

Smart Economic Production Quantity Model with Circularity Index, Shortages, and Waste Management by 3D Printing

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Abstract

In modern business, industries like electronics, aircraft, automobiles, etc., keep products in circulation through processes like reuse, remanufacture, and recycling to produce the original products while keeping environmental sustainability at the center. Therefore, circularity index directly affects the demand and selling price of the products. Further, these industries are also applying 3D printing techniques to reduce the level of waste from the process as much as possible. 3D printing continues to evolve, it promises to reshape manufacturing, healthcare, and various other sectors, unlocking new possibilities for innovation and customization. So, to address all these issues, a smart production inventory model is proposed in the current study considering shortages, 3D printing technique, production rate depended wastage, green investment technology, and a circularity index. Demand rate of product is considered as the function of the circularity index. Objective of current study is to obtain the optimal values of production rate, production period, and cycle time so that overall inventory cost is minimum. In current study, calculus-based optimization technique has been used to obtain the optimal solution. Finally, numerical analysis is provided to validate the proposed inventory model. The results show that circularity and 3D printing technique help to reduce waste from the system. In addition to this, emitted carbon level from the system is dropped from the production system. Managerial insights based on key parameters is also provided. At the end, future extension of the current model along with concluding remarks is incorporated.

Keywords: 3D Printing; Carbon Emission; Circular Economy; Green Investment; Waste Management.

1. Introduction

Environmental issues (EI) are the top concern for sustaining human civilization in the twenty-first century. Due to the population growth, non-renewable resources are declining day by day. Further, increase in non-biodegradable garbage has a terrible impact on the ecosystem and on the world's resources. Carbon emissions significantly contribute to climate change, threatening the environment and human well-being. Managing and reducing carbon emissions has become a global imperative. Due to this different organizations and governments across the globe are striving to adopt sustainable practices. Fig. 1 reflects the level of carbon emissions worldwide per capita (tonne/year) due to fuel consumption. Figure reflects that developed countries emit more CO₂. Organizations can adopt various approaches, such as investing in energy-efficient technologies, optimizing supply chain logistics, and promoting sustainable

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practices among employees to curb the carbon level in environment. Renewable energy adoption, including solar and wind power, can significantly reduce carbon emissions.

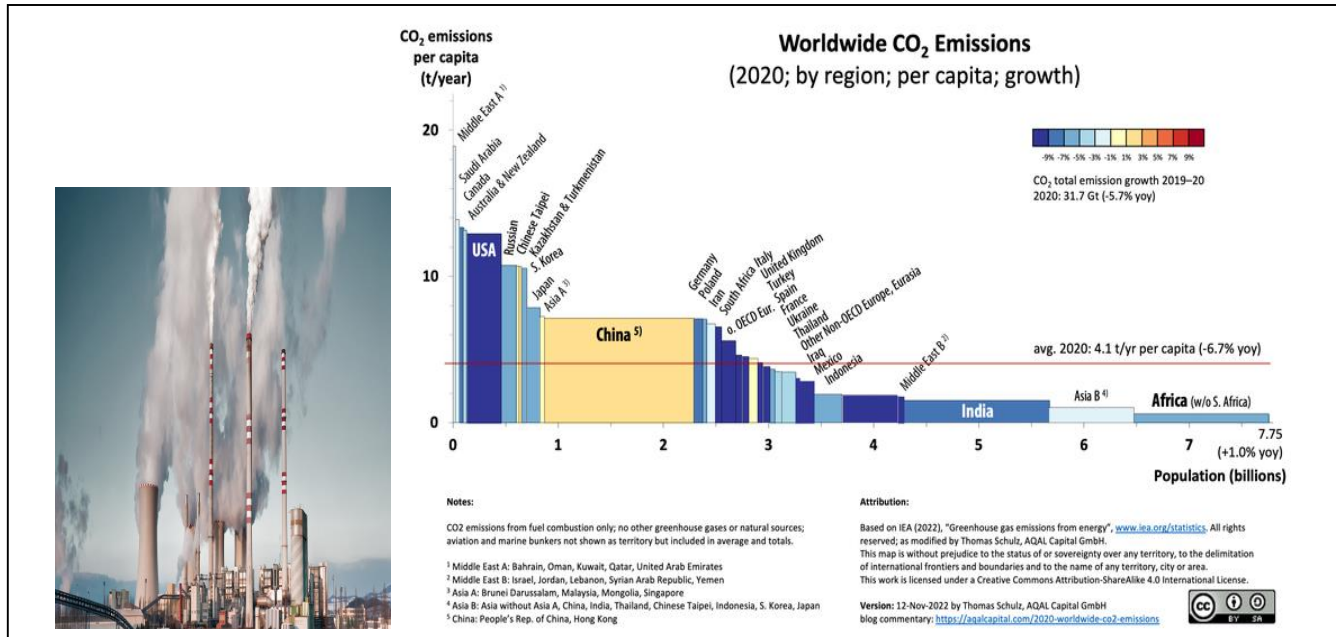


Figure 1. Worldwide CO₂ emissions by region, per capita growth of a particular year

Without prompt intervention in waste management, the worldwide yearly cost nearly quadruples to USD 640.3 billion by 2050. According to the report's analysis, implementing waste prevention and management strategies may reduce net yearly expenses to USD 270.2 billion by 2050. One of the main advantages of waste management is its capacity to reduce the environmental consequences of human actions. Effective waste disposal and recycling practices help minimize the volume of garbage deposited in landfills or incinerated, reducing greenhouse gas emissions and air pollution. 3D printing is the method of producing a three-dimensional item using additive manufacturing or computer-aided design. It can be done using a variety of procedures in which materials are deposited together, frequently layer by layer, and the deposition, joining, or solidification are subsequently controlled by computers. As per the study of Voulvoulis et al. (2020), using recycled plastic in 3D printing offers substantial environmental benefits, including a potential reduction in waste and energy consumption. Recycling and remanufacturing plastic can save between 30 % and 80 % of carbon emissions compared to processing and manufacturing virgin plastics. One of the main advantages of 3D printing is that 3D printing models have a lower mistake rate and error can be fixed before the printing process. Further, the circularity index is a factor which decides the percentage of the product that can be used or recycled again. John and Mishra (2023) suggested that circularity index is the most useful concept for the environment as well as the economic point of view.

Greening of industry is a strategy to achieve sustainable economic development and foster sustainable economies. The concept encompasses policy formation, enhanced industrial manufacturing methods, and resource-efficient productivity. The green industry initiative aims to raise awareness, disseminate information, and enhance capacities. Embracing sustainable practices is advantageous for the environment as well as for businesses. Businesses that adopt environmentally friendly practices may help lessen the effects of climate change, minimize their ecological footprint, and promote a more sustainable Earth for subsequent generations. Adopting eco-friendly supply chain strategies might result in financial benefits over time. Optimizing energy and resource use, minimizing waste, and increasing efficiency may reduce operating expenses and boost profitability. Green logistics reduce waste, expenses, and carbon emissions throughout delivery. Recognizing the benefits of green logistics may need an initial expenditure, but the long-term gains surpass the expenses. Nowadays, many NGOs and government organizations are motivating to make green products utilizing green technologies because of low greenhouse gas emission during manufacturing, and it makes use

of disposal, recyclability, biodegradability, etc. So, in this study, we have considered green investment to reduce environmental issues.

The aim of current research is to promote the achievement of sustainable development from the perspective of waste management using 3D printing technology, green investment, carbon emission tax, and circularity index. From the literature, it is observed that a small number of models were considered in earlier studies on the circular economy that explored the extent of circularity and how it affected demand. Mishra et al. (2022) considered 3D printing technology in CESC with carbon emissions. This study is an extension of Mishra et al. (2022) with the fact that a high production rate generates more waste, and hence the production rate of waste generation is dependent on the production rate. Objective of current work is to obtain the three different sustainability as follows:

Environmental Sustainability: Adopting the sustainable business practices like 3D printing, green technology, and circularity concept protect nature and helps to conserve natural resources wherever possible.

Social Sustainability: Reduction of waste with the help of 3D printing, and circularity index helps the people and society to live in good environment.

Economic Sustainability: In long run, customers are more inclined towards the industries whose business policy are more society centric. Sustainability also cuts business costs by reducing waste and carbon emissions.

1.1 Structure of Current Study

In Section 2, we have discussed review of literature, research gap and our contributions in the current study. Further, assumptions, notations, and problem definition are discussed in section 3. In the section 4, we have formulated the mathematical model of the proposed problem. Section 5 contained solution methodology to obtain the optimal solution. Section 6 covered numerical analysis, and sensitivity analysis with respect to different key parameters. Managerial insights, conclusion and future research are discussed in section 7 and 8 respectively.

2. Literature review

There are several research articles that have been focused on the circular economy, waste management, carbon emissions, and green investment. Now, in this section, literature review is carried out under the different key parameters.

2.1 Inventory models with circularity

With the goal of having a good effect on the environment while promoting economic progress, the circular economy method is gaining more and more attention as a substitute for the traditional linear economic model. In general, there are three different types of economic models: linear economies, recycling economies, and circular economies, as seen in figure 2. In these, the circular economy is more functional in the context of a friendly environment. Based on the data shown by a certain circularity indicator, the buyers decide what they will purchase. We must find a method for the economy and the environment to coexist in harmony. The circular economy aims to create a product or service such that there is no waste, with things that may be used in one form or another for a longer period, if not endlessly. Economic development and sustainability do not have to be mutually incompatible; it is only that the usual business model has divided them and frequently established them against each other. Lewandowski (2016) developed the circular economy business models in accordance with the conceptual structure. Rabta (2020) studied an indicator-based EOQ inventory model for the circular economy. In that model, Rabta made the supposition that the product can be manufactured with varied circularity, as determined by an index.

Circular economy based imperfect production model was presented by Su et al. (2021) considering traditional production system. In the model, they reduced the waste considering scrap for the production system. 3D printing technique was not applied by them. In this direction, Thomas and Mishra (2022) presented a sustainable circular model for the plastics reforming sector considering 3D printing technology to manage the waste. In that model they also considered traditional production model without shortages. Considering tradition production system, Peter et al. (2023) provided an inventory model that included sustainable technologies, carbon cap-and-trade regulations, and the notion of a product circularity index. Objective of developed model was to minimize carbon emissions and environmental destruction in LED enterprises.

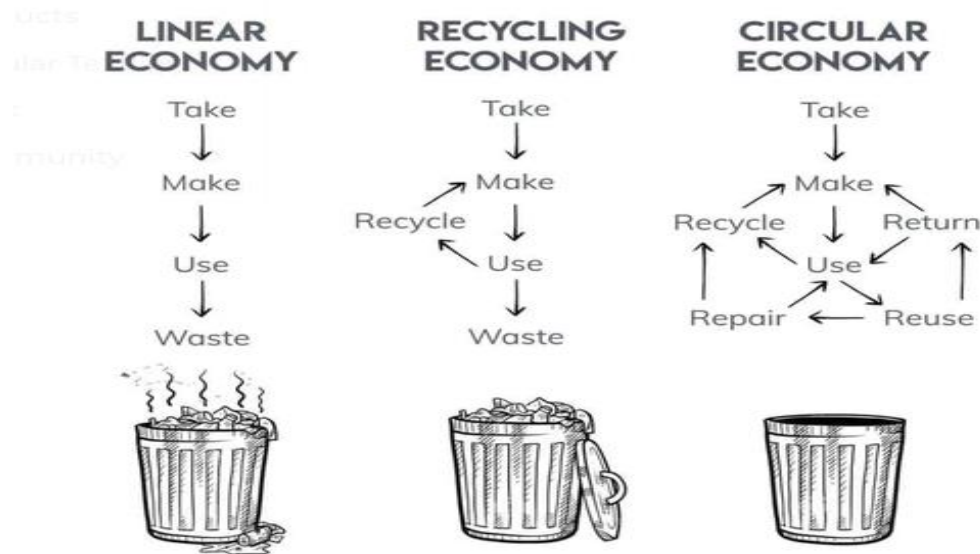


Figure 2. Types of economy with difference

2.2 Inventory models with carbon emission

The most crucial issue for sustaining human civilization in the twenty-first century is environmental issue. For instance, increased use of fossil fuels, plastics, and contaminated groundwater, air, and soil all contribute to the earth's groundwater issue and greenhouse gas emissions. The impact of carbon emissions is not limited to the environment alone, as it also has economic and social implications. It also has an impact on vulnerable communities, as climate change can exacerbate poverty, food insecurity, and displacement. Several publications are available that examine carbon emissions in the EPQ and EOQ models in the context of various environmental constraints. How the firms manage the carbon emissions through the carbon emissions trading was discussed by the Hua et al. (2011). They suggested that cap-and-trade mechanism forces the retailer to reduce carbon emissions and due to this total cost of the system increases. Benjaafar et al. (2012) highlighted that it is important to discuss the interactions between different firms in calculating overall emissions levels and corresponding costs. To minimize carbon emissions, Datta (2017) created an inventory model that included investments in environmentally conscious techniques and carbon tax regulations. Daryanto et al. (2019) highlighted the benefits of supply chain integration in term of carbon emissions reduction and total cost of system. Further, they suggested that incentive or compensation is required to 3PL to achieve a win-win solution. A CLSC model with a retailer, collector, and single manufacturer was explored by Jauhari et al. (2020). In that green technology, carbon cap-and-trade policy, imperfect manufacturing, and remanufacturing processes are also considered. Effect of the carbon emissions in the poultry-industry was discussed by Rana et al. (2021). They observed that carbon footprint increases due to holding, breeding, and deterioration. Jauhari et al. (2022) created a manufacturer-retailer inventory model considering remanufacturing, stochastic demand, and environmentally friendly investments. Dey et al. (2022) discussed how the green technology is good for environment. They discussed that with the effective use of green technology level carbon emissions can be reduced by 2.81%. To control the carbon emissions, carbon emissions cost is considered by them. Lok et al (2023) presented an inventory model for non-instantaneous deterioration with the objective to reduce the CO₂ emissions from the system so that total profit of the system is maximum. They observed that total profit is higher for non-instantaneous deteriorating items under the effect of preservation technology. To evaluate the impact of learning and forgetting, Singh et al. (2023) proposed an inventory model that included carbon emissions and agile manufacturing. Li et al. (2024) developed closed-loop supply chain with remanufacturing as well as low-carbon investment strategies in under multiple carbon policies.

2.3 Inventory models with waste management

Currently, every industry is working to reduce waste to combat global warming or safeguard the environment. There is currently no strategy in place to accomplish zero waste, but whatever the rules firms implemented, the amount of

rubbish created was reduced. That is why so many different sectors are working together to find a solution. Overall, waste management is a main aspect of inventory management that can help businesses reduce costs, improve efficiency, and minimize their impact on the environment. By adopting effective waste management strategies, businesses can ensure they are operating in a sustainable and responsible manner. This not only benefits the environment but also provides social and economic benefits by reducing costs and improving efficiency. Three shops system were considered by Alamri (2011) to reduce the waste from the system. They used first shop for remanufacturing of returned items; second shop for manufacturing new items; and third shop for collecting returned items. Jaber et al. (2014) extended the production model into two-level forward/reverse supply chain by considering remanufacturing and waste disposal system. They observed that collection rate and repairable rate have a significant impact on the batch sizes and hence on the total cost of the two-echelon system. Giri and Sharma (2015) examined a traditional production-inventory model in which the production process assumed to be defective and the defective items are repaired with a quality-affected return rate. Benkherouf et al. (2016) investigated a refurbishing inventory model to determine order and manufacturing quantities along with inventory levels of used goods. Christy et al. (2017) presented a three-echelon system considering remanufacturing and refurbishment. They considered price-and-quality dependent demand. They considered that remanufactured items are in like-new condition and sold on the primary market. Further, refurbished items are of poorer quality and are sold on the secondary market. To reconstitution of plastics into 3D-printer filament, Mohammed et al. (2017) developed a low-carbon footprint approach with waste reduction enhancement. With learning and reworking, a manufacturer-retailer integrated model was developed by Jauhari et al. (2018). Considering discounts on returned goods as well as social and environmental factors, Taleizadeh et al. (2019) examined costing and logistics in a multi-echelon CLSC. Shree et al. (2020) examined the impact of 3D printing as well as AM (additive manufacturing) on supply chain management. By applying 3D printing to minimize waste and reduce emissions in the plastic recycling industry, Thomas and Mishra (2022) established a circular economic supply chain model. Shahpasand et al. (2023) discussed a supply chain system using the example of tyre manufacture to examine the environmental and economic consequences of 3D printing.

2.4 Inventory models with green investment

The goal of green investment is to promote business activities that improve the environment. Green investments, which are frequently combined with socially responsible investing or environmental, social, and governance (ESG) criteria, prioritize organizations and initiatives that work to preserve natural resources, lessen environmental harm, and maximize productivity, or other environmentally conscious business practices, as well as enhanced sustainability. Another area where businesses can invest in green technology is in energy-efficient storage facilities. That can include the use of LED lighting, solar panels, and smart HVAC systems to reduce energy consumption and lower greenhouse gas emission. An integrated production-inventory model was developed by Chung et al. (2011) considering greening operations and remanufacturing process. They show that it is financially and socially viable to consider new technology evolution, higher rate of remanufacturing ability, and green handling ratio of take-back items. Tissayakorn et al. (2014) examine the use of green logistics management and the results for Thailand's logistics companies. Lou et al. (2015) investigated a two-stage supply chain inventory strategy with carbon cap-and-trade regulations and environmentally friendly technologies. Datta (2017) studied the impact of making investments in green technologies by considering production-inventory model. They observed that making financial investments in green technologies may lower emissions to some extent. Mishra et al. (2020) presented a sustainable EPQ model to reduce carbon emissions output, using investments in green technology and carbon cap and tax policies under different scenarios with and without shortages. Bhatnagar et al. (2022) presented a cleaner production model considering preservation technology. Numerically they observed that efficient preservation technology rises the profit by 1.6%. To reduce operational emissions, Jauhari and Wangsa (2022) design a closed-loop inventory model with remanufacturing and green technology. Singh et al. (2023) applied different mechanisms imposed by regulatory body to reduce carbon emissions. They observed that these mechanisms help to reduce the carbon from the system upto 20%. Kumar et al. (2023) presented an inventory model focussing the issues of society and environment. They observed that appropriate use of green technology reduces the footprint of carbon from the system by 4%. They also suggested that effective use of preservation technology also curb the waste from the system. To prevent backlogging, Bachar et al. (2023) developed an agile production system that reworks repairable, imperfect items with green investment. John et al. (2024) established a sustainable three-tier tuna fish supply chain model, including a farmer, processor, and retailer for two distinct quality grades of tuna fish. Green technology and integrated multi-trophic aquaculture techniques are used by farmers to decrease emissions and ocean pollution.

2.5 Research gap and contribution

Many authors given their contribution on circular economic model. Effect of the carbon emissions considered by Rana et al. (2021), with partially backloging in the poultry industry based on the acceptable delay in payment. They considered traditional production system only. Thomas and Mishra (2022) considered 3D printing, green investment, shortages, circularity index. They carried out this study considering traditional production system and without shortages. Many authors considered two or three factors from production rate dependent waste, circularity index, carbon emission, reduction of waste production due to 3D printing, volume flexibility, backloging and green investment. Here, we have considered all above factors together with labor or energy cost, tool cost, raw material cost dependent production cost. So, this model has more practical utility in comparison the quoted work in literature. To identify the novelty of current work, author's contribution table is presented as follows:

Table1. An overview of the relevant literature and the suggested model

Author(s)	Model Type	Circularity index	Carbon emission	Reduction of waste due to 3D printing	Volume Flexibility	Backlogging
Aseidu and Gu (1998)	EPQ	-	-	-	-	-
Richter and Dobos (1999)	EOQ	-	-	-	-	-
King et al. (2006)	EPQ	-	-	-	-	-
Chung et al. (2011)	SCM	-	-	-	-	-
Hua et al. (2011)	EOQ	-	√	-	-	-
Benjaafar et al. (2012)	SCM	-	√	-	-	Full
Jaber et al. (2014)	EPQ	-	-	-	-	-
Tissayakorn et al. (2014)	SCM	-	-	-	-	-
Giri and Sharma (2015)	SCM	-	-	-	-	-
Lou et al. (2015)	SCM	-	√	-	-	-
Benkherouf et al. (2016)	EPQ	-	-	-	-	-
Christy et al. (2017)	SCM	-	-	-	-	-
Datta (2017)	EPQ	-	√	-	-	-
Mazher et al. (2017)	EPQ	-	-	√	-	-
Jauhari et al. (2018)	SCM	-	-	-	-	-
Daryanto et al. (2019)	SCM	-	√	-	-	-
Taleizadeh et al. (2019)	SCM	-	-	-	-	-
Jauhari et al. (2020)	SCM	-	√	-	-	-
Mishra et al. (2020)	EPQ	-	√	-	-	Both
Rabta (2020)	EOQ	√	-	-	-	-
Shree et al. (2020)	SCM	-	-	√	-	-
Rana et al. (2021)	EPQ	-	√	-	-	Partial
Su et al. (2021)	EPQ	√	-	-	-	-
Jauhari et al. (2022)	SCM	-	√	-	-	-
Dey et al. (2022)	EPQ	-	√	-	-	-
Thomas and Mishra (2022)	SCM	√	√	√	-	Partial
Peter and Mishra (2023)	EPQ	√	√	-	-	-
Singh et al. (2023)	EPQ	-	√	-	-	-
This Paper	EPQ	√	√	√	√	Partial

As far as the authors knowledge, there is no researchers who considered the smart economic production quantity model with circularity index, carbon emission, waste reduction due to 3D printing, volume flexibility, backloging and generation of waste which depends on production rate.

3. Assumptions, Notations, and Problem Definition

3.1 Assumptions

- (i) 3D printing technology is considered to reduce the waste. Further, to minimize the waste, production rate is considered as a decision variable. (Shahpasand et al., 2023)
- (ii) Practically, it is observed that high production rate generates more waste. The raw material, labour experience, machine component, and manufacturing rate all affect the product's quality. In the current model, production rate is a variable. Due to this, it is assumed that generation of waste depends on production rate. Here, production rate of waste material is as $\theta = \left(\theta_0 - \frac{1}{P}\right)$, where θ_0 is constant. (Manna et al. 2017)
- (iii) Due to flexible production process, production cost depends on production rate. Expression for the production cost is as follows: $C_p = c_0 + c_1P + (c_2/P)$, where c_0 is raw material cost, c_1 is labor or energy cost, and c_2 tool cost. (Yadav et al., 2023)
- (iv) An acceptable level of shortage and backlogged which defined as a percentage of the total demand has been defined as $B(t) = \frac{1}{1+\varepsilon t}$, where $\varepsilon > 0$ is backlogging parameter and t is waiting time. (Rana et al., 2021)
- (v) The demand is the function of circularity index $D(\beta) = D_0 + f\beta$ where f is constant and represents additional demand factor, β is circularity index and $0 \leq \beta \leq 1$. (Thomas and Mishra, 2022)
- (vi) Carbon emissions from various activities connected to the manufacturing system have been considered to make the model environment sustainable. (Dey et al., 2022)
- (vii) $W(\mu) = 1 - e^{-\tau\mu}$, where μ is the cost to minimize waste using 3D printing and τ denotes the efficacy of 3D printing process to decrease waste. So, the percentage decline in waste generation rate is represented by the equation $1 - W(\mu) = e^{-\tau\mu}$. (Thomas and Mishra, 2022)
- (viii) The manufacturer creates a solution for a greener logistic system by spending money on modern technology (G), such as energy-efficient systems and renewable energy sources. A fraction of emission can be minimized by adopting green investment is $X = \xi(1 - e^{-\delta G})$, where ξ is the amount of the carbon emission after the implementation of green investment (GI) and δ is the efficiency parameter of green technology to minimize emission. (Datta, 2017)

3.2 Notations

μ	investment in 3D printing technique to minimize waste
τ	the waste reduction technology's efficiency parameter for reducing waste
K	setup cost for manufacturer per setup
P	production rate of manufacturing in unit per unit time (decision variable)
θ	waste material production rate in unit per unit time
T	the complete cycle length
t_1	time at which production stop
D_0	base demand rate in unit
d_0	rate of deterioration unit per unit of time
C_p	production cost (\$/unit)
C_d	deterioration rate (\$/unit/unit time)
C_b	backordering cost (\$/unit/unit time)
L_C	lost sale cost (\$/unit/unit time)
C_h	holding cost (\$/unit/unit time)
β	product circularity index and $0 \leq \beta \leq 1$
e_p	carbon emission by production process in kg/unit
e_{sp}	production setup process carbon emission in kg/unit

e_h	holding items carbon emission in kg/unit
e_{d_o}	deteriorating items carbon emission in kg/unit
e_w	carbon emission by waste in kg/unit
ψ	carbon tax per unit

3.3 Problem definition

In the current study, we have considered a smart economic production-inventory quantity model with circularity index, carbon emission, waste reduction due to 3D printing, volume flexibility, backlogging, generation of waste which depends on production rate and green investment. The cost of production is determined by the cost of raw materials, labor or energy, tool or die cost.

4. Mathematical Model

The model is described as: manufacturer start manufacturing process and continue till time t_1 . During the time interval $[t_1, t_2]$ inventory depleted due to deterioration and demand. Shortage occurs during the time interval $[t_2, T]$.

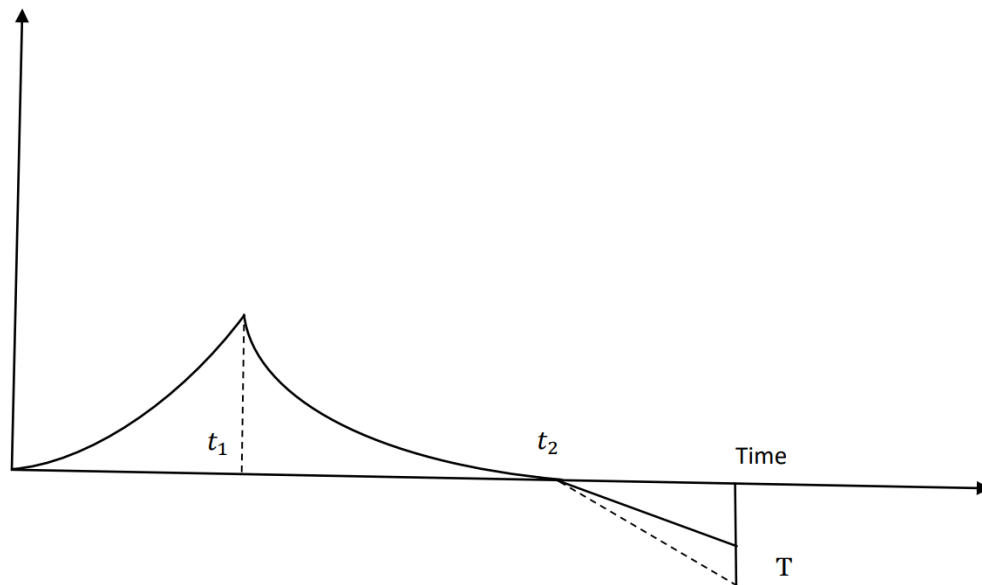


Figure 3. Manufacturer inventory level

$$\frac{dq_1(t)}{dt} + d_o q_1(t) = P(1 - \theta e^{-\tau u}) - D(\beta), \quad 0 \leq t \leq t_1, \quad q_1(0) = 0 \quad (1)$$

$$\frac{dq_2(t)}{dt} + d_o q_2(t) = -D(\beta), \quad t_1 \leq t \leq t_2, \quad q_2(t_2) = 0 \quad (2)$$

$$\frac{dq_3(t)}{dt} = -B(T - t) D(\beta), \quad t_2 \leq t \leq T, \quad q_3(t_2) = 0 \quad (3)$$

The solution of the above equations is given by

$$q_1(t) = \frac{P(1 - \theta e^{-\tau u}) - D(\beta)}{d_o} [1 - e^{-d_o t}] \quad (4)$$

$$q_2(t) = \frac{D_o + f\beta}{d_o} [e^{d_o(t_2 - t)} - 1] \quad (5)$$

$$q_3(t) = \frac{D_o + f\beta}{\varepsilon} (\log [1 + \varepsilon(T - t)] - \log [1 + \varepsilon(T - t_2)]) \quad (6)$$

Using the condition of continuity $q_1(t_1) = q_2(t_1)$, and Taylor's series expansion, we obtain the value of t_2 , in term of t_1 as follows:

$$t_2 = t_1 + \frac{P(1-\theta e^{-\tau\mu})-D(\beta)}{D(\beta)d_o} (1 - e^{-d_o t_1}) \quad (7)$$

Now, we evaluate different cost associated with manufacturing system.

Setup cost (SC) = K

Production Cost (PC) = $C_p \int_0^{t_1} P dt = C_p P t_1$

Cost due to waste is the sum of two factors (i) cost of the item produced due to defective production process (ii) cost due to deterioration. Therefore:

$$\begin{aligned} \text{Cost due to waste (WC)} &= C_p \int_0^{t_1} P \theta e^{-\tau\mu} dt + C_d \int_{t_1}^{t_2} q_2(t) d_o dt \\ &= C_p P \theta e^{-\tau\mu} t_1 + C_d (D_o + f \beta) \left[\frac{e^{d_o(t_2-t_1)} - 1}{d_o} - (t_2 - t_1) \right] \end{aligned}$$

$$\begin{aligned} \text{Holding cost (HC)} &= C_h \left[\int_0^{t_1} q_1(t) dt + \int_{t_1}^{t_2} q_2(t) dt \right] \\ &= C_h \left[\frac{P(1-\theta e^{-\tau\mu})-D(\beta)}{d_o} \{t_1 + \frac{1}{d_o} [e^{-d_o t_1} - 1]\} + \frac{D_o + f \beta}{d_o} \left[\frac{e^{d_o(t_2-t_1)} - 1}{d_o} - (t_2 - t_1) \right] \right] \end{aligned}$$

$$\begin{aligned} \text{Backordering Cost (BC)} &= -C_b \int_{t_2}^T q_3(t) dt \\ &= -C_b \frac{D_o + f \beta}{\varepsilon} \left[\frac{1}{\varepsilon} \{ (1 + \varepsilon(T - t_2)) (\log(1 + \varepsilon(T - t_2)) - 1) + 1 \} \right. \\ &\quad \left. - (T - t_2) \log(1 + \varepsilon(T - t_2)) \right] \\ &= -C_b \frac{D_o + f \beta}{\varepsilon} \left[\frac{1}{\varepsilon} \{ (\log(1 + \varepsilon(T - t_2)) - 1) \} - (T - t_2) \right] \end{aligned}$$

$$\begin{aligned} \text{Lost sale cost (LC)} &= L_c \int_{t_2}^T \left(1 - \frac{1}{1 + \varepsilon(T - t)} \right) D(\beta) dt \\ &= L_c D(\beta) \left[(T - t_2) - \frac{1}{\varepsilon} \{ \log(1 + \varepsilon(T - t_2)) \} \right] \end{aligned}$$

$$\text{Cost per cycle (CPC)} = \frac{1}{T} [K + PC + WC + HC + BC + LC] \quad (8)$$

Now, carbon emission costs due to setup, production, holding and waste can be calculated step by step as follows:

$$\text{Carbon emission cost per cycle due to setup (CE}_s) = \frac{\Psi}{T} e_{sp}$$

$$\text{Carbon emission cost per cycle by production (CE}_p) = \frac{\Psi}{T} \int_0^{t_1} e_p P dt = \frac{\Psi}{T} e_p P t_1$$

Carbon emission cost by holding inventory (CE_h)

$$= \frac{\Psi}{T} e_h \left[\frac{P(1-\theta e^{-\tau\mu})-D(\beta)}{d_o} \{t_1 + \frac{1}{d_o} [e^{-d_o t_1} - 1]\} + \frac{D_o + f \beta}{d_o} \left\{ \frac{e^{d_o(t_2-t_1)} - 1}{d_o} - (t_2 - t_1) \right\} \right]$$

$$\text{Carbon emission cost per cycle due waste (CE}_w) = \frac{\Psi}{T} [e_w C_p P \theta e^{-\tau\mu} t_1 + e_{d_o} (D_o + f \beta) \left[\frac{e^{d_o(t_2-t_1)} - 1}{d_o} - (t_2 - t_1) \right]]$$

$$\begin{aligned} \text{Total carbon emission cost (TCE)} &= \frac{\Psi}{T} \left\{ e_{sp} + e_p P t_1 + e_h \left[\frac{P(1-\theta e^{-\tau\mu})-D(\beta)}{d_o} \{t_1 + \frac{1}{d_o} [e^{-d_o t_1} - 1]\} + \frac{D_o + f \beta}{d_o} \right. \right. \\ &\quad \left. \left[\frac{e^{d_o(t_2-t_1)} - 1}{d_o} - (t_2 - t_1) \right] \right\} + \left[e_w C_p P \theta e^{-\tau\mu} t_1 + e_{d_o} (D_o + f \beta) \left[\frac{e^{d_o(t_2-t_1)} - 1}{d_o} - (t_2 - t_1) \right] \right\} \quad (9) \end{aligned}$$

In this study, we have used the green investment to reduce carbon emission. So, after green investment new reduced emission cost is

$$\text{TCE1} = \text{TCE} (1 - \xi(1 - e^{-\delta G})) \quad (10)$$

$$\text{Total Cost (TC)} = \text{CPC} + \text{TCE1} \quad (11)$$

We have to minimize the total cost TC given by equation (11). So objective functions given by

$$\text{Min TC} = \text{TC} (t_1, T, P) \quad (12)$$

$$\text{subject to } t_1 > 0, T > 0, P > 0$$

5. Solution Methodology

Equation (12) is non-linear function of t_1, T and P . So, closed form solution is not simple task.

Thus, the following solution procedure has been applied to obtain the optimum solution:

Step-(i): Put the values of all parameters in the equation (12).

Step-(ii): To find the stationary points, the derivatives of the objective functions w.r.t. t_1, T , and P are evaluated, and then each derivative is set to zero.

$$\frac{\partial(\text{TC})}{\partial t_1} = 0, \frac{\partial(\text{TC})}{\partial T} = 0, \frac{\partial(\text{TC})}{\partial P} = 0 \quad (13-15)$$

We obtain the values (t_1, T, P) by solving the system of equations mentioned above.

Step-(iii): Convexity of objective function is obtained with the help of Hessian matrix. Hessian matrix is defined as follows:

$$H = \begin{bmatrix} \frac{\partial^2 \text{TC}}{\partial (t_1)^2} & \frac{\partial^2 \text{TC}}{\partial t_1 \partial T} & \frac{\partial^2 \text{TC}}{\partial t_1 \partial P} \\ \frac{\partial^2 \text{TC}}{\partial T \partial t_1} & \frac{\partial^2 \text{TC}}{\partial (T)^2} & \frac{\partial^2 \text{TC}}{\partial T \partial P} \\ \frac{\partial^2 \text{TC}}{\partial P \partial t_1} & \frac{\partial^2 \text{TC}}{\partial P \partial T} & \frac{\partial^2 \text{TC}}{\partial (P)^2} \end{bmatrix}$$

Now, we compute the values of principal minors of Hessian matrix as follows:

$$M_{11} = \left(\frac{\partial^2 \text{TC}}{\partial (t_1)^2} \right)_{(t_1, T, P)}, \quad M_{22} = \begin{vmatrix} \frac{\partial^2 \text{TC}}{\partial (t_1)^2} & \frac{\partial^2 \text{TC}}{\partial t_1 \partial T} \\ \frac{\partial^2 \text{TC}}{\partial T \partial t_1} & \frac{\partial^2 \text{TC}}{\partial (T)^2} \end{vmatrix}_{(t_1, T, P)}, \quad M_{33} = \begin{vmatrix} \frac{\partial^2 \text{TC}}{\partial (t_1)^2} & \frac{\partial^2 \text{TC}}{\partial t_1 \partial T} & \frac{\partial^2 \text{TC}}{\partial t_1 \partial P} \\ \frac{\partial^2 \text{TC}}{\partial T \partial t_1} & \frac{\partial^2 \text{TC}}{\partial (T)^2} & \frac{\partial^2 \text{TC}}{\partial T \partial P} \\ \frac{\partial^2 \text{TC}}{\partial P \partial t_1} & \frac{\partial^2 \text{TC}}{\partial P \partial T} & \frac{\partial^2 \text{TC}}{\partial (P)^2} \end{vmatrix}_{(t_1, T, P)}$$

If M_{11}, M_{22} and M_{33} are all greater than zero i.e., Hessian matrix is positive definite, so the objective function is minimum at the point (t_1, T, P) .

Step-4: Thus, (t_1^*, T^*, P^*) is a point of extreme values and $\text{TC}(t_1^*, T^*, P^*)$ is the optimum value of the objective function.

6. Numerical analysis

To illustrate the proposed model, numerical example is provided here. For this, the data used here was taken from the relevant literature with some modification as per the requirement of current study.

Example: The industry spends 200 \$ for the setup (K), 2 \$ for holding (C_h), 5 \$ for deterioration (C_d), and emitted units of carbon by setup, production, holding, and wastage are 5 units, 1 unit, 0.9 unit, 1.2 units respectively and value of other parameters are $\delta = 0.5$, $\xi = 0.15$, $D_o = 200$, $f = 1000$, $\beta = 0.2$, $\tau = 0.34$, $\theta_0 = 0.5$, $K = 200$, $c_0 = 19$, $c_1 = 0.7$, $c_2 = 0.008793$, $C_d = 5$, $d_o = 0.1$, $C_b = 4$, $L_c = 4$, $\varepsilon = 0.04$, $\psi = 4$, $G = 20$, $\mu = 1$, $e_{d_o} = 1$;

Now, using the said methodology for optimal solution, equations (13–15) have the following solution:

$$t_1 = 94.7158, T = 243.954, P = 682.263$$

Now, the Hessian matrix's principal minors are assessed as follows:

$$M_{11} = .0114417 > 0, M_{22} = 27.1434 > 0, M_{33} = 21897.9 > 0,$$

The result above demonstrates the convexity of the goal function. Therefore, optimal solution is as follows:

$$t_1^* = 94.7158; T^* = 243.954; P^* = 682.263; TC^* = 270874.99$$

Remark: No existing research in the literature addresses environment and waste minimization issues with circularity index as discussed in the current study. So, it is not possible to compare quantitatively our results with the earlier work quoted in the literature. If we relax some assumptions in the present model, then this model and the previously mentioned models have the same base model.

(i) If we ignore circularity index, green investment, 3D printing, backordering and production rate dependent waste production then the base of the present model reduced to vendor's model proposed by Singh et al. (2023).

(ii) If we ignore green investment, 3D printing, backordering, and production rate dependent waste production in the current model then the base model of current study is same as the model proposed by Peter and Mishra (2023).

6.1 Effect of Green Investment on Optimal Solution

Fig.4 reflects the impact of green investment on the total optimal inventory cost. From Fig.4, it is observed that as the investment increases optimal inventory cost decreases significantly. This result indicates that as investment in green technology increases total carbon emissions from different operational activities associated with production system decreases and hence overall inventory cost of the system decreases.

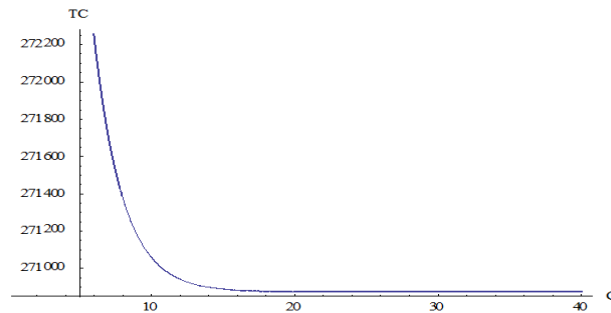


Figure 4. Effect on total cost with respect to green investment

6.2 Effect of Circularity Index on Optimal Inventory Cost

Fig.5 reflects the impact of the circularity index on the optimal inventory cost of the system. It observed that as the circularity index increases demand of product increases means customers believe that product have high reusability. This situation increases the faith of environment friendly customers in the product and hence the demand of the product increases. Overall impact of this situation is that the total inventory cost of the system declines significantly.

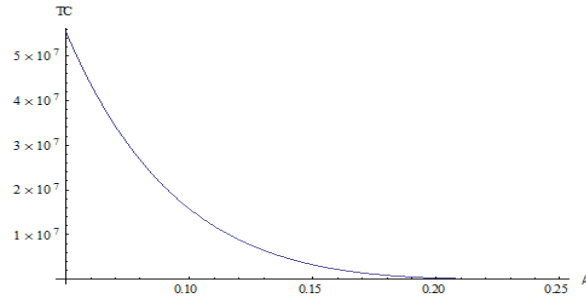


Figure 5. Total cost with respect to circularity index

6.3 Sensitivity Analysis w.r.t Different Key Parameters

Sensitivity analysis with respect to key parameters provides some important insights for decision-makers to understand the model's behavior. In this section, the sensitivity analysis of the model for key parameters and its consequences are covered. The impact of holding cost, back-order cost, setup cost, lost-sale cost, deterioration cost, carbon tax, production-related carbon emissions, deterioration-related carbon emissions, holding-related carbon emissions, waste-related carbon emissions, and setup-related carbon emissions have been observed in Table 2.

Table 2. Sensitive analysis with respect to key parameters

Parameters	% Change	TC	% Change in TC
Backordering cost (C_b)	+50	270993.79	+0.04386
	+25	270934.39	+0.02193
	-25	270815.59	-0.02193
	-50	270756.19	-0.04386
Lost sale cost (L_c)	+50	271053.19	+0.06579
	+25	270964.09	+0.03289
	-25	270785.89	-0.03289
	-50	270696.79	-0.06579
Holding cost (C_h)	+50	314119.67	+15.96481
	+25	292497.33	+7.98240
	-25	249252.65	-7.98240
	-50	227630.31	-15.96481
Deterioration cost (C_d)	+50	281501.52	+3.92292
	+25	276188.25	+1.96152
	-25	265561.72	-1.96152
	-50	260248.46	-3.92292
Carbon emission by holding (e_p)	+50	271325.31	+0.16624
	+25	271100.15	+0.08312
	-25	270649.83	-0.08312
	-50	270424.67	-0.16624
Carbon emission by setup (e_{sp})	+50	270875.03	+0.0000147
	+25	270875.01	+0.0000074
	-25	270874.97	-0.0000074
	-50	270874.95	-0.0000147
Carbon emission by holding (e_h)	+50	337039.88	+24.42635
	+25	303957.43	+12.21317
	-25	237792.54	-12.21317
	-50	204710.10	-24.42635
Carbon emission by wastage (e_w)	+50	274709.98	+1.41577
	+25	272792.49	+0.70789
	-25	268957.49	-0.70789
	-50	267039.99	-1.41577
	+50	278101.09	+2.66769

Carbon emission by deterioration (e_{d_o})	+25	274488.04	+1.48003
	-25	267261.94	-1.48003
	-50	263648.89	-2.66769
Carbon emission tax (ψ)	+50	348551.33	+28.67608
	+25	309713.16	+14.33804
	-25	232036.82	-14.33804
	-50	193198.65	-28.67608

- Carbon tax and carbon emissions because of holding the products in stock has great impact on the total inventory cost. If the carbon tax per unit emission per unit time increases, then the total inventory cost also increases and vice versa.
- From Table 2, it is observed that impact of holding cost is more on the total inventory cost in comparison to other costs.
- A slight change in total inventory cost is observed due to the change in the cost of deterioration. Total inventory cost also changed due to changes in the emitted unit of carbon because of deterioration.
- In comparison to other costs, the cost of backordering, lost sales, and carbon emissions by setup have a little impact on the total inventory cost.
- From Table 2, it is observed that as the carbon emission tax increases total inventory cost of the system also increases. Due to increase in tax, total tax paid by the production house increases and hence the total inventory cost of the system increases.

7. Managerial Insights

The present study's goal is to provide business leaders with some insightful and practical information. Among the most important management outcomes of the present study are the following:

- The carbon emissions tax rate, according to the study, significantly affects the total inventory cost of the system. As we have seen from Table 2, due to the change of 25% in carbon tax rate overall inventory cost changed by 14.33%. This result attracts the attention of decision-maker to work-out the mechanism so that emitted unit of carbon from the production system can be curbed.
- This research examined how the circularity index contributes to environmental benefits and product authenticity. It is evident from Fig. 5 that when the circularity index grows, the total cost decreases due to its positive impact on demand. This result gives direction to the decision-maker to increase the confidence of the customer about the product and its circularity index. This is helpful from an environmental point of view as well as an economic point of view.
- Carbon emissions from holding inventory have a big effect on total costs. From Table 2, it has been noticed that, due to a 50% change in carbon emission costs, the overall inventory cost changed by 24.42%. Thus, decision-makers must work in two different directions: (i) avoid long-term holding of products; and (ii) apply sustainable green warehouses for storage purposes.
- Study reflects that selection of green technology is very crucial for the decision-makers as it is helpful for the environment and for the economy.
- Study suggests that decision-makers must explore the 3D printing technique to reduce the waste from the production system as it is beneficiary for environment and for society.

8. Conclusion and future research

This paper considered the circularity index, carbon emission, green investment, and variable production rate with 3D printing technique, which is smart production technique and helpful to minimize waste. Comparative to other research, we designed a mathematical model for smart production system that examines the combined effects of the circularity index, carbon emission, green investment, variable production rate, production rate dependent waste production and 3D printing process. Developed model is illustrated with help of numerical example. Convexity of objective function is obtained with the help of Hessian matrix. In numerical illustrations, it is observed that total inventory cost can be reduced by effective management of carbon emissions and deteriorations. According to study, the environment and the economy are both negatively impacted by carbon emissions. The cost of carbon emissions has a large negative impact on inventory costs overall, with a variation in inventory costs approximately 28% because of this cost change.

Therefore, it is important to invest in green technology to curb the negative impact of carbon emissions on the environment and the economy. According to this research, the circularity index is advantageous for both the environment and the reliability of the product. From the analysis, it is observed that total inventory cost decreases due to rise in circularity index. High circularity index has positive impact of demand. Due to high demand, total inventory cost decreases. From analysis, it is observed that green investment and circularity index are big factor in modern industrial market system for environment and economic point of view. As a result, from an industrial perspective, this study is pertinent when considering the circularity index of products and green investment. The suggested model can be expanded to incorporate an inflationary environment (Yadav et al., 2023), and carbon cap-and-tax policy (Jauhari et al., 2020) to enhance practical utility of the current research. Another option for extending the model is to include a two-echelon supply chain model (Sarkar et al., 2023), specifically a vendor-buyer model with a trade credit policy (Kumar et al., 2023).

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