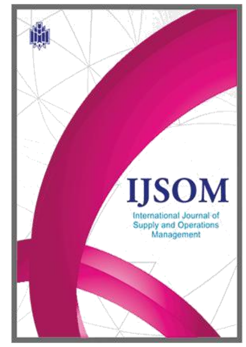
International Journal of Supply and Operations Management

IJSOM

2025, Volume 12, Issue 3, pp. 318-335
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com



Solving The Capacitated Hybrid Vehicle Routing Problem Using a Random General Variable Neighborhood Search: Computational Experiments

Amira Belhadj Ammar [a*], Mohamed Cheikh [a] and Taicir Moalla Loukil [a]

[a] *Faculty of Economics and Management, University of Sfax, Sfax, Tunisia*

Abstract

Green logistics is developed due to the rise of the negative effect of logistics on the environment; and notably the transportation sector. To find advantageous solutions to both business and the environment, some governments require incorporating new environmental limits and objectives studying the vehicle routing problems. In this context, a particular NP-hard problem in this paper is studied called the Capacitated Hybrid Vehicle Routing Problem. We proposed an integer linear program for the capacitated case and we developed a Random General Variable Neighbourhood Search algorithm, which can handle the problem with a significant number of customers. We solved small instances with mutually the standard solver CPLEX and with our proposed algorithm. Outcomes show that our approach provided the same results for 55 % of the cases and superior solutions for 40 % of the instances compared to the results provided by CPLEX. In parallel, we modify a large set of benchmark instances proposed in previous works by introducing the product demand of each customer and we solve it by the Random General Variable Neighbourhood Search algorithm. Results of numerical testing show that our method delivers the best-known solutions in computation times of just 3 seconds on average.

Keywords: Green Vehicle Routing Problem; Capacitated Vehicle Routing Problem; Variable Neighborhood Search; General Variable Neighborhood Search.

1. Introduction

For many years, diminishing the total cost of transportation was a significant task for companies that led to enhanced performance and increased profit. Hence, the vehicle routing problem (VRP) has become an attractive topic and one of the most considered combinatorial optimization problems. A significant modification in human life using technologies, especially in the transportation sector, provides many advantages regarding time, safety, and comfort. Although, this causes enormous damage to the environment. Some researchers have tried to demonstrate the importance of adopting Green Logistics practices including transportation, packaging, and inventory management, as shown by Akubia et al. (2025). Consequently, some governments require logistics companies to incorporate new environmental constraints and objectives into vehicle routing problems in order to achieve both business and environmentally friendly solutions, which has led to the emergence of the Green VRP concept (G-VRP) as a progression from the classical VRP by Erdogan and Miller-Hooks (2012). Various works have been studied on the G-VRP among others, including those by Jamshidian et al. (2021), Alinaghian et al. (2021), Luo et al. (2023), Shan et al. (2025) and Wu et al. (2025). Further details about the G-VRP can be found in the work of Garside et al. (2024).

*Corresponding author email address: amira.belhaj.ammar@fsegs.usf.tn
DOI: 10.22034/ijsom.2025.109882.2712

Mancini (2017) introduced a new continuation of the classical G-VRP, named the Hybrid VRP (H-VRP) respects the same objectives as the G-VRP however; it uses Hybrid Vehicles (HV) a type of vehicle that can overcome the constrained number of charging stations since it uses hybrid sources of energy. In his study, Mancini considered vehicles without capacity limits, which is unrealistic. Hence the idea of integrating this capacity constraint into the mathematical formulation.

Therefore, in this paper, we studied the Capacitated Hybrid Vehicle Routing Problem (CH-VRP). It is a development of one of the core combinatorial optimization problems, the Capacitated Vehicle Routing Problem (C-VRP). As per ,Sajid et al. (2023), the C-VRP is a significant variant of the classical VRP since capacity constraint plays a crucial role in the routing networks and it can be optimally solved only for small cases. For example, Simić et al. (2024) modelled a real case problem for the Serbian company CARNEX as a C-VRP with a homogeneous fleet, and used a hybrid nearest neighbours - Tabu search model to find solutions. Tirkolaee and Aydın (2021) approached the issue of transportation planning and outsourcing of medical waste management problems during pandemics as a C-VRP. They considered the capacity limitation of vehicles as the primary restriction. They solved the problem with a CPLEX solver. For large-scale instances researcher used heuristics and metaheuristics to solve the C-VRP. According to Suza et al. (2023), current literature on C-VRP covers many metaheuristics approaches known for their high efficiency. Ongoing with the literature, they proposed a robust algorithm based on the differential evolution to solve the C-VRP. Many extensions and variants of the C-VRP have been found in the literature such as the two-echelon C-VRP (2E-CVRP). Li et al (2022) solved the 2E-CVRP with grouping constraints and simultaneous pickup and delivery. Compared with the classical VRP, the interaction and interdependence between the two levels add complexity and challenge to the problem. In turn, the authors developed an exact heuristic to solve the problem. Küçük and Yildiz (2022) solved another variant of the C-VRP that is the three-dimensional loading capacitated vehicle routing problem (3L-C-VRP) by developing an integrated and decomposed constraint programming-based method for the vehicle routing part and an evolutionary algorithm for the loading part. As alternatives to previous works, Chi and He (2023) introduced the pickup-capacitated VRP (3L-PC-VRP) with three-dimensional loading constraints as another variant of the C-VRP. They developed an improved tree search algorithm, an improved greedy heuristic algorithm, and an improved branch-and-price-based algorithm as a solution for the problem. Shan et al. (2025) presented a case study on rural transportation in Guizhou Province, where they addressed a C-VRP with soft capacity constraints. After formulating the model, they solved it using a Genetic Algorithm with a self-adaptive mechanism and a novel initial solution generation method. Their metaheuristic achieved good results compared to other studies, from both economic and environmental perspectives.

In our work, we study the CH-VRP, which involves two well-known complex problems that are H-VRP and C-VRP. Since it is an NP-hard problem a Random General Variable Neighbourhood Search (R-GVNS) based metaheuristic is proposed to solve the CH-VRP. The Variable Neighbourhood Search (VNS) is a widely recognized local search algorithm known for its adaptation of neighbourhood search structures. Mladenović and Hansen (1997) proposed it for the first time and then it has been used as a solution of various optimization problems and also machine learning tasks. Recently, Kalatzantonakis et al. (2023) solved the C-VRP by reinforcement learning based on the VNS metaheuristic. We chose the VNS because it has great results in the literature and can extend to various optimization problems.

In our strategy, three neighbourhood structures are used for the proposed random variable neighbourhood descent (RVND) and the shaking step which are swap, insertion, and 2-opt* structures. To showcase the performance of our proposed approach, first, we extend the problem formulation proposed in Mancini (2017) as an integer linear program to the capacitated case, we modify a large set of benchmark instances proposed in the literature by introducing the product demand of each customer and we solve the small instances with the standard solver CPLEX and with the proposed algorithm. The outcomes show that our approach provided the same results for 55 % of the cases and superior solutions for 40 % of the instances compared to the results provided by CPLEX. In parallel, we modify a large set of benchmark instances proposed in the literature by introducing the product demand of each customer and we solved it by our proposed R-GVNS algorithm. Extensive experiments of the proposed metaheuristic on small and large-sized instances demonstrate that the suggested approach finds optimal solutions in small cases and good-quality solutions in larger ones in reasonable computational time. To summarize, this paper contributes to the literature as follows:

- We introduced a novel G-VRP variant, the CH-VRP, which is an H-VRP with capacity constraint.
- We proposed a mathematical model for small instances to tackle the problem and resolved it using the CPLEX solver.
- We developed an efficient R-GVNS metaheuristic that produced superior solutions for small and large instances.

The paper is organized as follows: Section 2 extends a summary of the most relevant work related to the HVRP problem; Section 3 presents the problem formulation; Section 4 explains the proposed algorithm. Section 5 reports the results and discussion. Finally, section 6 gives a conclusion.

2. Related work

This section illustrates a brief literature review of the related work to our problem. The G-VRP is a classical VRP that incorporates environmental aspects and objectives. Erdogan and Hooks (2012) conducted the first work related to this problem, then, many researchers concentrated on the G-VRP using various types of vehicles. We mention three types of vehicles used.

The first ones are the Internal Combustion Engines Vehicles (ICEVs): types of vehicles that use traditional petroleum fuels. The common objective between almost all the studies of G-VRPs, that used homogenous ICEV in the literature, is minimizing fuel consumption which also means minimizing CO2 emission [(Jabir et al. (2017) and Dukkanci et al. (2019)].

The second one is the Electric Vehicles (EVs): type of vehicles that use pure electric engines in which minimizing the total, travel time/cost is the main objective of the EVRPs since they use an eco-friendly vehicle that has zero emission [(Bruglieri et al. (2015), Ding et al. (2015) and (Jie et al. (2019). In some papers, authors considered heterogeneous fleets (EV and ICEV) where minimizing the travel-cost time is a common objective. [(Goeke et al. (2014), (Hermann et al. (2019) and (Kopfer et al. (2019), Li et al. (2020), Lu et al. (2020), Baso et al. (2021), Fan et al. (2023), Comert and Yazgan (2023), Zheng et al. (2024), Souza et al. (2024) , Lera-Romero et al. (2024)].

Abdallah (2013) mentioned that the mixed type of vehicles that use electricity and fuel had not been investigated in the literature. He conducted the first work that studied homogenous HVs. Limited research has explored the optimization of the H-VRP. In his work, Abdullah tried to answer a simple but important question «Which alternative among the ones that exist in the world is the best for both economic and environmental aspects? ». He focused their discussion on the ICEVs, which are a source of high greenhouse gas emissions, EVs, which help improve the urban air quality since they use renewable energy but require a long charging time, and HVs, which overcome the disadvantages of both ICE and EV. HVs are a type of vehicles that use both pure electric engines and traditional petroleum fuels. The H-VRP takes into consideration both the economic objectives of classical VRP and the environmental objectives of G-VRP such as minimizing CO2 emission and fuel consumption. Therefore, Abdallah tried to study this problem with homogenous vehicles and proposed a model for the H-VRP with time windows that minimize the HV's emission while minimizing the time and meeting the demand during a time window. He presumed that there is a linear relationship between electricity and gasoline consumption. The problem is solved using a Lagrangian relaxation model and a Tabu search algorithm.

Juan (2014) conducted the second work that considered the H-VRP. Instead of homogenous fleets, they concentrated on heterogeneous fleets. Authors supposed that they need to consider multiple driving ranges (long and short ranges) since the range is an important limitation for vehicles. They inspired their work from the classical C-VRP where each vehicle has its distance to cover. They solved this problem using an integer programming formulation and a multi-round heuristic algorithm. These works are followed by the one conducted by Arslan et al. (2015) who noticed that the long distance of HV was not well studied in the previous related works. Therefore, they decided to concentrate their study on long-distance HV trips that require numerous refuelling and researching stops. They approach this challenge using the dynamic programming model-based heuristic and the shortest path heuristic. Doppstadt et al. (2016) endeavored to address this novel challenge by proposing a new optimization problem investigating the distribution of goods using HV. They extended from the well–known traveling salesman problem to the HV-traveling salesman problem and solved it by mathematical problem formulation and a heuristic based on the Tabu search algorithm. Doppstadt et al. (2019) resolved their problem by adding the time window constraint.

Following the previous works, Nejad et al. (2017) study the optimal routing of HV s by introducing the Energy Efficient Routing problem (EERP). It was the first study to acknowledge the energy efficiency variance among different operating modes of HVs during route planning. The authors considered the EERP as a subset of shortest-path problems (SPP). They tried to solve it by designing exact and approximation algorithms. They first presented the EERP integer programming model, they proved the hardness of this problem after that they proposed a routing algorithm in which arcs of the road network are assumed to be pre-divided into distance segments with consistent energy efficiency conditions for the various operating modes. Vincent et al. (2017) believe that plug-in hybrid vehicles (Plug-In HV), which are HVs that can recharge themselves from external and internal sources, are the coming generation of vehicles. They tried to minimize the total travel cost using this type of vehicle by presenting a mathematical model and implementing a simulated annealing heuristic with a restart strategy with the Boltzmann function (SARSBF) and the Cauchy function (SARSCF) as acceptance criteria for a worse solution.

The H-VRP was introduced by Mancini (2017) as an expansion of the G-VRP. Our model in this paper is derived from her work in which we add the capacity constraint, a significant constraint in reality. (More details are presented in the next sections). To solve the problem Mancini proposed a large-scale neighbourhood search-based mathematical algorithm (MH).

Subsequently, Karak and Abdelghany (2019) studied the routing for pick-up and delivery services using a hybrid vehicle drone. They noted that the investigation of drone technologies during the last years has attracted many sectors such as logistics. Therefore, they tried to find a solution to the hybrid vehicle-drone routing problem (HVDRP) for pick-up and delivery. They considered this problem as an extension of three known problems. The HVDRP is similar to the C-VRP given the drone's constrained flight range and load capacity. It is also, similar to the two-echelon location and routing problem in which two-level routing interdependent decisions are involved. Moreover, it is similar to the truck and trailer routing problem because it requires routing the vehicle and the drone. The authors sought to solve this challenge by formulating a mixed-integer program and suggesting an innovative solution approach that builds upon the classic Clarke and Wright algorithms.

Zhen al. (2019) remarked that in the previous studies, researchers neglected the mode selection related to recharging and refuelling, which contributes to achieving optimal energy consumption. Consequently, they focused on solving the H-VRP mode selection by formulating the problem as a mixed integer linear programming model and developing an improved particle Swarm Optimization algorithm. Correspondingly, Bahramia et al. (2020) aimed to find the ideal use of power from the two energy sources along the vehicle’s route. They solved the problem using both the exact pseudo-polynomial algorithm and the fully polynomial time algorithm. Hiermann et al. (2019) Studied how a VRP's vehicles are made up, considering ICEVs, EVs, and P-HVs. They presumed that PHEVs alternate between the two energy sources at any point and that the quantity of charging determines how long it takes to recharge them. The problem is solved with the use of a hybrid genetic algorithm. The numerical examples demonstrate that an optimal mixed fleet has lower operational costs than a homogeneous fleet. Inspired by the previous research, Li et al. (2020) proposed a new hybrid metaheuristic to solve the H-VRP that combines the memetic algorithm with the sequential variable neighbourhood descent. Alsumairat and Alrefaei (2021) utilized the constant temperature simulated annealing SA-CT algorithm to solve the H-VRP and compared the findings of the suggested approach with those of the simulated annealing with decreasing temperature technique utilizing SA-DT.

All previous research and studies considered a deterministic environment; Yin and Zhao (2022) examined a nonlinear and ambiguous H-VRP. They divided the cost of transportation into two types of driving and linearized the nonlinear variable, where the fuel consumption, the cost of driving on electricity, and the cost of driving on traditional fuel are all taken to be arbitrary fuzzy variables with ambiguous probability distributions. Their study has put forth a distributional robust equilibrium optimization strategy for H-VRP, along with an ambiguous equilibrium risk value objective function and an ambiguous equilibrium chance constraint.

Zhu et al. (2024) investigated a real-world case based on the downtown Austin network, formulating the problem as a traveling salesman problem (TSP) that integrates plug-in hybrid electric vehicles (PHEVs) with drones. This study is among the few that address electric vehicle and drone routing simultaneously, providing a comprehensive framework to optimize hybrid delivery systems. The authors proposed a mixed-integer linear programming (MILP) model for this problem and developed an adaptive large neighbourhood search (ALNS) algorithm to solve it. The results demonstrate that this method outperforms other approaches, achieving promising performance in terms of solution quality.

More details about the H-VRP can be found in the work of Ammar et al. (2022).

A summary of the H-VRP-related studies is presented in Table 1.

Table 1. Related work to the H-VRP

	Vehicles type	Objective Function	References
Homogeneous Vehicles	Plug-in-Hybrid Vehicles	Reduce the total travel duration.	Abdallah (2013)
		Reduce the total distance.	Arslan et al. (2015)
		Reduce the total travel cost.	Nejad et al. (2017), Vincent et al. (2017), Li et al. (2020)
		Reduce the total cost of energy consumption	Zhen et al. (2019)
		Reduce the total cost of gasoline consumption	Bahramia et al. (2020)
	Hybrid Electric Vehicles	Reduce the total travel cost	Doppostadat et al. (2016), Doppostadat et al. (2019)
	Hybrid Vehicles	Reduce the total travel distance.	Mancini (2017)
		Reduce the total travel cost	Alsumairat and Alrefaei (2021)
		Reduce the ambiguous equilibrium risk value.	Yin and Zhao (2022)
		Reduce the total travel distance.	Our work
	Hybrid-Drone V	Reduce the total travel cost	Karak and Abdelghany (2019)
Heterogeneous Vehicles	ICEV Plug-in-Hybrid Vehicles	Reduce the total distance	Juan et al. (2014)
	ICEV Plug-in-Hybrid Vehicles Electric vehicles	Reduce the total travel cost.	Hiermann et al. (2019)
	Plug-in-Hybrid Vehicles Drone	Reduce the operational time	Zhu et al.(2024)

3. Problem description and formulation

3.1. The problem description

The CH-VRP is a network model. The problem description is as given: Let $G = (V, A)$ be a complete undirected graph where V denotes the list of vertices that include a depot, the customers list I, and the recharge stations set F. Each edge $(i, j) \in A$ is characterized by a distance d_{ij} and a travel time t_{ij}. The CH-VRP problem consists of visiting customers set, with M identical vehicles with max load capacity C. Any vehicle must start and end its trip at the depot and can program visits t recharging points if it is needed. Recharge stops are unrestricted, and at each stop, the battery will fully charge. No multiple trips are permitted, so at most M routes can be programmed. Each node has a specified service time p_j. The routes have a time limit T_{max} . Speeds are consistent across a link. It would be better to travel using only the electric mode, but, if necessary, the vehicle can travel part of the route using conventional fuel propulsion. This distance that joins the customer j is defined as W_j. When we use fuel propulsion a unitary distance penalty ρ is added to the cost to encourage the use of the electric mode.

For more clarity on the CH-VRP problem, we report in Fig.1 a small example with two customers and a depot. In this figure, the travel distances are above each edge. In a classical VRP with a single traditional vehicle, we choose the trip (0-1-2-0) with a total travel distance of 120 km. Using now the electric vehicle and assuming that the battery capacity is 100 KWh with a discharging rate is 1KWh/1km. Departing from the depot with a fully charged battery, we proceed to customer 1 and then to customer 2. The remaining battery level does not allow reaching the depot but allowing us to go to the charging station. Thus, the next destination will be the recharge station before heading back to the depot. The total covered distance is 40+30+40+70=180 Km.

If we utilize the Hybrid Vehicle, there are three scenarios: the path will be the same as that of a traditional vehicle using the traditional mode, the same as that of an electric vehicle using the electric mode, or the path will follow the sequence (0-1-2-0) with switching between modes as shown in Fig.1. Any distance that is not covered by the electric mode will incur a penalty ρ=3 to boost the use of the electric mode. Therefore, the total distance when using the traditional fuel only is 40+30+50+ ((40+30+50) *3) =120+ (120*3) = 480 km. The total distance when using the electric mode is 40+30+40+70= 180 km without an additional penalty value. The total distance when switching between modes (when using the hybrid mode is 40+30+50+ (U_j*3) = 40+30+50+ (50 *3) = 270 Km with U_j= 50 km is the distance covered by conventional mode to reach the depot.

Hence, the electric mode is encouraged to be used to serve the costumers (180 Km < 270 Km < 480 Km).

The HV will use the traditional mode if and only if the total travel distance using the electric mode, including visits to recharging stations, is greater than the distance using traditional fuel with the penalty considered.

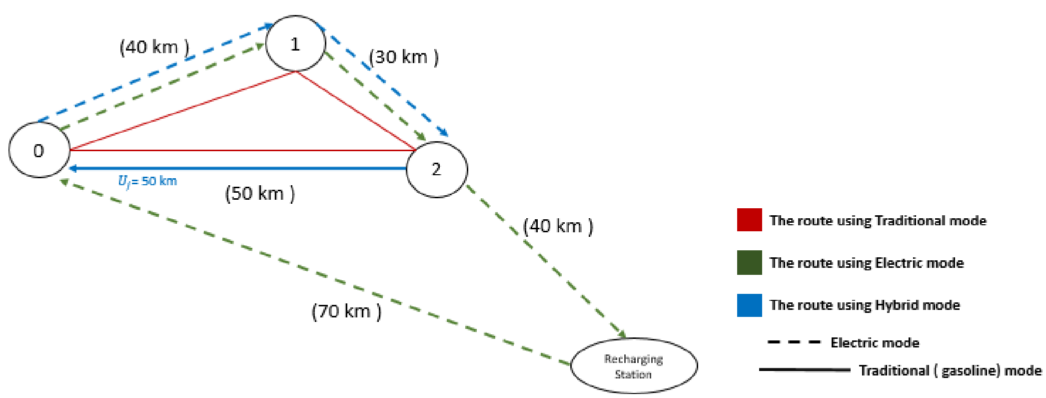


Figure 1. H-VRP: Problem description

To resume, we consider a problem having the following features:

- M identical Hybrid Vehicles with identical capacities. Each one of them has a load capacity L_j that varies conditional on the product to be transported.
- I is the set of customers where each one of them has a demand denoted by q_i and a service time denoted by p_j.
- A penalty ρ (unitary distance) is counted when traveling with traditional fuel.

3.2. Problem Formulation

we summarize the definitions of parameters and the variables in Tables 2 and 3 to help the reader.

Table 2. The set of parameters

I	List of customers
I_0	List of both customers and depot
F	List of refuelling and recharging stations
V	List of nodes without including depot I_0
V_0	List of all nodes
r	Battery charge consumption rate of the electric mode (for km)
C	Vehicles capacity
Q	Battery capacity
M	Maximum number of Hybrid Vehicles
μ_j	Smallest distance from node j to the nearest refuelling stations or to the depot
d_{ij}	Distance between node i and node j
ρ	Distance unitary penalty when using traditional fuel
t_{ij}	travel time between node i and node j
p_j	Service time of node j
T_{max}	The maximum duration of the route
q_i	The demand of customer i

Table 3. The definitions of the variables

X_{ij}	Binary variable equal to 1 if a vehicle travels from node i to node j and 0 otherwise
W_j	Distance completed with conventional fuel to arrive at customer j
U_j	Distance completed with conventional fuel to get to the depot from customer j
T_j	Arrival time at node j
Y_j	Battery level upon arrival at node j
L_j	Load of the vehicle upon arrival at node j

The mathematical description of the CH-VRP is detailed below:

$$Min \sum_{i\in v_0} \sum_{j\in v_0} d_{ij} X_{ij} + \sum_{j\in V_0} \rho (W_j + U_j) \quad (1)$$

$$\sum_{\substack{j\in V \\ i\neq j}} X_{ij} = 1 \qquad \forall\, i \in V \quad (2)$$

$$\sum_{\substack{j\in V_0 \\ i\neq j}} X_{ij} \leq 1 \qquad \forall\, i \in F \quad (3)$$

$\sum_{\substack{j\in V_0 \\ K\neq j}} X_{jk} = \sum_{\substack{j\in V_0 \\ i\neq j}} X_{ij}$	$\forall k \in V_0, j \in V_0$	(4)
$\sum_{\substack{j\in V_0 \\ i\neq j}} X_{0j} \leq M$		(5)
$\sum_{\substack{j\in V_0 \\ i\neq j}} X_{j0} \leq M$		(6)
$T_j \geq T_i + (t_{ij} + p_j)X_{ij} - T_{max}(1 - X_{ij})$	$\forall i \in V_0\ j \in V\ and\ i \neq j$	(7)
$0 \leq T_0 \leq T_{max}$		(8)
$t_{0j} \leq T_j \leq T_{max} - (t_{j0} + p_{ij})$	$\forall j \in V$	(9)
$Y_j \leq Y_i - r(d_{ij} - W_j) + Q(1 - X_{ij})$	$\forall j \in l\ i \in V_0\ i \neq j$	(10)
$Y_j = Q$	$\forall j \in F \cup V_0$	(11)
$Y_j \geq r(d_{j0}X_{j0} - V_j)$	$\forall j \in I$	(12)
$0 \leq L_j \leq L_i - q_i X_{ij} + C(1 - X_{ij})$	$\forall i \in V\ j \in V_0\ i \neq j$	(13)
$0 \leq L_0 \leq C$		(14)
$X_{ij} \in 0,1$	$\forall i \in V_0\ \forall j \in V_0$	(15)
$Y_j \geq 0,\ L_i \geq 0$	$\forall i \in I$	(16)

The aim (1) is to reduce the overall distance driven with hybrid vehicles. Constraint (2) ensures that each customer receives only one visit; (3) allows multiple visits per station. Constraint (4) ensures arrivals at each node match departures for all vehicles. A maximum of M vehicles can depart from and return to the depot on any given day thanks to constraints (5) and (6). Constraint (7) studies the arrival time at each node. Each vehicle must reach the depot by the time limit T_{max}, according to constraints (8) and (9). Constraint (10) tracks the battery charging level as it arrives at each node. Constraint (11) ensures that the battery level is reset to Q when a vehicle leaves the depot or charging station. According to constraint (12), traditional fuel propulsion is used to reach the depot or a charging station with a higher cost if there is insufficient battery level to get there. No capacity restriction applies to fuel tanks because they provide a much greater range than electric batteries and because fuel stations are more widely distributed throughout the road network. Constraint (13) tracks the vehicle load upon arrival at each customer. Constraint (14) ensures that the vehicle's load at the depot stays within the vehicle's capacity limit. Constraints (15) and (16) specify the domain of the variables.

4. The proposed algorithm

The CH-VRP, a variant of the classical VRP, is classified as an NP-hard problem. This categorization is supported by the exponential increase in potential paths and combinations as the number of customers and vehicles grows, as well as by the inclusion of capacity constraints. These factors contribute significantly to the complexity of finding an optimal solution within polynomial time, reinforcing its NP-hard status. To solve this problem, we propose the R-GVNS algorithm, a well-known metaheuristic method that is a variant of the classical VNS. Hansen et al. (2008) presented the VNS as a systematic neighbourhood change made during the descent phase to identify a local minimum and the perturbation phase to escape from local optima. Since its initial proposal in 1997, VNS has significantly progressed in terms of techniques and applications.

According to Jarboui et al. (2014), VNS is a metaheuristic that exploits the idea of systematic neighbourhood changes during a search to navigate away from valleys containing local optimal solutions. Following this, many researchers have used VNS to solve routing problems. For example, Cheikh et al. (2015) used VNS to explain the VRP with multiple trips. Cendani et al. (2024) developed a VNS-genetic algorithm hybrid to crack the VRP with simultaneous delivery and pickup of materials. Alrashidi and Al Ghamdi (2024) presented a hybrid approach combining VNS with the reinforcement learning paradigm to effectively resolve the G-VRP. These studies demonstrate that VNS is an effective algorithm for solving VRP problems.

The proposed method calls for two distinct neighbourhood structure sets. The shaking (perturbation) phase is done with the first set, while the second set performs the random variable neighbourhood descent (RVND) procedure. There are four input parameters for the method. l_{max} denotes the maximum number of successive shaking steps to apply to the current solution, and S_{max} is the number of neighbourhood structures used in the RVND procedure. Cpu_{max} represents the maximum amount of time allowed for the search, while x is the initial solution.

4.1. Solution representation

The CH-VRP network involves three node sets, depot, customers set, and recharge stations set. To encode the solution, we propose to use two arrays with variable lengths associated with each vehicle. The first table represents the sequences of visited customers while the second adds to the first a recharge station when needed to ensure the whole tour. Fig.2 illustrates the solution representation.

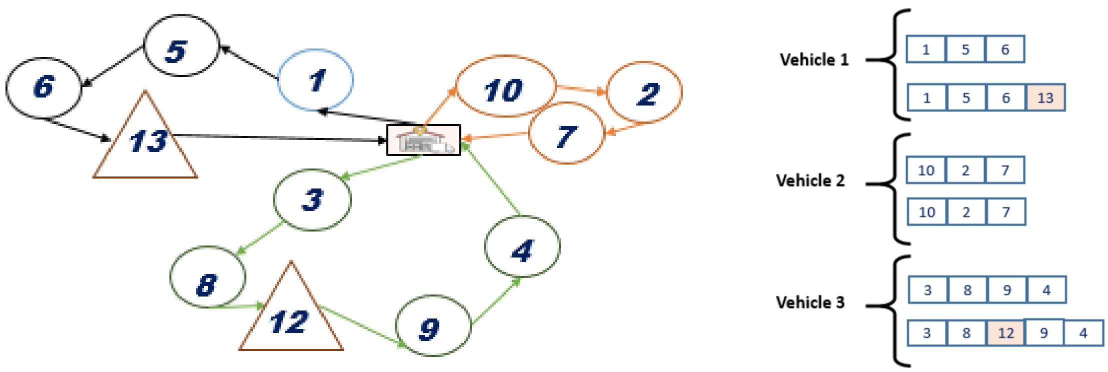


Figure 2. Solution Representation

4.2. Neighbourhood structures

In this method, we adopt three neighbourhood structures described as follows:

The Insert move; is used to delete a selected customer i from its current position and insert it at a randomly chosen position following a randomly selected customer j in the same or different route. The example in figure (Figure 3) illustrates relocating the customer (7). This one is deleted from route (2) and inserted in route (3) between the visits to the customer (9) and customer (4).

The Swap move; consists of a random selection of two customers i and j belonging to the same or different routes and swapping their positions. The example in figure (Figure 4) illustrates the swap position of customers (1) and (5) in the same route (1).

The 2-opt* move; this local search is an exchange heuristic. It consists of cutting two different routes (1) and (2) into two parts and recomposing them. The first route builds with the first part of (1) merged with the second part of (2). The second route is constructed by the first part of (2) assembled with the second part of (1). The example of (Figure 5) shows crossing the two routes in customer (2) in route (2) and customer (9) in route (3).

We notice that, after each applied neighbourhood, we evaluate the resulting route and insert a recharging station if necessary, using a heuristic that determines the best position for inserting the best station.

4.3. R-GVNS algorithm

The R-GVNS Algorithm proceeds through three steps. In the first step, the algorithm generates the initial solution randomly, therefore, there are no conditions for the first solution. More precisely, we assign a random sequence of customers and recharging stations (if necessary) to each available HV. In the second step (the shaking step), one of the three known neighbourhood structures (the Insert move, the Swap move, and the 2-Opt move), chosen randomly, creates a new solution x' from the initial solution x. In the third step (the improvement step), the RVND method, considering a random order of the neighbourhood structures, tries to improve a current solution x'. Algorithms 1 and 2 describe respectively the steps of the R-GVNS and the RVND mentioned above.

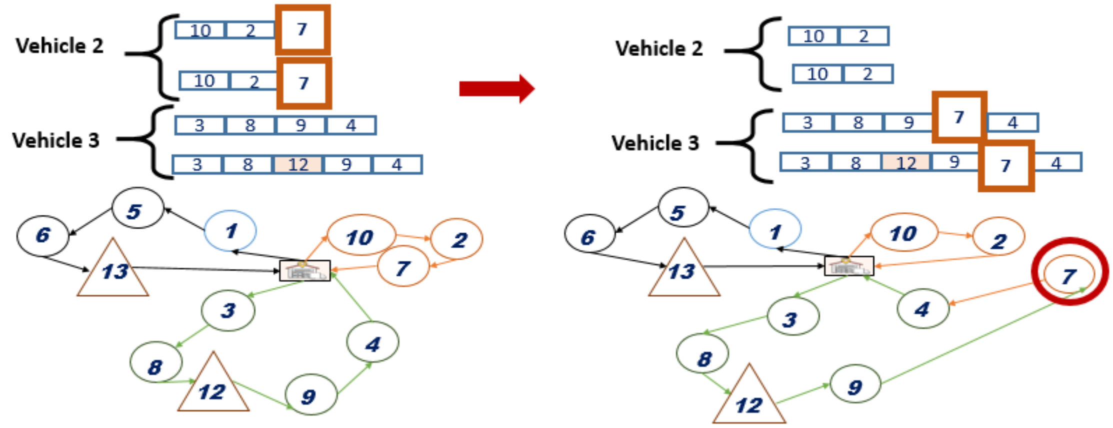


Figure 3. Insert Moves

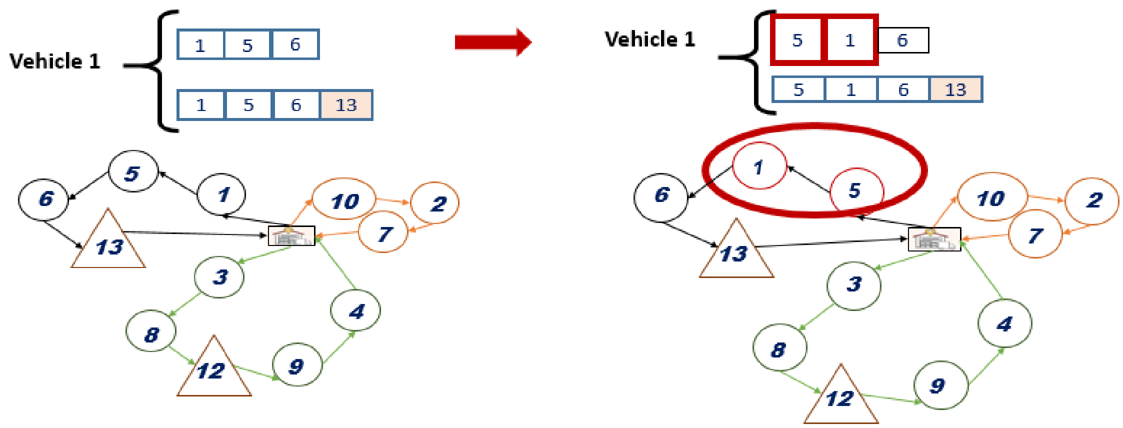


Figure 4. Swap Moves

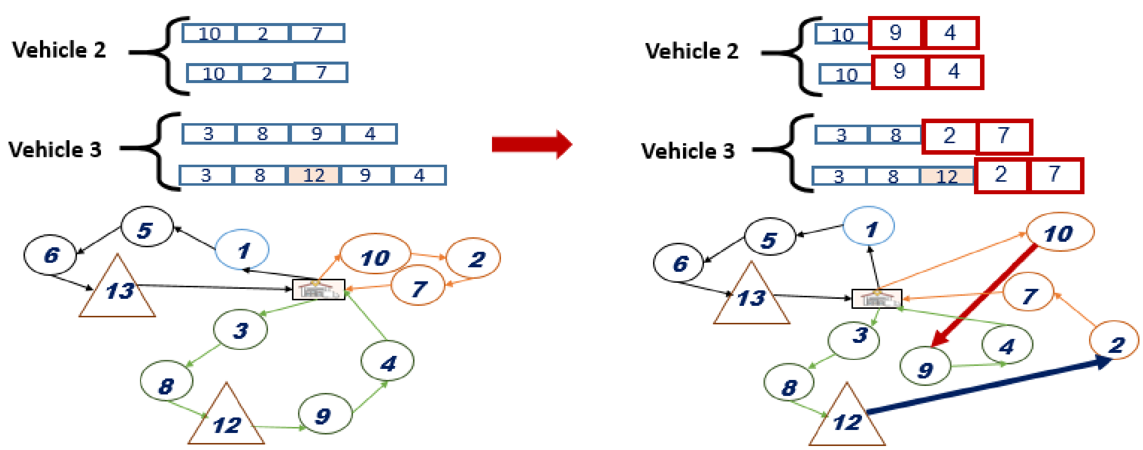


Figure 5. 2-Opt* Moves

Table 4. The steps of the R-GVNS

Algorithm1: R-GVNS Procedure
Input: x; $S_{\max}$; $l_{\max}$; Cpu_max
while ($time$ < Cpu_max) **do**
$l = 1$;
$\boldsymbol{while}$ ($l \le l_{\max}$) **do**
$x' = Shaking(x;\ l)$;
$x'' = \boldsymbol{RVND}(x';S_{\max})$;
$\boldsymbol{if}$ $((f(x'') < f(x))$ $\boldsymbol{then}$
$x = x''$; $l = 1$;
$\boldsymbol{else}$;
$l = l + 1$;
$\boldsymbol{end\ if}$;
$\boldsymbol{end\ while}$;
$time = CpuTime(\quad)$;
$\boldsymbol{end\ while}$; $\boldsymbol{return}$ x;

Table 5. The steps of the RVND

Algorithm 2: Random variable neighbourhood descent
$\boldsymbol{Input}$: x; S_{max}; L
$set\ L' = L$;
$s = rand(L')$ // select a random neighbourhood in list L';
$\boldsymbol{while}$ $(\lvert L' \rvert > 0)$ $\boldsymbol{do}$
$\boldsymbol{if}$ $(x\ is\ not\ a\ local\ optimum\ in\ neighborhood\ N_s)$ $\boldsymbol{then}$
$Let\ x'\ be\ the\ first\ solution\ in\ N_s(x)\ such\ that\ f(x') < f(x)$;
$x = x',\ L' = L,\ s = rand(L')$;
$\boldsymbol{else}$;
$L' = L'_{/\{s]}$, $s = rand\ (L')$
$\boldsymbol{end\ if}$;
$\boldsymbol{end\ while}$;
$\boldsymbol{return\ x}$;

5. Computational results

To validate the effectiveness and the efficiency of our proposed algorithm, we modify a large set of Erdogan benchmark instances by introducing the product demand of each customer, Erdogan and Miller-Hooks (2012).

First, we solve the small instances by implementing code with the standard solver CPLEX. Then we test the R-GVNS proposed algorithm on small instances. Finally, we use the algorithm to solve large instances. Computational tests have been carried out on Intel(R) Core (TM) i7-5500U, 2.40GHz, 8Gb of RAM.

5.1. Small size instances

Small-size instances introduced by Erdogan and Miller-Hooks (2012), are categorized into four groups: (S1); 10 instances randomly generated in which 20 customer locations by Uniform distribution with three fixed Recharging stations. (S2); 10 instances of 20 clustered customers with 3 recharging station locations. (S3); 10 instances, combined with five from the (S1) group and the rest from (S2), each one has six recharging stations randomly generated. S4; 10 instances, five from (S1) instances and five from (S2) instances, with several recharging stations gradually from 2 to 10 in increments of 2. We add to these instances the customer's demand generated using the uniform distribution between 1 and 20.

In small and large sets of experiments, unless otherwise stated, the battery capacity Q is constant at 60, with a mileage consumption rate of 0.2, a 60-gallon fuel tank with a fuel consumption rate of 0.2. It is assumed that the vehicle travels at an average speed of 40 miles per hour (mph), and the maximum allowable tour time is 11 hours. The presumed service durations are 30 minutes at customer locations and 15 minutes at recharge points. The cost/miles for distances driven with conventional fuel are 3 times greater than the cost/km for distance reached with electric engine ($\rho = 3$). In small-size instances, the max load capacity is set to 50, whereas in the large ones is fixed to 80.

Tables 6-9 illustrate details of the obtained results for small-size instances. The tables are organized as follows. The first column reports the name of the instance, while column 2 reports the best-known objective function obtained from the literature for the G-VRP problem proposed by Erdogan and Miller-Hooks (2012). Columns 3 and 4 report the number of customers and the number of used vehicles, respectively. Columns 5 and 7 give the objective function of CPLEX and R-GVNS, respectively. While Columns 6 and 8 show the trues covered distance results for CPLEX and R-GVNS, respectively. The computational time of the best-found solutions (expressed in seconds) for R-GVNS, is reported in column 9.

Note: The compilation by CPLEX is stopped after 3 hours. The optimality is not guaranteed and all the results are the best solutions that can be found by CPLEX in three hours.

Table 6. Results on set S1

Instance	***G-VRP***	***CH-VRP***						
				With CPLEX		***With R-GVNS***		
		n	***m***	***Fitness***	***Distance***	***Fitness***	***Distance***	***T (s)***
20c3sU1new	1818.35	20	6	**1688,63**	1545,77	**1688,63**	1545,77	1,35
20c3sU2new	1614.15	20	6	1628,86	1539,57	**1621,45**	1621,45	0,91
20c3sU3new	1969.64	20	5	**1560,11**	1395,32	**1560,11**	1395,32	0
20c3sU4new	1508.41	20	5	**1468,29**	1458,96	**1468,29**	1458,96	0,565
20c3sU5new	1752.73	20	6	**1644,94**	1640,26	**1644,94**	1640,26	0
20c3sU6new	1668.16	20	6	**1633,03**	1633,03	**1633,03**	1633,03	1,05
20c3sU7new	1730.45	20	6	**1667,77**	1629,85	**1667,77**	1629,85	0,98
20c3sU8new	1718.67	20	6	1725,69	1725,69	**1709,41**	1709,41	0
20c3sU9new	1714.43	20	5	**1595,56**	1498,51	**1595,56**	1498,51	0
20c3sU10new	1309.52	20	5	**1318,78**	1318,78	**1318,78**	1318,78	0,438

Table 7. Results on set S2

Instance	***G-VRP***	***CH-VRP***						
				With CPLEX		***With R-GVNS***		
		n	***m***	***Fitness***	***Distance***	***Fitness***	***Distance***	***T(s)***
20c3sC1new	1300.62	20	5	**1265,5**	1265,5	**1265,5**	1265,5	0,05
20c3sC2new	1553.53	19	5	**1367,95**	1273,14	**1367,95**	1273,14	0
20c3sC3new	1083.12	12	3	**878,51**	876,39	**878,51**	876,39	0
20c3sC4new	1135.9	18	5	**1109,73**	1109,73	**1109,73**	1109,73	1,44
20c3sC5new	2190.68	19	5	**1878,74**	1577,98	**1878,74**	1577,98	1,29
20c3sC6new	2883.71	17	6	**2356,03**	1840,34	**2356,03**	1840,34	0,126
20c3sC7new	1701.4	6	3	1407,37	1170,51	**1204,65**	1030,28	0,07
20c3sC8new	3319.74	18	7	2862,97	2267,41	**2573,90**	2430,61	0,19
20c3sC9new	1811.05	19	6	**1482**	1301,75	**1482,00**	1301,75	0,275
20c3sC10new	2648.84	15	4	**1711,66**	1336,2	**1711,66**	1336,2	0

Table 8. results on set S3

Instance	*G-VRP*	*CH-VRP*						
				With CPLEX		*With R-GVNS*		
		n	*m*	*Fitness*	*Distance*	*Fitness*	*Distance*	*T(s)*
S1_2i6s	1614.15	20	6	**1578,12**	1578,12	1579,19	1489,9	6,32
S1_4i6s	1561.3	20	5	**1405,06**	1405,06	**1405,06**	1405,06	0,267
S1_6i6s	1616.2	20	6	1600,78	1600,78	**1585,88**	1585,88	0,389
S1_8i6s	1902.51	20	6	1735,55	1730,95	**1719,31**	1719,31	0,122
S1_10i6s	1309.52	20	5	1354,55	1354,55	**1331,49**	1331,49	0,615
S2_2i6s	1645.8	20	6	1748,96	1623,06	**1696,62**	1696,62	0
S2_4i6s	1505.06	19	6	1619,76	1462,24	**1505,07**	1505,07	0,174
S2_6i6s	3115.1	20	7	2875,78	2341,52	**2380,34**	2339,75	0,684
S2_8i6s	2722.55	16	6	2085,66	1934,56	**1990,96**	1933,95	0,853
S2_10i6s	1995.62	16	5	1616,77	1616,77	**1585,46**	1585,46	0,294

Table 9. results on set S4

Instance	*G-VRP*	*CH-VRP*						
				With CPLEX		*With R-GVNS*		
		n	*m*	*Fitness*	*Distance*	*Fitness*	*Distance*	*T(s)*
S1_4i2s	1582.2	20	6	**1443,68**	1401,88	**1443,68**	1401,88	0,202
S1_4i4s	1580.52	20	5	**1443,68**	1401,88	**1443,68**	1401,88	0,765
S1_4i6s	1561.29	20	5	**1405,06**	1405,06	**1405,06**	1405,06	0
S1_4i8s	1561.29	20	6	**1401,47**	1401,47	1405,06	1405,06	0,292
S1_4i10s	1536.04	20	5	**1405,06**	1405,06	**1405,06**	1405	0,881
S2_4i2s	1135.89	18	5	**1109,81**	1103,68	**1109,81**	1103,68	0,116
S2_4i4s	1522.72	19	6	1611,36	1388,69	**1496,64**	1490,5	0,087
S2_4i6s	1786.21	20	6	1712,22	1486,66	**1484,7**	1478,56	0,287
S2_4i8s	1786.21	20	6	1707,6	1441,37	**1448,03**	1441,37	2,51
S2_4i10s	1783.63	20	5	1730,71	1505,46	**1444,74**	1438,08	0,526

As shown in the results reported in Table 6-9, RGVNS gives the best-known solutions in 38 out of 40 instances with computational times of about at most 3 seconds. Compared with solutions obtained by CPLEX, our algorithm finds the same results in 22 instances and gives better solutions for 16 ones. We note that the compilation with CPLEX stops after 3 hours, which means that optimality is not guaranteed, and all the results are the best solutions found.

Solving the small size instance with CPLEX solver takes almost 3 hours to find a solution while using R-GVNS takes only a few seconds. Therefore, the algorithm is not time-consuming.

Upon examining the results of each instance, noticing that some instances have the same fitness value and distance value, while others show a difference that can be either low or high. This difference reflects the additional cost associated with using traditional fuel (penalty). Furthermore, the large difference between the fitness value and the distance value indicates that routes are heavily penalized due to the use of traditional fuel.

In conclusion, R-GVNS outperformed CPLEX in terms of distance and fitness and the obtained results are slightly better (a percentage ranging from 0.5% up to 35% in some instances), indicating its effectiveness in finding optimal or near-optimal solutions quickly.

5.2. Large size instances

After proving the effectiveness of the R-GVNS we solve the modified large-size Erdogan benchmark instances. The large-size instances account for 12 instances with several customers ranging from 100 to 500. The capacity of each vehicle is assumed 80. Details of the results obtained are presented in Table 10. The results show that the R-GVNS is not long time-consuming (Maximum 5 minutes).

Table 10 illustrates details of the obtained results for large-size instances. It is arranged as follows; the first column reports the instance's names. Column 2 reports the number of trips. Column 3 illustrates the number of visited recharge stations. The number of trips made when the vehicle is running on conventional fuel (traditional fuel) is shown in Column 4. The best-penalized objective function discovered is shown in columns 5 and 6, along with the average outcomes of three runs, respectively. The R-GVNS covered distances of the best-found solution and the average outcomes from three runs are shown in columns 7 and 8, respectively. Whereas Columns 9 and 10 display, respectively, the average outcomes from three runs and the computing time of the best-found solutions (given in seconds).

Table 10. Results for large-size instances

				Fitness		**Distance**		**Time(s)**	
Instances	**# Trips**	**# Visited RS**	**# Trad Fuel**	**Best**	**Avg.**	**Best**	**Avg.**	**Best**	**Avg.**
111c21s	19	5	1	5012,06	5046,05	5008,55	5039,6	3,22	2,34
111c22s	18	5	1	4968,67	4983,66	4952,95	4972,95	4,31	3,73
111c24s	18	5	1	4944	4963,29	4940,95	4946,95	3,82	3,49
111c26s	18	4	3	4941,56	4967,41	4827,07	4933,21	3,87	3,93
111c28s	18	6	1	4881,98	4994,09	4874,57	4984,11	4,49	4,56
200c21s	32	11	3	8843,19	8901,55	8817,66	8647,97	17,34	17,08
250c21s	39	13	5	10559,16	10614,95	10288,79	10321,26	31,05	30,83
300c21s	48	14	5	12900,32	13008,88	12579,37	12656,83	39,41	48,1
350c21s	55	17	6	14832,11	14917,99	14411,16	14518,77	103,89	95,21
400c21s	64	19	7	17307,32	17373,5	16849,71	16944,94	108,2	156,67
450c21s	70	21	7	18929,12	19063,18	18461,36	18528,99	255,49	218,8
500c21s	79	26	10	21136,78	21256,47	20638,92	20763,89	196,2	293,49

According to the results of 111c instances, the overall travelled distance decreased by 130 miles (or about 3%), depending on the increase in the number of recharging stations from 21 to 28 (approximately a 33% increase). Fleet operational expenses can be decreased by increasing the recharging station's availability, although cost savings largely rely on where the additional stations are placed.

Based on the best-found results for small instances and the averages of the results for large instances, we can see that our algorithm achieves good results according to the different iterations and those in a very acceptable execution time according to the increase of instance sizes.

6. Conclusion

This paper presents a capacitated hybrid vehicle routing problem. It is an extension of the Hybrid-VRP, recently advanced in the literature. In this problem, vehicles have the option to use either electric propulsion or a traditional fuel engine, paying a higher unitary cost for distances travelled in the latter mode serving as a penalty to mitigate air pollution and increase operational expenses. To address this problem, we suggested the R-GVNS method. This method effectively explores the various search space regions by employing three neighbourhood structures in a random order utilizing a random variable neighbourhood search RVND. To address the capacity constraint, we extended the problem formulation from Mancini (2017) and solved the small cases using the CPLEX standard solver. The outcomes are contrasted with a set of Erdogan and Miller-Hooks (2012) adjusted examples that take the needs of the customer into account. The outcomes of numerical testing show that our method delivers the best-known solutions in 38 of the 40 scenarios with computation times of just 3 seconds on average.

Our approach finds the same results in 22 instances and provides superior solutions in 16 cases compared to the results provided by CPLEX. In particular, and in large-sized instances, our method gives a good solution quality in acceptable computational time. Such performance indicates that adapting R-GVNS for solving other variants of the routing problem constitutes a promising research avenue. While this work is theoretical, it has significant implications for decision-makers. The use of capacitated hybrid vehicles and the application of the R-GVNS metaheuristic can lead to substantial savings in travel distances, particularly over long distances. These reductions in distance not only enhance operational efficiency but also contribute positively to environmental sustainability by reducing CO2 emissions. Therefore, decision-makers should consider adopting these strategies to achieve both economic and ecological benefits.

Regarding recommendations for future investigations, we propose incorporating time window constraints into our model as a primary step. Furthermore, applying our approach to a real-world case problem would provide valuable insights. Another significant question to address is whether hybrid vehicles serve as a viable alternative to electric vehicles in developing countries, including Tunisia. This could potentially open up a promising research area for discussion.

References

Abdallah, T. (2013). The plug-in hybrid electric vehicle routing problem with time windows (Master's thesis, University of Waterloo).

Akubia, G. K., Gaffar, V., Sultan, M. A., & Andriana, D. (2025). The Impact of Green Logistics Management Practices on Manufacturing Firms' Sustainability Performance in Ghana and Indonesia. International Journal of Supply and Operations Management (IJSOM), 12(2), 215–235.

Alsumairat, N., & Alrefaei, M. (2021). Solving hybrid-vehicle routing problem using modified simulated annealing. *International Journal of Electrical and Computer Engineering (IJECE)*, 11(6/12), 4922-4931.

Alinaghian, M., Tirkolaee, E. B., Dezaki, Z. K., Hejazi, S. R., and Ding, W. (2021). An augmented Tabu search algorithm for the green inventory-routing problem with time windows. *Swarm and evolutionary computation*, 60(2), 2210-6502.

Alrashidi, M., and Al Ghamdi, M. (2024, February). Variable Neighbourhood Search Based on Reinforcement Learning for Green Vehicle Routing Problem. In 2024 International Conference on Artificial Intelligence in Information and Communication (ICAIIC), 530-537. IEEE.

Ammar, A. B., Loukil, T. M., and Cheikh, M. (2022, May). A survey on the hybrid vehicle routing problem: Literature review. In 2022 14th International Colloquium of Logistics and Supply Chain Management (LOGISTIQUA), 1-6. IEEE.

Arslan, O., Yıldız, B., and Karaşan, O. E. (2015). Minimum cost path problem for plug-in hybrid electric vehicles. *Transportation Research Part E: Logistics and Transportation Review*, 80(8), 123-141.

Babaee Tirkolaee, E., & Aydın, N. S. (2021). A sustainable medical waste collection and transportation model for pandemics. *Waste Management & Research*, 39(1/3), 34-44.

Bahrami, S., Nourinejad, M., Amirjamshidi, G., and Roorda, M. J. (2020). The Plugin Hybrid Electric Vehicle routing problem: A power-management strategy model. *Transportation Research Part C: Emerging Technologies*, 111(2), 318-333.

Basso, R., Kulcsár, B., and Sanchez-Diaz, I. (2021). Electric vehicle routing problem with machine learning for energy prediction. *Transportation Research Part B: Methodological*, 145(3), 24-55.

Bruglieri, M., Pezzella, F., Pisacane, O., & Suraci, S. (2015). A variable neighbourhood search branching for the electric vehicle routing problem with time windows. Electronic Notes in Discrete Mathematics, 47(2), 221-228.

Chi, J., and He, S. (2023). Pickup capacitated vehicle routing problem with three-dimensional loading constraints: Model and algorithms. *Transportation Research Part E: Logistics and Transportation Review*, 176(8), 103-208.

Cheikh, M., Ratli, M., Mkaouar, O., and Jarboui, B. (2015). A variable neighbourhood search algorithm for the vehicle routing problem with multiple trips. *Electronic Notes in Discrete Mathematics*, 47(2), 277-284.

Comert, S. E., and Yazgan, H. R. (2023). A new approach based on hybrid ant colony optimization-artificial bee colony algorithm for multi-objective electric vehicle routing problems. *Engineering Applications of Artificial Intelligence*, 123(8), 106375.

Ding, N., Batta, R., and Kwon, C. (2015). Conflict-free electric vehicle routing problem with capacitated charging stations and partial recharge. SUNY, University of Buffalo.

Doppstadt, C., Koberstein, A., and Vigo, D. (2016). The hybrid electric vehicle–traveling salesman problem. *European Journal of Operational Research*, 253(3/9), 825-842.

Doppstadt, C., Koberstein, A., and Vigo, D. (2020). The hybrid electric vehicle—Traveling salesman problem with time windows. *European Journal of Operational Research*, 284(2/7), 675-692.

Dukkanci, O., Kara, B. Y., and Bektaş, T. (2019). The green location-routing problem. *Computers & Operations Research*, 105(5), 187-202.

Erdoğan, S., and Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation research part E: logistics and transportation review*, 48(1/1), 100-114.

Fan, L., Liu, C., Dai, B., Li, J., Wu, Z., and Guo, Y. (2023). Electric vehicle routing problem considering energy differences of charging stations. *Journal of Cleaner Production*, 418(9), 138-184.

Garside, A. K., Ahmad, R., and Muhtazaruddin, M. N. B. (2024). A Recent Review of Solution Approaches for Green Vehicle Routing Problem and its variants. *Operations Research Perspectives*, 12(6),100303.

Goeke, D., and Schneider, M. (2015). Routing a mixed fleet of electric and conventional vehicles. *European Journal of Operational Research*, 245(1/8), 81-99.

Hansen, P., Mladenović, N., and Moreno Perez, J. A. (2010). Variable neighbourhood search: methods and applications. *Annals of Operations Research*, 175(11), 367-407.

Hiermann, G., Hartl, R. F., Puchinger, J., and Vidal, T. (2019). Routing a mix of conventional, plug-in hybrid, and electric vehicles. *European Journal of Operational Research*, 272(1/1), 235-248.

Jabir, E., Panicker, V. V., and Sridharan, R. (2017). Design and development of a hybrid ant colony-variable neighbourhood search algorithm for a multi-depot green vehicle routing problem. *Transportation Research Part D: Transport and Environment*, 57(12), 422-457.

Jamshidian, M., Alinaghian, M., and Tirkolaee, E. B. (2022). The time-dependent multi-depot fleet size and mix green vehicle routing problem: improved adaptive large neighbourhood search. *Optimization*, 71(11/9), 3165-3193.

Jarboui, B., Sifaleras, A., and Rebai, A. (2015). 3rd International Conference on Variable Neighbourhood Search (VNS'14).

Jie, W., Yang, J., Zhang, M., and Huang, Y. (2019). The two-echelon capacitated electric vehicle routing problem with battery swapping stations: Formulation and efficient methodology. *European Journal of Operational Research*, 272(3/2), 879-904.

Juan, A. A., Goentzel, J., and Bektaş, T. (2014). Routing fleets with multiple driving ranges: Is it possible to use greener fleet configurations? *Applied Soft Computing*, 21(8), 84-94.

Kalatzantonakis, P., Sifaleras, A., and Samaras, N. (2023). A reinforcement learning-variable neighbourhood search method for the capacitated vehicle routing problem. *Expert Systems with Applications*, 213(3), 118812.

Karak, A., and Abdelghany, K. (2019). The hybrid vehicle-drone routing problem for pick-up and delivery services. *Transportation Research Part C: Emerging Technologies*, 102(5), 427-449.

Kopfer, H., and Vornhusen, B. (2019). Energy vehicle routing problem for differently sized and powered vehicles. *Journal of Business Economics*, 89(7/6), 793-821.

Küçük, M., and Yildiz, S. T. (2022). Constraint programming-based solution approaches for three-dimensional loading capacitated vehicle routing problems. *Computers & Industrial Engineering*, 171(9), 108505.

Lera-Romero, G., Bront, J. J. M., and Soulignac, F. J. (2024). A branch-cut-and-price algorithm for the time-dependent electric vehicle routing problem with time windows. *European Journal of Operational Research*, 312(3/2), 978-995.

Li, X., Shi, X., Zhao, Y., Liang, H., and Dong, Y. (2020). SVND enhanced metaheuristic for plug-in hybrid electric vehicle routing problem. *Applied Sciences*, 10(2/1), 441.

Li, J., Wang, F., and He, Y. (2020). Electric vehicle routing problem with battery swapping considering energy consumption and carbon emissions. *Sustainability*, 12(24/11), 10537.

Li, J., Xu, M., and Sun, P. (2022). Two-echelon capacitated vehicle routing problem with grouping constraints and simultaneous pickup and delivery. *Transportation Research Part B: Methodological*, 162(8), 261-291.

Lu, J., Chen, Y., Hao, J. K., and He, R. (2020). The time-dependent electric vehicle routing problem: Model and solution. *Expert Systems with Applications*, 161(12), 113593.

Luo, H., Dridi, M., and Grunder, O. (2023). A branch-price-and-cut algorithm for a time-dependent green vehicle routing problem with the consideration of traffic congestion. *Computers & Industrial Engineering*, 177(3), 109093.

Mancini, S. (2017). The hybrid vehicle routing problem. *Transportation Research Part C: Emerging Technologies*, 78(5), 1-12.

Nejad, M. M., Mashayekhy, L., Grosu, D., and Chinnam, R. B. (2017). Optimal routing for plug-in hybrid electric vehicles. *Transportation Science*, 51(4/3), 1304-1325.

Sajid, M., Singh, J., and Rajak, R. (2023). Capacitated vehicle routing problem using algebraic particle swarm optimization with simulated annealing algorithm. *In Artificial Intelligence in Cyber-Physical Systems* (pp. 225-239). CRC Press.

Shan, Y., Li, Y., Liang, X., & Yue, L. (2025). Optimisation of multi-type vehicle routing considering soft capacity constraint and carbon emissions. International Journal of Systems Science: Operations & Logistics, 12(1).

Simić, D., Calvo-Rolle, J.L., Villar, J.R., Ilin, V., Simić, S.D., Simić, S. (2024, October). An Approach of Optimization in Capacitated Vehicle Routing Problem in Logistics Distribution. In The 19th International Conference on Soft Computing Models in Industrial and Environmental Applications. (SOCO 2024). Vol 888, *Lecture Notes in Networks and Systems*.

Souza, I. P., Boeres, M. C. S., & Moraes, R. E. N. (2023). A robust algorithm based on Differential Evolution with local search for the Capacitated Vehicle Routing Problem. *Swarm and Evolutionary Computation*, 77(3), 101245.

Souza, A. L. da S., Penna, P. H. V., & Souza, M. J. F. (2024). A variable neighbourhood search-based algorithm for the electric vehicle routing problem with time windows. *In Proceedings of the 2025 International Conference on Smart Transportation* (pp. 100–110). IEEE.

Vincent, F. Y., Redi, A. P., Hidayat, Y. A., & Wibowo, O. J. (2017). A simulated annealing heuristic for the hybrid vehicle routing problem. *Applied Soft Computing*, 53(4), 119-132.

Wu, Y., Huang, M., Yu, J., & Wang, Q. (2021). A VNS solution and objectives trade-offs for real-life large-scale vehicle routing problems. *International Journal of Industrial and Systems Engineering*, 39(4/1), 565-590.

Wu, Z., Lou, P., Hu, J., Zeng, Y., & Fan, C. (2025). An Adaptive Large Neighbourhood Search for a Green Vehicle Routing Problem with Depot Sharing. Mathematics, 13(2), 214.

Yin, F., & Zhao, Y. (2022). Distributionally robust equilibriums hybrid vehicle routing problem under twofold uncertainty. *Information Sciences,* 609(9), 1239-1255.

Zachariadis, E. E., and Kiranoudis, C. T. (2011). A local search metaheuristic algorithm for the vehicle routing problem with simultaneous pick-ups and deliveries. *Expert Systems with Applications*, 38(3/3), 2717-2726.

Zhang, S., Zhou, T., Fang, C., & Yang, S. (2024). A novel collaborative electric vehicle routing problem with multiple prioritized time windows and time-dependent hybrid recharging. *Expert Systems with Applications*, 244(6), 122990.

Zhen, L., Xu, Z., Ma, C., & Xiao, L. (2020). Hybrid electric vehicle routing problem with mode selection. *International Journal of Production Research*, 58(2/3), 562-576.

Zheng, Z., Liu, S., & Ong, Y.-S. (2024). Hybrid Memetic Search for Electric Vehicle Routing with Time Windows, Simultaneous Pickup-Delivery, and Partial Recharges. Computer Science: Neural and Evolutionary Computing, 1-13.

Zhu, T., Boyles, S. D., & Unnikrishnan, A. (2024). Plug-In Hybrid Electric Vehicle Traveling Salesman Problem with Drone. Transportation Research Record.