



An Economic Order Quantity Model for Deteriorating Items with Trade Credit Financing for Quadratic Demand

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ABSTRACT

Objective: Researchers established an EOQ model for degrading goods with trade credit policy under invariant, stock-linked, exponential, and linearly time-dependent demand. The analysis imposed a condition for quadratic time-dependent demand. The mathematical model is developed to obtain total profit by considering two different cases. One common observation is that the demands for the goods that are on display in the supermarket fluctuate. In this study demand is measured to be quadratic time-sensitive. The EOQ is generally applied to locate the most favorable order quantity in order to maximize the total supply cost.

Methods: The EOQ model considers that the total order for an article is received into inventory at one specified time which is when the EOQ model assumes that the products are produced.

Results: There are numerous costs acquired in the existent practice such as ordering cost, sales revenue, carrying cost, interest earned, and interest charged, etc.

Conclusion: The implementation of the sensitivity test and an optimal solution helps to confirm how the mathematical model will generate total profit in two different ways. The EOQ method is used to identify the order quantity that maximizes total supply costs in the order's viewpoints. This should assist with future management of degraded products under a trade credit scheme, as well as advance the accuracy and reliability of making Inventory-related decisions due to demand fluctuations.

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1. Introduction

Economic order quantity (EOQ), a fundamental concept in inventory management, helps businesses determine the ideal order quantity to lower overall inventory costs. It balances the costs of maintaining inventory and placing orders. Businesses can prevent under stocking, which can result in lost sales because of unfulfilled demand, and overstocking, which raises holding costs, by determining EOQ.

Important EOQ features from a business standpoint: Reducing Total Inventory Costs: EOQ seeks to identify the optimal point at which ordering and inventory holding expenses are the lowest. Keeping Ordering and Holding Costs in Balance: While holding costs include things like storage, insurance, and possible obsolescence, ordering costs include things like processing purchase orders.

Optimizing Inventory Levels: EOQ assists companies in determining how much to order each time in order to maintain ideal inventory levels, guaranteeing that they can satisfy demand without incurring undue expenses.

Enhancing Cash Flow: Companies can release funds that would otherwise be invested in surplus inventory by optimizing inventory levels. Increasing Efficiency: By streamlining the inventory management procedure and lowering the possibility of stock outs, EOQ raises overall operational effectiveness.

Helping to inform reordering decisions: The reorder point, which indicates when to replace a fresh order to refill stock, can be found using EOQ calculations. To sum up, EOQ is a useful tool for companies of all sizes to efficiently manage inventory, cut expenses, and boost overall operational effectiveness. It assists companies in striking the best possible balance between ordering and holding expenses, which improves inventory and boosts profitability. It assists companies in striking the best possible balance between ordering and holding expenses, which improves inventory control and boosts profitability.

The purpose of the EOQ formula is to determine the optimal number of product units to order. If successful, a company can lower the costs related to buying, transporting, and keeping units. An algorithm in computer software is used by businesses with large supply chains and variable costs to determine EOQ; the formula can be modified to produce different production levels or order intervals. EOQ is a vital cash flow tool. The formula can be used by a business to control the amount of cash locked up in the inventory balance.

For many businesses, inventory is their most valuable asset aside from human capital, and they must maintain enough stock levels to meet customer requests. The money saved by EOQ can be invested or used toward other business requirements if it helps lower inventory levels.

The EOQ formula determines the inventory reorder point for a business. When inventory reaches a certain level, the EOQ formula will force the need to order more units if it is applied to company activities. The company can continue to fulfil client orders and prevent inventory shortages by establishing a reorder point. A shortfall cost occurs when a business runs out of inventory. This is the money lost because the business does not have enough inventory to fulfil an order. A lack of inventory could also result in a client stopping business or placing fewer orders in the future.

The cost of placing an order, the cost of storing goods, and the time of restocking are all taken into consideration by EOQ. Higher ordering expenses and more storage space are required if a business consistently places minor orders to maintain a certain inventory level.

Economic Order Quantity (EOQ) Restrictions: The EOQ calculation assumes a constant level of customer demand. It is also assumed that ordering and holding costs remain constant in the computation. This means that the method might not account for business events like changing client demand, seasonal fluctuations in inventory costs, lost sales revenue due to inventory shortages, or discounts a company might get for buying more inventories.

Economic order amount is an approach for inventory management that helps in making wise decisions. It outlines the optimal amount of inventory that a company should purchase in order to meet demand and reduce holding and storage costs. The economic order quantity's assumption that the market for the company's products will stay stable over time is one of its many serious disadvantages. Increases in product demand or setup costs for the business will result in a larger economic order quantity. However, if the company's holding costs rise, it will be lower. Economic order quantity is crucial since it aids businesses in effectively managing their inventories. Businesses would typically store too much inventory during times of low demand and too little inventory during times of high demand if inventory management strategies like these are not used. Missed opportunities result from either issue. Effective inventory management is essential to a business's success. Due to the expense of managing inventory and occasionally needing to get rid of outdated stock, having too much inventory raises expenses and reduces profitability. A business that has too little inventory is missing out on revenues because it cannot keep up with demand. Efficient inventory management is ensured by economic order quantity.

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Quadratic demand, which is frequently depicted by a quadratic equation, is a situation in which the demand for an item varies over time in a parabolic fashion in an Economic Order Quantity (EOQ) model. The conventional EOQ model, on the other hand, makes the assumption that demand is constant. When quadratic demand is incorporated into EOQ calculations, changes must be made to account for the shifting demand pattern. This could result in different optimal order quantities and reorder points than when demand is constant. A constant demand rate is assumed by the basic EOQ model, which means that during a given time period, the amount of products required stays constant. These results in a simple formula that may be used to determine the ideal order quantity (EOQ) based on annual demand, holding costs, and ordering expenses.

In actuality, there is fluctuating demand for many things. Seasonality, promotions, and other variables may cause it to fluctuate; occasionally, it will exhibit a parabolic or quadratic pattern. Demand may, for instance, rise quickly at the beginning of a season, peak in the middle, and then fall toward the conclusion. A quadratic equation with the demand rate as a function of time can be used to illustrate this. The conventional EOQ calculation must be adjusted when demand fluctuates. The EOQ determined under a constant demand assumption is probably going to be different from the EOQ for a quadratic demand model. Depending on the particular demand pattern, the ideal order quantity may need to be revised to take the shifting need into account. This could result in bigger or smaller order quantities. It's also necessary to modify the reorder point, which initiates a new order. Depending on whether demand is rising or falling at the time the reorder is placed, the reorder point for quadratic demand may need to be higher or lower. Other variables, such as quantity discounts, time-varying holding costs, or even shortages (backordering), might be added to the EOQ model. Although these variables make the computations much more difficult, they can also result in a more accurate and realistic depiction of the requirements for inventory management.

Consider a product whose demand is at its highest in the summer. Stock outs could result from a standard EOQ model underestimating the ideal order quantity in the summer. This seasonal variation would be better captured by a quadratic demand model, which would lead to smaller orders at other times of the year and a greater EOQ in the summer. It can be more difficult to implement EOQ models with quadratic demand. It may entail more intricate computations and calls for precise demand pattern forecasts. Nonetheless, it might be worth it given the possible advantages of better inventory control, lower expenses, and enhanced customer support.

Only two types of time-varying demands have been considered so far by the researchers: linear and exponential. A linear time-varying demand implies a consistent change in the product's demand rate per unit of time. This is a somewhat implausible event that rarely happens in the actual market. Some academics have recently dealt with demands that fluctuate exponentially over time. The demand for some products, such as new computer chips, spare parts for new airplanes and other machinery, etc., is rising quickly, whereas the demand for computers and obsolete machine parts is falling quickly. This kind of demand was shown by some modelers as an exponentially rising or falling function of time.

The actual market demand for any particular product is unlikely to change at a rate as high as an exponential one, and an exponential rate of change is incredibly high. Another (perhaps useful) tactic is to look at a time-quadratic demand, which may show both accelerated and retarded demand growth.

The order-level inventory problem for a decaying item with a time-dependent quadratic demand is examined in this study. No shortage is permitted, and it is believed that the inventory would degrade at a steady pace. The model has an unlimited time horizon and is solved analytically. It is discussed how to obtain the solution for a finite time horizon. A numerical example is used to demonstrate the conclusions, and the model's sensitivity is examined.

A quadratic time-varying demand rate serves as the foundation for the current model. In the beginning, it was explained why a quadratic demand should be taken into consideration rather than a linear or exponential demand. Researchers typically assume that the demand rate is a linear function of time when working with time-varying demand patterns. It is uncommon to observe a constant increase ($b > 0$) or drop ($b < 0$) in the demand rate, as implied by demand rate functions of the form $D(t) = a + bt$, $a > 0$, $b \neq 0$. Quadratic holding costs in an EOQ (Economic Order Quantity) model indicate that the cost of keeping inventory rises in direct proportion to the square of the quantity maintained. In contrast, holding costs are assumed to be constant in the standard EOQ model. The ideal order quantity is impacted when holding costs are quadratic because the EOQ computation must be modified to take into consideration this non-linear relationship.

Assumes that the cost of storing inventory per unit of time is constant. Ordering costs, which go down with larger orders, and holding costs, which go up with larger orders, are balanced to create the EOQ formula. The holding cost rises in proportion to the square of the inventory level rather than remaining constant. This suggests that when levels rise, holding costs increase more quickly. For instance, the holding cost quadruples rather than just doubles when the amount of inventory doubles. When employing a quadratic holding cost, the EOQ will often be lower because the holding cost increases more quickly with increasing volumes. This is due to the fact that the holding costs are disproportionately greater for larger orders. Holding costs are not always may result from variables include storage capacity, insurance, and possible obsolescence. The EOQ model can be made more precise and realistic by using a quadratic holding. The quadratic relationship must be taken into consideration while adjusting the EOQ formula. This usually entails minimizing a total cost function that incorporates the quadratic holding cost term using calculus. The basic EOQ formula must be modified in order to include quadratic holding costs in the EOQ model, which produces more realistic optimal order quantities and more accurate depiction of inventory management issues.

Demand is divided into three categories in the EOQ (Economic Order Quantity) model: constant, time-dependent, and stock-dependent. While stock-dependent demand fluctuates according to inventory levels, time-dependent demand changes over time (e.g., seasonally), and constant demand assumes a stable pace. It is also possible to suppose that holding costs are either constant or quadratic, meaning that they rise in proportion to inventory levels. The ideal order quantity (EOQ) and general inventory management techniques are impacted by these variances. Because it is far more expensive to store bigger quantities, quadratic holding costs result in a higher EOQ than constant holding costs. In summary, the ideal EOQ and general inventory management choices are greatly influenced by the assumptions made about demand and holding costs. While time-dependent and stock-dependent demands—especially when combined with quadratic holding costs—offer a more accurate depiction of real-world inventories, constant

demand and holding costs simplify the model. Both stock-dependent and linearly time-dependent demand have an effect on optimal order quantities and overall inventory costs in EOQ (Economic Order Quantity) models, but they do so in different ways. Order timing and the ideal cycle length are impacted by linearly time-dependent demand, in which the rate of demand fluctuates at a steady pace over time. The ideal order quantity and total inventory holding costs are impacted by stock-dependent demand, in which demand is impacted by the existing inventory level. When calculating the ideal order quantity and order cycle length, the EOQ model must take the fluctuating demand rate into consideration. Depending on the time period taken into consideration, the ideal order amount may change.

In inventory management, a quadratic time-dependent demand is one in which a quadratic function describes how the demand for a product varies over time. This indicates that demand is neither growing nor decreasing at a steady pace, but rather at a faster or slower pace. The demand rate, $D(t)$, at time 't' is specifically given by the formula $D(t) = a_1 + b_1t + c_1t^2$, where ' a_1 ', ' b_1 ', and ' c_1 ' are constants and ' c_1 ' is not zero. This study aims to explore how time – varying quadratic demand influences the EOQ model.

According to the ordering quantity, Chung et al. (2004) established lot-sizing assessment under permissible delay. The best inventory approach for non-instantaneous receipt under trade credit was created by Ouyang et al. (2005). Teng et al. (2006) highlighted the significance of lot-sizing guidelines and ideal manufacturer pricing in trade credit financing. In a DCF analysis, Chung et al. (2006) suggested the best ordering strategy for deteriorating items under trade credit depend on the quantity of the order. The design of optimal ordering planning for a vendor in a supply chain with fluctuating trade credits was devised by Teng and Goyal (2007). Liao (2008) introduced an EOQ model that involves non-instantaneous delivery and exponentially decaying products while using two-stage trade credit. Chang et al. (2008) provided an inventory of lot-size models with permissible delay: A review. Tsao (2009) offered sellers optimal ordering and concessional strategies under a price cut and trade credit. Teng (2009) designed optimal ordering strategies for a trader who offers discrete trade credits to its products and clientele with poor credit clientele. Shah et al. (2010) created an EOQ model in fuzzy environments and allowed for delay. The optimal strategies for manufacturers to use in a supply chain with fluctuating trade credit were devised by Chang et al. (2010). Balkhi (2011) designed an optimal fiscal ordering strategy that utilizes diverse dealer trade credits to fade products for fixed horizon cases. Thangam (2012) planned strategies for reducing prices and sizing lots for unpreserved articles in a supply chain using move – on payment and two – stages trade credits. Two storehouses were used by Liao et al. (2012) to recognize lot-sizing conclusions for fading products by using order -size-associated trade credits. Inventory models were analyzed by Chung et al. (2013) in a supply chain system with conditional trade credits. Singh and Sharma (2014) presented optimal trade-credit course of action for fragile objects. Several researches are available in the literature like, Ouyang et al. (2015), Tyagi (2016), Majumdar et al. (2016), Tripathi et al. (2017), Wang and Lui (2018), Huang et al. (2019), Tripathi and Pandey (2020), Tripathi and Mishra (2021), Tripathi and Pandey (2023), Tripathi and Kaur (2024) etc. Das et al. (2024) designed a space valued EOQ model with diverse payment approaches for green items under break valued grey wolf optimizer algorithm strength. Tripathi (2024) presented an EOQ model for time sensitive demand with decline, inflation, shortages and permitted delay.

While theoretical models for quadratic time-dependent demand exist, there's a lack of extensive empirical studies validating these models in diverse real-world contexts. More research is needed to understand when and how quadratic demand functions best represent actual demand patterns in various industries. While it is acknowledged that quadratic demand is possible, there is a significant lack of empirical studies that demonstrate its widespread use in real world scenarios.

Linear or exponential demand is the primary focus of most inventory models, while quadratic demand is often considered as a secondary consideration or extension. Complex mathematical formulations and analytical challenges are more difficult to develop inventory models with quadratic time- dependent demand than linear or constant demand.

2. Notations and Assumption

2.1 Notations

Notations are as follows:

$q(t)$	inventory level at time 't'
$D(t)$	time associated demand rate
c	unit purchase cost
θ	deterioration rate, $0 < \theta < 1$
s	unit selling price ($s > c$)
$\Omega(T)$	retailer's profit/unit time
h	unit holding cost/unit time
Q	retailer's order quantity
M	seller's trade credit period in years
K	ordering cost/order
T	cycle time
T^*	optimal T
I_e and I_c	interest earned and charged/\$/year
SR	sales revenue
OC , PC and HC	ordering, purchase and holding cost
IP_i and IE_i	interest payable and earned, $i = 1-2$

2.2 Assumptions

1. Demand rate is quadratic time-linked i.e., $D(t) = a_1 + b_1t + c_1t^2$ where $a_1 > b_1 > c_1$.
2. Retailers can add returns and earn interest by the opening of the inventory cycle.
3. Shortages are not permitted.
4. Lead time is negligible
5. The retailer would decide the account at $t = M$ and pay for the interest charges on commodities in stock with rate I_c in $[M, T]$ as $T \geq M$.

3. Mathematical Formulation

The quadratic time-associated demand and, to a lesser extent, decline determine the pace of inventory change over time. The following is the differential equation:

$$\frac{dq(t)}{dt} + \theta q(t) = -(a_1 + b_1 t + c_1 t^2); 0 \leq t \leq T \quad (1)$$

$$\text{with the condition } q(T) = 0 \quad (2)$$

Solution of (1) with the condition (2), yields

$$q(t) = \frac{1}{\theta} \left[\left\{ a_1 + b_1 T + c_1 T^2 - \frac{b_1}{\theta} - \frac{2c_1 T}{\theta} + \frac{2c_1}{\theta^2} \right\} e^{\theta(T-t)} - \left\{ a_1 + b_1 t + c_1 t^2 - \frac{b_1}{\theta} - \frac{2c_1 t}{\theta} + \frac{2c_1}{\theta^2} \right\} \right] \quad (3)$$

and the order quantity is:

$$Q = q(0) = \frac{1}{\theta} \left[\left\{ a_1 + b_1 T + c_1 T^2 - \frac{b_1}{\theta} - \frac{2c_1 T}{\theta} + \frac{2c_1}{\theta^2} \right\} e^{\theta T} - \left\{ a_1 + b_1 t + c_1 t^2 - \frac{b_1}{\theta} - \frac{2c_1}{\theta} + \frac{2c_1}{\theta^2} \right\} \right] \quad (4)$$

The total profit contains the sales revenue, order cost, purchase cost, holding cost, interest earned, and interest charged:

$$\text{(i). The ordering cost} = K \quad (5)$$

$$\text{(ii) HC.} = \frac{hT^2}{\theta} \left\{ (a_1 + b_1 T + c_1 T^2) \frac{\theta}{2} - \frac{c_1 T}{3} \right\} \quad (6)$$

$$\text{(iii). PC} = cQ = cT \left\{ \left(a_1 + \frac{b_1 T}{2} \right) + (a_1 + b_1 T + c_1 T^2) \frac{\theta T}{2} \right\} \quad (7)$$

$$\text{(iv) SR} = sT \left\{ \left(a_1 + \frac{b_1 T}{2} \right) + (a_1 + b_1 T + c_1 T^2) \frac{\theta T}{2} \right\} \quad (8)$$

Note: The truncated Taylors series have been used in the exponential terms.

Case I: $T \leq M$

Since 'T' is less than 'M', there is no interest for financing the inventory in stock, thus interest payable in this case is zero, and interest earned is:

$$\begin{aligned} IE_1 &= sI_e \left\{ \int_0^T \int_0^t D(u) du dt + \int_0^T D(u) du (M - T) \right\} \\ &= sI_e T^2 \left\{ \left(\frac{a_1}{T} + \frac{b_1}{2} + \frac{c_1 T}{3} \right) M - \left(\frac{a_1}{2} + \frac{b_1 T}{3} + \frac{c_1 T^2}{4} \right) \right\} \end{aligned} \quad (9)$$

Case II: $T \geq M$

In this situation, dealer sells the products, builds up sales revenue, and make interest at rate I_e until time M . Thus, the interest earned from time 0 to M per cycle is

$$IE_2 = sI_e \int_0^M \int_0^t D(u) du dt = \frac{sI_e M^2}{2} \left(a_1 + \frac{b_1 M}{3} + \frac{c_1 M^2}{6} \right) \quad (10)$$

Also, M is the seller's trade credit phase accessible by contractors, and T is the inventory cycle.

The trader begins paying interest for the merchandise in-store after time M with rate I_e when the credit period M is less than or equal to cycle time T .

$$\begin{aligned} \text{Thus, IP}_2 &= cI_c \int_M^T q(t) dt \\ &= \frac{cI_c(T-M)}{\theta} \left\{ (a_1 + b_1 T + c_1 T^2) \frac{\theta(T-M)}{2} + M \left(b_1 + c_1 T - \frac{2c_1}{\theta} \right) - \frac{c_1}{3} (T - M)^2 \right\} \end{aligned} \quad (11)$$

From the above result, the profit per unit time can be communicated as

$$\Omega(T) = \frac{(SR - OC - PC - HC - IP_i + IE_i)}{T}$$

$$\Omega(T) = \begin{cases} \Omega_1(T), & T \leq M \\ \Omega_2(T), & T \geq M \end{cases}$$

Thus

$$\Omega_1(T) = \left[(s - c) \left\{ \left(a_1 + \frac{b_1 T}{2} \right) + (a_1 + b_1 T + C_1 T^2) \frac{\theta T}{2} \right\} - \frac{K}{T} - \frac{hT}{\theta} \left\{ (a_1 + b_1 T + C_1 T^2) \frac{\theta}{2} - \frac{c_1 T}{3} \right\} + I_e T \left\{ \left(\frac{a_1}{T} + \frac{b_1}{2} + \frac{c_1 T}{3} \right) M - \left(\frac{a_1}{2} + \frac{b_1 T}{3} + \frac{c_1 T^2}{4} \right) \right\} \right] \quad (12)$$

Now

$$\Omega_2(T) = \left[(s - c) \left\{ \left(a_1 + \frac{b_1 T}{2} \right) + (a_1 + b_1 T + C_1 T^2) \frac{\theta T}{2} \right\} - \frac{K}{T} - \frac{hT}{\theta} \left\{ (a_1 + b_1 T + C_1 T^2) \frac{\theta}{2} - \frac{c_1 T}{3} \right\} - \frac{cI_c}{\theta T} (T - M) \left\{ (a_1 + b_1 T + C_1 T^2) \frac{\theta(T-M)}{2} + M \left(b_1 + c_1 T - \frac{2c_1}{\theta} \right) - \frac{c_1}{3} (T - M)^2 \right\} + \frac{sI_e M^2}{2T} \left(a_1 + \frac{b_1 M}{3} + \frac{c_1 M^2}{6} \right) \right] \quad (13)$$

To find the optimal solution differentiating (12) and (13) two times w.r.t. T , yield

$$\frac{d\Omega_1(T)}{dT} = \left[\frac{(s-c)}{2} \{ b_1 + (a_1 + 2b_1 T + 3c_1 T^2) \theta \} + \frac{K}{T^2} - \frac{h}{6\theta} \{ (a_1 + 2b_1 T + 3c_1 T^2) 3\theta - 4c_1 \} + sI_e \left\{ \left(\frac{b}{2} + \frac{2c_1 T}{3} \right) M - \left(\frac{a_1}{2} + \frac{2b_1 T}{3} + \frac{3c_1 T^2}{4} \right) \right\} \right] \quad (14)$$

$$\frac{d^2\Omega_1(T)}{dT^2} = \left[(s - c)(b_1 + 3c_1 T)\theta - \frac{2K}{T^3} - \frac{h}{\theta} \left\{ (b_1 + 3c_1 T)\theta - \frac{2c_1}{3} \right\} + sI_e \left\{ \frac{2c_1 M}{3} - \left(\frac{2b_1}{3} + \frac{3c_1 T}{2} \right) \right\} \right] \quad (15)$$

Differentiating (13) two times with respect to ' T ', yield

$$\frac{d\Omega_2(T)}{dT} = \left[\frac{(s-c)}{2} \{ b_1 + (a_1 + 2b_1 T + 3c_1 T^2) \theta \} + \frac{K}{T^2} - \frac{h}{2\theta} \{ (a_1 + 2b_1 T + 3c_1 T^2) 3\theta - 4c_1 T \} - \frac{cI_c}{\theta T^2} \left\{ (b_1 + 2c_1 T) \frac{\theta T}{2} - \frac{c_1 M}{3} - \frac{2c_1 T}{3} \right\} (T - M)^2 + (a_1 + b_1 T + c_1 T^2) \frac{\theta}{2} (T^2 - M^2) + M C_1 T^2 + M^2 \left(b_1 - \frac{2c_1}{\theta} \right) - \frac{sI_e M^2}{2T^2} \left(a_1 + \frac{b_1 M}{3} + \frac{c_1 M^2}{6} \right) \right] \quad (16)$$

$$\frac{d^2\Omega_2(T)}{dT^2} = \left[(s - c)(b_1 + 3c_1 T)\theta - \frac{2K}{T^3} - \frac{h}{\theta} \left\{ (b_1 + 3c_1 T)\theta - \frac{2c_1}{3} \right\} - \frac{cI_c}{\theta T^2} \left\{ \frac{2M}{T^2} \left((b_1 + 2c_1 T) \frac{\theta T}{2} - \frac{c_1 M}{3} - \frac{2c_1 T}{3} \right) \left(1 - \frac{M}{T} \right) + \frac{(T-M)^2}{T^2} \left((b_1 + 4c_1 T) \frac{\theta}{2} - \frac{2c_1}{3} \right) + (a_1 + b_1 T + c_1 T^2) \frac{\theta M^2}{T^3} - \frac{(T^2 - M^2)}{T^2} (b_1 + 2c_1 T) - \frac{2M^2}{T^3} \left(b_1 - \frac{2c_1}{\theta} \right) \right\} \frac{sI_e M^2}{2T^2} \left(a_1 + \frac{b_1 M}{3} + \frac{c_1 M^2}{6} \right) \right] \quad (17)$$

The optimal solution is obtained on putting $\frac{d\Omega_1(T)}{dT} = 0$ and $\frac{d\Omega_2(T)}{dT} = 0$, we get

$$\frac{(s-c)}{2} \{ b_1 + (a_1 + 2b_1 T + 3c_1 T^2) \theta \} + \frac{K}{T^2} - \frac{h}{6\theta} \{ (a_1 + 2b_1 T + 3c_1 T^2) 3\theta - 4c_1 \} + sI_e \left\{ \left(\frac{b}{2} + \frac{2c_1 T}{3} \right) M - \left(\frac{a_1}{2} + \frac{2b_1 T}{3} + \frac{3c_1 T^2}{4} \right) \right\} = 0, \text{ or}$$

$$9c_1 \theta \{ 2\theta(s - c) - 2h - sI_e \} T^4 + 4 \{ 3b_1 \theta^2 (s - c) + (2b_1 - 2c_1) h - 2\theta sI_e (b_1 - c_1 M) \} T^3 + 6\theta \{ (s - c)(b_1 + a_1 \theta) - a_1 h - sI_e (a_1 - b_1 M) \} T^2 + 12K\theta = 0 \quad (18)$$

and

$$\frac{(s-c)}{2} \{b_1 + (a_1 + 2b_1T + 3c_1T^2)\theta\} + \frac{K}{T^2} - \frac{h}{2\theta} \{(a_1 + 2b_1T + 3c_1T^2)3\theta - 4c_1T\} - \frac{cI_e}{\theta T^2} \left\{ (b_1 + 2c_1T) \frac{\theta T}{2} - \frac{c_1M}{3} - \frac{2c_1T}{3} \right\} (T - M)^2 + (a_1 + b_1T + c_1T^2) \frac{\theta}{2} (T^2 - M^2) + Mc_1T^2 + M^2 \left(b_1 - \frac{2c_1}{\theta} \right) - \frac{sI_eM^2}{2T^2} \left(a_1 + \frac{b_1M}{3} + \frac{c_1M^2}{6} \right) = 0 \quad (19)$$

We will discuss some properties of the optimal solution.

Eq. (18) can be rewritten as

$$A_1T^4 + A_2T^3 + A_3T^2 + 12K\theta = 0 \tag{A}$$

Where $A_1 = 9c_1\theta\{2\theta(s - c) - 2h - sI_e\}$

$A_2 = 4\{3b_1\theta^2(s - c) + (2b_1 - 2c_1)h - 2\theta sI_e(b_1 - c_1M)\}$

$A_3 = 6\theta\{(s - c)(b_1 + a_1\theta) - a_1h - sI_e(a_1 - b_1M)\}$

Property 1: The optimal cycle time ‘T’ is a decreasing function of ‘K.’

Proof: Differentiating (A) with respect to ‘K’ taking ‘T’ as the dependent variable, we obtain

$$\frac{dT}{dK} = -\frac{12\theta}{T(4A_1T^2 + 3A_2T + 2A_3)} < 0.$$

This shows that the optimal ‘T’ is a decreasing function of ‘K’.

Property 2: The optimal ‘T’ is an increasing function of ‘h’.

Proof: Differentiating (A) with respect to ‘h’ taking ‘T’ as the dependent variable, we obtain

$$\frac{dT}{dh} = -\frac{2T(9c_1\theta T^2 + 2(2c_1 - 3b_1\theta) + 3a_1\theta)}{(4A_1T^2 + 3A_2T + 2A_3)} > 0.$$

Hence, optimal ‘T’ is an increasing function of unit carrying cost ‘h’.

Property 3: The optimal ‘T’ is a decreasing function of credit period ‘M’.

Proof: Differentiating (A) w.r.t. ‘M’ taking ‘T’ as the dependent variable, yields

$$\frac{dT}{dM} = -\frac{6b_1\theta T sI_e}{(4A_1T^2 + 3T + 2)} < 0.$$

Hence, optimal ‘T’ is a decreasing function of ‘M’.

Property 3: The optimal ‘T’ is an increasing function of ‘s’.

Proof: Differentiating (A) w. r. t. ‘s’ taking ‘T’ as the dependent variable, we get

$$\frac{dT}{ds} = \frac{T[9c_1\theta(I_e - 2\theta)T^2 + 4\{2\theta I_e(b_1 - c_1M) - 3b_1\theta^2\}T + 6\theta\{I_e(a_1 - b_1M) - (b_1 + a_1\theta)\}]}{(4A_1T^2 + 3A_2T + 2A_3)} > 0$$

Therefore, the optimal ‘T’ is an increasing function of ‘s’.

4. Numerical Examples

Example 1: Case 1: Let us consider the parameter values $a_1 = 1500, b_1 = 500, c_1 = 75, c = 80/unit, s = 100/unit, \theta = 0.02, h = 1\$/unit/year, I_e = 0.08/\$/year, K = 10\$/order$ and $M = 1/7$ years in the proper units. On substitution of these parameter values in (18) we get $T^* = 0.0910603, Q^* = 135.266, HC = \$5.40374, PC = \$15212.5, SR = \$13879.2, IE_1 = \$107.662$, and $\Omega_1(T) = \$31495.9$. On substitution of these in (18) we get, and $\frac{d^2\Omega_1(T)}{dT^2} = -26991.4 < 0$. It can also be shown by graph that the profit function is concave with respect to cycle time (Figure A1).

Example 2: Case 2: Let's look at $a_1 = 1500, b_1 = 500, c_1 = 50, c = 60/unit, s = 80/unit, \theta = 0.09, h = 1\$/unit/year, I_e = 0.08/\$/year, K = 10\$/order$ and $M = 1/12$ years in the proper units. On substitution of these parameter values in (16) we get $T^* = 0.185612, Q^* = 289.414, HC = \$26.283, PC = \$17370.2, SR = \$23160.2, IE_2 = \$33.3734$, and $\Omega_2(T) = \$34434.2$ and $\frac{d^2\Omega_2(T)}{dT^2} = -6441.37 < 0$. It can also be shown by graph that the profit function is concave with respect to cycle time (Figure A2)

5. Sensitivity Analysis

Case-I $T \leq M$ (See Table A1)

From the Table A1 (Appendix A) it can be easily seen that

1. The rise in s, M and Q outcomes the rise in $T^*, Q^*, PC^*, HC^*, SR^*, IE_1^*$ and $\Omega_1(T)$.

It means that the direction of parameters is the same.

2. Increase in 'c' the decrease in $T^*, Q^*, PC^*, HC^*, SR^*, IE_1^*$ and $\Omega_1(T)$.

The data indicates that parameter 'c' moves against cycle time, order quantity, purchase cost, holding cost, sales revenue, interest earned, and total profit.

3. A rise in h , outcomes causes a fall in $T^*, Q^*, PC^*, SR^*, IE_1^*$ and $\Omega_1(T)$, but an increase in HC^*

Case II $T \geq M$ (See Table A2)

From the Table A2 (Appendix A), it is seen that

1. The increase in 's' results decrease in T^*, Q^*, PC^*, HC^* and SR^* , while increase in IE_2^* and $\Omega_2(T)$.

The data indicates that parameter 's' moves against cycle time, order quantity, purchase cost, holding cost, sales revenue, while augmenting interest earned, and total profit.

2. The increase in 'c' results increase in $T^*, Q^*, PC^*, HC^*, SR^*, IE_2^*$ while decrease in $\Omega_2(T)$.

In other words, where 'c', cycle time, order quantity, purchase cost, holding cost, sales revenue and interest earned all follow the same path; total profit follows a different one.

3. Increase in h , results increase in T^*, Q^*, PC^*, SR^* and $\Omega_2(T)$, while IE_2^* keeps constant.

4. Increase in M results decrease in T^*, Q^*, PC^*, HC^* and $\Omega_2(T)$, while increase in IE_2^*

5. Although there is an increase in Q result in $T^*, Q^*, PC^*, HC^*, SR^*$, the decrease in $\Omega_2(T)$ and IE_2^* remains constant.

6. Conclusion and Future Research Directions

This paper examined the inventory model for demand that is quadratic and has a deterioration and trade credits. Two different cases resulted in total profit, and the optimal solution was discussed using a mathematical model. It has been demonstrated that the total profit function is concave when compared to cycle time. The discussion revolves around the variation of key parameters in sensitivity analysis. Additionally, there is a discussion of important properties related to optimal solutions. Cases I and II have opposing solutions for the unit holding cost variation.

Weibull distribution deterioration and exponential demand may be included in this paper's extension. The paper has been extended to include linear time-dependent holding costs. Fuzzy demand and fuzzy deterioration can be used to generalize the model. Future research on time – sensitive demand EOQ models can incorporate more complex demand pattern, considering imperfect quality items, and extending the model to multi – echelon inventory systems.

Author Contributions

All authors contributed equally to the conceptualization of the article and writing of the original and subsequent drafts.

Data Availability Statement

“Not applicable”

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Ethical considerations

The authors avoided data fabrication, falsification, and plagiarism, and any form of misconduct.

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Conflict of interest

The authors declare no conflict of interest.

Declaration of Generative AI and AI-assisted technologies in the writing process

In this section, there is no need to add a statement.

Appendix A

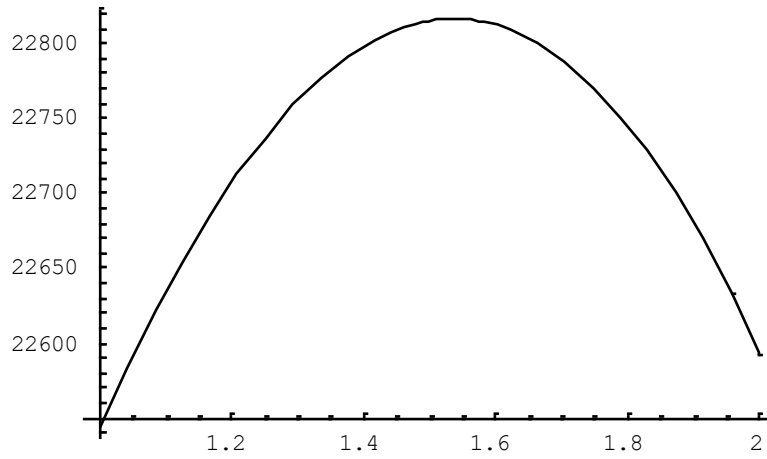


Figure A1. Ω_1 versus T

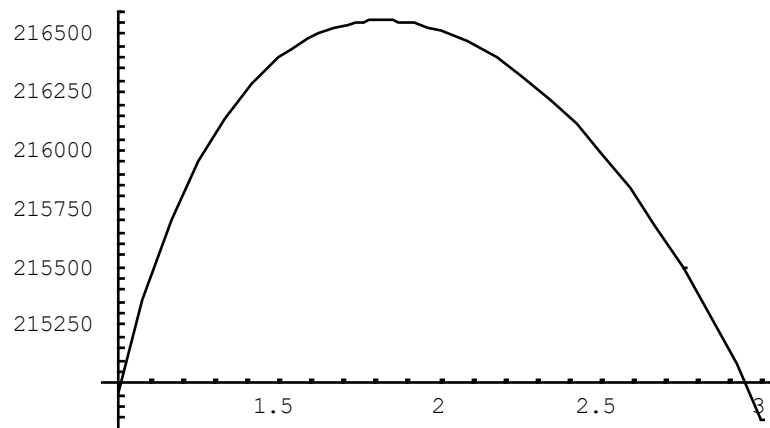


Figure A2. Ω_2 versus time

Table A1. Variation of optimal solution on the key parameters

Parameter (s)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
101	0.099790	148.71	12186.4	6.4786	15385.4	113.91	33032.8	-20655.4
102	0.111495	166.51	13643.5	7.9431	17395.5	120.55	34571.7	-14988.1
103	0.128230	192.08	15737.0	10.233	20261.5	126.67	36113.5	-10077.8
104	0.154527	232.59	19051.0	14.240	24766.3	128.38	37659.7	-6056.86
105	0.201807	306.38	25083.6	22.387	32922.3	132.41	39213.1	-3136.88

Parameter (c)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
80	0.091060	135.25	11103.3	5.4664	13879.2	107.66	31495.9	-26991.4
85	0.062988	93.245	8120.51	2.7263	9553.54	84.936	23897.8	-80554.9
90	0.051019	75.222	6949.72	1.8196	7721.91	72.377	16322.0	-15116.0
95	0.044000	64.913	6318.91	1.3670	6651.48	64.224	8759.64	-23538.0
100	0.039255	57.863	5929.25	1.0953	5929.25	58.383	1204.61	-33121.0

Parameter (h)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
2.0	0.074549	110.59	9063.92	7.5101	11329.9	95.444	31441.1	-46775.7
3.0	0.064051	94.837	7773.27	8.4444	9716.59	85.969	31394.1	-72614.4
4.0	0.056799	83.989	6884.42	8.9471	8605.53	78.655	31352.6	-10364.0
5.0	0.051459	76.019	6231.30	9.2501	7789.12	72.869	31315.0	-13928.0
6.0	0.047336	69.876	5727.90	9.4479	7159.88	68.171	31280.4	-17875.0

Parameter (M)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
0.15	0.091608	136.32	11171.2	5.5279	13964.0	116.04	31583.7	-26516.5
0.16	0.092386	137.49	11267.6	5.6156	14084.5	127.86	31705.5	-25862.8
0.17	0.093183	138.70	11366.5	5.7061	14208.0	139.88	31827.4	-25212.8
0.18	0.094002	139.94	11468.0	5.7996	14335.0	152.11	31949.3	-24569.5
0.19	0.094842	141.21	11572.2	5.8962	14465.2	164.57	32071.2	-24432.3

Parameter (θ)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
0.03	0.094219	142.29	11500.3	6.1729	14441.1	109.62	31506.7	-25148.0
0.04	0.099307	150.90	12138.6	7.0328	15171.2	112.52	31519.7	-21975.0
0.05	0.105996	161.69	112979.1	8.1333	16223.9	115.86	31534.3	-18499.2
0.06	0.114555	175.34	14058.1	9.5980	17527.7	119.32	31550.4	-15078.3
0.07	0.125616	192.94	15458.6	11.631	19323.3	122.46	31568.3	-11887.1

Table A2. Variation of optimal solution on the key parameters

Parameter (s)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
85	0.159570	247.44	14851.9	19.3765	21040.2	35.4593	41854.6	-15616.5
90	0.142135	219.592	13180.9	15.3481	19771.2	37.5451	49258.9	-27142.6
95	0.129542	199.603	11981.4	12.7338	18970.6	39.6309	56662.1	-40832.2
100	0.119955	184.56	11072.4	10.9090	18454.0	41.7168	64069.4	-56538.9
105	0.112372	172.519	10356.0	9.56656	18122.9	43.8026	71482.6	-74143.7
Parameter (c)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
65	0.241880	381.669	22903.3	44.8822	30537.7	33.3734	27187.4	-9327.11
70	0.347378	560.517	33621.4	93.5846	44828.5	33.3734	19544.8	-3556.67
75	0.547867	922.319	55260.2	238.030	73680.3	33.3734	10727.4	-15697.5
80	0.841739	1507.79	90126.8	582.599	12169.0	33.3734	6827.30	-9670.80
85	1.180170	2270.19	135229.0	1199.64	18030.0	33.3734	2587.40	-3787.20
Parameter (h)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
2.0	0.203984	319.202	19157.0	63.5632	25676.1	33.3734	34461.9	-10888.3
3.0	0.228043	358.783	21530.9	119.517	28876.0	33.3734	34463.6	-9266.38
4.0	0.261255	413.936	24837.9	209.851	33340.3	33.3734	34417.9	-7263.32
5.0	0.309460	495.342	29717.0	369.871	39940.5	33.3734	34281.1	-4980.32
6.0	0.383248	623.108	37369.3	686.083	50324.8	33.3734	33962.2	-2635.28
Parameter (M)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
0.084	0.184610	287.790	17272.8	25.9975	2330.40	34.1862	34434.1	-2635.28
0.085	0.183542	286.061	17169.0	25.6949	22892.0	35.0089	34431.9	-7237.96
0.086	0.182405	284.220	17058.6	25.3747	22588.6	35.8415	34427.6	-7791.72
0.087	0.181198	282.267	16941.5	25.0371	22588.6	36.6839	34420.9	-8462.11
0.088	0.179918	280.197	16817.3	24.6815	22423.0	37.5363	34411.8	-9261.14
Parameter (θ)	T^*	Q^*	PC	HC	SR	IE	$\Omega(T)^*$	$\frac{d^2\Omega(T)}{dT^2}$
0.10	0.228821	360.528	21633.3	40.3362	22844.4	33.3734	34165.2	-7717.15
0.11	0.252244	400.004	23998.5	40.5178	31998.0	33.3734	33804.6	-11842.7
0.12	0.266246	424.124	25443.1	55.3879	33924.1	33.3734	33505.4	-15136.5
0.13	0.274819	439.270	26350.0	59.3147	35133.3	33.3734	33275.4	-17686.1
0.14	0.279960	448.679	26913.2	61.8102	35884.3	33.3734	33104.3	-19708.3

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