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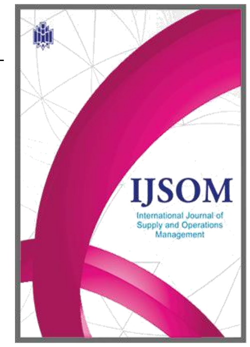

A Robust Optimization Model for Determining Optimal Diets for Food and Beverage (F&B) Items

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Abstract

This paper presents a simple and easy-to-use methodology for designing the menu in Food and Beverage (F&B) enterprises over a period of time, considering that certain elements of the problem are subject to uncertainty. The methodology considers both nutritional and financial aspects and allows the decision makers to explore the effect of the uncertainty on the final solutions, according to their perception of risk.
The proposed methodology is based on multi-objective mixed integer programming and in particular the Almost Robust Optimization (ARO) approach introduced by Baron et *al.* (2019). In contrast to conventional Robust Optimization techniques, the ARO approach is more flexible and offers the decision makers the possibility to express their attitude towards risk through appropriate parameters and obtain a series of solutions corresponding to different levels of risk.
The proposed model is applied in a case study concerning F&B enterprises from the island of Crete, Greece, using real data that was collected in collaboration with nutritionists and managers employed in F&B enterprises.
Decision-makers in F&B enterprises may use the proposed model as a decision support tool to incorporate the inherent uncertainty into the decision-making process. Through appropriate parameters, they may select optimal diets that are feasible for most realizations of the uncertain parameters, without incurring significant increases in cost. The model is flexible and produces a series of alternative solutions based on the decision makers' preferences and perception of risk.

Keywords: Robust optimization model; hospitality management; decision making under uncertainty; diet problem; food and beverage items.

1. Introduction

The idea of Robust Optimization (R.O.) is developed to deal with challenge the uncertainty of the data by defining some possible scenarios for non-deterministic parameters to obtain a robust solution that can guarantee all scenarios set at near optimal levels (Mirzapour Al-E-Hashem et al., 2011).
The objectives are to maximize the supply chain profit and customer satisfaction at the same time. Moreover, the carbon footprint is included in the first objective function in terms of cost (tax) to affect the total profit and treat the environmental aspect (Alinezhad et al. 2021).

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The objective of the diet problem is to select food items to be included in the diet of an individual or a group to minimize the total cost while meeting the prescribed nutritional requirements. Simultaneously, it aims to satisfy certain environmental constraints and sustainability conditions. Typically, dietary requirements are expressed as both a minimum and a maximum allowable level for each nutrient. Other restrictions, such as the minimum and/or maximum number of portions served per meal, may also be included to improve the quality of the menu.
This paper discusses an adaptation of the well-known diet problem, focusing on the perspective of food enterprises, particularly F&B items of tourism enterprises, such as hotels and restaurants.
The rest of the paper is organized as follows: In this section 2, we identify the research gap in the existing literature and present the unique contributions of our proposed model to address this gap. and in section 3 literature review the theoretical framework for the diet problem and explore its evolution over the years, considering various studies and approaches that have been undertaken. In section 4, we present and clarifies the details of the proposed methodology for addressing the food enterprise perspective of the diet problem. In section 5, we provide a comprehensive description of the model developed to tackle the diet problem from the viewpoint of food enterprises, with a focus on the F&B items of tourism enterprises. In section 6, we present the computational experiments conducted to test the proposed model and showcase the results obtained from its application. In section 7 engages in a comprehensive discussion of our research, analyzing the findings, and interpreting the results in the context of food enterprises and the tourism industry. In this final section 8, we draw conclusions based on the research findings, highlight any practical limitations, and indicate potential future research directions to further enhance the model and address any remaining gaps.

1.1 Problem Description

The diet problem has been expanded and enriched with new criteria to aid in making rational decisions in tourist enterprises like hotels or restaurants. These criteria include parameters such as the daily nutrition costs per person, the percentage of lunch or dinner expenses relative to the total daily diet cost per person, raw material yield during cooking, and nutrient content in F&B items of tourism enterprises. These additions enhance the problem of nutrition in enterprise function and make it more intriguing.
Moreover, the study of these new criteria under conditions of uncertainty and fuzziness allows for more effective measurement of parameters. It sheds light on the challenges faced by this developing industry and aims to design methods that can effectively tackle inaccuracies, which classical arithmetic methods often fail to address and provide reliable results.
While the daily minimum food cost remains an important goal, this paper introduces an additional objective, which is to determine the percentage of the cost of lunch and dinner relative to the total daily cost per person. To handle the excessive fuzziness of the data, a robust counterpart of the basic model has been developed.
By incorporating these advancements, this research contributes to the field of nutrition in tourist enterprises, facilitating better decision-making processes and addressing uncertainties to improve overall efficiency.
Creating tools to minimize costs in tourism enterprises is crucial for their financial viability and ensuring that tourism can bring tangible economic benefits to the host country. This paper presents such a tool, an optimization model aimed at helping food and tourism enterprises control the costs of the products and services they offer. The model proposed is a multi-objective Mixed Integer Programming (MIP) formulation specifically tailored for the F&B items of tourism enterprises, where dealing with strong fuzziness of data related to the nutritional contents of various food items is essential.
The primary objective of the optimization model is to design an indicative menu for the F&B items of a tourist enterprise over a two-week period while considering two economic goals: the daily cost per person and the percentage of lunch and dinner costs relative to the total daily cost per person. To handle the challenges posed by excessive data fuzziness, the paper introduces a robust counterpart of the basic model.
By providing a robust optimization approach, this research aims to empower food and tourism enterprises to make informed decisions that minimize costs while maintaining the quality of their offerings. This can contribute significantly to the financial success of tourism enterprises and ensure that tourism becomes a beneficial economic force for the host country.
The indicative menu is addressed to customers of tourism enterprises (hotels and restaurants). The proposed model provides an indicative proposal with ready meals per person. The user of the model is free to define equivalent ready meals to satisfy all the alternative preferences of potential customers. The food equivalents are a useful tool that enables exchanges and variety. Foods are divided into 6 groups containing the same proportions of basic nutrients

(carbohydrates, fat, protein). The system of equivalents divides foods into six groups, 1. Milk and Dairy Products, 2. Vegetables, 3. Bread – Cereals, 4. Starchy and Legumes, 5. Meat and Substitutes and 6. Fat (see https://health.gov/sites/default/files/2019-09/2015-2020_Dietary_Guidelines.pdf). General Rule: 1 equivalent of carbohydrate = 3 equivalents of vegetables = 3 equivalents of fat = 4 equivalents of meat = 5 equivalents of fish. Food equivalents enable the user to adapt the model to the specifics of the enterprise and reap the useful benefits of the model by defining which ready meals will achieve the minimum cost of feeding their customers. The flexibility to change parameters and constraints of the model is another important advantage for any user. We have followed scientific bases in our model in terms of parameters (nutrients from USDA, https://fdc.nal.usda.gov/ndb/Foods and others) and nutritional constraints from the Institute of Preventive, Environmental and Occupational Medicine (2014) (see http://www.prolepsis.gr/en/content/the-institute#scientific-publications).
The MIP model is presented from the perspective of an enterprise the purpose of which, besides satisfying its customers' needs (service, nutrition satisfaction and taste), is its financial sustainability (minimum cost per customer, etc.).
The method we chose to use is the basis model along the lines of the Almost Robust Optimization (ARO) approach introduced by Baron et *al.* (2019) to account for the ambiguity of the data while avoiding some of the problems associated with the purely robust optimization problem, such as infeasibility or a high degree of conservatism in the final solution. We adapted this method to the diet problem.
The ARO approach essentially allows the decision maker to explore the trade-offs between changes in the values of the objective function and different allowable levels of uncertainty.

2. Research Gap and Contributions

The present paper fills the research gap of an ARO methodology approach that is adapted for the first time to the dieting problem.
The main contributions of this paper can be outlined as follows:

1. We introduce a novel optimization model, formulated as a Mixed-Integer Programming (MIP) problem, to determine the optimal diet in F&B enterprises. The model is based on Goal Programming (GP) principles.

2. In our GP model, we consider two distinct goals: a) Daily nutrition cost per person: This represents the cost per person for the available food options provided by the enterprise. b) Participation rate of the two basic meals (lunch and dinner) in the overall daily diet cost per person: We establish the percentage of participation of the two main meals, lunch and dinner, in the total daily food cost per person. These meals are typically offered at hotels and often involve higher costs.

3. We integrate the ARO (Adaptive Robust Optimization) methodology, proposed by Baron et al. (2019), to address data uncertainties related to F&B items. This approach offers flexibility, simplicity, and effective communication with managers, empowering them to experiment with different strategies for handling uncertainty.

4. To demonstrate the applicability of our proposed model, we conduct a real case study using data from F&B enterprises in Greece.

By combining the GP model with the ARO methodology, we provide a robust and practical framework for optimizing food selection that takes into account dietary needs, cost considerations, and uncertainties in F&B data. Our study showcases the potential of this approach in real-world scenarios, paving the way for more efficient and informed decision-making in the food industry.

3. Literature Review

The diet problem was originally formulated by Stigler (1945), with the goal of finding the minimum cost of a diet that meets certain dietary requirements. The current forms of the linear programming model, as well as the basic technique for solving linear optimization problems, the Simplex method, are due to Dantzig (1947). In essence, Dantzig (1947) formally introduced the diet problem as an optimization problem and at the same time proposed an algorithm for solving it. Despite its relatively straightforward formulation, the diet problem has been extended in several ways, to reflect different aspects of reality. In fact, several years since the initial formulation, Dantzig himself stated that he was impressed by the fuzziness of the diet problem, as the various foodstuffs differ greatly from each other in terms of their nutrients (Dantzig, 1990; Buttriss *et al.,* 2014).

Over the years, the diet problem has been addressed by nutrition and health scientists, in various applications where the primary concern is the nutrition of people that they are monitoring. In most of these applications, the cost is treated as a secondary goal.

In the tourism industry, food enterprises are those that offer Food and Beverage (F&B) to tourists (Laloumis and Stefanakidis, 2014).

Tourism, as an important industry, has a major economic impact on many sectors, such as accommodation, food and transport. Tourism is an economic activity of global interest and interest, and as a social phenomenon has significant effects on the social, cultural and economic life of the various countries or regions. The multidimensional nature of tourism and the increase in tourist "flows" have resulted in its long-term evolution into a dynamic manufacturing sector in the modern economy (Zacharatos, 1999).

Generally, although optimization techniques make a considerable contribution and deliver practical plans in a reasonable time (Golpîra and Javanmardan, 2021), it is still challenging to cover uncertainties and risks caused by random data while optimizing multi-level programming in the economy.

- Robust optimization in the diet problem was first examined by Mulvey *et al.* (1995), who considered the results of nutritional ingredients uncertain. In recent decades, the theory of robust optimization has been introduced as a powerful tool for optimizing uncertain processes (Hoseinpour and Jahromi, 2019).
- In contrast to other robust optimization models that become less tractable when using binary decision variables, we demonstrate that by incorporating specific cuts, the Adapted Robust Optimization (ARO) model remains tractable. We apply this model to the diet problem, which has significant practical applications in the F&B items of tourist enterprises.
- Over the years, numerous researchers have worked on solving diet problems, either by replicating or expanding upon the foundational structure introduced by Stigler in 1945. Initially, the problem's basic structure was enhanced by updating food product prices based on their nutritional content or by introducing additional constraints.
- As time progressed, nutritionists became not only concerned with designing a healthy diet for their clients but also with defining the most cost-effective approach. Furthermore, the diet problem was studied and enriched by considering specific age or population groups. It was adapted for patients with various health conditions, such as diabetes, heart disease, or obesity, where finding an optimal diet plan is critical.

The optimization of diets remains a continually important problem, as food is an integral part of our daily lives, impacting our health and well-being. Additionally, food incurs costs, making it essential to find efficient and economical diet plans that cater to specific nutritional needs and constraints (Amin et al., 2019). Table 1 presents some applications of optimization in diet problem.

Most papers and studies have used nutritional and cost limitations in the analysis of dietary problems and solutions, but this paper expands both the small number of dietary and nutritional restrictions and the parameters that are important to consider in a food and beverage tourism enterprise. Only twelve papers and models consider environmental constraints.

Furthermore, most research on the problem of nutrition focuses on at least one of the following goals: the cost of nutrition, the similarity of nutrition, the environmental sustainability of nutrition, the prevention of health effects from diet, and the taste / satisfaction of diet. There are some gaps in the literature on eating problems. There are several papers in the literature that have addressed multiple goals in nutritional problems.

The purpose of this paper is to show how the diet problem, formulated as a MIP problem and extended to incorporate the fuzziness inherent in realistic applications, may become a useful tool for enterprises that offer ready meals and products (food enterprises). In order to be able to control the cost of the nutrition of their clients, as well as properly evaluate their cash break-even point at the same time.

The formulation of the model is a first attempt to apply the ARDO (Almost Robust Discrete Optimization) approach to the problem of multi-objective dieting in nutritional problems.

Table 1. Some Applications of Optimization in Diet Problem

Reference	**Description**
Stigler (1945)	First formulation of the diet problem, with the aim of finding the minimum cost of a diet that meets certain nutritional requirements.
Dantzig (1947)	Introduction of the diet problem as an optimization problem and at the same time proposal of an algorithm for its solution and also the current form of the linear programming model, as well as the basic technique for solving linear optimization problems and the Simplex method.
Fletcher *et al.* (1994)	Development of software, called "Microdiet System, 1990", which was used in some of the UK's leading hospitals and which was considered a computational method capable of creating individually acceptable diets, with a maximum capacity of one hundred foods and thirty dietary restrictions through linear programming.
Darmon *et al.* (2006)	A linear programming application using ‘’Microdiet System 1990’’ software (development by Fletcher *et al.*, 1994), showed that it was possible to meet the nutritional requirements for children aged 3-6 years in Malawi.
Lino's team (2007)	Developing a Microsoft Excel application (which, allowed her to better evaluate the USDA's official diet plans or create a new diet plan that meets her own selected nutritional goals.
Ferguson *et al.* (2009)	Using linear programming to develop complementary diets for specific populations with micronutrient deficiencies.
Maillot *et al.* (2010)	Application for the first time of linear programming to determine the optimal diet for large numbers of people, with the result that the calculation of the minimum cost of a nutritious diet should consider socio-cultural factors.
Macdiarmid *et al.* (2011)	Application of dietary trials supported by WWF in the UK to optimize the nutritional quality of dietary ingredients while reducing dietary greenhouse gas emissions.
Macdiarmid *et al.* (2012)	First application of the solver utility R software to solve the diet problem. The mathematical method optimizes an outcome which is a linear function of several controllable variables (e.g., the amount of food consumed) and subject to certain constraints (e.g., nutritional requirements).
Macdiarmid (2013)	Notice that healthier diets do not always have a lower environmental impact.
Corné van Dooren and Aiking (2016)	Confirmation that costs increase when only dietary restrictions are used, while costs decrease with the addition of environmental constraints.
Wilson *et al.* (2013) and Corné van Dooren and Aiking (2016)	Publication of studies using all three dimensions: nutrients, greenhouse gas emissions (environment) and costs. Suggestion that future studies of linear diets should combine all three factors.
Herforth *et al.* (2016)	Suggesting a model of an extended nutrition problem based on three domains: nutritional quality, economic well-being, and environmental sustainability.
Corné van Dooren (2018)	Literature review (from year 2000 onwards) of fifty-two scientific papers on the problem of nutrition, for the solution of which computers were used.
Hernández *et al.* (2021)	Description the mathematical optimization models proposed to find a set of diets for the Spanish population.

4. Methodology

Traditional optimization models often assume that the parameters and data are known and certain. In practice, however, such assumptions are usually unrealistic as the data may often be noisy or incomplete. An important issue when incorporating uncertainty into optimization models is the tractability of the resulting models. The problem of intractability is amplified when the decision variables are binary (such requirements on decision variables are ubiquitous in practice, especially in operational and planning models, e.g., logistics problems including facility location, vehicle routing, and manufacturing problems such as production planning and scheduling). A popular technique to capture data uncertainty in linear and mixed-integer linear programs is to use scenarios (Escudero *et al.*, 1993).

Traditional approaches for dealing with uncertainty include conventional Robust Optimization (RO) models, stochastic programming, and chance-constraint programming. However, all these approaches have certain drawbacks, as explained in Baron et *al.* (2019). Briefly speaking, in RO models all constraints are hard i.e., they must be satisfied under all scenarios, which is very restrictive. Also, stochastic programming penalizes any constraint violation in the objective function via a recourse function, while in our problem we seek to find solutions that are less sensitive to the uncertain data and at the same time allow the decision maker to adjust the amount of constraint violation. Finally, very often, chance-constraint models are intractable, which makes them less applicable in realistic situations.

In contrast to these approaches, the ARO (Almost Robust Optimization) approach allows the user to have a small deviation of certain constraints to ultimately lead to an optimal solution. The flexibility of the user to define this deviation or override of the constraints is the main advantage of the proposed method and it was considered the most suitable of the two mentioned above to be used and developed in this paper.

The ARO model trades off the solution value with robustness to find solutions that are almost robust (feasible under most realizations). Unlike the basic robust optimization which does not allow the optimal solution to violate any of the constraints under any realization, in ARO, infeasibility of the uncertain constraints under some of the scenarios may be allowed at a penalty.

The ARDO model (of the uses the same procedure as that used in ARO to calculate infringements. The only difference is that in ARDO all decision variables are binary (Baron *et al.*, 2019).

Robust discrete optimization is a highly active field of research where a plenitude of combinations between decision criteria, uncertainty sets and underlying nominal problems are considered. Usually, a robust problem becomes harder to solve than its nominal counterpart, even if it remains in the same complexity class (Goerigk and Khosravi, 2022).

There are cases where decision makers may have to deal with multiple goals, without a realistic solution to serve all goals. In this case, the concept of multi-objective programming can be used to resolve this issue. Each criterion is described as a "goal", in our model we use various criteria as "goals", such as daily cost per person, percentage of meal costs and dinner at total daily cost per person and nutrients. In the following sections, we present the deterministic version of the proposed multi-objective optimization model and then extend it along the lines of the ARO approach.

5. Mathematic Model

- In this section we introduce the proposed mathematical model whose objective is to determine the optimal indicative menu for a tourist enterprise over a period of four weeks. The model is essentially a goal programming formulation that comprises two objectives: i) minimization of the total cost of the menu, and ii) minimization of the cost of lunch and dinner, since these two meals account for a significant part of the total cost.
- We initially present the deterministic version of the model and then develop a robust counterpart long the lines of the ARDO approach proposed by Baron *et al.* (2019). The basis for the formulation of the model bridges the gaps in the literature review as for the first time the ARDO approach is applied to the multi-objective diet problem in nutritional problems.

5.1 Deterministic Model

Sets

f = Food Types, indexed by f.

g = Food Groups indexed by g.

m = Meals indexed by m.

n = Nutrients indexed by n.

d = Time Periods indexed by d.

Parameters

$\mathrm{NV}_{\mathrm{fn}}$: maximum allowed deviation per unit value for nutrient n of food f

$\mathrm{FT}_{\mathrm{fg}}$: 1 if Food Type f is included in Food Group g, 0 otherwise

$\mathrm{FM}_{\mathrm{fm}}$: 1 if Food Type f may be included in Meal m, 0 otherwise

$\mathrm{MAXNV}_{\mathrm{n}}$: maximum allowance of nutrient n

$\mathrm{MINNV}_{\mathrm{n}}$: minimum requirement of nutrient n

NP_{f} : net price of Food Type f (calculated and including the loss by cooking)

$\mathrm{MinPerCal}_{\mathrm{m}}$: minimum percentage of calories for Meal m

$\mathrm{MaxPerCal}_{\mathrm{m}}$: max percentage of calories for Meal m

RC_{n} : Robust Coefficient of nutrient n

$\mathrm{UNV}_{\mathrm{fn}}$: $+\ \mathrm{Q}_{\mathrm{nd}}$ Nutrient Value Per Unit Limit

DPPC: Daily Per Person Cost Goal
(expresses the desired cost per person, as determined by the F&B manager)

LDC: Lunch and Dinner Cost Goal
(expresses the desired cost per person of lunch and dinner, as determined by the F&B manager)

DP_{g} : Daily Number of Portions of Food Group g

WP_{g} : Weekly Number of Portions of food group g

Decision Variables

The essence of the proposed model is to select which food types will be served during each meal on each day of the planning horizon. Therefore, the main decision variables may be defined as follows:

$$x_{fdm} = \begin{cases} 1, \text{ if food type } f \text{ is served in meal } m \text{ on day } d \\ 0, \text{ otherwise} \end{cases}$$

In order to express the deviations from the desired value of each goal, the following deviation variables are necessary:

PTC_d: underachievement of the desired per person daily total cost

NTC_d: overachievement of the desired per person daily total cost

$PLDC_d$: underachievement of the desired per person daily cost of lunch and dinner

$NLDC_d$: overachievement of the desired per person daily cost of lunch and dinner

The underachievement reflects the amount by which the resulting cost of the diet (or of lunch and dinner) is lower than the desired value whereas the overachievement is the amount by which the cost exceeds the desired value. Since both objectives correspond to cost, the overachievement variables NTC_d and $NLDC_d$ are undesirable and need to be minimized.

Proposed Model (M1)

Constraints

Allowance Constraint

$$\sum_f \sum_m NV_{fn} \cdot X_{fdm} \cdot FM_{fm} \leq MaxNV_n \qquad \forall\ n,d \qquad (1)$$

Requirements Constraint

$$\sum_f \sum_m NV_{fn} \cdot X_{fdm} \cdot FM_{fm} \geq MinNV_n \qquad \forall\ n,d \qquad (2)$$

Constraints (1) and (2) relate to the maximum and minimum nutrients that must be provided to the average customer by the indicative menu per day.

Min Calories per Meal

$$\sum_f X_{fdm} \cdot FM_{fm} \cdot NV_{fn} \geq MinPerCal_m \sum_f \sum_{m'} X_{fdm'} \cdot NV_{fn} \cdot FM_{fm'} \qquad \forall\ d,m \ \ (n=\text{'calories'}) \qquad (3)$$

Max Calories per Meal

$$\sum_f X_{fdm} \cdot FM_{fm} \cdot NV_{fn} \leq MaxPerCal_m \sum_f \sum_{m'} X_{fdm'} \cdot NV_{fn} \cdot FM_{fm'} \qquad \forall\ d,m \ \ (n=\text{'calories'}) \qquad (4)$$

Constraints (3) and (4) relate to the maximum and minimum calories that must be provided by each individual meal per day.

Food Groups Constraint per Day

$$\sum_f \sum_m FT_{fg} \cdot FM_{fm} \cdot X_{fdm} \leq DP_g$$

∀ d and g∈{*Snacks, Fruits, Cereals and potatoes, Milk and Dairy, Vegetables/Salads*} (5)

Food Groups Constraint per Week

$$\sum_f \sum_{d \in D_w} \sum_m FT_{fg} \cdot FM_{fm} \cdot X_{fdm} \leq WP_g$$

for w=1,…4, $D_w = \{d: 7(w-1) \leq d \leq 7w\}$
and g∈{Red meat, White meat, Fish and Seafood, Legumes, Sweets / Desserts, Cooked foods, Eggs, …, Fast Food} (6)

Note that the set D_w denotes the days in week w of the 4-week planning horizon.

Constraints (5) and (6) stipulate that certain food groups cannot be offered more than a certain number of times per day or per week respectively. For instance, it is not beneficial to include more than a certain number of *Snacks* items in the daily dietary plan. Similarly, other food groups e.g., Red Meat, may only be offered a certain number of times per week. These constraints are included in the model following recommendations by nutritionists.

Achieve Daily per Person Cost in F&B

$$\sum_f \sum_m NP_f \cdot X_{fdm} + PTC_d - NTC_d \ = \ DPPC \qquad \forall\ d \qquad (7)$$

Achieve Daily per Person Cost for Lunch and Dinner in F&B

$$\sum_f \sum_{m=\,Lunch\ or\ m=Dinner} NP_f \cdot X_{fdm} + PLDC_d - NLDC_d \;=\; LDC \qquad \forall\, d \qquad (8)$$

In addition, constraints (7) and (8) define the deviations (underachievement and overachievement) from the specified target values of the two goals, namely the daily per person cost goal and the daily per person cost of lunch and dinner. These two equalities are formulated in the standard manner of constraints in Goal Programming (GP).

Non-negative Decision Variables

$$X_{fdm} \in \{0,1\}\ ,\ PTC_d \geq 0\ ,NTC_d \geq 0\ ,\ PLDC_d \geq 0\ ,\ NLDC_d \geq 0$$

Finally, conditions (12) express the nature of the decision variables.

Objective Function (deterministic model)

$$\text{Min z}= \sum_d \left(\frac{NTCd}{DPPC} + \frac{NLDCd}{LDC}\right) \qquad (9)$$

Finally, the objective function (9) expresses the sum of the undesirable percentage deviations from the desired values of the two goals, namely the amount by which the two cost functions exceed the target values of the two objectives.
In order to check the efficiency of the solution, after solving the above optimization model, we also perform the test introduced by Masud and Hwang (1981). More specifically, as a second stage of the process, we maximize the wanted deviation variables subject to conditions that the solution obtained by the original model is not degraded.
More simply, we solve the following optimization problem:

$$\text{Max z}= \sum_d \left(\frac{\mathrm{PTCd}}{\mathrm{DPPC}} + \frac{PLDCd}{\mathrm{LDC}}\right) \qquad (10)$$

Subject to

$$\mathrm{NTC_d} \leq \mathrm{NTC_d^*} \qquad \forall\, d \qquad (11)$$
$$\mathrm{NLDC_d} \leq \mathrm{NLDC_d^*} \qquad \forall\, d \qquad (12)$$

Constraints (1) – (8)

Where $\mathrm{NTC_d^*}$ and $\mathrm{NLDC_d^*}$ are the optimal solutions of variables $\mathrm{NTC_d}$ and $\mathrm{NLDC_d}$ respectively, given by model (M1).

5.2 Almost Robust Optimization Model

As mentioned, we adopt the model proposed by Baron et al. (2019), called ARO model, and apply it to the dieting problem to accommodate some of the uncertainty inherent in real life instances. In general, when some of the parameters of the problem are subject to uncertainty, resulting in a number of data realizations (scenarios), the essence of the ARO model is to find solutions that are nearly robust (feasible under most scenarios). Unlike the basic set-based formulation, which does not allow the optimal solution to violate any of the constraints under any realization, in ARO, the infeasibility of uncertain constraints under any of the scenarios (realizations) can be allowed with a penalty (deviation).
In our model, we assume the food content of different food items to be uncertain. In particular, we assume that the food content is an uncertain parameter within a range $[Lower_{fn}, Upper_{fn}]$ where $Lower_{fn}$ and $Upper_{fn}$ denote the lower and the upper bound respectively on the amount of nutrient n contained per unit quantity of food item f.
As a result, the constraints that are affected by the uncertain parameters are constraints (1) and (2), namely the maximum allowance and the minimum requirements constraints.

We further define P_{nds} and Q_{nds} as new sets of variables expressing the amount by which the resulting diet exceeds the maximum allowance or falls short with respect to the minimum requirements of nutrient n respectively per scenario s.

Hence, these deviations are expressed as follows:

$$P_{nds} = \max\{0,\ MaxNV_n - \sum_f \sum_m NV_{fn} \cdot X_{fdm} \cdot FM_{fm}\} \qquad \forall\ n,d,s \qquad (13)$$

$$Q_{nds} = \max\{0,\ MinNV_n - \sum_f \sum_m UNV_{fn} \cdot X_{fdm} \cdot FM_{fm}\} \qquad \forall\ n,d,s \qquad (14)$$

Following the approach suggested by Baron et *al.* (2019), we employ two alternative penalty functions to penalize these uncertain constraint violations over all the scenarios.

1. Max penalty function: this penalty is applicable when the decision maker does not have knowledge of the probabilities p_s of each scenario s:

$$P_{nd} = \max_s \{a_{nds}\ P_{nds}\} \qquad (15)$$

$$Q_{nd} = \max_s \{b_{nds}\ Q_{nds}\} \qquad (16)$$

Where a_{nds} and b_{nds} are the per unit constraint violation penalty of the maximum allowance and the minimum requirements constraints respectively under scenarios s.
This penalty function determines the maximum (worst-case) penalty over all scenarios and is most suitable for applications where high risks are involved but less suitable for low and medium risk applications as it is very conservative (Mulvey et *al*., 1995).

2. Expected penalty function: this penalty is suitable when the decision maker has full probabilistic knowledge of the scenarios:

$$P_{nd} = \sum_s p_s \cdot a_{nds} \cdot P_{nds} \qquad (17)$$

$$Q_{nds} = \sum_s p_s \cdot a_{nds} \cdot Q_{nds} \qquad (18)$$

Let ℓ denote the maximum allowable risk specified by the decision maker for violating constraints (1) and (2). Specifically, ℓ incorporates the decision maker's attitude towards risk, i.e., lower values of ℓ imply that the decision maker is less tolerant and more conservative.

Given a specific value of ℓ, the following robust optimization problem yields the diet that corresponds to the decision maker's perception of risk:

Objective Function (ARO)

$$\text{Min } z = \sum_d \left(\frac{NTCd}{DPPC} + \frac{NLDCd}{LDC}\right) \qquad (19)$$

The objective function (19) expresses the sum of the undesirable percentage deviations from the desired values of the two goals, namely the amount by which the two cost functions exceed the target values of the two objectives.
The allowance and requirements constraints (constraints (4) and (5) in the deterministic model), are modified as below with the defined penalties (deviations) P_{nds} and Q_{nds} .

Subject to

$$\sum_f \sum_m NV_{fn} \cdot X_{fdm} \cdot FM_{fm} \leq MaxNV_n - P_{nds} \qquad \forall\ n,d,s \qquad (20)$$

Constraints (15) stipulate that

$$\sum_{f}\sum_{m} UNV_{fn} \cdot X_{fdm} \cdot FM_{fm} + Q_{nds} \geq MinNV_n \quad \forall\ n,d,s \quad (21)$$

In addition, the following constraints are inserted:

$$\frac{Pnds}{\text{MaxNVn}} \leq \ell \quad \forall\ n,d,s \quad (22)$$

$$\frac{Qnds}{\text{MinNVn}} \leq \ell \quad \forall\ n,d,s \quad (23)$$

As can be seen, this model allows the decision maker to control the magnitude of the penalties (violations) by changing the value of ℓ. In the following section we present an application of this approach in a case study concerning F&B enterprises in the island of Crete, Greece.

6. Computational Experiments and Results

In this section we present an adaptation of the ARO model proposed by Baron et *al.* (2019), aimed at food and beverage enterprises, such as hotels and restaurants, which aim to optimize both the cost and economic viability of the meals offered to their clients over a specific time period.
We defined about 250 Food Types, and ranked them in ten (10) different food groups (see table 2). In addition, these 250 Food Types were allocated to five (5) different Meals (Breakfast, Brunch, Lunch, Supper and Dinner).

We use in the model real dietary recommendations for healthy adults, as presented in detail in the Greek nutrition guide (Institute of Preventive, Environmental and Occupational Medicine, 2014), with dietary restrictions per group of ready meals, per serving on a daily and weekly basis. These recommendations are summarized in Table 2.

Table 2. Aggregate Presentation of Nutritional Recommendations

▼ Food Groups	▼ Recommendation
Vegetables / Salads	4 portions / day (1 portion: 150-200 grams cooked or raw)
Fruits	3 portions / day (1 portion: 120-200 grams)
Cereals (and Potatoes)	5-8 portions / day (1 portion: 1 slice of bread, 4 cup cooked rice / pasta, etc.) of which, potatoes about 3 portions / week (1 portion: 1 potato cooked, 120 -150 grams)
Milk and Dairy	2 portions / day (1 portion: 1 glass of milk, 1 yoghurt, 30 grams hard cheese, etc.)
Red Meat	Up to 1 serving / week (1 portion: 120-150 grams cooked)
White Meat	1-2 portions / week (1 portion: 120-150 grams cooked)
Eggs	Up to 4 / week (1 portion: 1 egg)
Fish and Seafood	2-3 portions / week (1 portion: 150 grams cooked)
Legumes	At least 3 portions / week (1 portion: 150-200 grams cooked drained)
Sweets / Desserts	Up to 1 serving / week
Source: Institute of Preventive Medicine, Environmental and Occupational Health Prolepsis, Greece (2014)	

We applied our model to enterprises on the island of Crete, as we worked with Chefs who assisted us with the definition of Food Types as well as other details e.g., weight reduction of food items due to cooking. This assistance was crucial as it is necessary to have access to reliable data such as prices and nutritional ingredients per portion of food items for our proposed model to produce meaningful results.
The model was implemented using the AIMMS software, with CPLEX employed as the underlying solver (AIMMS, 2023). In addition to F&B managers and nutritionists, we also consulted financial advisers and enterprise chefs to provide data concerning prices as well as to determine appropriate target values for the objectives.
Below, we present some computational results applying our model. We conduct the following two experiments:

1. We use our software and we run it without penalty (with regard to nutrients) and we see the proposed results.

2. We use our software and run it with a penalty (with regard to nutrients) and we see the different proposed results that arise. We employ both penalty functions mentioned in section 5, namely the maximum penalty function and the expected penalty function. We assume that $a_{nds} = 1$ for all n, d, s. Also, in the case of the expected penalty function we assume that all scenarios have equal probability, i.e., $p_s = {}^{1}\!/_{|S|}$, where $|S|$ is the total number of generated scenarios.

In this section, we show how the violation limit can be used by decision makers to choose an appropriate penalty limit that best fits their risk preferences and thus compute more optimal solutions than this tolerance of constraints in terms of minimum nutritional requirements. We also highlight the benefits of being almost robust as opposed to fully robust.
In table 3 we present indicative results of the model, setting the daily cost per person in F&B as 4.5 monetary units (goal 1) and ultimate goal 2, the percentage of the two main meals (lunch and dinner) to be 40% of the total daily cost (i.e., 1.8 monetary units). The resulting solutions are feasible and integer.
We observe the resulting optimal solution and we have an average Daily per Person Food Cost in F&B of about 3.7 monetary units (indeed below the goal of 4.5 monetary units) and a share of lunch and dinner in the daily food cost per person of about 40%.

Regarding the ARDO model, we set the lower and the upper bound on the amount of nutrient n contained per unit quantity of food item f at 90% and 110% respectively of the corresponding deterministic value and assume that the value is distributed uniformly in this interval. Based on these assumptions, we develop a total of randomly generated 30 possible scenarios.

Table 3. Deterministic Model

Daily Per Person Food Cost In F&B (28 days)	**Cost Lunch and Dinner (28 days)**
119	49

In Table 4 we report the mean and the maximum of the two objective functions with respect to the penalty limit ℓ over the 30 randomly generated instances (scenarios).
As expected, we observe that as the value of ℓ increases, the two objective function values decrease, giving each time better optimal solutions compared to the deterministic model.
Table 4, summarizes the results of all 30 scenarios for each value of ℓ. We show the results for both objectives, namely the total per person cost and the cost of lunch and dinner over the whole-time horizon (28 days). For both objectives we show the average value and maximum value of each objective over all the 30 scenarios. The first observation is that all instances yield feasible solutions, which is not always the case for Robust Optimization problems, as stated in Baron *et al.* (2019). Secondly, as the penalty limit increases, the incremental improvement in the cost values increases at a slower rate compared to the incremental increases in the penalty. More simply, it may not be necessary for the decision makers to accept a very high penalty limit since the relative benefit (with respect to the improvement in the objective function values) may not grow as fast as the penalty.
Moreover, by allowing a very small amount of infeasibility, the ARDO model yields substantially improved solutions. By merely increasing the penalty limit from $\ell = 0$ to $\ell = 0.05$, the value of the total cost is reduced by more than 30% which is a substantial reduction for a large-scale enterprise such as the F&B item of a hotel.

Hence, the decision makers may use the model as a tool for exploring alternative solutions and for selecting optimal diets that are feasible for almost all realizations without incurring significant increases in cost.

Table 4. ARDO model

	Total Per Person Food Cost In F&B (28 days)		**Total Cost Lunch and Dinner (28 days)**	
Penalty limit (ℓ)	*Mean*	*Maximum*	*Mean*	*Maximum*
0.00	135.33	146.18	54.36	58.78
0.05	90.19	92.22	35.80	37.50
0.10	76.66	76.77	31.77	29.78
0.15	67.76	77.43	31.15	33.17
0.20	69.81	72.70	28.91	30.93
0.25	67.93	70.72	25.94	27.78

7. Discussion

This paper extends the diet problem by incorporating the logic proposed by Baron et al. (2019) and the ARDO approach. It marks the first implementation of this logic for the food service enterprises' dieting problem.

Several previous studies have focused on addressing people's nutritional needs with minimum cost, primarily using linear programming. For instance, Lino's team (2007) developed software used in leading UK hospitals capable of generating individually acceptable diets. Darmon et al. (2006) created a linear programming software to meet the nutritional requirements of children aged 3-6 years in Malawi. Maillot et al. (2010) applied linear programming to determine optimal diets for a large number of individuals, considering socio-cultural factors. Recently, Hernández et al. (2021) described mathematical optimization models to find diets for the Spanish population.

However, this research introduces a novel optimization model, the Mixed Integer Programming (MIP) nutrition problem, based on Goal Programming (GP). The uniqueness of this model lies in its direct focus on food service enterprises' operations, aiming to minimize the cost of food per person to ensure financial sustainability.

While the model also considers people's nutritional needs, it does so indirectly. The primary emphasis is on supporting decision-makers in exploring alternatives and selecting optimal diets that are both feasible and do not lead to significant cost increases for the food service enterprises.

By addressing the financial aspect of food service enterprises' operations while still considering nutritional requirements, this research provides a valuable tool for decision-makers to make informed choices and achieve their financial objectives.

8. Conclusions

In this paper, we present a mixed-integer programming model for determining the optimal diet in Food and Beverage Items. Acknowledging the uncertainty prevalent in real-life problems, we adopt the Almost Robust Discrete Optimization (ARDO) approach, which allows decision-makers to consider solutions feasible under various uncertain parameters.

The proposed model considers two criteria and employs a two-stage Goal Programming (GP) approach, ensuring that the resulting solutions are Pareto optimal, meaning improvements in one criterion cannot be achieved without deteriorating another.

This model can be applied by any firm in the food industry that offers prepared food, such as hotels, restaurants, and catering services. It serves as a valuable costing tool and can be adapted to suit the specific goals and expectations of each enterprise. The model's core idea is to estimate costing per person since revenue in any food enterprise is generated on a per-person basis.

As a potential future research direction, integrating a specific off-the-shelf food database that provides up-to-date prices and nutritional information for various food types per serving could enhance the model's practicality and accuracy.
Reviewing recent literature reveals a growing trend of expanding the parameters considered in food-related problems to include not only socio-cultural factors but also environmental sustainability aspects. Therefore, another potential future research direction is to extend our model to incorporate these environmental factors, making it more holistic and aligned with sustainable practices.
Additionally, exploring the use of interval analysis to model selected parameters and variables can lead to an optimization problem where revenue and cost estimates are represented by intervals, allowing for a more robust and flexible decision-making process.

Practical Limitations

A practical limitation that was inferred in our work is finding reliable databases. The most reliable sources of dietary ingredients from the United States Department of Agriculture (U.S.D.A., https://fdc.nal.usda.gov/index.html) are defined as servings per 100 grams, not per person. Therefore, this creates difficulty in calculating the optimal solution of the proposed model on a per person basis (minimum daily food cost). To overcome this problem, after processing, we defined nutrients (energy, protein, sugars, sugars, lipids, carbohydrates, fats and saturated fats) per serving and not per 100 grams as provided by our nutritionist-partners.

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