm was improved compared to the first algorithm.

References

Boffey, B., Yates, D., & Galvao, R. D.(2003). An algorithm to locate perinatal facilities in the municipality of Rio de Janeiro. *Journal of the Operational Research Society*, Vol. 54, pp. 21-31.

Calvo, A.B., & Marks, D. H. (1973). Location of health care facilities: an analytical approach. Socio-Economic Planning Sciences, Vol. 7, pp. 407-422.

Cao, E., & Lai, M. (2010). The open vehicle routing problem with fuzzy demands. *Expert Systems with Applications, 37*(3), 2405-2411.

Farahani, R.Z., Hekmatfar, M., Fahimnia, B., & Kazemzadeh, N.(2014). Hierarchical facility location problem: Models, classifications, techniques, and applications. *Computers & Industrial Engineering*, Vol. 68, pp. 104-117.

Galvão, R.D., Espejo, L. G. A., & Boffey, B. (2006). Practical aspects associated with location planning for maternal and perinatal assistance in Brazil. *Annals of Operations Research*, Vol. 143, pp. 31-44
.

Geem, Z.W., Kim, J.H., Loganathan, G.V. (2001). "A new heuristic optimization algorithm: harmony search''. *J. Simulations* 76, pp.60–68
.
Gerrard, R.A., & Church, R. L.(1994). A generalized approach to modeling the hierarchical maximal covering location problem with referral, *Papers in Regional Science*, Vol. 73, pp. 425-453.

Ghaffari-Nasab, N., Ahari, S. G., & Ghazanfari, M. (2013). "A hybrid simulated annealing based heuristic for solving the location-routing problem with fuzzy demands''. *Scientia Iranica*, *20*(3), 919-930.

Hodgson, M.J., & Jacobsen, S. K.(2009). A hierarchical location-allocation model with travel based on expected referraldistances. *Annals of Operations Research*, Vol. 167, pp. 271-286.

Landa-Torres, I., Del Ser, J., Salcedo-Sanz, S., Gil-Lopez, S., Portilla-Figueras, J. A., & Alonso-Garrido, O.(2012). A comparative study of two hybrid grouping evolutionary techniques for the capacitated P-median problem. *Computers & Operations Research*, Vol. 39, pp. 2214-2222.

Lee, K.S., Geem, Z. W., Lee, S. H., & Bae, K. W.(2005). The harmony search heuristic algorithm for discrete structural optimization. *Engineering Optimization*, Vol. 37, pp. 663-684.

Liu, B.(2004). Uncertain Theory: An Introduction to its Axiomatic Foundations, *Springer, Berlin*.

Megiddo, N. ,Supowit KJ.(1984). On the complexity of somecommon geometric location problems. , *SIAM J Comput*, Vol. 13, pp. 182-196.

Narula, S.C.(1984). Hierarchical location-allocation problems: a classification scheme, *European Journal of Operational Research*, Vol. 15, pp. 93-99
.
Narula, S.C., & Ogbu, U. I. (1979). An hierarchal location—allocation problem." *OMEGA, The International Journal of Management Science*, Vol. 7, pp. 137-143.

Okabe, A., Okunuki, K. I., & Suzuki, T. (1997). A computational method for optimizing the hierarchy andspatial configuration of successively inclusive facilities on a continuous plane. *Location Science*, Vol. 5, pp. 255-268.

Tien, J.M., & El-Tell, K. H. A. L. A. F. (1984). A quasihierarchical location-allocation model for primary health care planning. Systems, Man and Cybernetics, *IEEE Transactions on*, Vol. 3, pp. 373-380.

Yan, M. (2003). Impact of Remote Sensing & GIS in Management of Cities Futures, *Translated by EsmailYousefi, Urban Management Quarterly*, Vol., pp. 15-16.

Yasenovskiy, V., & Hodgson, J. (2007). Hierarchical location-allocation with spatial choice interaction modeling. *Annals of the Association of American Geographers*, Vol. 97, pp. 496-511
.
Zadeh, L.A.(1965) . Fuzzy sets, *Information and Control*, 8, pp. 338–353.