IJSOM International Journal of Supply and Operations Management

Robust Multi-Objective Optimization for Debris Removal During the Response Phase of Unpredictable Natural Disasters under Uncertainty

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Article Info	ABSTRACT

Article type:
Research Article

Article history:
Received March 9, 2025
Received in revised form June 5, 2025
Accepted December 9, 2025
Available online February 10, 2026

Keywords:
humanitarian logistics
debris removal
response phase
natural disaster
robust multi-objective optimization

Objective: This study aims to address post-earthquake emergency response challenges by emphasizing the critical role of timely debris removal operations in ensuring rapid accessibility for the rescue team thereby reducing casualties, and mitigating the operational risks faced by rescuers in post-disaster environments under uncertain conditions. The objective is to develop a decision-making approach to determine the visiting order of critical nodes, the travel path between consecutive critical nodes, and the blocked edges to be cleared during debris removal operations, whose effectiveness remains stable across all plausible realizations of uncertain parameters while dealing with multiple objectives.

Methods: To deal with uncertainty, a robust routing mathematical model is presented to help debris removal teams to find suitable routes subject to three objective functions including minimizing debris removal team's travelling time plus debris removal operations time, minimizing the risk of rescuers in critical regions and maximizing the total benefit gained by accessing to damaged and critical regions of the city thereby reducing the loss of lives. To solve the proposed multi-objective model while simultaneously handling the uncertainty of parameters, a robust multi-objective optimization approach with augmented epsilon constraint is proposed in this paper. To test the efficiency of the proposed model of this study, real data taken from Rudbar-Manjil devastating Earthquake (20 June 1990, Iran) is used as a case study. The results identified the most effective routes and operational sequences for debris removal teams under uncertainty, with a fuzzy decision-making method selecting the preferred Pareto-optimal solution.

Results: The analysis determined the optimal visiting sequence of critical nodes for debris removal operations. For each pair of consecutive critical nodes, the most efficient routes were identified for the debris removal teams. Additionally, the specific road segments on which debris clearance should be performed were mapped and prioritized. Sensitivity analysis confirmed the robustness of the proposed model across different budgets of uncertainties.

Conclusion: This research provides a practical framework for optimizing debris removal operations under real-world uncertainties and supporting robust decision-making, which can improve the efficiency of disaster response and inform planning for future emergency management scenarios. The findings indicate that the model is versatile and can be adapted to other disaster scenarios by adjusting geographical parameters, resource constraints, and uncertainty modeling.

Cite this article: Soltani, R., & Nazari, A. (2026). Robust Multi-Objective Optimization for Debris Removal During the Response Phase of Unpredictable Natural Disasters under Uncertainty. *International Journal of Supply and Operations Management*, *13*(1), 80-108. https://doi.org/10.22034/ijsom.2025.110728.3343

Publisher: Kharazmi University
DOI: https://doi.org/10.22034/ijsom.2025.110728.3343

1. Introduction

According to the Emergency Events Database (EM-DAT), on average, 341 climate-related disasters have occurred per year since year 2000. These events cause significant damage to both lives and property. By intelligent planning and well-organized deployment of available resources, known as humanitarian logistics, the resulting damage can be reduced. Humanitarian logistics comprises four main phases: Preparation, Response, Recovery and Reconstruction phases. The Preparation phase takes place during the pre-disaster stage, while the remaining phases—Response, Recovery, and Reconstruction—occur in the post-disaster stage. The Preparation phase involves taking precautions before a disaster occurs to minimize risks and adverse outcomes. The Response phase begins immediately after the disaster and typically lasts up to three day (Feng et al., 2003). During this phase, rescue operations are conducted and emergency services are provided to victims as quickly as possible, with the aim of saving lives and alleviating suffering. Both time and available resources are extremely limited during this stage.

The Recovery phase begins after the Response phase and may last for several months. This phase focuses on restoring disaster-affected areas, including transportation routes and other critical infrastructure. Finally, the Reconstruction phase can extend over several years and aims to fully rehabilitate the affected area and restore normal living conditions for the victims.

Two important components of humanitarian logistics are debris collection and debris removal operations. Following a disaster, debris often accumulates on road surfaces. Although related, debris collection and debris removal are fundamentally different processes. Debris collection operations, associated with the Recovery and Reconstruction phases, primarily aim to gather debris from roads for recycling and reuse in the rehabilitation of disaster-affected areas.

In contrast, debris removal operations, associated with the Response phase, focus on rapidly clearing debris from blocked roads to facilitate faster rescue efforts and the efficient distribution of humanitarian aid. Mavroulis et al. (2023) highlighted the challenges encountered in earthquake debris management and the associated threats to public health and the environment based on operational experience from Turkey’s earthquake of February 6, 2023. Also, Aydin et al. (2024) highlighted the importance of earthquake debris waste barriers for policymakers following Turkey’s earthquake in 2023. Debris removal plays a critical role in the aftermath of various types of disasters such as earthquakes, flood, wildfires, landslides, volcanic eruptions, tsunami, hurricanes and tornados. Efficient debris removal is essential for restoring access to essential infrastructure, supporting rescue operations, and facilitating the overall recovery process. In sum, key insights regarding debris removal which are common across different disaster types are as follows:

- **Rapid and Efficient Clearing:** Regardless of the disaster type, debris removal ensures emergency access and safety, enabling rescue and recovery teams to reach affected areas promptly.
- **Restoration of Infrastructure:** In many cases, restoring critical infrastructure such as transportation networks, communication systems, and public utilities is essential for long-term recovery.
- **Risk Mitigation:** Debris can create secondary risks, such as fire hazards (in the case of wildfires), flooding (in the case of hurricanes), or further landslides (in the case of heavy rainfall). Prompt debris removal can reduce these risks.
- **Economic and Social Recovery:** Clearing debris from commercial areas, residential neighborhoods, and public spaces is essential for enabling economic recovery and social stability after a disaster.

This paper focuses on earthquakes as a type of natural disaster and aims to identify optimal routes for the debris removal team to access critical areas (i.e., districts where hospitals, schools and other essential facilities are located) by clearing debris from blocked roads. Since the debris to be removed is located on road surfaces, the problem is formulated as a routing problem, following the approach of Sahin et al. (2016), in which reaching a required node necessitates traversing an arc.

This paper considers three objectives: minimizing the total time spent by the debris removal team, including both travel and operation time, minimizing the risk faced by rescuers in critical regions, and maximizing the total benefit gained from accessing damaged and critical regions, thereby reducing the loss of lives. In addition, real-world uncertainties are taken into account to support robust decision-making. For this purpose, a multi-objective robust optimization approach is employed, simultaneously addressing both the uncertainty and multi-objective nature of the model. A combination of the lexicographic optimization and augmented e-constraint method is used to handle multiple objectives, ensuring that the range of the objective functions in the payoff table is optimized and only efficient solutions are generated. To account for uncertainty, the robust approach proposed by Bertsimas and Sim (2004) is employed which is more tractable and does not require probability distributions. Furthermore, the model remains linear after robustification.

The main contributions of the present work are as follows:

- Formulating a debris removal problem with multiple objectives- including travel and operation time, risk faced by rescuers, and the benefit of accessing critical areas- to better reflect real-world conditions.
- Employing a robust multi-objective optimization approach to address intrinsic uncertainties of the multi-objective debris removal problem.
- Investigating the proposed model on a real case study.

The remainder of this paper is organized as follows: Section 2, reviews related literature. The developed mathematical model is presented in section 3 along with the proposed robust optimization approach and the proposed solution method to solve the robust multi-objective model. In Section 4, the case study and the computational results are presented. Finally, concluding remarks and future researches are presented in Section 5.

2. Related literature on debris removal problem in the response phase

- ***Deterministic Single-objective debris removal problem***

Aksu and Özdamar (2014) considered a planning problem for the urgent restoration of the transportation network during the response phase. The main objective of their study was to maximize network accessibility to all critical points through road restoration process, thereby facilitating rapid victim evacuation and debris removal operations. To achieve this, a dynamic path-based mixed-integer mathematical model was proposed to identify blocked and critical routes and to reopen the routes using limited resources. Finally, the performance of the proposed model was evaluated using two experimental instances based on Istanbul’s earthquake. Akbari and Salman (2014) presented a mixed integer routing model to immediately clean and reconnect the transportation network using a specified number of servers (i.e. the debris clearing teams) following a disaster. They considered a set of blocked roads in the transportation network, where debris clearing teams start their operations from supply nodes to clear these roads. The goal of their study was to determine suitable routes for debris-clearing teams, aiming to minimize the maximum time required for each team, including travel time, road clearance time, and waiting time. They developed a heuristic method to solve the proposed model and evaluated the efficiency of the model through numerical examples. Kasaei et al. (2016) presented two mixed-integer arc routing problems with deterministic parameters to clear blocked roads and restore road accessibility. The first problem aimed to minimize the time required to reconnect the road network, including both travel time and debris removal time. The second problem sought to maximize the total benefit gained by reconnecting network components within a given time limit, calculated as the difference between the total collected prizes and travel time. Both models were solved using heuristic methods, and the results were compared with the results of exact methods. Berktaş et al. (2016) presented two single-objective mixed-integer programming models with deterministic parameters for debris removal from road surfaces during the response phase. The first model, minimizes the total time required to access critical nodes, including both debris removal team travel time and operation time. The second model minimizes the weighted sum of visiting times, where weights indicate the priorities of the critical nodes. Both models were solved using heuristic methods. Sahin et al. (2016) proposed a single-objective routing model aimed at minimizing the total time of debris removal teams, including both travel and operations times, to rapidly remove debris from road surfaces and to provide relief to critical areas as quickly as possible. All parameters of this model were deterministic, and a heuristic method based on the Dijkstra's shortest path algorithm was developed to solve large-scale instances of the problem. Akbari and Salman (2017) presented a single-objective mixed-integer programming model with deterministic parameters to restore the road network following an earthquake during the response phase. The model minimizes the maximum total travel time of each vehicle, including time spent traversing edges, the time for road clearance and waiting time. To solve the proposed model, the authors proposed an MIP relaxation-based heuristic method and a local search algorithm. Subsequently, they compared the heuristic results with those obtained from the exact method. Akbari et al. (2017) presented an exact mixed-integer arc routing model for Multi-vehicle- Prize Collecting Arc Routing for Connectivity Problem, aiming to provide rapid accessibility to damaged and sensitive areas. The objective function maximizes the total prize gained by reconnecting disconnected network components within a specific time constraint, thereby determining the coordinated routes for each relief team. To solve multiple single-vehicle MIP models, they developed a metaheuristic algorithm based on lagrangian relaxation, called the successive single-vehicle algorithm, and evaluated its performance using the Istanbul road network dataset generated according to predicted earthquake scenarios. Ajam et al. (2019) studied a mixed-integer road clearing model that minimizes the total time from the depot to each critical node and presented a heuristic method, a lower-bound approach and a metaheuristic method based on a combination of Greedy Randomized Adaptive Search Procedure (GRASP) and Variable Neighborhood Search (VNS), to solve the model. They evaluated the efficiency of these methods using a case study from Istanbul and compared the algorithms with existing approaches in the literature. Akbari et al. (2021) modelled a road restoration problem on an undirected, edge-weighted graph with blocked edges, where the unblocking time of a blocked edge is revealed online once the road restoration team visits one of its end-nodes. In the proposed model, the locations of disrupted roads can be identified using drone or satellite imagery; however, an accurate estimation of the time required to restore a road segment can only be made after expert observations in the field. The objective function minimizes the time at which the road network becomes reconnected. Finally, the worst-case performance of the online algorithms is evaluated against the offline optimal solutions using the competitive ratio, and the performance of the model is examined through a real case study in Istanbul, Turkey. Ajam et al. (2022) proposed algorithms to determine the routes and schedules of multiple work teams, aiming to minimize the total latency in reaching critical locations. The performance of these heuristics was evaluated using datasets from the literature.

- ***Non-deterministic single-objective debris removal problem***

Çelik et al. (2015) presented a single-objective planning model for debris removal from road surfaces under uncertainty in debris volumes and the corresponding clearance times. The goal of their study was to determine the sequence of debris removal in each period so that the total weighted flow from supply nodes to demand nodes is maximized. To solve the model, a heuristic method was developed and its performance under different scenarios was evaluated and validated through numerical examples. Aydin et al. (2018) proposed a methodology to evaluate four different road recovery strategies (proximity, time/proximity, hierarchy/proximity, time) for restoring connectivity of blocked roads following an earthquake. They investigated the effects of these strategies on the recovery behaviour using three criteria (i.e. mean recovery time, recovery process efficiency and the magnitude of uncertainty). They also tested the methodology on a case study in Nepal and concluded that sequencing road segments, based on the required restoration time, is the best strategy when the decision-makers aim to maximize efficiency during the recovery process. Yaşa et al. (2022) developed a stochastic mathematical model to address the debris removal scheduling problem with multiple crews. The model generates schedules that account for all possible clearing time scenarios, with the goal of maximizing network accessibility throughout the clearance operation while considering uncertainty in clearance times. Metaheuristic methods were also employed to solve the problem. Akbari et al. (2022) proposed a new mathematical routing model that determines the schedules and routes of both a relief distribution team and a road restoration team to provide faster access to critical nodes for the relief distribution team. The objective function minimizes the time by which the last demand node is served by the relief distribution team, considering that the unblocking times are stochastic. Finally, the performance of the model is demonstrated through a detailed case study from Harris County, Texas. Hosseini et al. (2024) presented a probabilistic method for predicting urban road blockage and developed a network restoration plan through resilience optimization. The proposed methodology was implemented in Region 2 of the Tehran metropolitans. Kheildar et al. (2025) developed a two-step approach based on reinforcement learning methods for debris removal operation aiming to minimize debris operation completion time. In the first step, the damaged area was initially assessed using an unmanned aerial vehicle (UAV) and a state-action-reward-state-action (SARSA) algorithm to estimate the damage and in the second step, debris removal groups removed debris from all blocked roads based on the information reported by the UAV. Then a heuristic algorithm was developed to solve the model and finally, performance of the proposed model indicated through a case study from Rudbar earthquake.

- ***Deterministic multi-objective debris removal problem***

Feng and Wang (2003) developed a multi-objective scheduling model for highway emergency rehabilitation following earthquake attacks considering the constraints of time and resources. Three objectives are formulated in this paper including maximizing the performance of emergency rehabilitation, minimizing the risk of rescuers, and maximizing the savings of life. Finally, a case study presenting the Chi-Chi earthquake, a disaster happened in central Taiwan, is conducted. Heydari et al. (2022) proposed a debris removal model aimed at minimizing access time to critical areas, such as hospitals, while maximizing coverage of these areas. The model was solved using the AUGMECON2 method, and the Grasshopper Optimization Algorithm (GOA) was employed as a metaheuristic to solve selected test problems. Finally, the efficiency of the solution method was evaluated through a case study in Tehran, Iran, and the results were analyzed. Cheng et al. (2022) developed a Multi-Period Two-Echelon Location-Routing Problem in which the main decisions involve the locations of temporary Disaster Waste Management Sites and the routing of vehicles in both echelons. The model aims to minimize the cost and duration of disaster waste clean-up by utilizing Temporary Disaster Waste Management Sites, which can store and process waste before it is sent to the final disposal sites. To solve the model for large instances, a Genetic Algorithm (GA) was employed, and the performance of the model was evaluated using several random samples.

- ***Non-deterministic multi-objective debris removal problem***

Özdamar et al. (2014) proposed a multi-objective routing-allocation model for debris removal operations following an earthquake. The first objective function maximizes networks accessibility, while the second objective function minimizes the total time required for debris removal operations. The authors also considered stochastic uncertainty in debris removal times and proposed a heuristic method to solve the multi-objective model. Sayarshad et al. (2020) proposed a dynamic relocation debris removal model with queuing formulation to optimally position clearance equipment considering uncertainty in equipment requests and response times. The objective function maximizes the total weighted flow from supplier nodes to demand nodes while minimizing total travel costs and clearance equipment relocation costs. The latter refers to the costs of allocating clearance equipment to teams at each node. They also employed the partial observable Markov decision process (POMDP) model and a value function to determine the best decision in each period during the online debris removal operation. Finally, they evaluated the efficiency of their approach through a case study based on Hurricane Harvey. Nabavi et al. (2022) presented a novel bi-objective mixed-integer programming model to synchronize multiple vehicles for victim evacuation and debris removal processes, and to restore road connectivity during the response phase. The first objective function aims to minimize the total costs of the relief logistics network, while the second objective minimizes the total operation times of the vehicles. Moreover, a novel two-stage data-driven approach, including a new hybrid machine learning model and robust optimization with φ-divergence, was employed to address the unreliability of travel and service times. Finally, to illustrate the performance of the proposed model and solution approach, a real case study from Kermanshah, Iran, was used. Amirsahami et al. (2025) addressed the challenges of post-earthquake road disruptions in urban relief logistics and proposed a hybrid decentralized system combining trucks and drones for effective disaster response. They also introduced a simulation-based bi-objective fuzzy chance-constrained programming model to manage uncertainty in truck travel times.

According to the literature review and to the best of the authors` knowledge, although uncertainty is the intrinsic nature of debris removal problems and an approach is needed to present solutions which won't change against any realizations of uncertainties, none of the researches have proposed robust multi objective optimization considering Bertsimas and Sim (2004) robust approach for debris removal problems. On the other hand, few multi-objective models in the literature consider risk of landslides and fall of debris in the response phase which can be dangerous for debris removal team.

Therefore, this paper seeks to address the following research question:

- What are the best multi-objectives debris removal decisions- regarding the visiting order of critical nodes, the travel path between consecutive critical nodes, and the blocked edges to be cleared during debris removal operations- that remain robust under all realizations of uncertain parameters?

Summary of the literature review is presented in Table 1.

Table1. Literature review

Authors	**Routing problem**	**Objective function**							**Uncertainty**	**Case study**
		Travel time	**Debris removal time**	**Accessibility**	**Risk**	**Benefit (Coverage)**	**Cost**	**Others**		
Feng and Wang (2003)					*			- Performance - Saving of life		Chi-Chi Earthquake (Taiwan)
Özdamar et al. (2014)	*	*	*	*					Stochastic	Istanbul (Turkey)
Aksu and Özdamar (2014)	*			*						Istanbul (Turkey)
Akbari and Salman (2014)	*	*	*							Istanbul (Turkey)
Çelik et al. (2015)	*			*					Stochastic	Boston (US)

Authors	Routing problem	Objective function							Uncertainty	Case study
		Travel time	Debris removal time	Accessibility	Risk	Benefit (Coverage)	Cost	Others		
Sahin et al. (2016)	*	*	*							Kartal (Istanbul)
Berktaş et al. (2016)	*	*	*							Kartal and Bakırköy (Istanbul)
Kasaei et al. (2016)	*	*	*			*				Istanbul (Turkey)
Akbari and Salman (2017)	*	*	*							Istanbul (Turkey)
Akbari et al. (2017)	*					*				Istanbul (Turkey)
Aydin et al. (2018)	*			*					Stochastic	Sindhupalchok (Nepal)
Ajam et al. (2019)	*	*								Istanbul (Turkey)
Sayarshad et al. (2020)	*			*			*		Stochastic	Hurricane Harvey (US)
Heidari et al. (2021)	*	*	*			*				Tehran (Iran)
Akbari et al. (2021)	*	*								Istanbul (Turkey)
Yaşa et al. (2022)	*			*					Stochastic	Random Sample
Ajam et al. (2022)	*	*								Real Network
Akbari et al. (2022)	*	*	*						Stochastic	Texas (USA)
Cheng et al. (2022)	*	*					*			Random Samples
Nabavi et al.	*	*					*		Robust	Kermanshah (Iran)

(2022)

Authors	**Routing problem**	**Objective function**							**Uncertainty**	**Case study**
		Travel time	**Debris removal time**	**Accessibility**	**Risk**	**Benefit (Coverage)**	**Cost**	**Others**		
Hosseini et.al (2024)	*	*							Scenario-based	Tehran (Iran)
Amirsahami et al. (2025)	*	*							Fuzzy	Tehran (Iran)
Kheildar et al. (2025)		*	*							Rudbar earthquake (Iran)
The current study	*	*	*		*	*			**Robust**	**Rudbar-Manjil (Iran)**

3. Problem definition and model formulation

3.1 Problem description

In this study, a debris removal operations problem following an earthquake is studied, and a multi-objective routing model is proposed to efficiently reach the critical and damaged areas of a city after a disaster. The disaster-affected region is represented as an undirected, complete graph in which nodes correspond to various city locations and arcs represent roads. Locations that are of higher importance and require relief operations, are referred to as critical nodes, while those of lesser importance are defined as intermediate nodes. Furthermore, the location from which the debris removal team starts its operations is referred to as the supply node. It should be noted that some paths may contain blocked arc, where an additional time, referred to as debris removal time, is required in addition to the travel time to traverse those arcs. The aim of this study is to ensure efficient access to critical nodes by navigating routes that may include blocked edges requiring clearance.

To achieve this, the debris removal team departs from the supply node, access all critical nodes by clearing debris along the critical path, and then returns to the supply node. It should be noted that visiting all critical nodes is necessary, whereas visiting intermediate nodes is not required. In fact, some intermediate nodes may be visited, while others may not. Let $G = (N,A)$ denote the network where N represents the set of nodes and $A=\{(i,j)\cup(j,i)\}$ represents the set of arcs. It is worth noting that even if the arcs are directed, the parameter settings of arcs (i,j) and (j,i) are assumed to be symmetric. The node set N includes the supply node, critical nodes and intermediate nodes.

Some arcs in the arc set are assumed to be blocked, and are represented by the binary parameter I_{kl} . This parameter takes the value of 0 if the arc (k,l) is blocked by debris, and 1 otherwise. The parameter t_{kl} denotes the travel time required to traverse arc (k,l)∈A and w_{kl} represents the required effort in terms of time to remove debris from arc (k,l)∈A if this arc is blocked. Since the parameter settings for arcs (i,j) and (j,i) are symmetric, oncee debris is removed from one of them, the other arc becomes available as well. Let DL⊂N denote the set of critical nodes and SL⊂N\DL represent chosen supply node, where DL∪SL= L.

3.2 Mathematical model

The proposed model determines the visiting order of critical nodes and the travel path between consecutive critical nodes, while accounting for blocked roads. This model incorporates three objective functions: minimizing the total effort, defined as the debris removal team's travel time plus debris removal operation time, minimizing the risk faced by rescuers in critical regions, and maximizing the total benefit obtained by accessing damaged and critical regions of the city.

The assumptions of the proposed model are as follow:

1. Only a subset of the nodes is visited by debris removal team. (This means the team targets critical locations- such as hospitals, shelters, emergency services, fire stations, and other high-priority zones- to ensure efficient recovery and only those nodes that are prioritized based on their criticality or need for debris removal are restored first. In fact, the goal is to first ensure that evacuation routes and essential paths connecting strategic and critical locations are accessible (as considered in Cardoni et al., 2022 and Sahin, 2016).
2. The disaster-affected region, is represented as an undirected, complete graph, which is typically the case in geohazard-related emergencies (Aydin et al., 2018). (This implies that there are no restrictions on connectivity between nodes, and all travel paths are bidirectional. In fact, if a route is open in one direction, it is equally accessible in the opposite direction.)
3. Only one debris removal team and a single supply node are considered, for simplicity. This assumption reduces the model's complexity (Sahin,2016). Under this assumption, the debris removal team departs from the supply node, performs the removal operations, and then returns to the same supply node after completing its tasks.
4. The condition of roads are assumed to remain stable throughout the removal operations. That is, if an arc is open at the start, it will remain open until the end of operations. Similarly, if an arc is restored by debris removal operations, it will remain available until the end of operations. This assumption allows for a static network structure and simplifies the optimization problem for tractability (Çelik et al., 2015).
5. Uncertainty is considered for key model parameters such as travel time, debris removal time, risk levels associated with debris removal operations, and the benefits gained from accessing nodes. Accounting for these uncertain factors captures the inherent unpredictability of post-disaster conditions and supports robust decision-making in disaster response and recovery models. Ignoring such uncertainties can adversely affect the quality of the optimization decisions made based on imperfect information (Özdamar et al. 2014).

The parameters and the decision variables are defined as follows:

Parameters

$\tilde{\boldsymbol{t}}_{\boldsymbol{kl}}$	Required time to traverse arc (k,l) ∈ A.
$\tilde{\boldsymbol{w}}_{\boldsymbol{kl}}$	Required effort in terms of time to remove debris from blocked arc (k,l) ∈ A.
$\tilde{\boldsymbol{R}}_{\boldsymbol{kl}}$	The risk value of debris removal operations at blocked arc (k,l) ∈ A.
$\tilde{\boldsymbol{D}}_{\boldsymbol{l}}$	The benefit value gained by accessing node l ∈ N.

Decision Variables

$\boldsymbol{TT}$	Total travel time required to visit all critical nodes.
$\boldsymbol{Y_{ij}}$	Binary variable which is 1, if the debris removal team visits the critical node j ∈ L right after the critical node i ∈ L; 0 otherwise.
$\boldsymbol{I_{kl}}$	Binary variable which is 0, if arc (k,l) ∈ A is blocked by debris; 1, otherwise.
$\boldsymbol{B_{kl}}$	Binary variable which is 1, if the debris on arc (k,l) ∈ A is removed; 0 otherwise.
$\boldsymbol{P_i}$	Visiting time of critical node i ∈ DL (excluding the debris removal time).
$\boldsymbol{H_l}$	Binary variable which is 1, if node l ∈ N is visited; 0, otherwise.

The proposed mathematical model is as follows:

$$Minimize\ \ f_1 = \sum_{i \in DL\ \&\ j \in SL\ \&\ k,l \in N} X_{ijkl}\,\tilde{t}_{kl} + \mathrm{TT} + \sum_{k,l \in N\ \&\ k<l} B_{kl}\,\widetilde{w}_{kl} \tag{1}$$

$$Minimize\ \ f_2 = \sum_{k,l \in N\ \&\ k<l} B_{kl}\,\tilde{R}_{kl} \tag{2}$$

$$Maximize\ \ f_3 = \sum_{l \in N} H_l\,\widetilde{D}_l \tag{3}$$

$$Subject\ to\ \ \sum_{j \in DL \cup SL} Y_{ji} = 1 \qquad \forall\, i \in DL \tag{4}$$

$$\sum_{j \in DL \cup SL} Y_{ij} = 1 \qquad \forall\, i \in DL \tag{5}$$

$$\sum_{j \in DL} Y_{ij} = 1 \qquad \forall\, i \in SL \tag{6}$$

$$\sum_{l \in N} X_{ijil} - \sum_{l \in N} X_{ijli} = Y_{ij} \qquad \forall\, i,j \in L \tag{7}$$

$$\sum_{l \in N} X_{ijjl} - \sum_{l \in N} X_{ijlj} = -Y_{ij} \qquad \forall\, i,j \in L \tag{8}$$

$$\sum_{l \in N} X_{ijkl} - \sum_{l \in N} X_{ijlk} = 0 \qquad \forall\, i,j \in L\,, k \in N\,, k \neq i\,, k \neq j \tag{9}$$

$$P_i = 0 \qquad \forall\, i \in SL \tag{10}$$

$$P_j \geq P_i + \sum_{k,l \in N} X_{ijkl}\,\tilde{t}_{kl} - \mathrm{M}\big(1 - Y_{ij}\big) \qquad \forall\, i \in L\,, j \in DL \tag{11}$$

$$\mathrm{TT} \geq P_i \qquad \forall\, i \in DL \tag{12}$$

$$\sum_{k,l \in N} X_{ijkl} \leq Y_{ij}|N||N| \qquad \forall\, i,j \in L \tag{13}$$

$$\sum_{i,j \in DL \cup SL} X_{ijkl} + \sum_{i,j \in DL \cup SL} X_{ijlk} \leq (B_{kl} + I_{kl})|L||L| \qquad \forall\, k,l \in N \tag{14}$$

$$B_{kl} \leq 1 - I_{kl} \qquad \forall\, k,l \in N \tag{15}$$

$$\sum_{i,j \in DL \cup SL \,\&\, k \in N} X_{ijkl} \geq H_l \qquad \forall\, l \in N \,\&\, k \neq l \tag{16}$$

$$\mathrm{TT} \geq 0 \tag{17}$$

$$P_i \geq 0 \qquad \forall\, i \in L \tag{18}$$

$$X_{ijkl} \in \{0,1\} \qquad \forall\, i,j \in L,\; k,l \in N \tag{19}$$

$$B_{kl} \in \{0,1\} \qquad \forall\, k,l \in N \tag{20}$$

$$Y_{ij} \in \{0,1\} \qquad \forall\, i,j \in L \tag{21}$$

$$H_l \in \{0,1\} \qquad \forall\, l \in N \tag{22}$$

Objective function (1) minimizes the total time spent by debris removal team to visit all critical nodes and return to the supply node. The first part of the objective function (1) minimizes the total time spent by debris removal team to return to the supply node after visiting the last critical node. The second and the third parts of the objective function (1) minimizes debris removal team's travel time and debris removal operations time, respectively. Objective function (2) minimizes the risk of rescuers in critical regions. Finally, objective function (3) maximizes the total benefit obtained by accessing damaged and critical regions of the city.

Constraints (4) and (5) together establish a visiting order for the critical nodes, starting and ending at the supply node, and ensuring that each critical node is visited consecutively. Constraint (6) ensures that the debris removal team visits exactly one critical node immediately after departing from the supply node. Constraints (4), (5) and (6) together form a closed path. Finally, Constraints (7), (8), and (9) define a directed path between two consecutive critical nodes, which may include intermediate nodes. Constraint (10) indicates that the debris removal team begins at the supply node. Constraint (11) assigns the visiting time of critical nodes without accounting for the time required to remove debris from blocked edges, if any. Debris removal efforts are taken into account by the third part of the objective function (1). Additionally, Constraint (11) eliminates subtours between critical nodes, and it is worth noting that subtours are allowed between intermediate nodes appearing on different critical path segments. The second part of the objective function (1), together with Constraint (12) minimizes the most disadvantageous node's visiting time. Constraint (13) ensures that if a pair of critical nodes is not visited, no directed path exists between them. Constraint (14) ensures that travel along an arc is possible if it is already open or if the debris on its surface has been removed. Constraint (15) ensures that only a blocked arc can be cleared. Constraint (16) ensures that if the benefit of node l is collected, then that node is visited at least once. It should be noted that benefit values are assigned to all nodes- including critical nodes, intermediate nodes and the supply node- based on their importance. Once each node is visited, its benefit is collected which is shown by a binary variable in objective function (3) and constraint (16). Constraints (17)-(22) define the domain restrictions.

3.3 The proposed robust model for debris removal problem

In general, parameter uncertainty arises from situations where data and information are incomplete. This uncertainty can impact both the optimality and feasibility of problems. According to Ben-Tal et al. (2009), uncertainty stems from prediction error, measurement error and implementation error. To address uncertainty, various approaches have been proposed in the literature, including stochastic programming, probabilistic programming, fuzzy programming and robust optimization. One approach to handling uncertainty is stochastic programming, which often requires solving a large number of scenarios and can be computationally expensive. This is particularly challenging given the time-sensitive nature of debris removal operations after an earthquake. When multiple parameters are uncertain, the number of possible scenarios increases exponentially, leading to scalability issues. On the other hand, Probabilistic programming and fuzzy programming respectively require precise distribution and membership functions for uncertain parameters, which are typically estimated from historical data-a resource that is rarely available in disaster situations. The uncertainty set robust optimization approach, however, can manage uncertainty without relying on exact probability distribution, which are hard to estimate in disaster contexts.

The robust optimization approach was first introduced by Soyster (1973). His method yields solutions that are feasible for all realizations of uncertain data within a convex set. This approach was later expanded by Ben-Tal and Nemirovski (1998, 1999, 2000), El-Ghaoui and Lebret (1997), El-Ghaoui et al. (1998) and Bertsimas and Sim (2004). Amongst the uncertainty set robust approaches, Bertsimas and Sim's approach is linear and more tractable, allowing the level of conservatism to be flexibly adjusted in terms of probabilistic bounds of constraint violation. Therefore, in this paper we adopt the robust optimization proposed by Bertsimas and Sim (2004) to deal with uncertain parameters. In the proposed model of this paper, the uncertain parameters $\widetilde{w}$, $\tilde{R}$ and $\widetilde{D}$ appear in objective functions (1), (2) and (3), respectively, while the uncertain parameter $\tilde{t}$ is included in both objective function (1) and constraint (11).

The robust optimization approach is primarily designed for models in which the constraints are ”less than or equal to” inequalities. However, since the objective functions in the proposed model of this paper, are not of this form, these three objective functions must first be transformed into equivalent forms suitable for robust optimization. Once transformed, their robust counterparts can be derived. The procedures for these transformations are presented below.

The objective function (1) can be equivalently converted to (23) and (24), by defining an auxiliary variable f_1.

$$Minimize\ f_1 \tag{23}$$

$$f_1 \geq \sum_{i \in DL\ \&\ j \in SL\ \&\ k,l \in N} X_{ijkl}\, \tilde{t}_{kl} + \mathrm{TT} + \sum_{k,l \in N\ \&\ k<l} B_{kl}\, \widetilde{w}_{kl} \tag{24}$$

The robust counterpart of constraint (24) is presented in (25).

$$f_1 \geq \sum_{i \in DL\ \&\ j \in SL\ \&\ k,l \in N} X_{ijkl}\, t_{kl} + \mathrm{TT} + \sum_{k,l \in N\ \&\ k<l} B_{kl}\, w_{kl} + \sum_{k,l \in h} e_{kl}$$

$$+ \sum_{k,l \in u\ \&\ k<l} q_{kl} + Z_1 \Gamma_1$$

$$e_{kl} + \mathrm{Z}_1 \geq \hat{t}_{kl} X_{ijkl} \qquad \forall\, k,l \in h\ \&\ i \in DL\ \&\ j \in SL$$

$$e_{kl} \geq 0 \qquad \forall\, k,l \in h$$

$$q_{kl} + \mathrm{Z}_1 \geq \widehat{w}_{kl} B_{kl} \qquad \forall\, k,l \in u\ \&\ k<l$$

$q_{kl} \geq 0 \quad \forall k,l \in u \;\&\; k < l$

$Z_1 \geq 0 \quad (25)$

In which Γ_1 is the protection level and denotes the number of uncertain parameters and e_{kl}, q_{kl}, Z_1 denote the dual variables of the linear equivalent model of the protection function. Similarly, the objective function (2) can be equivalently transformed into (26) and (27) by defining an auxiliary variable f_2.

$$Minimize\ f_2 \quad (26)$$

$$f_2 \geq \sum_{k,l \in N \,\&\, k<l} B_{kl}\, \tilde{R}_{kl} \quad (27)$$

The robust counterpart of constraint (27) is presented in (28).

$$f_2 \geq \sum_{k,l \in N \,\&\, k<l} B_{kl}\, R_{kl} + \sum_{k,l \in v \,\&\, k<l} g_{kl} + Z_3 \Gamma_3$$

$g_{kl} + Z_3 \geq \hat{R}_{kl} B_{kl} \quad \forall k,l \in v \;\&\; k < l$

$g_{kl} \geq 0 \quad \forall k,l \in v \;\&\; k < l$

$Z_3 \geq 0 \quad (28)$

In which Γ_3 is the protection level and denotes the number of uncertain parameters and g_{kl}, Z_3 denote the dual variables of the linear equivalent model of the protection function in formulation (28).

Similarly, the objective function (3) can be equivalently converted to (29) and (30), by defining an auxiliary variable f_3.

$$Maximize\ f_3 \quad (29)$$

$$f_3 \leq \sum_{l \in N} H_l\, \tilde{D}_l \quad (30)$$

Constraint (30) is converted to robust counterpart (31).

$$f_3 \leq \sum_{l \in N} H_l\, D_l - \sum_{l \in c} WU_l - Z_4 \Gamma_4$$

$WU_l + Z_4 \geq \hat{D}_l H_l \quad \forall l \in c$

$WU_l \geq 0 \quad \forall l \in c$

$Z_4 \geq 0 \quad (31)$

In which Γ_4 represents the protection level and indicates the number of uncertain parameters, while WU_l, Z_4 denote the dual variables of the linearized equivalent model of the protection function in formulation (31).

And finally, constraint (11) has a robust counterpart according to (32).

$$P_j \geq P_i + \sum_{k,l \in N} X_{ijkl}\, t_{kl} - M(1 - Y_{ij}) + \sum_{k,l \in s} a_{kl} + Z_2 \Gamma_2 \quad \forall i \in L, j \in DL$$

$a_{kl} + Z_2 \geq \hat{t}_{kl} X_{ijkl} \quad \forall k,l \in s \;\&\; i \in L \;\&\; j \in DL$

$a_{kl} \geq 0 \quad \forall k,l \in s$

$Z_2 \geq 0$ (32)

Where Γ_2 is the protection level and indicates the number of uncertain parameters in formulation (32) and a_{kl}, Z_2 denote the dual variables of the linearized equivalent model of the protection function in formulation (32).

Also, parameters of the robust counterparts are defined as follows:

$\hat{t}_{kl} = 0.5\ t_{kl}$, $\hat{w}_{kl} = 0.5\ w_{kl}$, $\hat{R}_{kl} = 0.5\ R_{kl}$, $\hat{D}_l = 0.5\ D_l$

$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4 \in [0\ , number\ of\ all\ parameters]$

In summary, the robust counterpart of the proposed model is presented in (33).

$Minimize\ f_1\ , Minimize\ f_2\ , Maximize\ \ f_3$

s.t.

(4) – (10) , (12) – (22) , (25) , (28) , (31) , (32) (33)

4. Solution methodology

The debris removal problem is inherently NP-hard because the case in which there are no blocked edges and all nodes are critical is equivalent to the travelling salesman problem (Sahin et al, 2016). This implies that its computational complexity grows significantly with the number of nodes, arcs, and uncertain parameters. Scalability can be achieved by employing heuristics, decomposition approaches, parallel computing, and approximation methods which allow the problem to be tackled effectively- even for larger case studies- while balancing the trade-off between computational efficiency and solution quality. However, in this paper, the DRP is investigated on a small-scale case study, which can be tackled by commercial optimization packages. The literature already presents various methodologies for solving multi-objective problems via optimization packages, with one of the most common being the epsilon-constraint method. Nevertheless, some issues remain with the traditional epsilon-constraint approach, addressed in the following.

4.1 Lexicographic Augmented epsilon constraint method

There are two main issues associated with the traditional epsilon-constraint method; the optimality of the ranges of objective functions over the efficient sets obtained from the payoff table, and the efficiency of the generated Pareto-optimal solutions. To address the first issue, lexicographic optimization method has been used in the literature to construct the payoff table consisting solely of efficient solutions. To overcome the second issue, Mavrotas (2009) proposed the "augmented epsilon-constraint method" in which the objective function constraints are transformed into equalities by incorporating appropriate slack or surplus variables. These variables are then incorporated as a secondary term (with lower priority in a lexicographic manner) in the objective function, forcing the program to produce only efficient solutions.

$Minimize\ (f_1\,(\bar{x}) - epsilon \times (s_2/r_2 + s_3/r_3 + \cdots + s_p/r_p))$

Subject to all constraints,

$f_2\,(\bar{x}) + s_2 = e_2\ , \ldots , f_p\,(\bar{x}) + s_p = e_p \qquad s_2, \ldots , s_p \in R^+$ (34)

Where $s_2,\ldots,s_p$ denote slack variables and r_i represent the range of the *i*th objective function which is calculated from the payoff table, *i.e.*, ($r_i = f_i^{SN} - f_i^{U}$). It should be noted that $epsilon$ is an adequately small number (usually between 10^{-3} and 10^{-6}) to establish the lexicographic manner (Mavrotas (2009)) . The formulation of model (34) is known as the augmented epsilon- constraint method that only generates efficient or non-dominated solutions. So, by combining the augmented epsilon constraint method with the lexicographic optimization method, only efficient solutions (not weakly efficient solutions) will be produced within the optimal ranges of the objective functions calculated from the payoff table. Therefore, in this paper, a combination of the lexicographic optimization and the augmented epsilon-constraint method is employed to solve the proposed multi-objective debris removal model.

4.2 Fuzzy decision-making method

To select the most preferred solution from the feasible Pareto-optimal solutions obtained by solving model (34), the fuzzy decision-making method proposed by Amjady et al. (2009) is employed. In this approach, a linear membership function is first calculated for each objective function corresponding to each Pareto-optimal solution. This membership function measures the relative distance between the objective function value of each Pareto-optimal solution and the corresponding values at the utopia and pseudo-nadir points. An objective function value closer to its utopia point (and farther from its pseudo-nadir point) results in a higher membership value, indicating a higher degree of optimality. Equations (35) and (36) present the membership functions for all objective functions in the set of Pareto-optimal solutions obtained by solving model (34).

$$\underset{n=1,2}{\mu_n^r} = \begin{cases} 1 & f_n^r \le f_n^U \\ \frac{f_n^{SN}-f_n^r}{f_n^{SN}-f_n^U} & f_n^U \le f_n^r \le f_n^{SN} \;, \quad n = 1,\dots,p \quad (Minimization) \\ 0 & f_n^r \ge f_n^{SN} \end{cases} \tag{35}$$

$$\underset{n=3}{\mu_n^r} = \begin{cases} 0 & f_n^r \le f_n^{SN} \\ \frac{f_n^r-f_n^{SN}}{f_n^U-f_n^{SN}} & f_n^{SN} \le f_n^r \le f_n^U \;, \quad n = 1,\dots,p \quad (Maximization) \\ 1 & f_n^r \ge f_n^U \end{cases} \tag{36}$$

The linear membership function given in Equation (35) is applied to minimization objective functions, while the one in Equation (36) is applied to maximization objective functions. Where f_n^r $(n = 1,\dots,p)$ denotes the value of *n*th objective function in the *r*th Pareto-optimal solution, and μ_n^r $(n = 1,\dots,p)$ represents the membership function value of the *n*th objective function in the *r*th Pareto-optimal solution (i.e. the degree of optimality). Afterwards, the individual membership function values are calculated. Then, the total membership function value (total degree of optimality) shown in Equation (37) is calculated for each Pareto optimal solution, which accounts for both the individual membership function values and the relative importance of the objective functions (w_n values).

$$\mu^r = \frac{\sum_{n=1}^{p} w_n . \mu_n^r}{\sum_{n=1}^{p} w_n} \tag{37}$$

Where μ^r, denotes the total membership function of the *r*th Pareto-optimal solution, and w_n represents the weight of the *n*th objective function, defined according to its relative importance. Finally, the Pareto-optimal solution with the highest value of μ^r is selected as the most preferred Pareto-optimal solution.

4.3 Robust multi-objective optimization

Since the proposed model in this paper is multi-objective and includes uncertain parameters in all objective functions as well as in one of the constraints, it constitutes a robust multi-objective optimization problem ($\mathrm{RMOP_U}$). Several approaches have been proposed in the literature to formulate robust multi-objective optimization models. Ehrgott et al. (2014) first formulated the robust counterparts of the uncertain model to address uncertainty, and then implemented the epsilon-constraint method to handle the multi-objective nature of the problem. Fliege and Werner (2014) addressed multi-objective optimization problems under uncertainty using the epsilon-constraint method and defined robust multi-objective counterparts for such problems. They further demonstrated that, when solving a multi-objective optimization problem with uncertainty (MOP_u), the order of applying the robust counterparts and the epsilon-constraint method does not affect the final results. In other words, whether the robust counterparts are first incorporated into the uncertain multi-objective model followed by the epsilon-constraint method (ECS), or the epsilon-constraint method is applied first and the robust counterparts are added afterward, the outcomes remain equivalent. That is, robustifying (MOP_u) and then applying ECS leads to the same family of scalar problems as applying ECS first to (MOP_u) and then robustifying the results. In other words: ECS (Robustification(MOP_u)) = Robustification (ECS(MOP_u)). These results are schematically shown in Figure (1) in which *R* denotes Robustification and *S* denotes Scalarization.

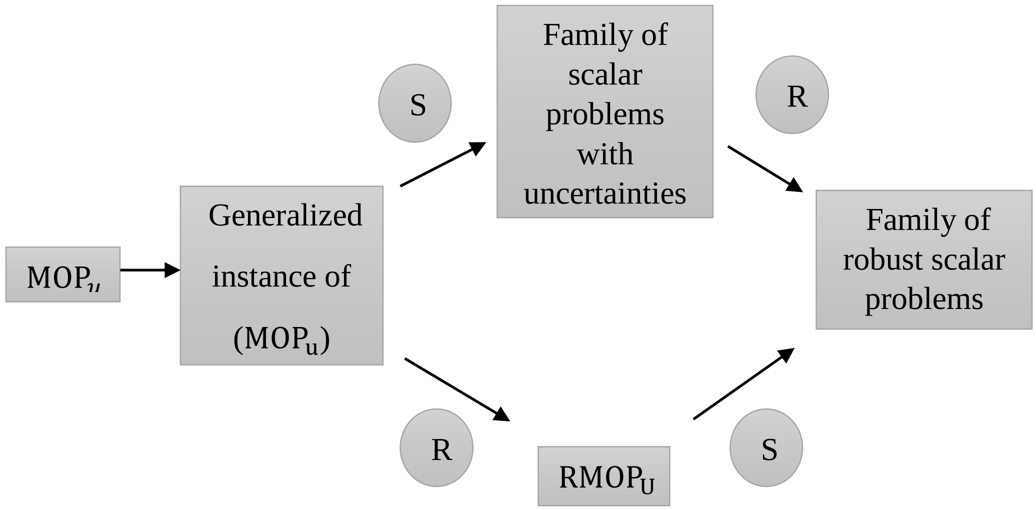

Figure 1. Robust multi-objective optimization (Fliege and Werner, 2014)

In this paper, the first approach is adopted to address the proposed robust multi-objective debris removal problem. Specifically, the robust counterpart of the model is first formulated according to Model (33) to handle uncertainty, and then an augmented lexicographic epsilon-constraint method is employed to manage the multi-objective nature of the model. For the latter, the payoff table is initially constructed using the lexicographic method. Based on the resulting objective function values and their corresponding ranges, the required epsilon values for the augmented epsilon-constraint method are determined. Using these epsilon values, the augmented epsilon-constraint method is then employed to generate a set of Pareto-optimal solutions. Finally, a fuzzy decision-making method is used to select the most preferred Pareto- optimal solution from this set. The detailed implementation of these steps is presented in the following section.

5. Case study implementation and computational results

To evaluate the performance of the proposed model, it is applied to a case study based on the Rudbar-Manjil Earthquake (occurred on June 20, 1990, Iran). The earthquake took place along a fault line between Rudbar and Manjil, two cities located in the northern Iran (Tavakoli et al., 1991). In this paper, Rudbar earthquake is selected as the case study due to the severity of the earthquake, the large number of casualties and damaged buildings in the region, and the availability of relevant data. Following the earthquake, access to hospitals and neighboring towns in Rudbar was severely restricted due to road blockages caused by rockfalls, building collapses, and landslides. As a result, emergency relief operations were hindered, and the evacuation of injured victims was delayed, leading to a significant loss of life. Additionally, uncertainties in travel time, debris removal time, risk levels associated with debris removal operations, and the benefit values of gaining access to certain nodes further complicated the efforts of relief and debris removal teams.

Therefore, given the unpredictable nature of earthquakes, this study examines the performance of the proposed robust multi-objective routing model for debris removal using this real case study. The primary objective is to determine the most effective routes for debris removal teams, thereby enabling faster access to damaged areas and critical locations within the town and helping to minimize casualties and property damage in future events.

Rudbar case is considered as an undirected, complete graph (Figures (4) and (5)) in which different areas and hospitals are represented by nodes, and roads are represented by arcs. As illustrated in Figures (4) and (5), nine nodes are considered in this graph. Node (1), represented by a yellow triangle, is designated as the supply node from which the debris removal team departs. Nodes (2), (3), (4), and (5), shown as red circles, are critical nodes that must all be visited by the debris removal team. Nodes (6), (7), (8), and (9), represented by blue circles, are intermediate nodes, some of which may be visited by the debris removal team.

As mentioned earlier, upon completing visits to all critical nodes and selected intermediate nodes, and performing debris removal operations where necessary, the debris removal team will return to the supply node. These nodes are introduced in Table 2.

Table 2. Nodes of Rudbar town

Node numbers	Node type	Node usage
1	Supply node	Red Crescent of Rudbar town
2, 3	Critical nodes	Critical roads to Ganjeh and Manjil towns, respectively
4, 5	Critical nodes	Hospitals
6, 7, 8, 9	Intermediate nodes	The other neighborhoods

5.1 Data values

- The travel time between nodes k and l (t_{kl})

To calculate the travel time between nodes k and l, traversed by debris removal team, the inverse of the team`s speed, which is 20 km/h, is multiplied by the corresponding node-to-node distance matrix of Rudbar. It should be noted that, the maps and distances are obtained through desk studies, questionnaires, field studies, available reports, the study by Bahreyni et al. (1991) and MicroStation software.

- Debris removal time between nodes k and l (w_{kl})

To calculate the required time to remove debris from the blocked edges, the following equation, proposed by Sahin et al. (2016), is used:

$$w_{kl} = SOE * t_{kl} \tag{38}$$

where, SOE denotes the severity of earthquake, and its value is set to seven for Rudbar case study.

- Risk faced by debris removal team between nodes k and l (R_{kl})

Regarding the risk values associated with debris removal team operations along arcs, three types of risks, including rock-fall risk, buildings collapse risk and landslide risk are considered. Generally, Rudbar town is a landslide-prone and high-risk region, and several landslides occurred there during the Rudbar-Manjil Earthquake (Haeri et al., 1996). Feng and Wang (2003) assigned risk values for work-troops in different areas on a scale from 1 to 9 using GIS. In this paper, all types of risks are assumed to have the same weight and the risk values range from 0 to 6. A zero risk value is assigned to open arcs, while a risk value of 6 is assigned to blocked arcs located in the most sensitive and hazardous areas for work-troops engaging in debris removal operations and face all kinds of risks.

- The amount of blockage between nodes k and l (I_{kl})

To find the value of arc blockages, we conducted desk research, questionnaire and field studies. We also utilized available reports, data from Bahreyni et al. (1991) and reports from the International Institute of Earthquake Engineering and Seismology in Iran.

- Benefit of node l (D_l)

The benefit values gained by having access to each node are assigned to that node based on its discoverability and importance. Naturally, nodes hosting hospitals or critical roads to nearby towns (such as Ganjeh and Manjil in this case study) play a crucial role after an earthquake. Ensuring rapid accessibility to these nodes, significantly reduces losses and casualties, thereby generating the highest benefit values. Conversely, regions of lesser importance correspondingly yield lower benefit values.

5.2 Analysis and computational results

To verify the proposed model, a small-size numerical example is considered, and the model is run using GAMS software. By getting sensible results, the proposed model is verified.

Specifically, the proposed robust multi-objective optimization problem is coded in GAMS software and run using CPLEX solver, which is an efficient solver for solving MIP problems. For this purpose, at first, the robust counterpart of the proposed model is formulated, and then the payoff table shown in Table 3 is calculated using the lexicographic optimization method.

Table 3. The payoff table (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

Objective functions	f_1**(Time)**	f_2**(Risk)**	f_3**(Prize)**
f_1(Time)	44529	11	88
f_2(Risk)	60074	8	88
f_3(Prize)	46828	18	120

After obtaining the payoff table and gaining the values and ranges of all objective functions, the required epsilon values for implementing the augmented-epsilon-constraint method are calculable by using Equations (39) and (40).

$$e_{p,np} = f_p^{SN} - \left(\frac{f_p^{SN} - f_p^{U}}{q_p}\right) \times np, \text{ (For minimization objective functions)} \quad (39)$$

$$e_{p,np} = f_p^{SN} + \left(\frac{f_p^{U} - f_p^{SN}}{q_p}\right) \times np, \text{ (For maximization objective functions)} \quad (40)$$

Where, $p = 1, \dots, n$ is the number of objective functions, and $np = 0, 1, \dots, q_p$.

The values of the objective function in the utopia and pseudo-nadir points are denoted by superscripts U and SN. By considering $q_1 = q_2 = q_3 = 2$ (Amjady et al., 2009) the following formula for calculating epsilon values of all objective functions can be obtained according to Equations (39) and (40).

$e_1 = 60074 - ((60074 - 44529)/2) \times n_1$, $n_1 = 0, 1, 2$

$e_2 = 18 - ((18 - 8)/2) \times n_2$, $n_2 = 0, 1, 2$

$e_3 = 88 + ((120 - 88)/2) \times n_3$, $n_3 = 0, 1, 2$

The epsilon values obtained by the above calculations are presented in Table 4.

Table 4. The epsilon values (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

n_p	e_1(Time)	e_2(Risk)	e_3(Prize)
0	60074	18	88
1	52301.5	13	104
2	44529	8	120

It is assumed that the third objective function (Maximizing the prize) is the main objective function for the augmented-epsilon-constraint method, and consequently the augmented- epsilon-constraint method is formulated according to model (41).

$Maximize\ (f_3\,(x) + epsilon \times (s_1/r_1 + s_2/r_2))$

s.t.

$f_1\,(x) + s_1 = e_1\ ,\ f_2(x) + s_2 = e_2\ ,\ epsilon = 10^{-3}$

Other constraints of the robust counterpart model hold. (41)

Model (41) is solved $(q_2 + 1) \times (q_1 + 1) = 9$ times considering epsilon values in Table 4, and consequently the set of Pareto-optimal solutions is obtained (Table 5) (Amjady et al., 2009). Infeasible solutions (6) and (9) are discarded.

To choose the most preferred solutions from Pareto-optimal solutions, a fuzzy decision-making method with membership functions presented in Equations (42)-(44) is employed. Individual membership function values and total membership function values are calculated and shown in Table 5. The solution with the highest total membership function value is known as the most preferred solution (see Table 6). It should be noted that Pareto-optimal solutions (7) and (8) which have been selected as the most preferred Pareto-optimal solutions, involve two different routes in the debris removal routing problem. Optimal visiting order for the nodes in Pareto-optimal solutions (7) and (8), are shown in Figures (2) and (3), respectively. Furthermore, the travel routes selected for the debris removal team in Pareto-optimal solutions (7) and (8), are shown in Figures (4) and (5), respectively. Note that in these figures, dashed lines indicate routes where debris removal operations are performed, while solid lines indicate routes where no debris removal occurs.

$$\underset{n=1,2}{\mu_n^r} = \begin{cases} 1 & f_n^r \le f_n^U \\ \dfrac{f_n^{SN} - f_n^r}{f_n^{SN} - f_n^U} & f_n^U \le f_n^r \le f_n^{SN}\ , \qquad (Minimization) \\ 0 & f_n^r \ge f_n^{SN} \end{cases} \tag{42}$$

$$\underset{n=3}{\mu_n^r} = \begin{cases} 0 & f_n^r \le f_n^{SN} \\ \dfrac{f_n^r - f_n^{SN}}{f_n^U - f_n^{SN}} & f_n^{SN} \le f_n^r \le f_n^U\ , \qquad (Maximization) \\ 1 & f_n^r \ge f_n^U \end{cases} \tag{43}$$

$$\mu^r = \frac{\sum_{n=1}^{3} w_n . \mu_n^r}{\sum_{n=1}^{3} w_n} \quad , \quad w_1 = 0.5, w_2 = 0.4, w_3 = 0.1 \tag{44}$$

Table 5. Pareto optimal solution values, membership function values and total membership function values (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

$(\boldsymbol{\mu^r})$	$\boldsymbol{f_3}$**(Prize)**	$\boldsymbol{f_1}$**(Time)**	$\boldsymbol{f_2}$**(Risk)**	solution number*(r)*
$\mu^1 = 0.73$	$f_3^1 = 108$ $\mu_3^1 = 0.625$	$f_1^1 = 45537$ $\mu_1^1 = 0.935$	$f_2^1 = 13$ $\mu_2^1 = 0.5$	1
$\mu^2 = 0.73$	$f_3^3 = 108$ $\mu_3^2 = 0.625$	$f_1^2 = 45537$ $\mu_1^2 = 0.935$	$f_2^2 = 13$ $\mu_2^2 = 0.5$	2
$\mu^3 = 0.4$	$f_3^3 = 88$ $\mu_3^3 = 0$	$f_1^3 = 60074$ $\mu_1^3 = 0$	$f_2^3 = 8$ $\mu_2^3 = 1$	3
$\mu^4 = 0.73$	$f_3^4 = 108$ $\mu_3^4 = 0.625$	$f_1^4 = 45537$ $\mu_1^4 = 0.935$	$f_2^4 = 13$ $\mu_2^4 = 0.5$	4
$\mu^5 = 0.73$	$f_3^5 = 108$ $\mu_3^5 = 0.625$	$f_1^5 = 45537$ $\mu_1^5 = 0.935$	$f_2^5 = 13$ $\mu_2^5 = 0.5$	5
$\mu^7 = 0.78$	$f_3^7 = 88$ $\mu_3^7 = 0$	$f_1^7 = 44529$ $\mu_1^7 = 1$	$f_2^7 = 11$ $\mu_2^7 = 0.7$	7
$\mu^8 = 0.78$	$f_3^8 = 88$ $\mu_3^8 = 0$	$f_1^8 = 44529$ $\mu_1^8 = 1$	$f_2^8 = 11$ $\mu_2^8 = 0.7$	8

As mentioned, solutions 7 and 8 have the highest value of membership functions and are selected as the best routes. Although the objective functions of both routes are the same, the resulted routes are different. By jointly minimizing team exposure to risk, reducing total time spent by debris removal team, and maximizing the benefits of restored access, the model captures the trade-offs that emergency managers must navigate in recovery operations. For example, the safest routes for debris removal crews (i.e. $f_2^3 = 8$) traverse longer distances or take more time (i.e. $f_1^3 = 60074$), whereas lesser time consuming routes (i.e. $f_1^4 = 45537$) may pass through structurally unstable zones with higher risk of secondary collapse (i.e. $f_2^4 = 13$). The selected route is the route with minimum time (i.e. $f_1^7 = 44529$) and medium risk level (i.e. $f_2^7 = 11$) while the collected prize is not necessarily the highest ($f_3^7 = 88$). These trade-offs help decision-makers prioritize which roads to open first, assign specialized teams to hazardous zones, and schedule operations to balance safety with speed. From a policy perspective, the quantified risk and benefit metrics can guide the development of standardized clearance protocols and inform investment in more resilient transport and utility networks in earthquake-prone regions.

Table 6. The most preferred and desired Pareto optimal solution values (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$).

Objective functions	**Objective function values**	$(\boldsymbol{\mu_n^r})$	$(\boldsymbol{\mu^r})$
f_1(Time)	44529	1	
f_2(Risk)	11	0.7	0.78
f_3(Prize)	88	0	

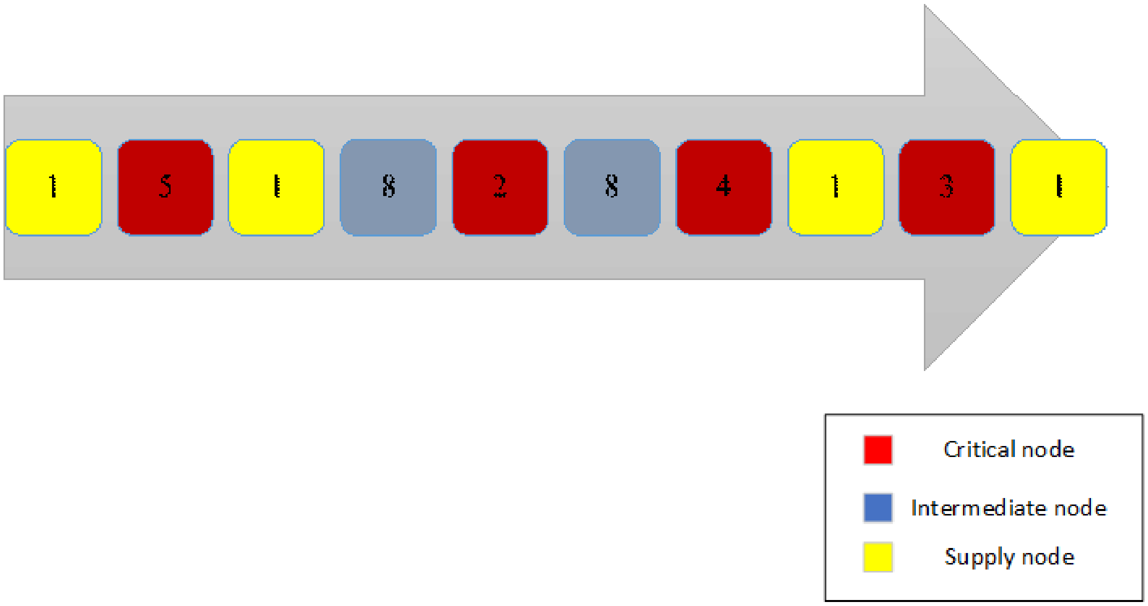

Figure 2. Optimal visiting order for the nodes in 7th Pareto optimal solution (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

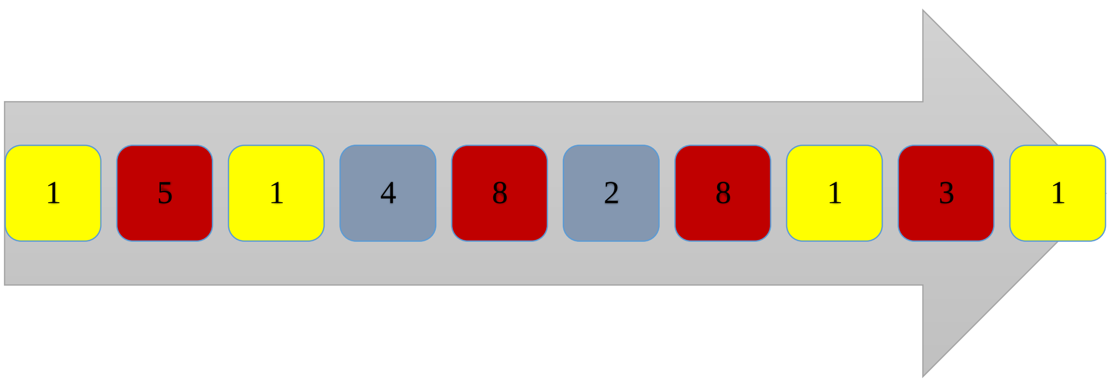

Figure 3. Optimal visiting order for the nodes in 8th Pareto optimal solution (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

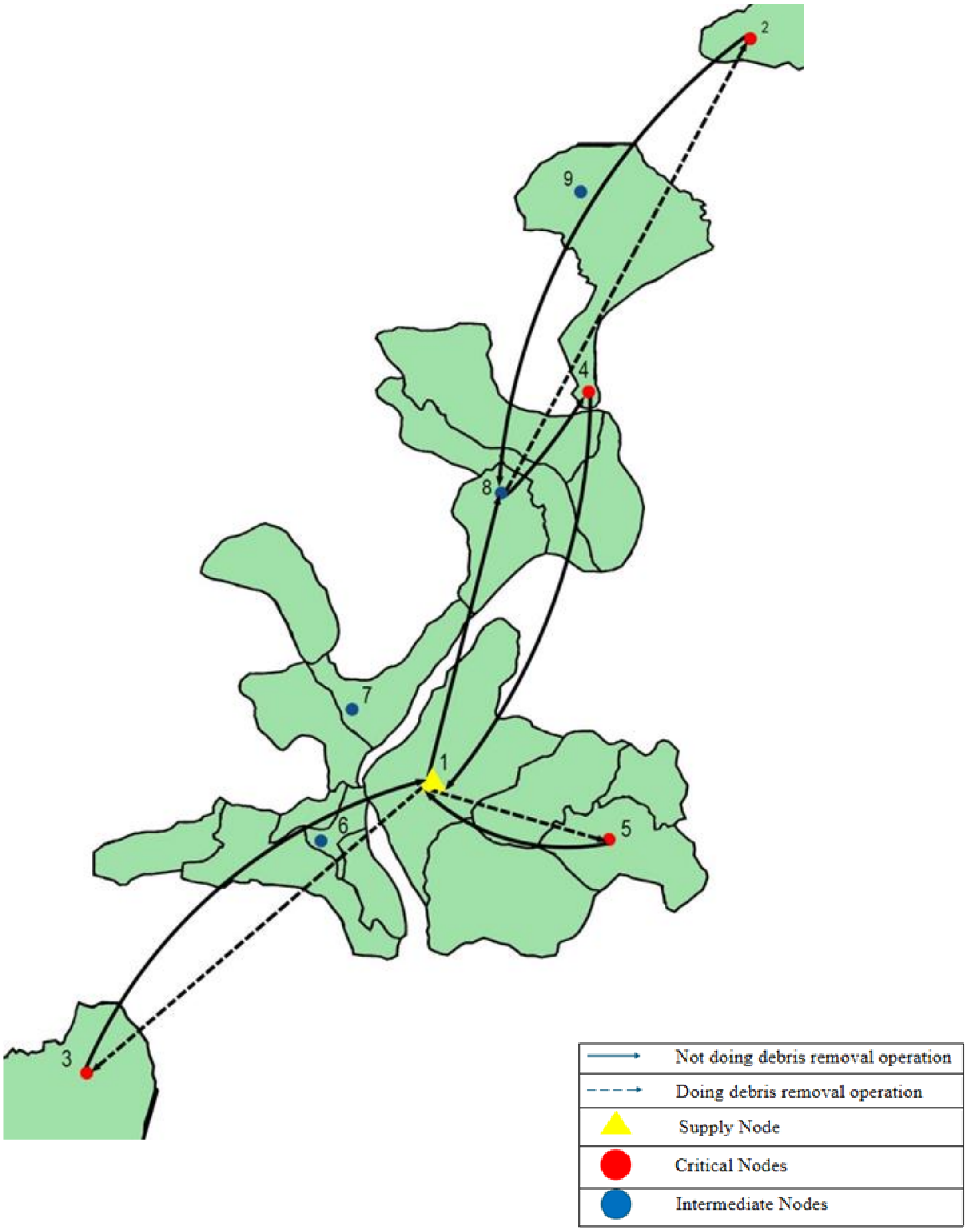

Figure 4. The travel routes to be traversed by debris removal team in the 7th Pareto optimal solution (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

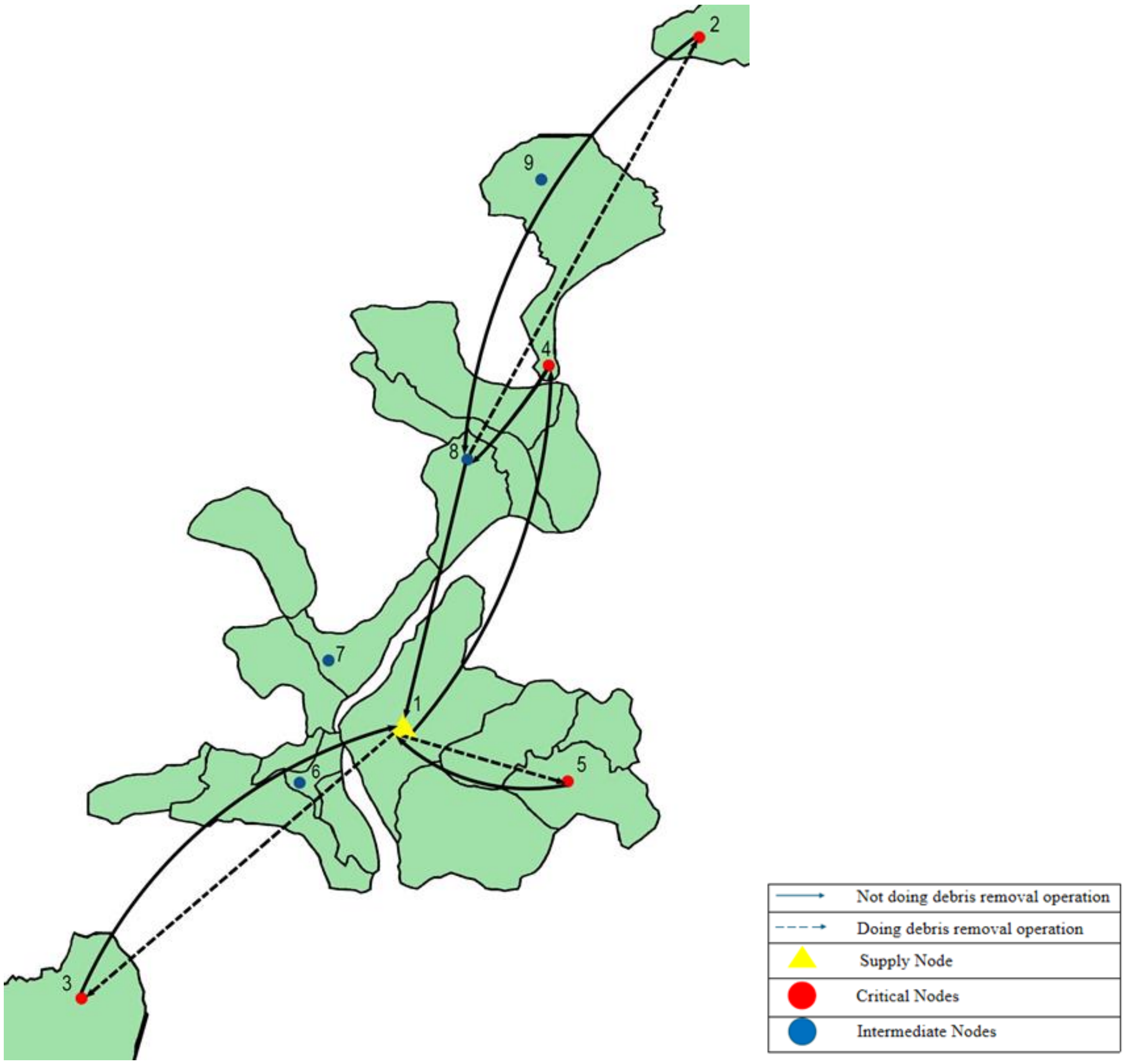

Figure 5. The travel routes to be traversed by debris removal team in 8th Pareto optimal solution (for $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$)

In the proposed model, the uncertainties exist in all objective functions and one of the constraints of the model which respectively affect optimality (solution) robustness and feasibility (model) robustness. If the uncertainty in times or risks is high, increasing the protection level can provide greater confidence in the optimization outcomes. This is particularly beneficial in emergency situations where timely access to critical areas (such as hospitals) is paramount to minimizing casualties. Managers can use this information to adjust their strategies based on available data about the severity of the disaster and the urgency of accessing key locations, such as hospitals, for life-saving operations. In the following, a sensitivity analysis is carried out on budgets of uncertainty ($\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$) with considering objective weights ($w_1 = 0.5, w_2 = 0.4, w_3 = 0.1$). The final robust solutions for different values of Γ_i are presented in Table 7. In Figures (6), (7) and (8), the values of time, risk and prize objective functions for different values of Γ_i are shown, respectively. For higher values of Γ_i, the results are the same, so they are discarded. With considering the results of sensitivity analysis, it is clear that by increasing the Γ_i values, objective function values are getting worst that it makes sense with considering the nature of protection function performance in objective functions.

- First protection level ensures timely access to hospitals and critical nodes. A higher protection level increases the time objective function but ensures its optimality and timely access to critical areas in the worst-case realization of time. This insight allows decision-makers to adjust protection levels to balance the reliability of the removal operation with the need for speed in accessing life-saving locations such as hospitals or other critical infrastructure. The ability to adjust the protection level gives manager's flexibility in making resource allocation decisions based on the urgency of the situation.
- Higher values of the second protection level ensure the model's feasibility in the worst-case scenario. Specifically, they guarantee that the visiting time of the next critical node is greater than that of the current node considering uncertainty in traverse time.
- Sensitivity analysis of the third protection level shows that a higher protection level increases the risk objective function as well as ensuring its optimality and certainty of the risk estimates in the worst case, enabling managers to make more informed decisions on where to send rescuers and have better control on the exposure of rescue teams to high-risk zones, thereby reducing the likelihood of accidents and fatalities. This is particularly important in ensuring the safety of personnel when working in unstable environments.
- Increasing the fourth protection level ensures the optimality of the benefit function, which are tied to reducing losses and casualties by ensuring rapid access to these critical nodes. A higher protection level allows managers to accurately predict which regions should be prioritized for debris removal in the worst case, ensuring that hospitals and emergency services receive timely access. This strategic prioritization is crucial in minimizing casualties by allowing rapid medical interventions.

However, it's important to note that while higher protection levels lead to more reliable outcomes, they also come at a higher operational time or risk, so a balance must be struck between speed/time, risk and benefit efficiency.

Table 7. The final robust solutions for different values of Γ_i

Objective functions/Γ_i values	**f_1(Time)**	**f_2(Risk)**	**f_3(Prize)**
$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 0$	30240	8	108
$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 2$	44529	11	88
$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 4$	51063	12	68
$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 6$	53287	12	54
$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 8$	54465	12	54
$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 = 10$	54465	12	54

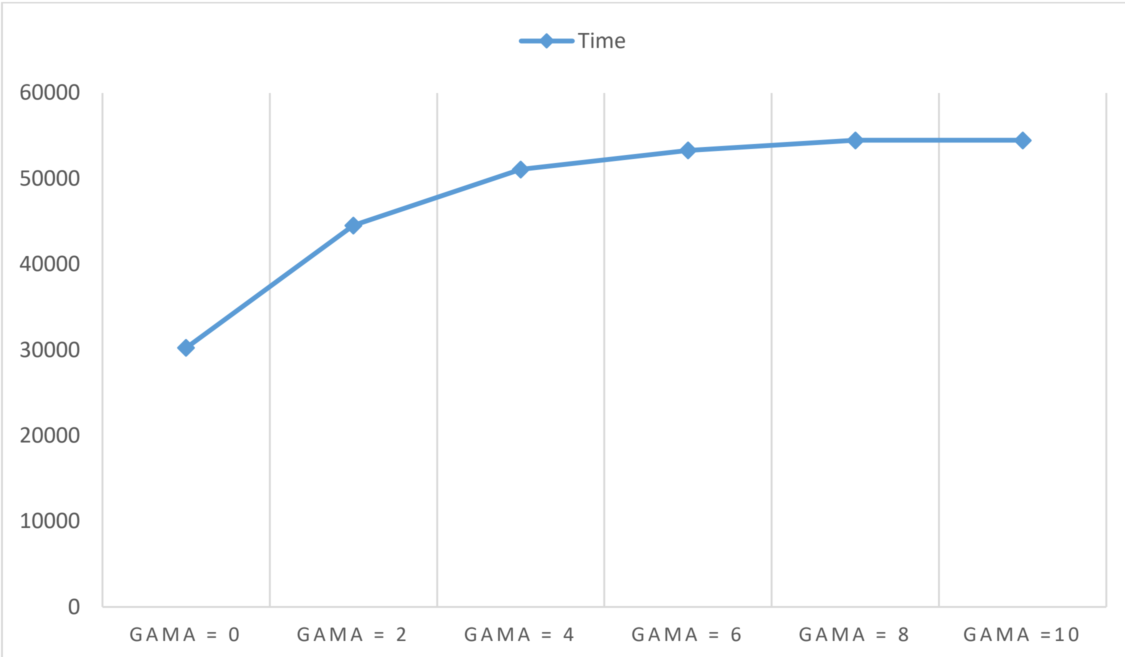

Figure 6. The values of time objective function for different values of Γ_i

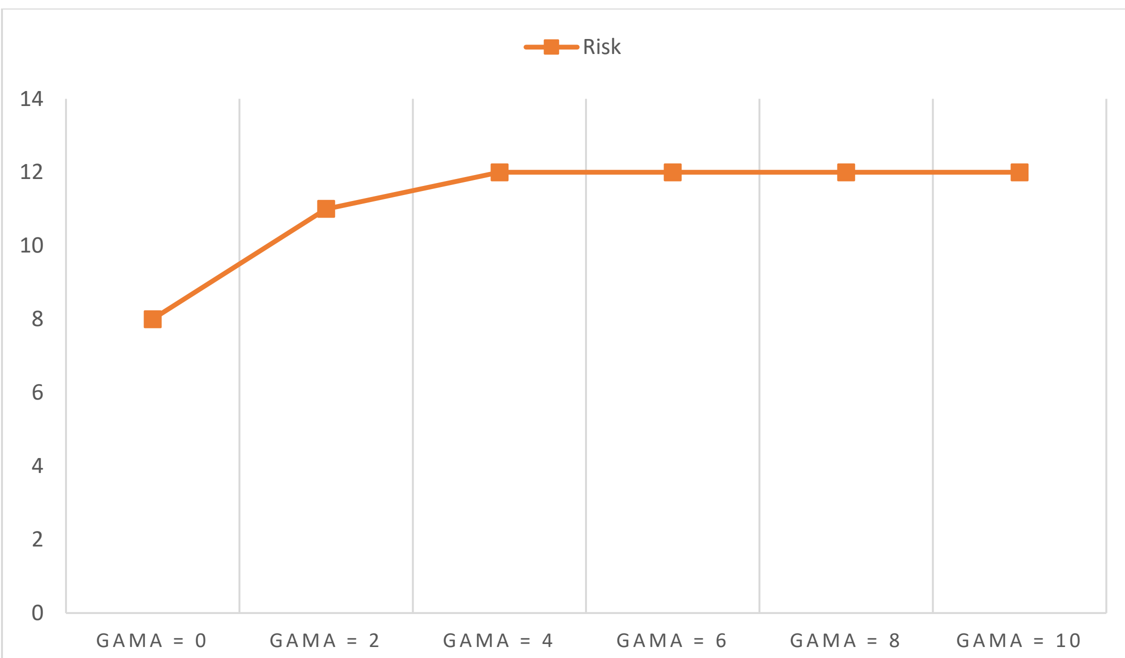

Figure 7. The values of risk objective function for different values of Γ_i

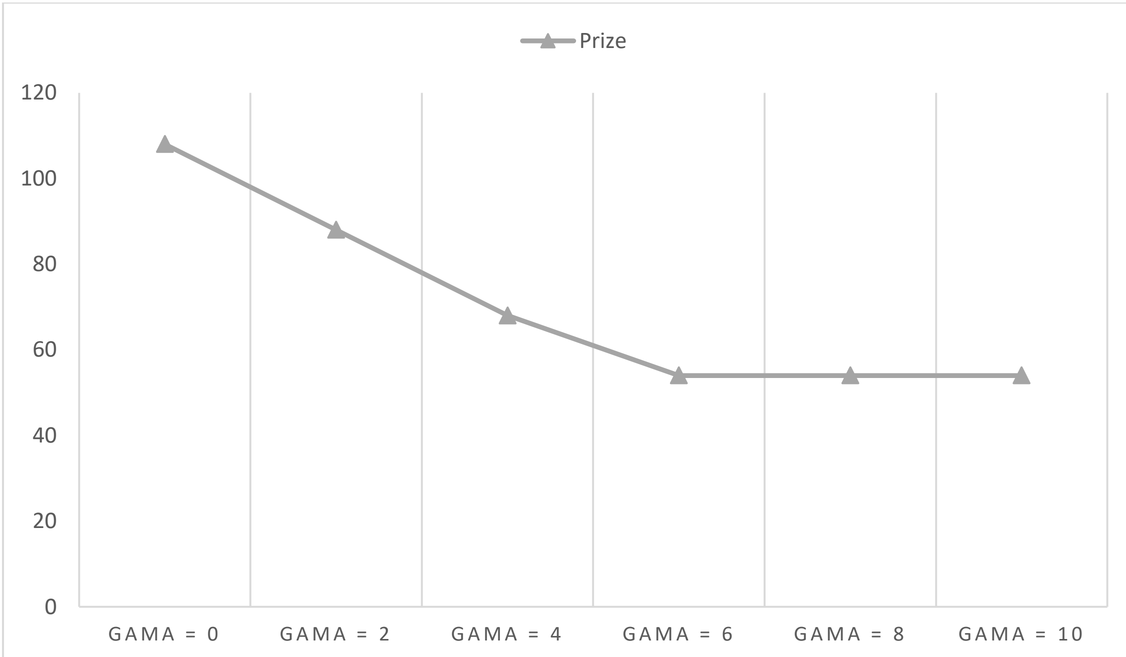

Figure 8. The values of prize objective function for different values of Γ_i

5. Conclusion

Earthquake debris on roads poses major challenges for rescue operations, making timely removal essential. This paper proposes a robust multi-objective optimization approach for debris removal under uncertainty, applied to the Rudbar-Manjil earthquake case study. A fuzzy decision-making method was used to identify the most preferred Pareto-optimal solution and the best routes for the debris removal team. Finally, a sensitivity analysis has been carried out on budgets of uncertainty (i.e. protection levels) which verified the proposed robust model. The proposed multi-objective robust optimization method is versatile and can be applied to other numerical cases with adjustments to the geographical parameters, uncertainty modelling, and resource constraints. Adapting the model to other cases may require relaxing some of the model's assumptions to enhance the model's realism and applicability across different disaster scenarios, such as considering multiple teams, dynamic road conditions, or more complex network topologies. To consider dynamic road condition, travel time, operating time and roads' availabilities are considered as a function of real time and event-driven optimization can be employed. Moreover, stochastic or robust version of the proposed model can be developed to consider road availability under some scenarios. As another extension to the proposed model, one can explore multi-agent systems where several teams with multiple vehicles, starting from different supply nodes, collaborate or compete for resources and routes. In case of multi-vehicle, parameters of the proposed model such as required time to traverse an arc and required effort to remove debris, may be different for each vehicle. Future studies could also enhance risk assessment during debris removal by accounting for hazards such as aftershocks, landslides, or structural collapses, and explore mitigation strategies, including safety protocols and real-time hazard detection, to reduce risks to teams.

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