

A Multi-depot Vehicle Routing Problem with Time Windows and Load Balancing: A Real World Application

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Abstract

This paper presents a mixed integer non-linear programming (MINLP) model for a bi-objective and multi-depot vehicle routing problem with time windows. The main goals of the paper are minimization of total cost and equitable distribution of commodities between vehicles. Two types of vehicle including delivery and installation vehicles are utilized in the network regarding customers' needs. Satisfying all demands of the customers is not obligatory and unmet demands are permitted which leads to extra cost. A presented model is applied for real life case study in different provinces of Iran. To tackle the small-size problems, the augmented ϵ -constraint method is utilized by linearization of the model. Because of the NP-hard nature of the problem, as the size of the problem increases, so does the complexity. As such, we develop multi-objective simulated annealing (MOSA meta-heuristic) algorithm for large scale problems. Then, several numerical experiments and sensitivity analyses are conducted to validate the presented model and the solution method, which indicate the efficiency of our proposed approach.

Keywords: Vehicle routing problem; E-constraint method; Mixed integer non-linear programming.

1. Introduction

Vehicle routing problem (VRP) is a significant concept in the transportation field of study to specify optimal routes originating from depots, serving the customers, and getting back to the depots with the consideration of limitations on the capacity of the vehicles (Azadeh and Farrokhi-Asl, 2019; Kaboudani et al., 2018). An extension for VRP is known as a multi-depot vehicle routing problem with time windows (MDVRPTW) in which multiple depots serve the customers by different vehicles under time-windows limitation. In multi-depot vehicle routing problem (MDVRP), the origin and ultimate destination of each vehicle is the same depot (Rabbani et al., 2015). Occasionally in real cases, various vehicles with different capacities are needed to meet customers' demands. This type of problem is known as fleet size and mix vehicle routing problem (FSMVRP). Since we consider two different kinds of vehicles including delivery and installation vehicles, our problem is categorized as the FSMVRP where the former is just for delivering the products and the latter is for responding to the installation demand of the delivered products. It should be noted that customers cannot have only the installation demands, however having only the delivery demand is allowed (Bae and Moon, 2016). In addition to the general type of time windows constraint, a time interval is assumed between the delivery time of the product and the installation time.

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In real conditions, the differences in workloads of drivers generally lead to the drivers' dissatisfaction, so considering a load balancing between the vehicles can solve the problem. According to the literature, balancing objective function might be equitable distribution of the delivered products, the time length of the routes, and the number of customers visited in each route (Lee and Ueng, 1999). Among different kinds of balancing indices, the load balancing introduced by Kritikos and Ioannou (2010) is applied in this paper to increase fairness between the drivers and their satisfaction. To the best of our knowledge, there are a few studies in the literature discussing the load balancing on VRP with time windows.

In VRP studies, all the demands of each customer usually should be met, but in reality, the transportation network might be unable to satisfy all the demands because of some reasons such as the limitation on the depots' capacity. In these cases, the unmet demands may lead to additional costs.

In this paper, we deal with a real case study in Iran. The objectives considered in this problem include minimization of total costs and minimization of load imbalances between different vehicles. Therefore, for solving the problem, a multi-objective approach should be applied. The size of the problem varies based on the populations in our case study. For the provinces with less population, an exact method for solving the multi-objective problem can be applied. The size of our problem increases for the provinces with high density of population leading to more complexity because of the NP-hard nature of the problems (Farrokhi-Asl et al., 2017). The multi-objective approach used for small-size problems is the augmented ϵ -constraint method (AUGMECON) for generating efficient solutions. Meta-heuristic approaches have been applied in various NP-hard optimization problems in many areas. Different algorithms of meta-heuristic such as Simulated Annealing, Variable Neighbourhood Search procedure, Tabu Search algorithms, and Genetic Algorithm have been used in the context of VRP (Farrokhi-Asl et al., 2018). In this study, for the large-size problems, because of an increase in complexity, Multi-Objective Simulated Annealing (MOSA) is applied as a meta-heuristic algorithm.

To the best of our knowledge, this is the first study investigates load balancing on the delivery and installation vehicles separately in the VRP with time windows by assuming that unmet demands.

The rest of this paper is organized as follows: in Section 2, a survey of the related literature is presented. Section 3 provides the problem sets based on the described assumptions by applying a mathematical model. Because of the bi-objective nature of the problem, two solution methods are illustrated in Section 3 for solving the problem including AUGMECON2 approach and the MOSA meta-heuristic. In Section 4, a real case study is implemented in Iran for a vehicle routing problem. Moreover, several sensitivity analyses of the model are done in Section 5. Finally, concluding remarks and future research directions are provided in Section 6.

2. Literature review

Vehicle routing problem (VRP) is an impressively studied optimization problem in the literature. VRPs attract a lot of attention in the distribution networks because it plays an important role in decreasing the costs of distribution and collection systems (Rabbani et al., 2018; Ghomi and Asgarian, 2019).

In addition to the classic VRP, different types of VRP has been investigated in the literature. Recently, the VRP with time windows (VRPTW) was investigated by Desaulniers, et al. (2016). They presented a VRPTW by considering the electric vehicles with a capacity constraint. Afterward, Bianchessi, et al. (2019) introduced split delivery VRP with customer inconvenience restrictions and time windows (SDVRPTW-IC). They proposed a developed branch-and-cut approach as a solution approach. Another type of time windows constraint was presented by Kim, et al. (2013). They considered service level constraint in a way that the time difference between the delivery of the products and installations of them should not be further from the service level. Also, Xiao et al. (2019) proposed a VRPTW with the objectives of reducing the number of vehicles and minimizing the time-wasting during the delivery process caused by early arrival. Majidi, et al. (2017) presented an uncertain green VRP with time windows and simultaneous pickup and delivery. The aim of their study was the minimization of the cost of fuel consumption and greenhouse gasses emissions. They tackled the uncertainty in their model by using fuzzy credibility measure theory. An adaptive large neighborhood search heuristic is used for solving the MINLP model.

In addition to cost or total distance minimization, load balancing is considered as an objective function. Lee and Ueng (1999) applied load balancing for the agricultural products' distribution. In the literature, most of the problem considering load balancing are multi-objective problems. Dharmapriya, et al. (2010) studied load balancing besides minimization of transportation cost, and total distance by considering a capacity for all the vehicles. Moreover, Kritikos and Ioannou (2010) proposed a new approach based on data envelopment analysis for the balance cargo VRP with time windows.

Wang and Bian (2016) introduced the vehicle routing problem with simultaneous pick-up and delivery (VRPSPD) and considered two penalty coefficients for unsatisfied demand delivery and unmet pick-up. Kachitvichyanukul, et al. (2015) presented an MDVRPPD model intending to minimize the transportation cost and penalty cost for unmet demands. Molina and Salmeron (2020) applied an approach to perform the route design minimizing the total fixed vehicle costs and distribution costs and satisfying all problem constraints.

In the literature, a few studies are investigating the multi-objective VRPTW. Ghannadpour, et al. (2014) proposed a multi-objective dynamic VRPTW. The objectives of this study are minimizing the total distance traveled, the total number of vehicles, waiting time imposed on vehicles, and maximizing the customers' satisfaction rate. A genetic algorithm is used in this paper for solving a real-life case study of VRPTW. Ghoseiri and Ghannadpour (2010) investigated a multi-depot homogenous locomotive allocation problem as a VRPTW. In this paper, a multi-depot problem is changed into different single depot problems by clustering and then the routes are determined using a hybrid genetic algorithm.

According to Table 1, the contributions of this paper are presented as follows:

- Proposing a new bi-objective MDVRPTW
- Considering load balancing for two types of vehicles, separately
- Applying a penalty for unmet demands in the objective function
- Using two different methods for solving a bi-objective problem, AUGMECON2 and MOSA, based on the size of the real problem

Table 1 Features of this study versus other studies

Paper	VRPTW	MDVRP	FSMVRP	Objective function							Solution method				Case Study	Complexity
											Objective		Mechanism			
											single	multiple	Heuristic	Exact		
				Depot establishment	Transportation cost / time	Vehicle cost	Labor cost	Unmet demand cost	Load balance							
This study	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓		✓		✓	$O(m \cdot n \cdot k)$
Guezouli and Abdelhamid (2017)	✓	✓	✓	✓	✓	✓				✓				✓		$O(g(mm + nm + n))$
Kritikos and Ioannou (2010)	✓				✓	✓			✓	✓		✓			✓	
Bianchessi, et al. (2019)	✓				✓	✓				✓			✓			
Koç, et al. (2016)	✓		✓	✓	✓	✓				✓				✓		$O(m \cdot n \cdot k)$

Table 1. Continued

Complexity	Case Study	Solution method		Objective function	FSMVRP	MDV/RP	VRPTW	Paper
		Objective	Mechanism					
		single	multiple	Heuristic	Exact	Metaheuristic		
		Load balance	Unmet demand cost	Labor cost	Vehicle cost	Transportation cost / time	Depot establishment	

3. Problem description

In this study, a vehicle routing problem is proposed where the commodities are mailed from multiple depots to customers' locations with two different types of demand satisfied by delivery or installation vehicles. The delivery vehicle is used to deliver each customer's requested product. On the other hand, the installation vehicle meets the customers that their delivery needs are satisfied already. The schematic illustration of our network is shown in Fig. 1. There is an interval between the arrival time of delivery and installation vehicles which is called service level. To increase the satisfaction level of the drivers, minimization the imbalance level is one of our objective functions. This imbalance level is defined as the difference between total load carried by a vehicle and the predetermined average load (delivery vehicle), and the difference between the total installation and the predetermined average installation (installation vehicle) (kritikos and Ioannou, 2010). In the situation that all of the customer demands are not satisfied, a penalty is considered for unmet demands. Our goal is to minimize the cost of these penalties (Kachitvichyanukul, et al. 2015).

The assumptions of the model are considered as follows:

- The origin and destination of each vehicle should be the same depot.
- Each depot has a limited capacity.
- Each customer's need should be satisfied just by one vehicle.
- The load carried by each delivery vehicle should not exceed the capacity of the vehicle.
- The duration of each vehicle trip consists of transportation time, waiting time, and service time (service time is just for installation needs and is the time of installing the delivered products) should satisfy the time window constraint.
- There is no need to satisfy all the customer's demands and unmet demand is allowed in this problem.
- The unmet demand is just considered for delivery of the product because of the capacity restriction of the depots.
- All the customers with known demands should be allocated to the vehicles.
- The average load of all the delivery vehicles and the average installation done for all the installation vehicles are known as parameters.

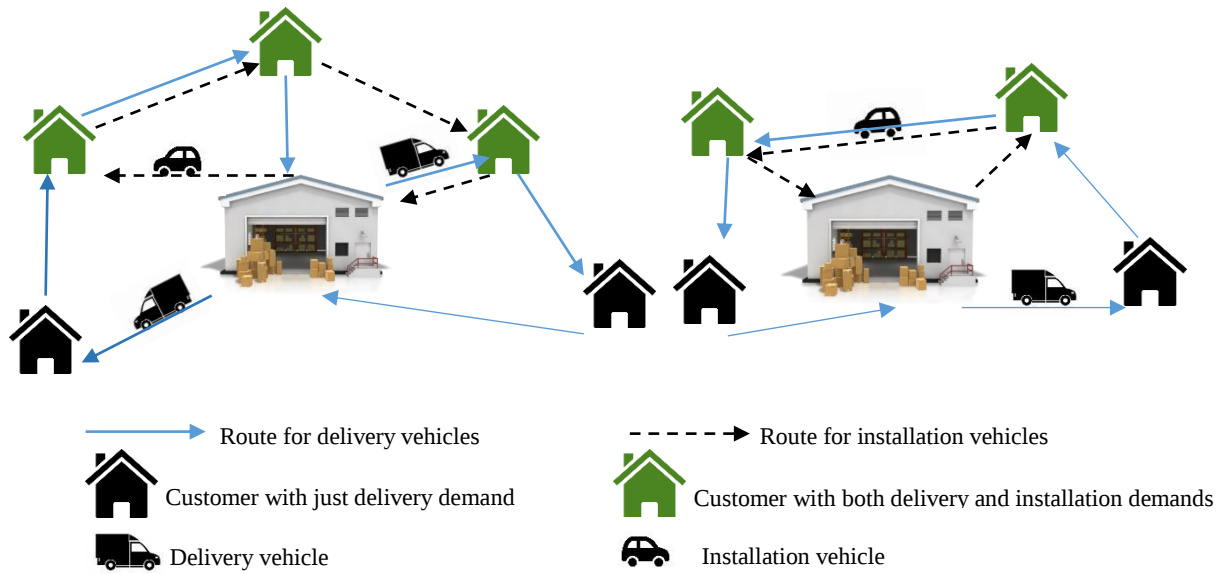


Figure 1. Illustrations of an MDVRP with delivery and installation vehicle

The notations used in this problem are as follows:

Sets

I Set of delivery vertices

$$p \subseteq I$$

P Set of installation vertices

J Set of depot vertices

V Set of all vertices

$$V = I \cup J$$

R Set of installation and depot vertices

$$R = P \cup J$$

K Set of delivery vehicles

S Set of installation vehicles

A Set of all vehicles

$$A = K \cup S$$

Parameters

C_j	Fixed cost for depot j	$\forall j \in J$
FC_k	Fixed cost for vehicle k	$\forall k \in K$
FC_s	Fixed cost for vehicle s	$\forall s \in S$
TC_k	Transportation cost for the vehicle k per unit time	$\forall k \in K$
TC_s	Transportation cost for the vehicle s per unit time	$\forall s \in S$
WC_k	Wage cost for the driver of vehicle k per unit time	$\forall k \in K$
WC_s	Wage cost for the driver of vehicle s per unit time	$\forall s \in S$
SC_i	Unit shortage cost for non-delivered products in customer i	$\forall i \in I$
$t_{v_1v_2}$	Transportation time between vertices v_1 and v_2	$\forall v_1, v_2 \in V$
Rt_p	Installation time for customer p	$\forall p \in P$
E_i	Earliest time of customer i	$\forall i \in I$
L_i	Latest time of customer i	$\forall i \in I$
SL	Service level for installation vertices	
d_i	Demand of customer i	$\forall i \in I$
Q_K	Capacity of vehicle k	$\forall k \in K$
CAP_j	Total amount of commodity available at depot j	$\forall j \in J$
$\overline{Ka_k}$	The predetermined average load per vehicle k	$\forall k \in K$
$\overline{Sa_s}$	The predetermined average installation done by vehicle s	$\forall s \in S$
γ_k	A weight coefficient showing the load imbalance in the objective function for vehicle k	$\forall k \in K$
β_s	A weight coefficient showing the load imbalance in the objective function for vehicle s	$\forall s \in S$
M	A large number	

Decision variables

$\lambda 1_{ik}$	Reaching time to customer i for delivery vehicle k	$\forall i \in I$
$\mu 1_{is}$	Reaching time to customer i for installation vehicle s	$\forall i \in I, s \in S$
$\lambda 2_{jk}$	Reaching time to depot j for delivery vehicle k	$\forall j \in J, k \in K$

$\mu 2_{js}$	Reaching time to depot j for installation vehicle S	$\forall j \in J, s \in S$
WK_i	Time of waiting of delivery vehicle at customer i	$\forall i \in I$
WS_p	Time of waiting of installation vehicle at customer p	$\forall i \in P$
$H_{v_1v_2k}$	All load delivered by vehicle k in the route from vertice v_1 to vertice v_2	$\forall v_1, v_2 \in V, k \in K$
$L1_K$	Total load delivered by vehicle k	$\forall k \in K$
$L2_s$	Total installation done by vehicle S	$\forall s \in S$
$x_{v_1v_2k}$	1 if delivery vehicle k travels from vertice v_1 to vertice v_2 ; 0, otherwise	$\forall v_1, v_2 \in V, k \in K$
y_j	1 if depot j is used; 0, otherwise	$\forall j \in J$
u_i	1 if demand of customer i is not satisfied; 0, otherwise	$\forall i \in I$
$z1_k$	1 if vehicle k is active; 0, otherwise	$\forall k \in K$
$z2_s$	1 if vehicle S is active; 0, otherwise	$\forall s \in S$

The mathematical modeling of the problem can be formulated as follows:

$$\text{Min} \sum_{j \in J} C_j y_j + \sum_{v_1 \in V} \sum_{v_2 \in V} \sum_{k \in K} TC_k t_{v_1v_2} x1_{v_1v_2k} + \sum_{v_1 \in V} \sum_{v_2 \in V} \sum_{s \in S} TC_s t_{v_1v_2} x2_{v_1v_2s} + \sum_{k \in K} \sum_{i \in J} \sum_{j \in I} FC_k x1_{ijk} \quad (1)$$

$$+ \sum_{s \in S} \sum_{i \in J} \sum_{j \in P} FC_s x2_{ijs} + \sum_{i \in I} \sum_{k \in K} WC_k \lambda 1_{ik} + \sum_{j \in J} \sum_{s \in S} WC_s \mu 2_{js} + \sum_{i \in I} SC_i (1 - u_i) \quad (2)$$

$$\text{Min} \sum_{k \in K} \gamma_k |L1_k - \overline{Ka}_k| z_k + \sum_{s \in S} \beta_s |L2_s - \overline{Sa}_s| z_s$$

$$\sum_{v_1 \in V} x_{v_1v_2k} = \sum_{v_1 \in V} x_{v_2v_1k} \quad \forall v_2 \in V, k \in K \quad (3)$$

$$\sum_{v_1 \in R} x_{v_1v_2s} = \sum_{v_2 \in R} x_{v_2v_1s} \quad \forall v_2 \in R, s \in S \quad (4)$$

$$\sum_{k \in K} \sum_{v_1 \in V} x_{v_1v_2k} = 1 \quad \forall v_2 \in I \quad (5)$$

$$\sum_{s \in S} \sum_{v_1 \in R} x_{v_1v_2s} = 1 \quad \forall v_2 \in P \quad (6)$$

$$\sum_{v_1 \in J} \sum_{v_2 \in I} x_{v_1v_2k} \leq 1 \quad \forall k \in K \quad (7)$$

$$\sum_{v_1 \in J} \sum_{v_2 \in P} x_{v_1v_2s} \leq 1 \quad \forall s \in S \quad (8)$$

$$\sum_{v_1 \in V} x_{v_1v_2k} \leq y_{v_2} \quad \forall v_2 \in J, k \in K \quad (9)$$

$$\sum_{v_1 \in R} x_{v_1v_2s} \leq y_{v_2} \quad \forall v_2 \in J, s \in S \quad (10)$$

$$x_{v_1v_2k} \leq z1_k \quad \forall v_1, v_2 \in V \quad (11)$$

$$x_{v_1 v_2 s} \leq z_s \quad \forall v_1, v_2 \in V \quad (12)$$

$$\sum_{v_1 \in I} \sum_{v_2 \in V} d_{v_1} x_{v_1 v_2 k} \leq Q_k \quad \forall k \in K \quad (13)$$

$$\lambda 2_j \geq \lambda 1_i + WK_i + t_{ij} + M \left(\sum_{k \in K} x_{ijk} - 1 \right) \quad \forall i \in V, j \in I \quad (14)$$

$$\mu_j \geq \mu_i + Rt_i + WS_i + t_{ij} + M \left(\sum_{s \in S} x_{ijs} - 1 \right) \quad \forall i \in R, j \in P \quad (15)$$

$$\lambda 2_{jk} \geq \lambda 1_{ik} + WK_i + t_{ij} + M(x_{ijk} - 1) \quad \forall i \in I, j \in J, k \in K \quad (16)$$

$$\mu 2_{js} \geq \mu 1_{ik} + Rt_i + WS_i + t_{ij} + M(x_{ijs} - 1) \quad \forall i \in P, j \in J, s \in S \quad (17)$$

$$E_i \leq \lambda 1_{ik} + WK_i \leq L_i \quad \forall i \in I, k \in K \quad (18)$$

$$E_i \leq \mu_{ik} + WS_i + Rt_i \leq L_i \quad \forall i \in P, k \in K \quad (19)$$

$$\lambda 1_{ik} + wk_i \leq \mu 1_{ik} + ws_i \leq SL + \lambda 1_{ik} + wk_i \quad \forall i \in P, k \in K \quad (20)$$

$$\sum_{v_1 \in I} d_i \left(\sum_{v_2 \in J} x_{v_1 v_2 k} \right) z_k = L_k \quad \forall k \in K \quad (21)$$

$$\sum_{v_1 \in P} d_{v_1} \left(\sum_{v_2 \in J} x_{v_1 v_2 s} \right) z_s = L_s \quad \forall s \in S \quad (22)$$

$$\sum_{v_2 \in I} \sum_{k \in K} x_{v_1 v_2 k} H_{v_1 v_2 k} \leq CAP_{v_1} \quad \forall j \in J \quad (23)$$

$$\sum_{v_1 \in V} \sum_{k \in K} x_{v_1 v_2 k} = u_{v_2} \quad \forall v_2 \in I \quad (24)$$

$$\lambda 2_{js} = \mu 2_{js} = Rt_j = WK_j = WS_j = 0 \quad \forall j \in J, s \in S \quad (25)$$

$$x_{v_1 v_2 k}, y_j, u_i, z_k, z_s \in \{0, 1\} \quad \forall i \in I, j \in J, k \in K, s \in S \quad (26)$$

$$\lambda 1_{ik}, \lambda 2_{jk}, \mu 1_{ik}, \mu 2_{jk}, WK_i, WS_i, H_{ijk}, L_k, L_s \geq 0 \quad \forall i \in I, j \in J, k \in K, s \in S \quad (27)$$

The objective function (1) includes the summation of fixed costs of the vehicles and the depots, transportation costs, and labor costs. Also, the last part of the objective function calculates the shortage cost caused by undelivered demands. The objective function (2) minimizes the imbalance of loading for delivery and installation vehicles.

Constraints (3), (4) guarantee that the same vehicle leaves and enters each customer's location. Constraints (5), (6) ensure that each customer can serve by only one delivery vehicle, and each customer with installation need can serve by only one installation vehicle. In other words, the demand of each customer is only met by one vehicle, otherwise, the shortage happens. Constraints (7), (8) specify that a vehicle is allocated to a depot or not. Constraints (9), (10) state that if a depot is open, then a vehicle can be allocated to it. Constraints (11), (12) are used to ensure that just active vehicles can serve the customers. Constraint (13) determines the limitation on the capacity of delivery vehicles. Constraints (14)-(17) are the logical time constraints for the time of service, waiting, and transportation between vertices (a time for service is considered for just the nodes with installation requirements). Constraints (18), (19) impose the time window constraints for each customer. The interval between delivery of products to customer and installation service is considered as the service level which is shown in constraint (20). Constraint (21) is related to the calculation of total loads for delivery vehicles and constraint (22) is related to the calculation of total installation needs satisfied by each installation vehicle. Constraint (23) ensures that all the loads of vehicles leaving each depot shouldn't exceed the capacity of that depot. Constraint (24) specify the demands of a customer is satisfied or not. Constraint (25) determines the times related to the depots. Furthermore, constraints (26) and (27) are domain constraints.

3.1 Model Linearization

The second objective function that is for load imbalance in Eq. (2) is nonlinear because of the existence of an absolute value in this objective function. For linearizing this objective, a technique of variable transformation is applied. Eqs. (28)-(42) demonstrate the linearization form of this objective function.

$$L2_s - \overline{Sa_s} = q_s - p_s \quad (28)$$

$$q_s \leq Mw_s \quad (29)$$

$$p_s \leq M(1-w_s) \quad (30)$$

$$Ja_s \leq q_s + p_s \quad (31)$$

$$Ja_s \leq Mz_s \quad (32)$$

$$Ja_s \geq q_s + p_s - M(1-z_s) \quad (33)$$

$$t_s = \sum_{s \in S} \beta_s Ja_s \quad (34)$$

$$z_s, w_s \in \{0,1\} \quad (35)$$

$$q_s, p_s \geq 0 \quad (36)$$

$$L1_k - \overline{ka_k} = q_k - p_k \quad (37)$$

$$q_k \leq Mw_k \quad (38)$$

$$p_k \leq M(1-w_k) \quad (39)$$

$$Ja_k \leq q_k + p_k \quad (40)$$

$$Ja_k \leq Mz_k \quad (41)$$

$$Ja_k \geq q_k + p_k - M(1-z_k) \quad (42)$$

$$z_k, w_k \in \{0,1\} \quad (43)$$

$$q_k, p_k \geq 0 \quad (44)$$

Eq. (2) could be replaced with Eq. (45).

$$\text{Min} \sum_{s \in S} \beta_s Ja_s + \sum_{k \in K} \beta_k Ja_k \quad (45)$$

3.2 An improved version of the augmented ϵ -constraint method

This paper deals with a bi-objective MDVRPTW with delivery and installation. The nature of the problem motivates us to apply the multi-objective approaches. In the multi-objective mathematical programming instead of an optimal solution, there is a set of Pareto frontier of the efficient solutions because an improvement in one of the objective functions may cause the other objectives to get worse. Pareto solutions are the compromise solutions for the conflicting objectives with reasonable trade-offs among the objectives. The set of all the Pareto solutions is known as the Pareto frontier. Two approaches used in this paper are the augmented ϵ -constraint method (AUGMECON2) for the small size cases problems and the MOSA meta-heuristic for the large size cases. AUGMECON2 provides efficient solutions for multi-objective problems with conflicting objectives; however, when the size of the problem increases, because of its complexity, the elapsed time increases exponentially and it takes a lot of time for running. As such, for the large size problems, MOSA is

applied. It is worth noting that MOSA is an efficient algorithm for solving VRPTW problem which is applied in different studies (Ariyani et al., 2018; Chen and Shi, 2019; Rabbouch et al., 2020)

For solving a multi-objective problem by ε -constraint method, the aim of the model is the optimization of one of the objective functions and the residual objective functions are treated as constraints (Khalili-Damghani et al., 2014). For avoiding the weakly efficient solutions in the ε -constraint method, the improved version of it (AUGMECON) is proposed by Mavrotas (2009). Another version of the augmented ε -constraint method presented by Mavrotas and Florios (2013) is AUGMECON2 that provides the exact Pareto front. In the AUGMECON2 the multi-objective problem is formulated as follows:

$$\begin{aligned}
 & \text{Max } (f_1(x) + \text{eps} \times (S_2 / r_2 + 10^{-1} \times S_3 / r_3 + \dots + 10^{-(p-2)} \times S_p / r_p)) \\
 & \text{st} \\
 & f_2(x) - S_2 = e_2 \\
 & f_3(x) - S_3 = e_3 \\
 & \dots \\
 & f_p(x) - S_p = e_p \\
 & x \in S \\
 & S_i \in R^+
 \end{aligned} \tag{46}$$

The flowchart of the AUGMECON2 is shown in Figure 2.

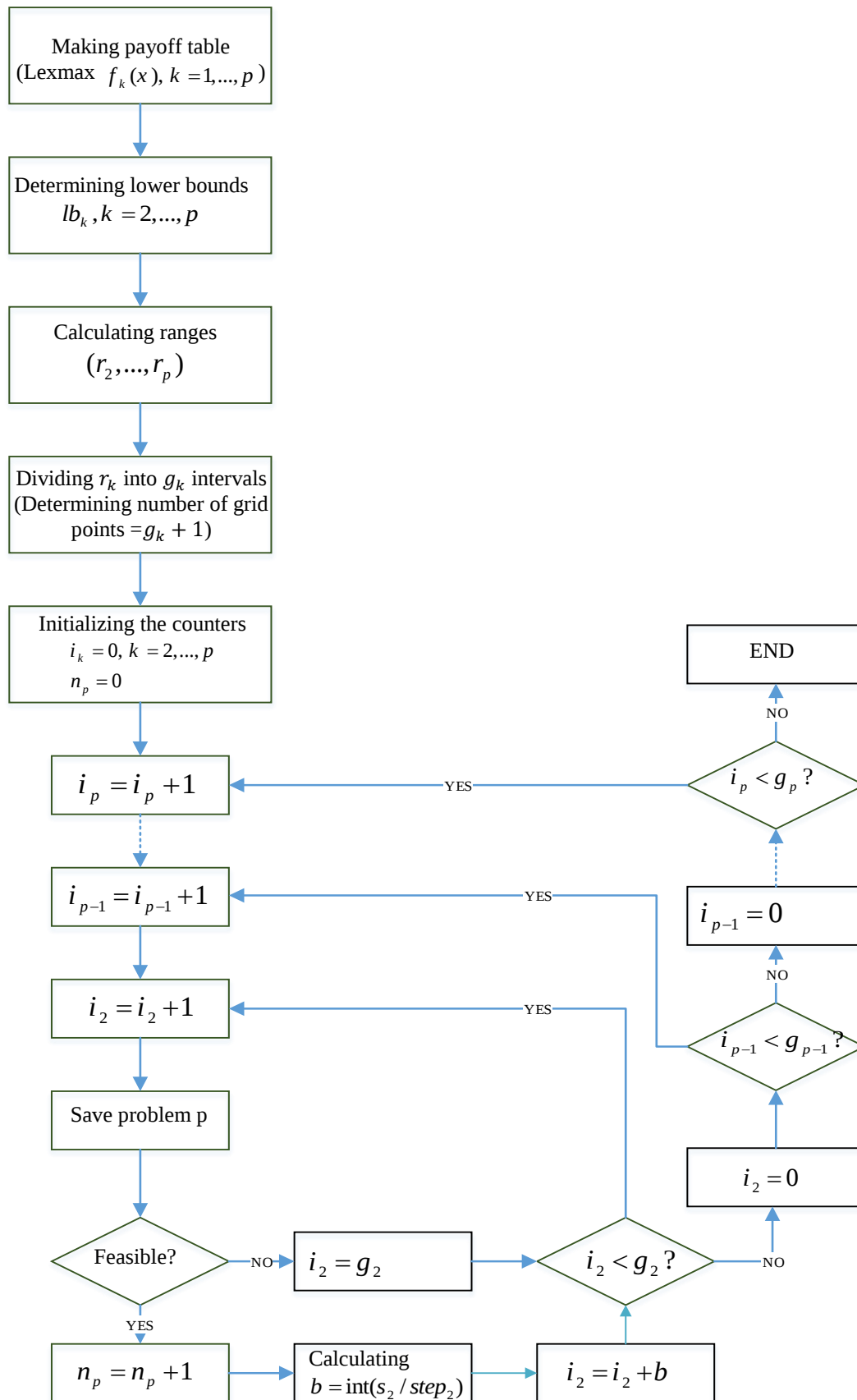


Figure 2. Flow chart of the AUGMECON2 method

3.3 Multi-objective simulated annealing (MOSA)

Simulated annealing is a fast local search evolutionary algorithm. The basis of this algorithm is the annealing process in the industry that the cost of the objective function is related to the free energy of the annealing process and searching the neighborhood space is related to the disturbance in changing states. The solutions are controlled by the algorithm like the cooling process, and the searching algorithm reaches the ultimate solution when the state is frozen (Farrokhi-Asl et al., 2019).

The multi-objective simulated annealing is utilized by Farrokhi-Asl et al. (2019). The standard procedure of this method is like the single objective SA, but it uses different pre-determined weight vectors related to different annealing processes (Rabbani et al., 2016).

3.3.1 Solutions representation and initialization

In the MOSA process, an initial solution generated randomly is needed to start searching for the neighborhood space. For example, if there are 8 customers (3 of them need installation), 2 depots, 3 delivery vehicles, and 1 installation vehicle; solution representation of the problem is shown in Figs. 3 to 5. We specify three strings for the problem, each of the strings has two parts. The first part of string 1 (q_1) is for assigning the customers with delivery needs to delivery vehicles, and its second part (q_2) is for the dedication of the customers with installation needs to the installation vehicle. String 2 is for assignment of the customers to depots, that its first and second parts (q_{01}, q_{02}) show dedicating delivery and installation customers to depots relatively. String 3 is for assigning the vehicles to the depots that its first part (q_{v1}) is for assignment of the delivery vehicle and its second part (q_{v2}) for installation vehicle. As an example, according to Figs. 3 and 5, the first vehicle starts its path from the second depot and services the customers {1, 3}. In the same way, the routing of the second and third vehicles that starts from the second depot, is {8, 4, 5, 7} and {6, 2} respectively. {11, 13, 12} are the customers with installation needs served by the installation vehicle.

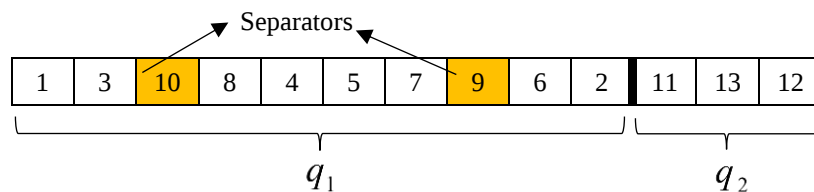


Figure 3. Example of a string for allocating customers to the vehicles

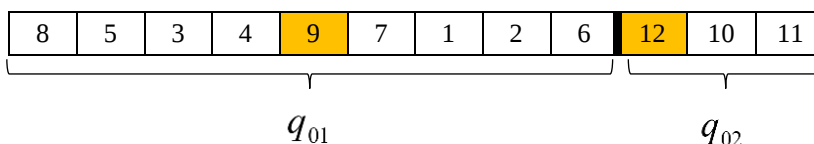


Figure 4. Example of a string for allocating customers to the depots

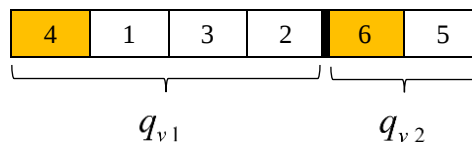


Figure 5. Example of a string for allocating vehicles to the depots

3.3.2 Creating new solutions

After generating the initial solutions, the MOSA algorithm pursues the following steps for searching the neighbourhood space and finding the new solutions. It is considerable that generating the new solutions from the current solution is done by swapping, inversion, and reversion operators.

- Step 1.* Initialize the algorithm (initial temperature, initial solution, $r=0$)
Step 2. Choosing a local search engine.
Step 3. Generating new solutions from the current solution.
Step 4. If the new solution has the acceptance condition, accept it.
Step 5. $r=r+1$. If $r < 100$ go to step 2, otherwise; go to step 6.
Step 6. Decline the temperature and put $r=0$.
Step 7. If the stopping condition is not satisfied, go to step 2; otherwise, stop

4. Real case study

The suggested approaches are customized for a home appliances company in Iran established in 1994, and now it has more than 1000 branches in different provinces. This company is an official representative for audio and video, health care, lighting, and solar power products. The logistic part of this company not only provides services such as storage and delivery, but also does it have after selling, installation services, and repair. As a result, this company tries to optimize transportation and product flow from different branches to the final customers. Different mechanized depots are utilized with characteristics such as 40,000 m^2 of total depot space, 27000 m^2 useful area of the depots, the possibility of loading and unloading of products by multiple vehicles, and the useful storage capacity of 60,000 m^3 of goods and products (equivalent to 26,000 pallets).

In this paper, 10 provinces are considered for investigation which is shown in Fig. 6. Characteristics of each case are given in Table 2. The statistics show the high demands of the customers in different provinces and the company has multiple depots in each province based on the population. The services of the company to the customer may be delivery of the home appliance and installing of them. The company aims to assign the customers to the appropriate depot and to find the optimal routes for each servicing vehicle.

For small size cases (i.e., cases 1 to 5) the bi-objective model is solved by AUGMECON2 proposed in section 3.2, but for provinces with a high population (cases 6 to 10), the size of the problem increases, so we use the MOSA algorithm proposed in Section 3.3 to deal with the complexity of the problem. To find the computational limits of the proposed solution approach, the model has been tested in the larger sizes to find out whether it could be solved in a reasonable computational time or not. It is observed that the running time depends more on the number of depots and vehicles. The proposed AUGMECON2 method has been coded in GAMS 24.8.3 environment using CPLEX solver, and the MOSA algorithm has been coded in MATLAB software. The codes are implemented on a standard core i7 PC with 2.40 GHz and 8.00 GB RAM.

Table 2. Characteristics of different cases

cases	1	2	3	4	5	6	7	8	9	10
Number of depots	2	2	2	3	3	3	4	4	4	5
Number of customers with delivery needs	8	10	14	20	25	30	40	50	60	70
Number of customers with installation needs	4	6	8	8	14	18	23	28	31	39
Number of delivery vehicle	3	3	4	4	5	5	6	7	7	8
Number of installation vehicle	1	1	2	2	3	3	4	4	5	5

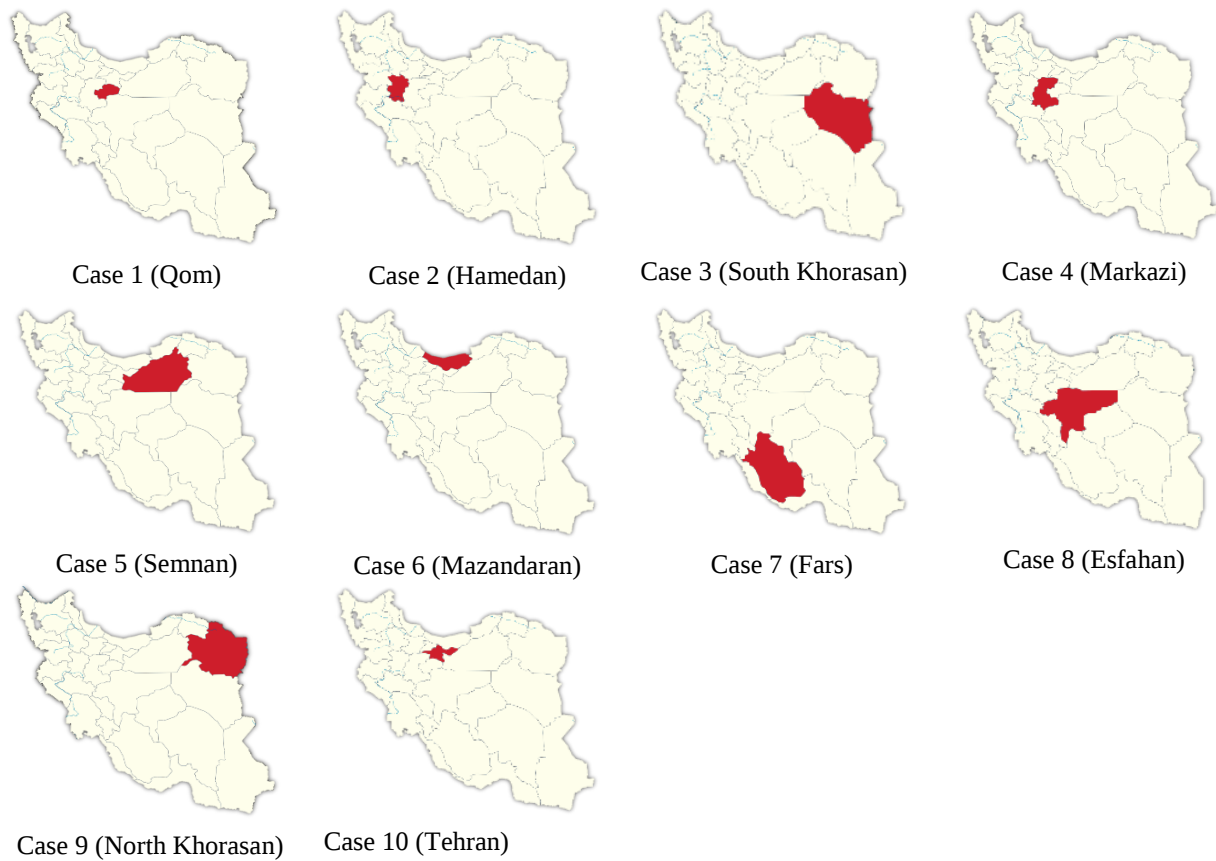


Figure 6. Different case studies of the problem

As an aforementioned, AUGMECON2 and MOSA are used as multi-objective methods for finding promising solutions of vehicle routing problem in small and large size case studies. The results are as follows. As an example, the generated efficient solution of the AUGMECON2 method for the first case (Qom province) is shown in Table 3. These efficient solutions (i.e., Pareto solutions) are shown in a Fig .7 as a Pareto front.

For cases 6 to 10, for the reason of large scales of the problems, the MOSA is applied as a multi-objective method for solving the vehicle routing problems of the proposed company. This method also gives us approximated Pareto front instead of the optimal solution. The Pareto front of the MOSA method for the case 10 (Tehran province) is shown in Fig. 8.

Table 3. Efficient solutions of the AUGMECON2 method for case 1

Efficient solutions	First objective function	Second objective function	Efficient solutions	First objective function	First objective function
1	83173.21	123	11	108424	85
2	83897.41	108	12	88052.43	100
3	85042.03	106	13	102823.8	91
4	87100.69	103	14	94183.68	93
5	90746.95	97	15	94153.68	92
6	91987.84	95	16	87520.88	103
7	96291.73	93	17	98392.63	90
8	100094.2	92	18	98782.63	89
9	104543	90	19	89758.43	97
10	105701.7	88			

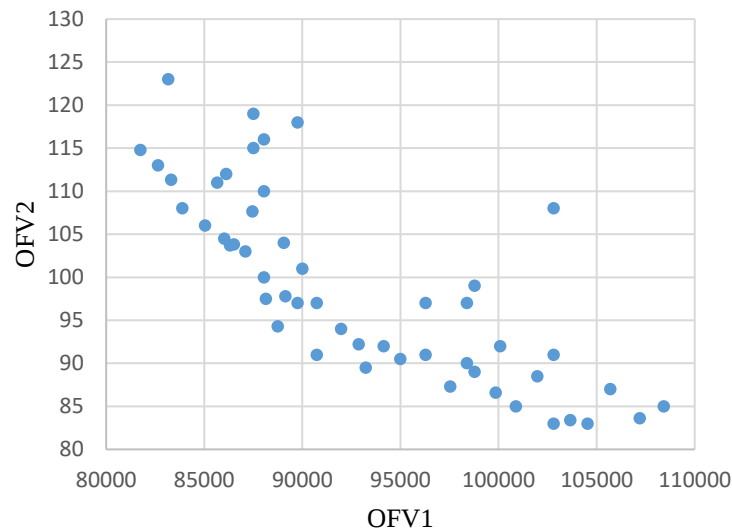


Figure 7. The Pareto front of the AUGMECON2 method for case 1

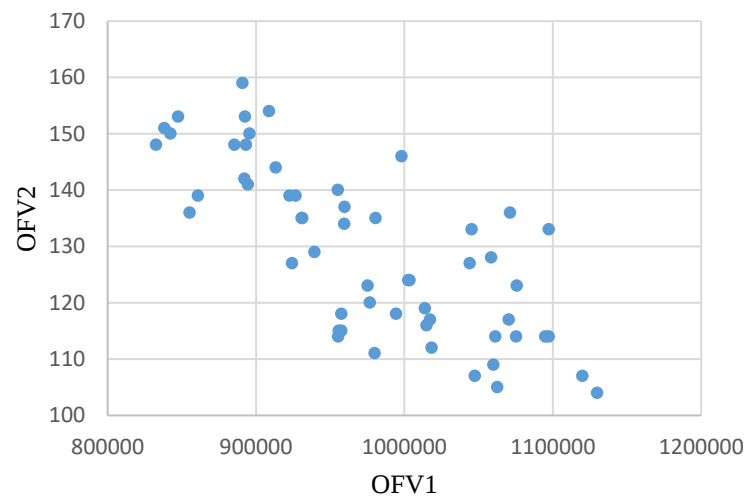


Figure 8. The Pareto front of the MOSA method for case 10

It should be noted a large number of efficient solutions for the bi-objective problem is generated by MOSA and AUGMECON2 methods that may be confusing for the decision maker to select the most promising ones. TOPSIS method as a multi attribute decision making (MADM) technique is applied for choosing the most preferred efficient solution. TOPSIS is a ranking MADM technique proposed by Yoon and Hwang (1995). Ranking the alternatives in TOPSIS technique is based on both the closeness of the alternatives to the positive ideal solution and their remoteness from the negative ideal solution. In the situation that there are m alternatives and n criteria, the steps of the TOPSIS technique is as follows:

Step 1. Making the decision matrix. In this problem, m alternatives are obtained efficient solutions and n criteria are the objective functions. x_{ij} Stands for the score of the i -th efficient solution with respect to j -th criterion.

$$DM = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{bmatrix} \quad (47)$$

Step 2. Making the normalized decision matrix (R).

$$R = [r_{ij}]_{m \times n} \quad (48)$$

where

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (49)$$

Step 3. Making the normalized weighted decision matrix (V).

$$V = [v_{ij}]_{m \times n} = r_{ij}(\cdot) w_j \quad (50)$$

where w_j is the weights of the criteria.

$$W = [w_1, w_2, \dots, w_n]$$

Step 4. Specifying the positive ideal solution (A^+) and the negative ideal solution (A^-) as (51) and (52) respectively.

$$A^+ = [v_1^+, v_2^+, \dots, v_n^+], \quad (51)$$

$$A^- = [v_1^-, v_2^-, \dots, v_n^-] \quad (52)$$

where $v_j^+ = \max v_{ij}$ for benefit criteria (53)

$$v_j^+ = \min v_{ij} \quad \text{for cost criteria}$$

and $v_j^- = \min v_{ij}$ for benefit criteria (54)

$$v_j^- = \max v_{ij} \quad \text{for cost criteria}$$

Step 5. Calculating the distance of each alternative from the most negative solution and positive ideal solution as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, i = 1, \dots, m \quad (55)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, \dots, m \quad (56)$$

Step 6. Calculating the closeness coefficient for each of the alternatives as follows:

$$CC_i = \frac{D_i^-}{D_i^- + D_i^+}, i = 1, \dots, m \quad (57)$$

The alternative with the highest closeness coefficient is the most preferred alternative. Applying the TOPSIS technique on the efficient solutions of our problems leads to the most preferred efficient solutions. For example for the case 1, its efficient solutions were in Table 3, that these efficient solutions are the first candidates for TOPSIS. The weights of the two objective functions are considered 0.6 and 0.4, respectively. Table 4 presents the rankings and distances from the negative and positive ideal solutions for the efficient solutions of the first case. Based on the TOPSIS method, the efficient solution 19 is the most preferred efficient solution.

Table 4. TOPSIS ranks

Efficient solutions	D^+	D^-	CC_i
1	0.035958131	0.0367628	0.043351
2	0.021789656	0.0384261	0.054723
3	0.020057003	0.0376515	0.055949
4	0.017966979	0.0363586	0.057393
5	0.015828077	0.0356041	0.059363
6	0.015944763	0.035702	0.059279
7	0.020544891	0.0334347	0.053115
8	0.025510371	0.0317423	0.047544
9	0.03147015	0.0317339	0.043056
10	0.032922036	0.0333556	0.043157
11	0.036762835	0.0359581	0.042402
12	0.01587237	0.0367878	0.059907
13	0.029167311	0.031359	0.04443
14	0.017727813	0.0351528	0.057006
15	0.017304497	0.0359465	0.057887
16	0.018170931	0.0358377	0.056902
17	0.022657582	0.0344733	0.051745
18	0.023038935	0.0351019	0.051773
19	0.014861368	0.0366579	0.061017

After applying TOPSIS method for small size case studies, the final solution of the cases 1 to 5 is shown in Fig. 9. For the large size cases, TOPSIS is applied on the MOSA efficient solutions and the routing plan for the case 10 (Tehran province) is shown in Fig. 10.

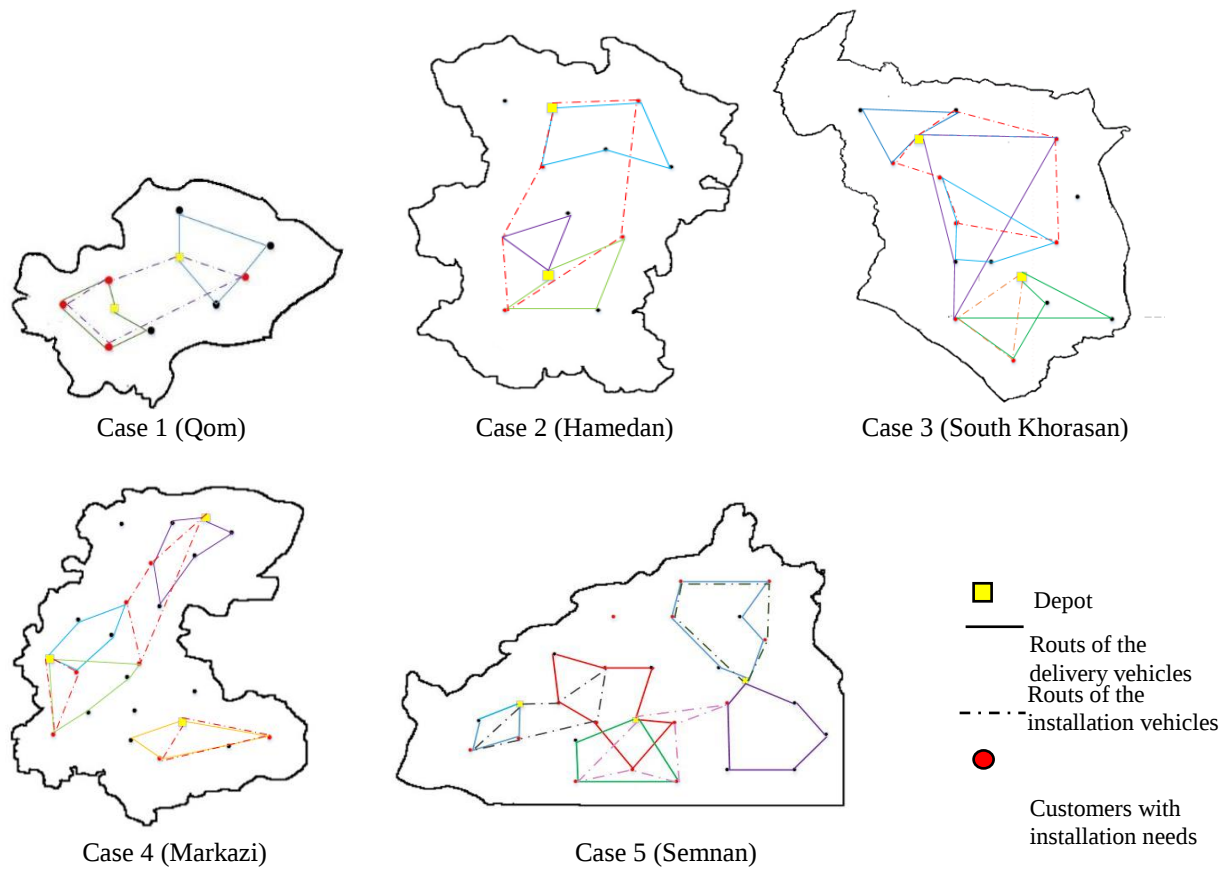


Figure 9. The final routing plan of the small size cases for MDVRPTW

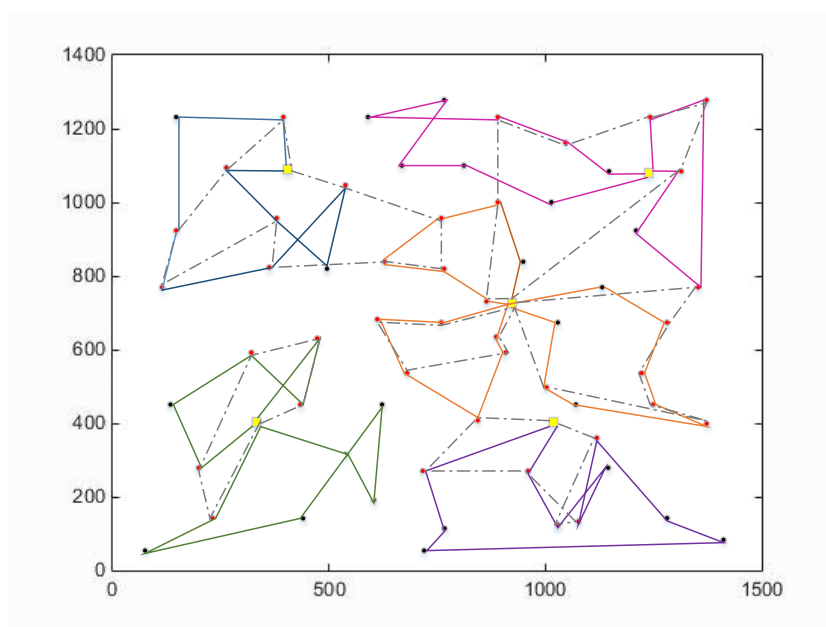


Figure 10. The final routing plan of the case 10 (Tehran province)

5. Conclusion and Implications

Several sensitivity analyses on the first case (Qom province) are executed to find the effective parameters on the objective functions. The effects of the average demand and shortage cost are investigated on the two objective functions and the number of unsatisfied demands as shown in Figs .11 to 18. Additionally, Figs. 11 and 12 demonstrate the variation of the objective functions when the average demand changes.

It can be seen that increase in average demand makes the first objective function worse, because of its influence on the transportation cost, shortage cost, and the number of active vehicles. Clearly, the second objective function gets worse too, because by increasing in demand points some of the vehicles may have an excessive load for satisfying customers' needs and this causes an imbalanced vehicle load. The increases in the objective functions are up to the point that none of the demands are satisfied, from this point, the objective functions are without change.

When the shortage cost increase, the vehicles try to satisfy the demands of more customers, so the transportation cost, active vehicle cost, and imbalance load between the vehicles increases. These variations continue until all the demands are satisfied. These influences of shortage costs on the objective function is shown in Figs. 13 and 14. Because the number of unsatisfied demands is an effective variable that has impressive influences on both objective functions, its variation versus changing the shortage cost and mean of demand are shown in Figs. 15 and 16.

As shown in Figs. 17 and 18, by considering a larger capacity for depots the number of satisfied demands increases that cause improvement in the first objective function.

There are some limitations in applying this research to real cases and different aspects of this study may be vague in real case studies, for example the proposed routes by this research may not be logical from the geographical point of view and the information of some depots in some branches was not available which caused difficulties in data gathering. This model can be applied for real logistics systems such as distribution of dairy products, home appliance, home healthcare problems and waste collection. By applying this model the managers will be able to decrease their shortage costs and help them better distribution of their products and facilitate their routing problems. There are some examples of real-life cases such as Tirkolaee et al. (2020) deal with a green vehicle routing problem with intermediate depots considering different urban traffic conditions, fuel consumption, time windows of services, and uncertain demand for perishable products. Besides, Babaei et al. (2019) applied a multi-trip vehicle routing problem with time windows specifically related to urban waste collection.

6. Conclusion

This paper presented a bi-objective multi-depot vehicle routing problem with time windows as an MINLP model. The objective functions of the model deal with the total cost and load imbalance between the two types of vehicles. In the situation that all of a customers' demands are not satisfied, a penalty is considered for unmet demands in the first objective function. Installation vehicle meets the customers that their delivery needs are satisfied already. Between the arrival time of delivery and installation vehicle, an interval known as the service level is considered. A case study for routing plan of a home appliance company in Iran is applied for 10 provinces. Based on the population of the provinces, the size of the problem differs. The size of 5 problems was small and the augmented ϵ -constraint method has been applied to the linearized model of them. Because of the NP-hard nature of the vehicle routing problems, when the cases get larger the complexity increased a lot and we have used multi-objective simulated annealing for large size problems. Also, large number of efficient solutions for the bi-objective problem was generated by MOSA and AUGMECON2 methods that were difficult for the decision-maker to select the most appropriate ones. TOPSIS method was applied for choosing the most preferred efficient solutions. After applying the TOPSIS method for case studies, the Pareto front of the non-dominated solutions and the final routing plans were demonstrated for small and large size cases. Moreover, several numerical experiments and sensitivity analyses were conducted to validate the presented model and the solution method.

There are some limitations in applying this research on real cases and different facets of this study may be vague in real case studies, for example the proposed routes by this research may not be logical from the geographical point of view and the information of some depots in some branches was not available which caused difficulties in data gathering. For the future researches applying clustering techniques for the customers to group them optimally based on some characteristics like their locations or demands is suggested.

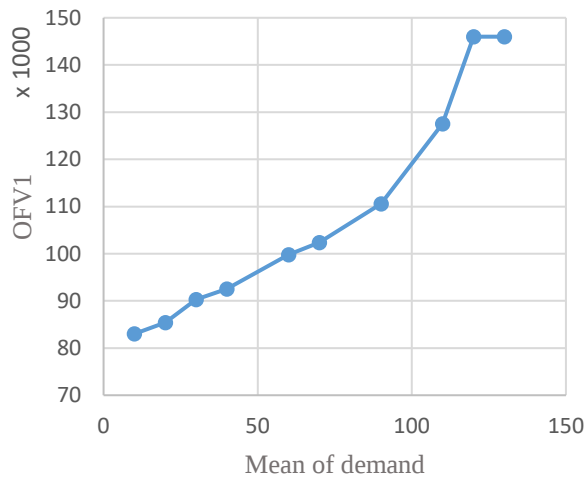


Figure 11. OFV1 vs. mean of demand

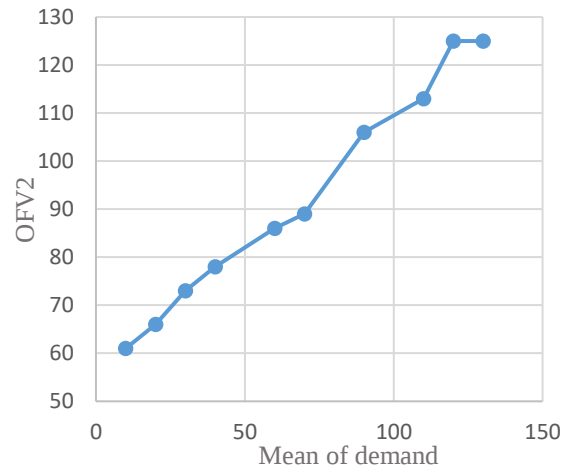


Figure 12. OFV2 vs. mean of demand

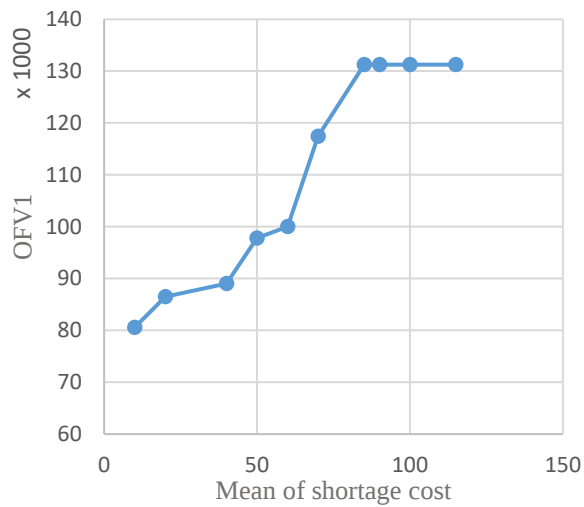


Figure 13. OFV1 vs. mean of shortage cost

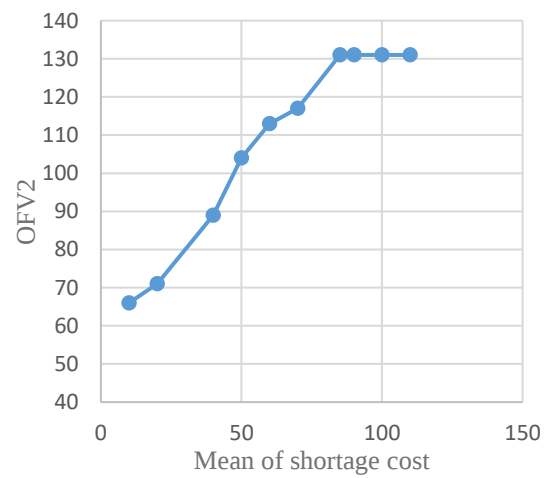


Figure 14. OFV2 vs. shortage cost

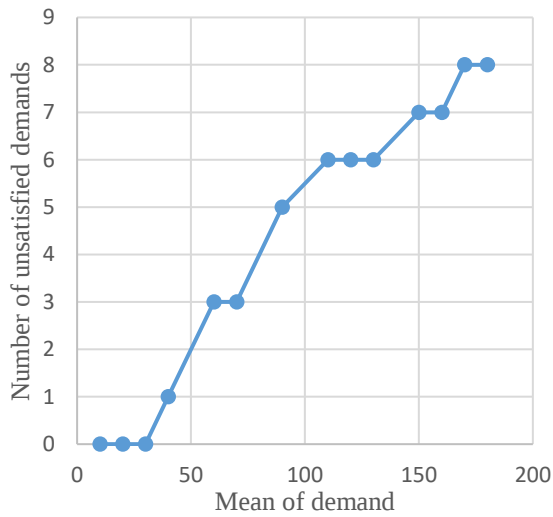


Figure 15. Number of unsatisfied demand vs. mean of demand

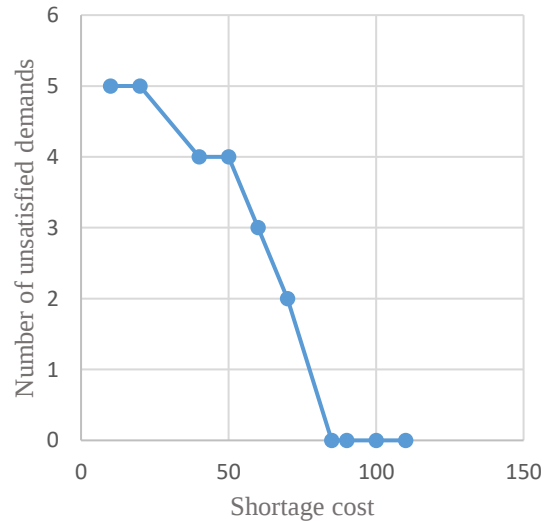


Figure 16. Number of unsatisfied demand vs. mean of demand

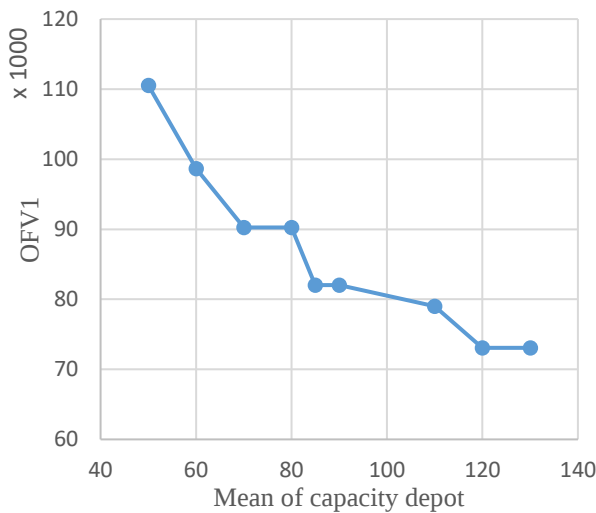


Figure 17. OFV1 vs. mean of capacity depot

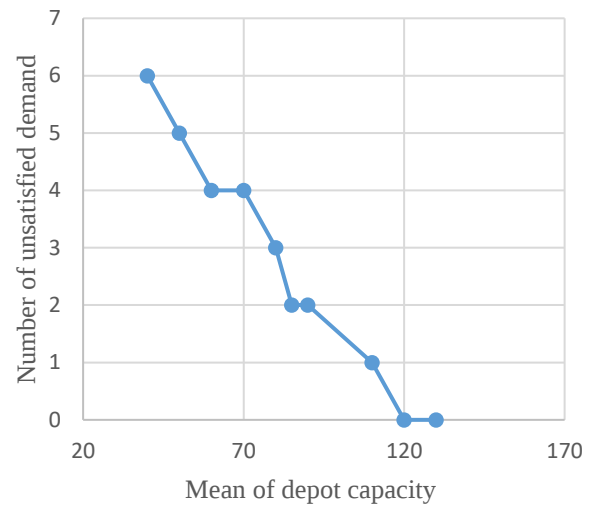


Figure 18. Number of unsatisfied demand vs. mean of depot capacity

References

- Afshar-Nadjafi, B., and Afshar-Nadjafi, A. (2017). A constructive heuristic for time-dependent multi-depot vehicle routing problem with time-windows and heterogeneous fleet. *Journal of king saud university-Engineering sciences*, Vol. 29(1), pp. 29-34.
- Ariyani, A. K., Mahmudy, W. F., and Anggodo, Y. P. (2018). Hybrid Genetic Algorithms and Simulated Annealing for Multi-trip Vehicle Routing Problem with Time Windows. *International Journal of Electrical & Computer Engineering (2088-8708)*, Vol.8, pp. 4713-4723.
- Azadeh, A., and Farrokhi-Asl, H. (2019). The close-open mixed multi depot vehicle routing problem considering internal and external fleet of vehicles. *Transportation Letters*, Vol. 11(2), pp. 78-92.
- Babaei Tirkolaee, E., Abbasian, P., Soltani, M., and Ghaffarian, S. A. (2019). Developing an applied algorithm for multi-trip vehicle routing problem with time windows in urban waste collection: A case study. *Waste Management & Research*, Vol. 37(1_suppl), pp. 4-13.
- Bae, H., and Moon, I. (2016). Multi-depot vehicle routing problem with time windows considering delivery and installation vehicles. *Applied Mathematical Modelling*, Vol.40(13-14), pp. 6536-6549.
- Bianchessi, N., Drexl, M., and Irnich, S. (2019). The split delivery vehicle routing problem with time windows and customer inconvenience constraints. *Transportation Science*, Vol.53(4), pp. 1067-1084.
- Chen, J., and Shi, J. (2019). A multi-compartment vehicle routing problem with time windows for urban distribution—A comparison study on particle swarm optimization algorithms. *Computers & Industrial Engineering*, Vol.133, pp. 95-106.
- Desaulniers, G., Errico, F., Irnich, S., and Schneider, M. (2016). Exact algorithms for electric vehicle-routing problems with time windows. *Operations Research*, Vol.64(6), pp. 1388-1405.
- Dharmapriya, U., Siyambalapitiya, S., and Kulatunga, A. (2010, January). Artificial intelligence computational techniques to optimize a multi objective oriented distribution operations. In *Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management Dhaka, Bangladesh*.
- Farrokhi-Asl, H., Makui, A., Ghousi, R., and Rabbani, M. (2019). Designing a sustainable integrated forward/reverse logistics network. *Journal of Modelling in Management*, Vol.14(4), pp. 896-921.

- Farrokhi-Asl, H., Makui, A., Jabbarzadeh, A., and Barzinpour, F. (2018). Solving a multi-objective sustainable waste collection problem considering a new collection network. *Operational Research*, Vol. 20, pp. 1-39.
- Farrokhi-Asl, H., Tavakkoli-Moghaddam, R., Asgarian, B., and Sangari, E. (2017). Metaheuristics for a bi-objective location-routing-problem in waste collection management. *Journal of Industrial and Production Engineering*, Vol.34(4), pp. 239-252.
- Ghannadpour, S. F., Noori, S., Tavakkoli-Moghaddam, R., and Ghoseiri, K. (2014). A multi-objective dynamic vehicle routing problem with fuzzy time windows: Model, solution and application. *Applied Soft Computing*, Vol.14, pp. 504-527.
- Ghomi, S. F., and Asgarian, B. (2019). Development of metaheuristics to solve a transportation inventory location routing problem considering lost sale for perishable goods. *Journal of Modelling in Management*, Vol. 14(1), pp. 175-198.
- Ghoseiri, K., and Ghannadpour, S. F. (2010). A hybrid genetic algorithm for multi-depot homogenous locomotive assignment with time windows. *Applied Soft Computing*, Vol. 10(1), pp. 53-65.
- Guezouli, L., and Abdelhamid, S. (2017). A new multi-criteria solving procedure for multi-depot FSM-VRP with time window. *International Journal of Applied Industrial Engineering (IJAIE)*, Vol. 4(1), pp. 1-18.
- Kaboudani, Y., Ghodsypour, S. H., Kia, H., & Shahmardan, A. (2018). Vehicle routing and scheduling in cross docks with forward and reverse logistics. *Operational Research*, Vol. 20, pp. 1-34.
- Kachitvichyanukul, V., Sombuntham, P., and Kunnapadeelert, S. (2015). Two solution representations for solving multi-depot vehicle routing problem with multiple pickup and delivery requests via PSO. *Computers & Industrial Engineering*, Vol. 89, pp. 125-136.
- Khalili-Damghani, K., Abtahi, A. R., and Tavana, M. (2014). A decision support system for solving multi-objective redundancy allocation problems. *Quality and Reliability Engineering International*, Vol. 30(8), pp. 1249-1262.
- Kim, K. C., Sun, J. U., and Lee, S. W. (2013). A HIERARCHICAL APPROACH TO VEHICLE ROUTING AND SCHEDULING WITH SEQUENTIAL SERVICES USING THE GENETIC ALGORITHM. *International Journal of Industrial Engineering*, Vol. 20, pp. 99-113.
- Koç, Ç., Bektaş, T., Jabali, O., and Laporte, G. (2016). The fleet size and mix location-routing problem with time windows: Formulations and a heuristic algorithm. *European Journal of Operational Research*, Vol. 248(1), pp. 33-51.
- Kritikos, M. N., and Ioannou, G. (2010). The balanced cargo vehicle routing problem with time windows. *International Journal of Production Economics*, Vol. 123(1), pp. 42-51.
- Lahyani, R., Coelho, L. C., and Renaud, J. (2018). Alternative formulations and improved bounds for the multi-depot fleet size and mix vehicle routing problem. *Or Spectrum*, 40(1), 125-157.
- Lee, T. R., and Ueng, J. H. (1999). A study of vehicle routing problems with load-balancing. *International Journal of Physical Distribution & Logistics Management*, Vol. 29(10), pp. 646-657.
- Majidi, S., Hosseini-Motlagh, S. M., Yaghoubi, S., and Jokar, A. (2017). Fuzzy green vehicle routing problem with simultaneous pickup–delivery and time windows. *RAIRO-Operations Research*, Vol. 51(4), pp. 1151-1176.
- Mavrotas, G. (2009). Effective implementation of the ϵ -constraint method in multi-objective mathematical programming problems. *Applied mathematics and computation*, Vol. 213(2), pp. 455-465.
- Mavrotas, G., and Florios, K. (2013). An improved version of the augmented ϵ -constraint method (AUGMECON2) for finding the exact pareto set in multi-objective integer programming problems. *Applied Mathematics and Computation*, Vol. 219(18), pp. 9652-9669.
- Molina, J. C., Salmeron, J. L., and Eguia, I. (2020). An ACS-based Memetic algorithm for the Heterogeneous Vehicle Routing Problem with time windows. *Expert Systems with Applications*, Vol. 157, pp. 113379.

- Rabbani, M., Mokhtarzadeh, M., and Farrokhi-Asl, H. (2018). A New Mathematical Model for Designing a Municipal Solid Waste System Considering Environmentally Issues. *International Journal of Supply and Operations Management*, Vol. 5(3), pp. 234-255.
- Rabbani, M., Ramezankhani, M. J., Farrokhi-Asl, H., and Farshbaf-Geranmayeh, A. (2015). Vehicle routing with time windows and customer selection for perishable goods. *International Journal of Supply and Operations Management*, Vol. 2(2), pp. 700-719.
- Capacitated Windy Rural Postman Problem with several vehicles: A hybrid multi-objective simulated annealing algorithm
- Rabbouch, B., Saâdaoui, F., and Mraïhi, R. (2020). Empirical-type simulated annealing for solving the capacitated vehicle routing problem. *Journal of Experimental & Theoretical Artificial Intelligence*, Vol. 32(3), pp. 437-452.
- Tirkolaee, E. B., Hadian, S., Weber, G. W., and Mahdavi, I. (2020). A robust green traffic-based routing problem for perishable products distribution. *Computational Intelligence*, Vol. 36(1), pp. 80-101.
- Wang, G., and Bian, W. (2016, July). Vehicle routing problem with simultaneous delivery and pick-up under different weights. In *2016 International Conference on Logistics, Informatics and Service Sciences (LISS)* (pp. 1-4). IEEE.
- Xiao, Y., Zuo, X., Kaku, I., Zhou, S., and Pan, X. (2019). Development of energy consumption optimization model for the electric vehicle routing problem with time windows. *Journal of Cleaner Production*, Vol. 225, pp. 647-663.
- Yoon, K. P., and Hwang, C. L. (1995). *Multiple attribute decision making: an introduction* (Vol. 104). Sage publications.