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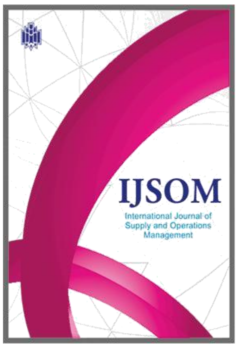

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Optimization under uncertainty: generality and application to multimodal transport

Naoufal Rouky [a*], Jaouad Boukachour [b], Dalila Boudebous [b], Ahmed El Hilali Alaoui [c]

[a] *Euromed Polytechnic School, Euromed Research Center, Euromed University of Fes, Fes 30030, Morocco*
[b] *Normandie Univ, UNIHAVRE, 76600 Le Havre, France*
[c] *Euromed Research Center, Euromed University of Fes, Fes 30030, Morocco*

Abstract

In a realistic environment, operational decision problems involve several sources of uncertainty, due to measurement errors, approximate parameters, or simply the unavailability of information at the time of decision-making. These disturbances are important in the optimization process and must be taken into consideration. To meet these needs, optimization under uncertainty has emerged as an important area of modern operations research and has gained increasing popularity in recent years by tackling complex optimization problems, such as multimodal chain management and container terminals management. In this regard, the present article provides a general overview of the different optimization paradigms and approaches used in the literature to support decision-making in the face of uncertainty. In particular, this article aims to present a state of the art on application of optimization under uncertainty in multimodal transport problems, with a particular focus on the application of Robust Optimization.

Keywords: Optimization, Uncertainty, Robust Optimization, Multimodal transport.

1. Introduction

In a real-world context, many optimization problems are subject to data uncertainties. One can think of all production, planning, transportation, and financial problems where decision-makers are faced with imperfect knowledge about future customer demands, resource availability, oil prices, or interest rates. Incorporating this lack of information into the initial modeling of the problem increases its complexity but neglecting it can make the model lose credibility. Indeed, an optimal solution obtained by a deterministic approach that does not consider the uncertainties may differ from the real optimum.

*Corresponding author email address: n.rouky@ueuromed.org
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In the literature, approaches dealing with optimization under uncertainty have followed a variety of modeling philosophies over the years, which can be classified into the following three categories (Roy, 2010):

- **A posteriori approaches:** in these approaches, solutions for the initial (deterministic) optimization problem are first computed, using classical deterministic optimization methods. Once the solutions have been calculated, the influence of input parameter perturbations on the quality of these solutions is studied. An example of the most well-known a posteriori approach is *Sensitivity Analysis*.
- **A priori approaches:** In these approaches, uncertainties about input data are incorporated into the initial formulation of the problem. These approaches aim to find the best solution that protects against disturbance. The uncertainties are considered during the optimization process and the solution found by these approaches is an optimal (or approximate) solution to the optimization problem under uncertainties**.**
- **Online approaches:** The optimization problem is solved when the data is known in this category of approaches.

In practice, online approaches are rarely used since decision-making must be established in advance to allow solutions to be implemented before when the data will be known. Thus, it is preferable to use an a priori or a posteriori approach to manage disturbances on the input data of the systems under study, to obtain prior solutions that will be effective in dealing with possible future disturbances. In this context, several types of these approaches have been developed in the literature to guide the decision-maker in making decisions under uncertainty such as stochastic optimization, sensitivity analysis, fuzzy optimization, belief functions, and robust optimization.

1. Stochastic Optimization

Stochastic optimization techniques are probabilistic approaches where uncertainties on the input data are modeled by probability distributions to describe the probability of occurrence of an event. Stochastic approaches assume that the probability distributions governing the data are known or can be estimated, and the objective is to find feasible solutions for all (or almost all) possible realizations of uncertain parameters while optimizing the expected value of the objective function (Shapiro et al., 2009). In the literature, three types of stochastic optimization paradigms can be distinguished:

- **Stochastic programming without recourse (SP):** generally, in stochastic programming, the expected value of the objective function is calculated by integrating over the set of uncertain parameters, which can be a difficult task. In the case of discrete or finite sets of uncertainties, realizations of uncertainty can be modeled using a finite set of scenarios, which simplifies the calculation of the expected value. As a result, in uncertainty-based optimization theory, stochastic programming is often considered a scenario-based approach and problem formulations are performed at several decision levels depending on the order in which the uncertainties arise. Each level involves a discrete temporal representation of the problem and establishes the information on the uncertain parameters available at the time of its occurrence. The simplest formulation considers only those decisions that are made before the uncertainty is revealed. This type of formulation is called stochastic programming with a single decision level or stochastic programming without recourse (Grossmann et al., 2016).
- **Two-stage stochastic programming (TSP):** the problem variables are divided into two decision levels in this type of formulation. The variables at the first level are those that need to be decided before the uncertain parameters are realized. Then, once the uncertainties are revealed, further improvements are made to the

resolution of the problem by selecting, at some cost, the values of the variables at the second level of decision or appeal. Second level variables are interpreted as corrective actions or remedies against any infeasibility arising from a particular realization of the uncertainties. Due to uncertainties, the costs of the second level are also considered random variables. The objective is to select the variables in the first level to minimize the sum of the costs in the first level and the expected random costs in the second level. This type of formulation is also called stochastic programming with recourse (Sahinidis, 2004).

- **Chance constrained programming (CCP):** another aspect of stochastic programming is that of programming with probabilistic constraints introduced by Charnes and Cooper (1959). Unlike the aspect of recourse models, this type of formulation focuses on the probability of constraint violation by ensuring that the probability of encountering a certain constraint is greater than a certain level. In other words, probabilistic constraint programming restricts the range of possible solutions so that the level of confidence provided by the final solution (i.e., the degree of protection against uncertainties) is high.

A more detailed description of the different stochastic optimization paradigms is available in the tutorial proposed by (Shapiro and Philpott, 2007) and in the work of (Birge and Louveaux, 1997) and (Sahinidis, 2004).

Table 1 presents some work that uses stochastic optimization to solve multimodal transport problems under uncertainty.

Table1. Overview of the use of stochastic optimization in multimodal transport problems

Reference	Stochastic optimization paradigm	Description
Min (1991)	CCP	Propose a stochastic scheduling model with probabilistic constraints to help distribution managers to choose the most efficient intermodal combination that minimizes transportation costs, risks, and meets various service requirements on time
Cheung and Chen (1998)	TSP	Effectively solve the problem of the dynamic allocation of empty containers to match customer demand over time. The goal is to minimize the total cost of leasing containers and determine the level of inventory required at ports.
Wu et al. (2006)	CCP	Optimize the selection of container shipping routes in a maritime transshipment problem with uncertain demand. The objective is to maximize profit during transport subject to capacity constraints and the balance of empty and heavy containers
Meng and Wang (2010)	CCP	Study the problem of short-term planning of shipping lines for a fleet of ships, taking into consideration the uncertainties about the demand for shipping containers between ports
Wang and Yang (2012)	CCP	Minimize the cost of repositioning empty containers, assuming that the number of containers available at the beginning of planning is uncertain
Meng et al. (2012)	TSP	Study the problem of short-term planning of shipping lines for a fleet of vessels with uncertain demands. Given a network of shipping lines services comprising several shipping routes, the problem is to determine the number and type of vessels required in the fleet and to assign each of

		these vessels to a particular route to maximize the value of the total profit.
Van Hui et al. (2014)	TSP	Study the problem of planning the shipment of goods with random processing times in intermodal terminals. Shipping activities are divided into two groups according to regional parameters. Processing times for activities in the first region are assumed to be random while those in the second region are deterministic.
Rouky et al. (2019)	TSP	Investigates the stochastic quay crane scheduling problem where the loading /unloading times of containers and travel time of quay cranes are considered uncertain. A simulation-based ant colony algorithm is proposed for the resolution of the problem.
Guo et al. (2021)	CCP	Investigates the dynamic and stochastic shipment matching problem faced by network operators in hinterland synchromodal transportation. The objective is to provide optimal matches between shipment requests and multimodal services within a finite horizon under spot request uncertainty. To solve the problem an anticipatory algorithm, with a sample average approximation method to address spot request uncertainties, was proposed.
Zhang et al (2022)	TSP	Study the incomplete multimodal hub location problem in many-to-many transportation and distribution systems with considering transportation costs. An improved Benders decomposition algorithm is implemented to solve the problem by adopting a sample average approximation approach and a dualization strategy.
Zweers and Mei (2022)	CCP	Study the intermodal transportation networks problem with stochastic travel times and overbooking to minimize the travel cost.

CCP: Chance Constrained Programing, TSP: Two-Stage Programming TSP

2. Fuzzy Optimization

Fuzzy set programming also deals with optimization under uncertainty. The main difference between stochastic and fuzzy approaches lies in the way uncertainties are modeled. In the case of stochastic programming, discrete or continuous probability functions are used to model uncertainties in the data. However, this type of modeling is not always appropriate, particularly when the information about the uncertain data is vaguely defined or described qualitatively; because of partial data, insufficient understanding of data, or when it relates to human behavior (Tang et al., 2004). Fuzzy set optimization considers random parameters as fuzzy numbers and constraints are treated as fuzzy sets (Buckley and Eslami, 2002). Some constraint violations are allowed and the degree of satisfaction of a constraint is defined as the belonging function of the constraint. Two types of fuzzy set programming can be distinguished: flexible programming devoted to the study of uncertainties on the second member of the constraints and possibilistic programming which deals with uncertainties in the coefficients of the objective function as well as in the coefficients of the constraints. In both types of fuzzy programming, the objective function is addressed as a constraint of the problem, and the lower and upper bounds of this constraint define the expectations of decision-makers. Belonging functions are used to represent the degree of satisfaction with constraints and the levels of expectations of decision-makers concerning the objective function.

The reader interested in more details on fuzzy set optimization can refer to (Luhandjula and Gupta, 1996), (Ross, 2009), and (Lodwick and Untiedt, 2010). Table 2 presents some applications of fuzzy set optimization in multimodal transport problems.

Table 2. Some Applications of Fuzzy Set Optimization in Multimodal Transport Problems

Reference	Description
Ding and Chou (2015)	Development of a multi-objective decision-making model to determine the best selection of transshipment ports in a maritime container transport chain
Ries et al. (2014)	Design of a decision support system based on the solution of the container storage problem in a container terminal. The objective is to provide real-time decisions to deal with a high degree of uncertainty affecting the arrival dates of containers at the storage areas
Bray et al. (2015)	Measurement of the efficiency of transport and service systems considering uncertainties in the input data, using an approach that combines Fuzzy Set Optimization (FSO) and Data Envelope Analysis (DEA). The accuracy of the measurements provided by the hybrid approach is compared to that of the traditional DEA approach
Expósito-Izquiero et al. (2016)	A joint resolution of the problems of berth allocation and scheduling of quay cranes under uncertainty about container ship arrival dates and handling times
Segura et al. (2017)	Proposal of a mathematical model to solve the problem of allocating continuous berths, considering the uncertainties on the dates of arrival of container ships
Wang et al. (2018)	Proposal of a mathematical model and a memetic algorithm for solving the Hub-and-Spokes network design problem in a Rail-Road intermodal transport system
Sheikhtajian et al. (2020)	Studied the uncertain Liquified Natural Gaz (LNG) inventory-routing problem. Vessel speed was modeled as a fuzzy parameter and a memetic genetic algorithm was proposed as a solution method.
Sharma et al. (2020)	Developed an efficient soft set-based approach for the multi-objective multimodel transportation problem. This approach was used to successfully perform multi-criteria decision-making for the choice of various modes of transport with different objectives and tested for real data set.
Cheemakurthy and Garme (2022)	Proposed a Fuzzy Analytic Hierarchic Process (AHP) in combination with particle swarm optimization for the evaluation of ferries in Stockholm.
Trikollaee and Aydin (2022)	Build up a useful fuzzy bi-level Decision Support System (DSS) for integrated design of sustainable supply chain and co-modal transportation network for perishable products under uncertain environments.
Wang et al. (2022)	Studied the uncertain petroleum supply chain design problem with the pipeline system. Uncertainties on resource cost, demand, and price were considered and a Fussy min-max goal programming model integrated with a heuristic approach was proposed for the resolution.

3. Sensitivity Analysis

Sensitivity analysis is one of the a posteriori approaches for optimization under uncertainty. It consists in studying the impact of perturbations in the input data of a problem on the quality of the solutions obtained by classical optimization methods. Sensitivity analysis does not aim to solve the problem under uncertainty but to analyze the behavior of the solutions already obtained in the face of data perturbations, highlighting the links between the inputs and outputs of the system under study. Indeed, it is an a posteriori analysis of the stability of the deterministic solutions to determine the input data whose uncertainties on their values generate the greatest degradation in the value of the objective function. In combinatorial optimization, two main types of approaches can be distinguished:

- **Quantitative sensitivity analysis:** which seeks to evaluate various measures of sensitivity for each solution and to propose efficient algorithms for their calculation.

- **Qualitative sensitivity analysis:** which aims to analyze the structural, combinatorial, or geometric properties of solutions.

Table 3 provides an overview of the applications of sensitivity analysis to multimodal container transport problems.

Table 3 Overview of Applications of Sensitivity Analysis to Multimodal Transportation Problems

Reference	Description
Vis (2006)	Comparison of the performance of straddle carriers and automated gantry cranes in container handling, considering the uncertainties on handling equipment travel times and container arrival dates.
Caris and Janssens (2010)	Solving a container transfer problem in an intermodal terminal using deterministic algorithms and analyzing the sensitivity of solutions found in the face of various data perturbations.
Lu and Park (2013)	Proposal of a sensitivity analysis approach to identify critical factors for improving the productivity of container terminals.
Jin et al. (2015)	Resolution of the congestion problem encountered in the berthing of container ships by introducing a proactive management strategy from a terminal perspective that adjusts the calling schedule of supply vessels to balance the distribution of workload at the wharf level.
Heggen et al. (2017)	Proposal of a multi-objective heuristic for the planning of train loading operations, and development of a sensitivity analysis approach to consider the different aspects that affect the capacity utilization of trains.
Munim and Haralambides (2018)	Proposal of a linear model to find the optimal economic conditions for efficient cooperation between the ports of Bangladesh and India, at the level of intermodal traffic. Sensitivity analysis considering uncertainties about the capacities and demands of the different ports is used to establish the robustness of the strategic decisions that could be taken.
Ertem et al. (2022)	Studied the intermodal transportation in humanitarian logistics within a Turkish network using retrospective analysis. Dynamic and capacitated models are proposed to analyze the sensitivity of the system to time and scarce resource environments.
Guo et al. (2022)	Investigated the effect of different cascading failure modes and attack strategies of the multimodal transport network. The uniqueness of the network was studied by complying with the percolation theory and a cascading failure model will consider recovery mechanisms and dynamics.

4. Belief Functions

The theory of belief functions, also known as evidence theory or Dempster-Shafer Theory (DST), is a general framework for reasoning under uncertainty, with understandable links to other fields such as probability, possibility theory, and imprecise probability theory. First introduced by Arthur Pentland Dempster (Dempster, 1967) in the context of statistical inference, the theory was then used by Glenn Shafer (Shafer, 1976) to model uncertainty. The theory of belief functions is a rich and flexible framework generalizing Bayesian inference to the treatment of uncertainty. It allows to explicitly represent uncertainties in the data by perfectly expressing what is already known and considering what remains to be known. The most widely used model in the theory of belief functions is the Transferable Belief Model (TBM) (Smets and Kennes, 1994), in which the aim is to monitor, record, and transmit new knowledge based on observed information. The Transferable Belief Model differentiates between two levels of reasoning:

- **The credal level:** from the Latin "credo" meaning "I believe", which consists of two parts: a static part for representing information and a dynamic part for observing and combining knowledge.
- **The pignistic level:** from the Latin "pignus" meaning "bet", also known as the decision-making level, where risks are assessed, and decisions are made.

The theory of belief functions has been applied in several works dealing with various fields, such as pattern recognition (Appriou, 1991) and diagnosis (Smets, 1998). But little work in the literature has been devoted to its applications to multimodal transport problems.

Although the first four approaches presented in this article have been widely used in the literature for the management of uncertainties, in particular the resolution of many multimodal transport problems, their application in practice is often very difficult, especially if the probability laws or degrees of membership associated with the uncertain data cannot be precisely determined. This is particularly important in new real-world applications, where there is a lack of historical data that can allow these estimates to be made. As alternatives to these approaches, robust optimization techniques are increasingly being used to deal with uncertainties when dealing with real-world problems. In what follows, we will present these techniques in more detail.

5. Robust Optimization

Robust optimization is an approach that seeks to find a so-called "robust" solution to an optimization problem in which the data are uncertain without the use of probabilistic analysis. Introduced as early as 1955 by Dantzig (Dantzig, 1955), the idea of robustness has experienced a resurgence of interest in recent decades with numerous applications by both practitioners and theorists. In contrast to stochastic approaches, robust optimization models uncertain data using continuous or discrete sets of possible values with no attached probabilities. In the continuous case, sets of uncertainties are often represented by intervals and convex sets (i.e. polyhedron, box, or ellipsoid), based on the minimum and maximum deviations of the uncertain parameters from their nominal values. The discrete case is referred to as scenario-based models, where the possible values of the uncertain parameters are modeled by a discrete finite set of scenarios with the same probability of occurrence. In both cases, it is the cartesian product of the sets of uncertainties considered that define the possible instances of the problem under consideration. Several definitions of a "robust solution" circulate in the literature; we will adopt the one proposed by Gabrel and Murat (2010). According to these authors, a solution is qualified as robust if it is "acceptable" in all possible scenarios and if its performance is never "too bad". A robust optimization approach then consists in defining the best strategy to protect against the various possible realizations of uncertainties, while minimizing the value of the worst case on all possible solutions. Therefore, when using a robust approach, it is important to identify the context in which the study is being conducted. According to Kouvelis and Yu (2013), the implementation of a robust optimization approach involves three main steps:

1. Selection of an appropriate robustness criterion (i.e. absolute robustness, maximum regret, relative regret, α-robustness, p-robustness, etc.).
2. Choice of the appropriate representation of uncertain parameters (i.e. discrete set of scenarios or convex set).
3. Proposal of the mathematical model and the algorithm for generating robust solutions.

In the remainder of this section, we will present the main robustness criteria applied in the literature. We will then describe the different sets used in robust optimization to model uncertainties in the data and we will explain the Pareto principle of robustness. Finally, some applications of robust optimization to the problems of multimodal transport will be discussed.

6.1 Robustness Criteria

Several robustness criteria have been developed in the literature to measure the quality of robust solutions. These criteria can be divided according to (Aloulou et al., 2005) into two parts big families:

- **Approaches based on the optimization of a criterion:** these approaches are based on the classical optimization criteria of worst-case scenarios such as absolute robustness (min-max), maximum regret (min-max regret), and relative regret (min-max relative regret).
- **Approaches based on robustness conditions:** these approaches impose conditions to be met for a solution to be considered robust such as the α-robustness and the ρ-robustness.

In the following subsections, we give a brief description of the most used criteria in the literature

a) Absolute robustness

A robust solution, according to this criterion, is the best worst-case solution in all possible scenarios. Consider a problem of minimizing an objective function f, with X the set of possible solutions to this problem, and S the set of all possible scenarios. The criterion of absolute robustness is defined by the following relationship:

$$\min_{x \in X} \max_{s \in S} f(x, s)$$

This criterion is often linked to the notion of risk where decision-makers seek to protect themselves against losses generated by a large change in data. Many works use this criterion as a robustness measure, we mention the works on the Shortest Robust Path (Gabrel and Murat, 2010) and the Robust Knapsack Problem (Kouvelis and Yu, 2013).

b) Maximum regret

Regret is the sense of loss experienced by a decision-maker when he learns of the existence of a preferable solution to a given scenario than the one that was applied. A robust solution, according to the maximum regret criterion, is the one that has the smallest worst deviation from the optimal solutions over all possible scenarios. Indeed, regret is defined in robust optimization as being the difference between the value of a solution and the value of the optimal solution in the same scenario, and maximum regret is the highest regret of a solution overall scenarios. A robust solution is then the solution with the smallest maximum regret. This criterion is defined by the following relation:

$$\min_{x \in X} \max_{s \in S} \left(f(x, s) - f(x^*, s)\right)$$

$x^* \in X$ is the optimal solution on the scenario $s \in S$ and $f(x^*, s)$ is its value in scenario s.

c) Relative Regret

The principle of relative regret is almost similar to maximum regret, the only difference resides in the fact that we replace, in the definition of regret, the minimization of the absolute deviations between the solutions and the optimal values by the minimization of the percentages of deviation from the optimal solution. The relation used to represent relative regret is as follows:

$$\min_{x \in X} \max_{s \in S} \left(\frac{f(x, s) - f(x^*, s)}{f(x^*, s)}\right)$$

The family of approaches based on the optimization of the worst-case criterion, presented above, is generally considered in the literature as a very conservative family. Indeed, studying the notion of robustness based on this criterion often leads to privileging a single aspect which is the protection against the worst scenarios, even if it is very improbable, in practice, that all the uncertain parameters reach their higher values simultaneously. On the other hand, a solution that protects against worst-case achievements is robust, but it is very bad when applied in other scenarios.

d) α- Robustness

This approach was presented for the first time by Aloulou et al. (2005), as a less conservative alternative to worst-case criterion-based optimization approaches, for problems where uncertain data are modeled using discrete scenarios. In this criterion, a parameter α defines a tolerance threshold to limit the degree of conservatism of a solution. The first step of this approach consists of ordering the values of each solution $x \in X$ over the set of scenarios S in descending order, the resulting vector is called the "disutility vector" of the solution x. Then, a fictitious disutility vector for a solution x' called "ideal solution", whose coordinates represent the minimum costs on each row of the disutility vector matrix, is calculated. A robust solution according to this approach is any solution in which the difference between its disutility vector and the fictitious disutility vector does not exceed the tolerance threshold α.

e) bw-Robustesse

For Roy (2010), robustness is defined as an ability to resist "roughly" or "areas of ignorance" to protect oneself from impacts deemed regrettable in all possible scenarios. According to the author, classical min-max approaches do not fully answer this question since they only focus on minimizing the worst case. To overcome this drawback, Roy proposed a new robustness criterion called bw -robustness. In this approach two parameters are used: a constant w which defines the value that the decision-maker does not want to exceed in all scenarios (compliance with this value must be guaranteed and represents a firm constraint of the problem), and a value b, with $b \leq w$ (for a minimization problem), which the decision-maker wants to achieve under the greatest number (or proportion) of scenarios. In other words, if $f(x, s)$ is the value of a solution x on the scenario s, then the robust solution according to the bw-robustness criterion is the solution that maximizes the number of scenarios where $f(x, s) \leq b$, while ensuring that $f(x, s) \leq w \; \forall s \in S$. Like the criteria of the min-max family, bw -robustness can also be presented in the form of absolute robustness, regret, or relative regret.

Other robustness criteria have also been used in the literature to determine the quality of a robust solution, such as *p*-robustness, lexicographic min-max, *pw*-robustness, etc. A state of art on these criteria is presented in (Coco et al., 2014).

6. Uncertainty Sets and Corresponding Robust Optimization Models

In robust optimization, a set of uncertainties represents how the disturbances in the data are modeled. The structure of a set of uncertainties strongly influences the existence and the traceability of solutions. In this section, the most important sets of uncertainties and the corresponding robust optimization models are briefly presented. Let's consider the following uncertain linear programming problem:

$$\begin{gathered} \text{minimize } c\ \mathrm{x} \\ subject\ to: A\mathrm{x} \le b \\ \mathrm{x} \in \mathrm{X} \end{gathered}$$

Without loss of generality we assume, in what follows, that only the coefficients of the matrix A are subject to uncertainties and that their values belong to a set U called the set of uncertainties. In the literature, four main forms of uncertainties set can be distinguished:

- **Finite and discrete set of scenarios:** the uncertainties are represented by a set of possible scenarios, in this case $U_S = \{s_1, s_2, \dots, s_i, \dots, s_q\}$, with s_i the values of possible realizations of uncertainties on the coefficients of matrix A in the scenario i.
- **Box:** in this type of set of uncertainties, the uncertain coefficients $\tilde{a}_{ij}$ of matrix A are assumed to belong to intervals of the form $[\underline{a}_{ij}, \hat{a}_{ij}]$ where $\underline{a}_{ij}$ represents the nominal value and $\hat{a}_{ij}$ the maximum disturbance. Thus, the values of the uncertain parameters can be given by $\tilde{a}_{ij} = \underline{a}_{ij} + \varepsilon_{ij}\hat{a}_{ij}$, with ε_{ij} a random variable such that $-1 \le \varepsilon_{ij} \le 1$. The interaction of disturbances forms a set of uncertainties in the form of a box. This set is often used to model worst-case realizations, and can be described as follows:

$$U_B = \{\tilde{a}_{ij} \mid \tilde{a}_{ij} = \underline{a}_{ij} + \varepsilon_{ij}\hat{a}_{ij}, |\varepsilon_{ij}| \le 1\}$$

- **Ellipsoidal:** to reduce the degree of conservatism in Box sets, Ben-Tal and Nemirovski (1998) proposed an alternative set called the ellipsoidal set of uncertainties. This set introduces a parameter Ω which makes it possible to reduce the uncertainty space by removing the extremities of the intervals and thus avoiding the worst case. This set is given by the following relation:

$$U_E = \{\tilde{a}_{ij} \mid \tilde{a}_{ij} = \underline{a}_{ij} + \varepsilon_{ij}\hat{a}_{ij}, \textstyle\sum_j \varepsilon_{ij}^2 \le \Omega_i^2\ \forall i\}$$

- **Polyhedral:** introduced by Bertsimas and Sim (2004), this type of set aims to ensure a compromise between the robustness and the performance of the solutions by proposing parameterizable modeling of uncertain data. This set uses a parameter Γ called the robustness budget to control the number of data that will be subject to uncertainties:

$$U_P = \{\tilde{a}_{ij} \mid \tilde{a}_{ij} = \underline{a}_{ij} + \varepsilon_{ij}\hat{a}_{ij}, \textstyle\sum_j |\varepsilon_{ij}| \le \Gamma_i\ \forall i\}$$

Figure I show the difference between the set of box, ellipsoidal and polyhedral uncertainties for a problem with two uncertain parameters. Based on these different sets, several robust modeling approaches have been proposed in the literature, namely: the Soyster approach, the Ben-Tal and Nemirovski approach, and the Bertsimas and Sim approach.

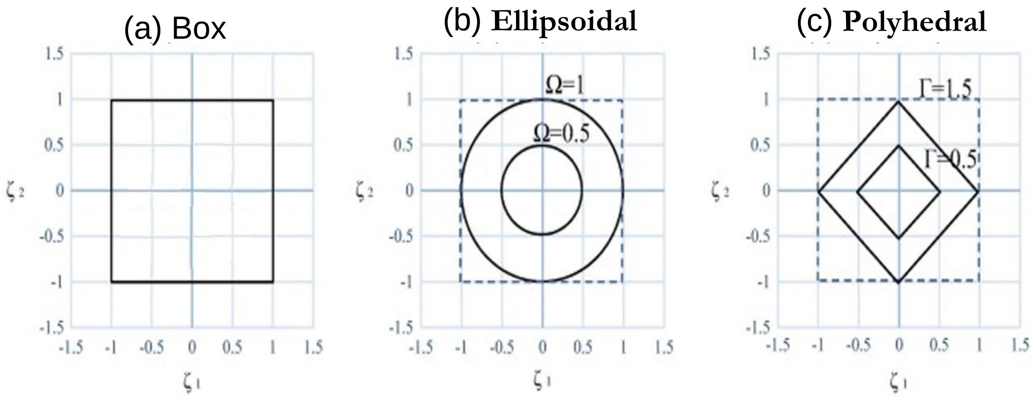

Figure I. Illustration of the most used sets of uncertainties.

7.1 Soyster's approach

Soyster (1973) proposed a linear optimization model to build a feasible solution for all the realizations of uncertainties belonging to the set box U_B. Soyster's robust model was formulated as follows:

$$\begin{aligned} &\text{Minimize } cx \\ &subject\ to: \sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b\ \forall\ \tilde{a}_{ij} \in \mathcal{U}_B, i = 1, \dots, m \\ &\mathrm{x} \in \mathrm{X} \end{aligned}$$

This formulation is also known as the column uncertainty formulation. This approach, like the min-max criteria, aims to protect against the worst-case scenario. Moreover, Soyster shows that for problems where the variables x_j are not negative the model is equivalent to:

$$\begin{aligned} &\text{Minimize } cx \\ &subject\ to: \sum_{j=1}^{n} (\underline{a}_{ij} + \hat{a}_{ij}) x_j \leq b, \forall i = 1, \dots, m \\ &\mathrm{x} \geq 0 \end{aligned}$$

Soyster's approach is an absolute guarantee against all possible realizations of uncertainties, it reduces the resolution of the problem under uncertainties to the resolution of a deterministic problem in which all the uncertain parameters take their values in the worst case. However, despite the guarantee of feasibility that this approach offers, it is often considered very conservative in the literature. Indeed, Ben Tal and Nemnirovski (1999) point out that because of the protection against the worst case there is a huge loss of optimality in the other scenarios.

7.2 Ben-Tal and Nemirovski's approach

To overcome the drawbacks of Soyster's approach, Ben-Tal and Nemirovski (1999) proposed a new approach based on the use of the ellipsoidal set of uncertainties U_E. This approach decreases the degree of conservatism by allowing a weak violation of the worst-case values on the constraints. Indeed, on each constraint *i* of the problem, the random parameters ε_{ij} which define the values of maximum deviations are supposed to belong to a set $E(\Omega_i) = \{\varepsilon_{ij} | \sum_j \varepsilon_{ij}^2 \leq \Omega_i^2, \varepsilon_{ij} \in [-1,1]\}$ and the corresponding robust model is given by:

$$\text{Minimize } c\mathrm{x}$$
$$subject\ to\text{: } \sum_{\mathrm{j}=1}^{\mathrm{n}} \underline{\mathrm{a}}_{ij}\mathrm{x}_{\mathrm{j}} + \max_{\varepsilon_{\mathrm{ij}} \in E(\Omega_i)} \{\hat{\mathrm{a}}_{\mathrm{ij}}\varepsilon_{\mathrm{ij}}\mathrm{x}_{\mathrm{j}}\} \leq b\ , \forall\ \mathrm{i} = 1 \ldots \mathrm{m}$$
$$\mathrm{x} \geq 0$$

This model of uncertainties is also known in the literature by the name "line model", because of parameters Ω_i that are defined to control the degree of conservatism on each constraint *i* of the studied problem.

7.3 Bertsimas and Sim's approach

Bertsimas and Sim (2004) proposed an approach similar to the approach of Ben-Tal and Nemirovski to find a compromise between the performance of solutions and their robustness. The difference between the two approaches lies only in the set of uncertainties considered. Indeed, the approach of Bertismas and Sim is based on the polyhedral set of uncertainties assuming that the random parameters ε_{ij} are defined on a set of the form $\phi_i(\Gamma_i) = \{\varepsilon_{ij} | \sum_{j=1}^{n} |\varepsilon_{ij}| \leq \Gamma_i\ , \varepsilon_{ij} \in [-1,1]\}$. The robust model corresponding to this approach is given by:

$$\text{Minimize } c\mathrm{x}$$
$$subject\ to\text{: } \sum_{\mathrm{j}=1}^{\mathrm{n}} \underline{\mathrm{a}}_{ij}\mathrm{x}_{\mathrm{j}} + \max_{\varepsilon_{\mathrm{ij}} \in\ \phi_i(\Gamma_i)} \{\hat{\mathrm{a}}_{\mathrm{ij}}\varepsilon_{\mathrm{ij}}\mathrm{x}_{\mathrm{j}}\} \leq b \quad \forall\ \mathrm{i} = 1 \ldots \mathrm{m}$$
$$\mathrm{x} \geq 0$$

This approach is also known as the "robustness budget approach" since it uses a parameter Γ which controls the amount of data that will be subject to uncertainty. Thus, a robust solution according to the approach of Bertsimas and Sim is defined as a solution that protects against all situations in which at most the values of Γ_i coefficients on each constraint *i* of the studied problem are disturbed.

7.4 Pareto Robustness

The standard robust optimization approaches, described previously, are based on the use of a single criterion, to determine a robust solution to an optimization problem over all the realizations of the possible uncertainties. However, in many applications, this solution is seldom unique, as there may be several robust optimal solutions, and some may confer more benefits to the decision-maker than others. Indeed, even if all the robustly optimal solutions have the same objective function value on the optimized criterion (min-max for example), these solutions do not necessarily have the same performance in the other scenarios. According to Iancu and Trichakis (2014), standard approaches are not desirable in practice because they lead to sub-optimal performance and do not facilitate the choice of the decision-maker. To solve these problems, Iancu and Trichakis proposed a new concept called Pareto robustness, which is inspired by the principle of Pareto optimality used in multi-objective optimization. The purpose of Pareto robustness is to find solutions that are robustly efficient in the sense of Pareto, i.e., robustly feasible solutions where no improvement on the value in a scenario can be made without sacrificing performance in another. In other words, robust Pareto solutions are solutions for which there are no other robustly feasible solutions that have a performance at least equal on all scenarios $s \in U_S$ and strictly better on at least one of these scenarios. The representation of robust Pareto solutions facilitates decision making, in the sense that it allows the decision-maker to explore the possible trade-offs between all the scenarios before deciding how they should be prioritized.

8. Overview of Robustness Applications in Multimodal Transport

To ensure the performance of the multimodal supply chain, analysis of the robustness of solutions proposed in the face of various disruptions in data of the system studied must be carefully examined at each decision level. Robust optimization has been applied extensively over the past few years. Ordóñez and Zhao (2007) studied the problem of designing transport networks with uncertainties on travel times and demands. The authors propose a robust formulation of the problem based on the use of the set of polyhedral uncertainties. Mudchanatongsuk et al. (2008) considered a similar problem with uncertain transport costs and demands. Two robust formulations, based on polyhedral and ellipsoidal sets, have been developed and a column generation approach has been proposed to solve lagrangian relaxation on large instances. In Pishvaee et al. (2011), the authors addressed a loop supply chain network design problem with uncertainties about costs and demands. A deterministic model was first developed to design the network, then the robust counterpart of the model was proposed using the approach of Ben-Tal and Nemirovski. The qualities of solutions obtained by the deterministic and robust models were compared in several scenarios with different datasets.

The problems of locating hubs under uncertainties have also been widely studied in the literature, a full state of the art on these problems is available in (Correia and da Gama, 2015). Merakh and Yaman (2016) studied the problem of P-hub with multiple allocation and uncertain demands. The authors use a pipe uncertainty model and a hybrid model to characterize demand uncertainties. The former considers that the only information available is an upper limit of the total rate adjacent to each node, while the latter incorporates lower and upper limits on each Origin / Destination flow. The authors present robust formulations and a Benders decomposition algorithm to solve the two problems for instances up to 150 nodes. Ghaderi and Rahmaniani (2016) have dealt with the problem of P-hub with unique allocation and uncertainties on requests and handling times. The robust problem is modeled to minimize the maximum regret criterion and hybrid metaheuristics are proposed to solve it. Zetina et al. (2017) presented robust counterparts to the problem of locating hubs based on the approach of Bertsimas and Sim. Three cases of realization of uncertainties were considered: (1) uncertainties on demands, (2) uncertainties on transport costs, and (3) both sources of uncertainties at the same time. For each case, a robust formulation was presented and solved using a Branch-and-Cut algorithm.

Abassi et al. (2019) studied the intermodal freight transport problem considering uncertainties in terminals' capacities and costs. Uncertainties were mitigated using a discrete set of scenarios and regret minimization criteria. To solve the problem a Simulated Annealing approach combined with an exact method is proposed. Mayer et al. (2020) investigate the sustainable food grain transportation problem with uncertain supply and intentional disruptions. A mixed-integer non-linear robust optimization model was proposed with as objective the minimization of total relative regret associated with the total cost and a Particle Swarm Optimization with a Differential Evolution approach was used for the resolution. The robust Location-Arc Routing (RLAR) with time windows and uncertain demand is studied in (Kahfi et al, 2021). A Multi-Objective Dragonfly Algorithm (MODA) and Non-dominated Sorting Genetic Algorithm (NSGA-II) were proposed to solve the problem.

At intermodal terminals, Bruns et al. (2014) studied the problem of determining train loading plans by considering different types of uncertainties. Two approaches have been presented to include uncertainties: strict robust load plans, in which it is assumed that the solution cannot be changed once implemented, and robust and adjustable load plans that allow the planner to react once the uncertain parameters become known. The robust formulations associated with the two approaches allow an efficient resolution of the problem. Fotuhi and Huynh (2017) proposed a new robust model based on the maximum regret criterion for the extension of an intermodal network. The objective of the model is to identify critical rail links to be upgraded, locations to establish new intermodal terminals, and existing terminals to be developed while considering uncertainties about demands. A Hybrid Genetic Algorithm that uses Column Generation to determine freight flows has been developed to solve the proposed model.

Robust modeling of the repositioning empty containers problem has been proposed in (Erera et al., 2009). The proposed model is based on the approach of Ben-Tal and Nemirovski and the uncertainties arise mainly from forecasts of future supplies and demands for assets. The authors established the conditions for the feasibility of a repositioning plan and the recovery measures in response to the uncertainties. Tsang and Mak (2015) addressed the problem of repositioning empty containers by assuming that container demands are uncertain. Robust modeling of the problem has been proposed and several tests have been carried out to evaluate its performance.

Shang et al. (2016) proposed an interesting application of robust optimization for the joint resolution of berth allocation and berth crane scheduling problems in a container terminal. The authors considered the uncertainties in ship arrival dates and container handling times. The problem was modeled following the approach of Bertsimas and Sim, and a genetic algorithm was proposed for the resolution of larges instances. Rouky et al. (2018) addressed the Rail Shuttle Routing Problem (RSRP) in Le Havre port with uncertainties on service and travel times. A robust formulation of the problem, based on the Bertsimas and Sim approach, was proposed and a Robust Ant Colony Optimization (RACO) was developed for the resolution. Park et al. (2021) studied the Berth Scheduling Problem to minimize the sum of the baseline schedule costs and the expected recovered schedule costs. Uncertainty on vessels' arrival times was modeled using a finite set of discrete scenarios and a Particle Swarm Optimization approach was proposed to solve the problem. Guo et al. (2021) proposed a Berth Allocation with vessel handling time uncertainty considering the impact of weather conditions. An efficient Particle Swarm Optimization algorithm embedded with a machine learning approach is devised for solving the berth allocation in large-scale problem cases. Rodrigues and Agra (2021) investigated the Integrated Berth Allocation and Quay Crane Scheduling with uncertain vessel arrival times. The problem is modeled as a two-stage robust mixed-integer program and a decomposition algorithm is proposed for solving the problem.

Table 4 Summary of Some Related Research and Applications of robust optimization to Multimodal Transportation Problems

Reference	Problem	Uncertainty Type	Uncertainty mitigation	Solution Method	Objective
Ordóñez and Zhao (2007)	Transport Network Expansion	Travel time, Demand	Polyhedral set	Solver	Decide capacity expansions for a transit network.
Mudchanatongsuk et al. (2008)	Transport Network Design	Transport cost, Demand	Polyhedral and Ellipsoidal sets	Column Generation	Whether increased or not the arcs capacity and the arcs flow to route commodities at minimum transportation and investment cost.
Erera et al. (2009)	Empty Containers Repositioning	Supply and Demand	Ben-Tal and Nemirovski model	Solver	Minimize the cost of repositioning empty containers.
Pishvaee et al. (2011)	Supply Chain Network Design	Transport cost and Demand	Ben-Tal and Nemirovski	Solver	Minimizes the total cost, which includes fixed opening centers costs and transportation costs
Bruns et al. (2014)	Train Loading Plans	Demand	Bertsimas and Sim, Ben-Tal and Nemirovski	Solver	Choose wagon settings and assign load units to wagons of a train such that the utilization of the train is maximized, and setup and transportation costs in the terminal are minimized.
Tsang and Mak (2015)	Empty Containers Repositioning	Demand	Absolute robustness	Solver	Identifying optimal repositioning schedule to rebalance empty containers with minimal cost

Merakh and Yaman (2016)	P-hub Location	Demand	Polyhedral set	Benders Decomposition	Minimize the total transportation cost.
Shang et al. (2016)	Integrated Berth Allocation and Quay Crane Assignment	Quay carne productivity	Bertsimas and Sim	Genetic Algorithm and an Insertion Heuristic	Minimize the total weighted handling time and the waiting time for all vessels within the planning horizon
Fotuhi and Huynh (2017)	Intermodal Network Extension	Demand and Supply	Maximum regret	Hybrid Genetic Algorithm	Identify locations for new intermodal terminals and existing terminals to expand so that the costs of total transportation and lost sales are minimized for normal and disrupted situations.
Zetina et al. (2017)	Uncapacitated Hub Location	Demand and Transportation cost	Bertsimas and Sim	Branch-and-cut Algorithm	Minimize the total setup cost of the hubs and the total transportation cost
Rouky et al. (2018)	Rail Shuttle Routing	Transfer time and Travel time	Bertsimas and Sim	Ant Colony algorithm	Improve the performance of the container transfer system in the Le Havre port, by minimizing the total empty travel time of locomotives and protecting against delays
Abasssi et al. (2019)	Intermodal Freight Transport	Capacity and transportation cost	Maximum regret	Simulated Annealing approach	Minimize the total cost which comprises the usage costs of selected terminals in addition to the unimodal and the intermodal transportation costs.

Mayer et al. (2020)	Sustainable Food Grain Transportatio n	Supply	Relative regret	Particle Swarm Optimization	Minimize the total intermodal transportation cost of food grain.
Kahfi et al. (2021)	Location-Arc Routing	Demand	Bertsimas and Sim	Multi-Objective Dragonfly Algorithm and Non-dominated Sorting Genetic Algorithm	find the best compromise on minimizing costs and waiting time of the vehicles.
Guo et al. (2021)	Berth Allocation	Vessel handling time	Finite set of discrete scenarios	Particle Swarm Optimization	Evaluation of vessel handling time under different weather conditions.
Park et al. (2021)	Berth Scheduling	Vessel arrival time	Finite set of discrete scenarios	Particle Swarm Optimization	Minimize the sum of the baseline schedule costs and the expected recovered schedule costs.
Rodrigues and Agra (2021)	Integrated Berth Allocation and Quay Crane Scheduling	Vessel arrival time	Bertsimas and Sim	Decomposition algorithm	Minimize the total completion time of the worst-case scenario.

7. Conclusion

Multimodal transport is a very complex system characterized by the diversity of its operations, lack of information and the presence of a multitude of actors often have conflicting goals. For these reasons, multimodal operators are nowadays in search of effective operational strategies, allowing them to reduce the impact of uncertainties and meet established performance and service targets. Deterministic optimization models cannot overcome the complexity because they are usually built on a very abstract level neglecting the uncertain and dynamic behavior of real-world systems. To meet these needs, optimization under uncertainty has emerged as an important area of modern operations research. This article was devoted to the study of the different optimization techniques under uncertainties used in the literature and their applications to multimodal transport systems. First, an overview of these different techniques was presented. Then, we focused on robust optimization approaches, for which a detailed description and a state of the art on their application for solving multimodal transport problems were presented. The analysis presented in this paper aims to help researchers to select the appropriate approach for a given uncertain multimodal problem.

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