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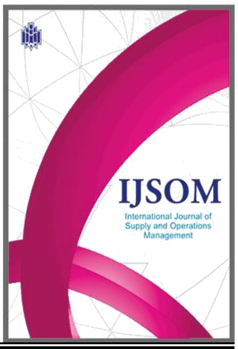

Coordinating a Socially Responsible Supply Chain with Random Yield under CSR and Price Dependent Stochastic Demand

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Abstract

Corporate social responsibility plays an important role in associating customers with socially responsible firms. Faithful consumers are willing to give extra money for commodities or services that incentive the firms to take corporate social responsibility (CSR). This article studies the coordination issue in a two-stage supply chain which is composed of a manufacturer and a retailer who sells a short shelf-life product in a single period. The manufacturer exhibits CSR and simultaneously determines its CSR investment and production quantity, as his production process is subject to random yield. On the other hand, the retailer decides the selling price and order quantity simultaneously while facing price and CSR sensitive stochastic demand. We construct an agreement between the retailer and the manufacturer which comprises a revenue-sharing along with cost-sharing contract. We show that the supply chain can perfectly coordinate under this composite contract and allow arbitrary allocation of total channel profit to ensure that both the retailer and the manufacturer are benefited. We further analyze the impact of randomness in production as well as the effect of CSR investment on the performance of the entire supply chain. A numerical example is provided to explain the developed model and gain more insights.

Keywords: Random yield; Demand uncertainty; Corporate social responsibility; Channel coordination; Pricing.

1. Introduction

One of the most essential concerns of today's supply chain management is to prevent the 'double marginalization' phenomenon (Spengler, 1950) because all the players want to take advantage of both competitive and cooperative relationships. Therefore, they individually seek to optimize their profits that usually lead to a situation where the players have different and sometimes conflicting objectives. For this reason, a supply chain needs collaboration of the members to remove the conflicting objectives among them. One of the interesting collaboration instruments to remove the conflictive objectives is a contract mechanism among the channel members. Contract mechanism induces the members in a decentralized supply chain to work as a centralized supply chain to improve the whole supply chain-wide profit. A contract with this efficiency has been called a 'perfect coordination contract' (Bernstein and Federgruen, 2005). A great amount of literature has discussed contract-based coordination with the help of popular contracts such as quantity discounts (Jeuland and Shugan, 1983; Mandal and Giri, 2019), quantity flexibility (Tsay, 1999; Xiong et al., 2011), buy-back policy (Pasternack, 1985; Ding and Chen, 2008), and so on. For a detailed survey on the contract mechanism, we refer readers to Cachon (2003) and Tsay et al. (1999).

A revenue sharing contract is commonly used in the video renting industries such as Hollywood Entertainment and Blockbuster Inc. (Giannoccaro and Pontrandolfo, 2009). It offers a buyer the right to buy a certain quantity of products at a comparatively lower wholesale price before the information on demand is settled, and gives a certain portion of his revenue to the supplier after selling season is over (Nezhad et al., 2015). Thus, by offering

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a revenue-sharing contract, the supplier bears some risks of uncertainty (like demand fluctuation) of the retailer. As a result, the contract incentives the retailer to purchase more products (Cachon and Lariviere, 2005). On the other hand, a cost-sharing contract is an agreement between supplier and buyer to share the cost of developing an intangible aid or service, such as promoting a product, production method, or advantage that affects both of them, directly or indirectly. Such an arrangement is used so that the entire cost of developing the aid may not have to be paid by a particular firm (He and Zhang, 2008;Fakhrzad et al., 2018).

To sustain in a competitive business environment, companies are adopting various modern technologies in their day to day business activities. These activities sometime may affect the environmental and social life of its stakeholders (Zamanian et al., 2020). Therefore, the companies should operate their businesses in a more socially responsible way to build up corporate goodwill of their stakeholders. Also, an ethical and environmentally conscious customer is willing to buy a product of a CSR company at a higher price. Therefore, CSR activities are becoming popular to both managers and the researchers. However, for companies, it is not easy to exhibit CSR activities into their business strategies. Pre-declared CSR activities may not catch all opportunities to benefit companies. On the other hand, postponed CSR approaches may lead to higher CSR costs when it is found that they have already violated social obligation. Exhibiting CSR activities on the upstream members influences sales and profit of the downstream members. For example, in 1996, sales and image of NIKE dropped down once it was found that a few of its subcontractors had been employed child labor (Gimenez and Tachizawa, 2012). However, the CSR activity is not a problem to a particular supply chain member. Only the morality of the chain members is not enough to exhibit CSR. It is, therefore, required to create an interest among the members to invest in CSR which ensures that each of them will be benefited from these activities.

In many industries with the same quantity input, the production process output varies significantly due to the influence of many factors. In most of the agricultural-based industries (where the weather is an important factor for production), accurate weather forecasting is impossible and as a result, production yield varies. In any high-tech manufacturing industry like LCD (liquid crystal display), semi-conductor, silicon chips, and so on, the quality of the product is uncertain due to small timing error or the presence of a small amount of dust contained in the air of the manufacturing area. In particular, almost all industries have somewhat the same phenomena concerning production randomness (Saha and Chakrabarti, 2018). However, it is not so clear how this yield randomness and secondary resources impact the decisions of the chain members.

Based on the practices mentioned above, in this paper, we study a two-tier supply chain comprising of a retailer and a manufacturer who exhibits social responsibility to increase company's goodwill. The retailer sells a single kind of perishable or seasonal product to satisfy stochastic market demand in a single period. The production process of the manufacturer is subject to random yield and the retailer faces a stochastic market demand which is price and CSR sensitive. Because of random production yield, a secondary source is introduced. A composite contract which has two components - revenue-sharing and cost-sharing is proposed to coordinate the supply chain. The primary objective of this study is to find the answer of the following questions:

- How do demand and yield uncertainties impact the decisions of the supply chain?
- Is only the morality of the chain members enough to exhibit CSR?
- How does the presence of secondary source affect the performance of the supply chain?
- How to remove the conflictive objectives of the channel members to coordinate the supply chain?

2. Literature review

Many studies have shown the advantages of CSR activities in business. Carter and Jennings (2002) pointed out the direct and positive effect of CSR activities on the performance of the supply chain. Cramer (2008) traced out a blueprint for the guidance of managers in selecting their suitable ways to implement CSR activities into their companies. Hsueh and Chang (2008) proposed that channel-wide efficiency can be archived by a suitable allocation of CSR investment among the chain members. But it is a critical problem to ensure that all the chain members exhibit their CSR activities in a responsible manner. In the scenario of implementing CSR initiatives, Boyd et al. (2007) claimed that unnecessary surveillance could be inefficient and damage supplier-buyer partnership and would not actually enhance compliance; rather, visibility and loyalty contribute to enhance supply chain performance. Amaeshi et al. (2008) indicated that it is the responsibility of the stronger firm in a supplier-buyer partnership to exhibit certain ethical influence on the weaker firm. They lighted up the use of organizational culture, code of ethics, anti-pressure community programs, employee training programs, and quality reorientation as potential sources of possessing good ethical impact within the supply chain.

Ni et al. (2010) investigated CSR investment in a three-level supply chain facing a retail price and CSR sensitive deterministic demand. They adopted game-theoretical analysis to obtain the optimal investment in CSR under the whole-sale price only contract. As an extension of the previous work, Ni and Li (2012) considered a supply chain where each firm determined its own CSR choice and bearded the corresponding CSR cost. They adopted a game-theoretical approach to obtain the firm's optimal ordering decisions under CSR commitment. They established a win-win situation depending on both profitability and CSR performance under a coordination contract. Goering (2012) regarded a two-echelon supply chain in which each member is socially concerned about its consumer's welfare to enhance profitability as well as reputation. In order to achieve system-wide profit in a decentralized setting, he introduced a two-part tariff contract and realized that the equilibrium tariff for coordination varies from the traditional two-part tariff. Panda (2014) examined a socio responsible supply chain in which the manufacturer exhibits CSR activities and the retailer faces linear retail price dependent deterministic demand. They investigated that under a Stackelberg game, the revenue-sharing contract could achieve supply chain coordination except for the scenario when the retailer exhibits CSR activities. Panda et al. (2017) investigated the effect of CSR activities on a closed-loop supply chain. They observed that CSR activity generates grater profit than traditional profit-maximizing objective, and channel coordination is achieved through revenue sharing contract. Modak et al. (2016) considered a three-level supply chain that consists of a manufacturer undertaking CSR, multiple distributors, and multiple retailers using a new revenue-sharing contract in the context of CSR.

A lot of studies have emphasized the issues of random production yield and utilization of emergency source under production risk. In fact, emergency source improves channel performance even when there is no supply uncertainty (Bulinskaya, 1964). Among the early researchers, Silver (1976) considered the buyer's received quantity to be stochastically proportional to his order quantity and explained some reasons for supply uncertainty such as human errors in counting, scrap in a production run, quantities being rounded, pilferage or damage during transit, *etc*. Kazaz (2004) investigated the production planning as well as emergency resourcing of an olive oil manufacturer who experiences both uncertain demand as well as random yield in production. He demonstrated that the optimal quantity of production decreases under the presence of a secondary resource, where both the retail price and the purchasing cost depend on production yield. He and Zhang (2008) investigated the impact of random production yield in a two-echelon supply chain under various risk-sharing agreements, which was later extended by Heand Zhang (2010) by analyzing the secondary market's effects on supply chain decisions. Xu (2010) analyzed the production and procurement management problems in a newsvendor model and showed that the channel members are better off in the presence of option contract, where the manufacturer can either buy option contract from a supplier before the customer demand is realized or place an instant order at a higher price after the demand is realized under both stochastic demand and random production yield.

There are various works on coordinating a supply chain under demand uncertainty (Petruzzi and Dada, 1999; Yao et al., 2006) but they do not consider CSR investment. Hsueh (2014) first considered a socially responsible supply chain under stochastic demand and found that a new revenue-sharing contract able to coordinate the chain for an exogenous retail price. Zhao and Yin (2018) extended the work of Hsueh (2014) by considering endogenous retail price. They were able to achieve the supply chain coordination by using a modified revenue-sharing contract under CSR investment and retail price dependent stochastic customer demand, having a linear CSR investment and retail price dependent deterministic demand. Although Zhao and Yin (2018) dealt with demand uncertainty but they assumed deterministic yield in the production process of the manufacturer.

One widely used way to tackle random yield is to use a secondary resource. But there may be a situation where a manufacturer can't access a secondary resource to mitigate his yield risk. To the best of the authors' knowledge, the issue of coordination of a socially responsible supply chain with price and CSR investment dependent stochastic demand along with random yield in production process till remains to be tackled.

To narrow the research gap, we investigate a socially responsible supply chain that consists of a retailer and a manufacturer facing production yield uncertainty. The retailer faces retail price- and CSR-dependent stochastic demand. The centralized model of the supply chain is first analyzed as a benchmark model. We investigate two scenarios in the decentralized supply chain model. One scenario assumes the risk of randomness in both demand and production and the cost of CSR investment is not shared among the chain members. The other scenario assumes that the chain members share both the risk of uncertainty and the cost of CSR investment. In both scenarios, the optimal pricing and ordering strategy of the retailer, and CSR investment and wholesale price of the manufacturer are analyzed. The contribution of the paper with respect to the relevant existing literature is three-fold:

- We incorporate both uncertain demand and random production yield in a socio responsible supply chain.
- We analyze the effects of the CSR activity and the secondary resource, which provide guidance for managers to take action under different market scenarios.
- We also investigate the proposed supply chain's coordination problem. We design a contract mechanism that improves CSR activities as well as the whole supply chain's expected profit.

The remaining paper is structured as follows: Section 3 presents notation and assumptions for developing the proposed model. Section 4 discusses the centralized supply chain. Section 5 describes the decentralized model under no risk and cost-sharing contract. Section 6 illustrates the decentralized model under risk and cost-sharing contract and discusses two contracts - standard revenue-sharing contract and the revenue-sharing along with the cost-sharing contract. Section 7 is devoted to numerical analysis for theoretical support and gaining more managerial insights. In Section 8, the paper is concluded with some future research directions.

3. Problem description

We study a supply chain that consists of a retailer and a manufacturer. The retailer trades a seasonal product in a single period to satisfy uncertain demand. The CSR activities like health and education development, investment for environment protection, insurance for workers, *etc.* are undertaken by the manufacturer who faces random yield in production. In general, a higher CSR investment results in higher market demand. We consider a non-decreasing function $k\sqrt{\eta}$ as reward market demand where k is the customer's CSR sensitivity which is affected by a huge number of socio-cultural parameters and η is the CSR investment of the manufacturer. The stochastic market demand x, experienced by the retailer is a non-negative continuous random variable with the general distribution. Over the region [l,u], $f(\cdot)$ and $F(\cdot)$ represent the probability density function and cumulative distribution function respectively of the random demand with mean $\bar{x}$ and standard deviation σ_x. After forecasting the market demand x and knowing the manufacturer's contract, the retailer decides to place an order to the manufacturer for Q units of the final product. Because of random yield in production, the manufacturer sets a higher lot size Q_m. Suppose that the produced quantity is yQ_m where y is a random variable having cdf $G(\cdot)$ and pdf $g(\cdot)$ with mean $\bar{y}$ and standard deviation σ_y over the region [a, b], $0 \leq a \leq b \leq 1$. If the produced amount is less than the amount ordered, then there is no emergency resource to fulfill the order. But if the produced amount is more, the excess amount can be salvaged in a secondary market at a lower wholesale price. Therefore, the manufacturer must be very careful when he sets his production lot size Q_m. The manufacturer hands over the produced units to the retailer before the start of the selling period. Based on the contract agreement, the transfer payment is made. We assume symmetric information *i.e.,* at the start of the selling season, both the players have the full information. All the members associated in the supply chain are neutral and takes a rational decisions. Also, reordering is not possible. The variables used in our model are listed as given below:

x	stochastic customer demand with mean $\bar{x}$ and variance σ_x^2
y	random yield with mean $\bar{y}$ and variance σ_y^2
c_m	unit production cost of the manufacturer
c_r	unit handling cost of the retailer
g	unit goodwill loss of the retailer for unmet customer demand
v_r	unit salvage value of a residual product at the retailer
v_m	unit salvage value of leftover at the manufacturer
η	CSR expenditure of the manufacturer
p	unit retail price of the final product at the retailer
Q	order quantity of the retailer
Q_m	aimed production lot size of the manufacturer
w_m	unit wholesale price of the manufacturer

We will introduce more symbols whenever needed. To avoid trivial cases, the following restrictions are made: $v_m < c_m < w_m$; $v_r < w_m + c_r < p$; $c_r + \frac{Cm}{\bar{y}} < p$. The first two restrictions prevent the manufacturer and the retailer respectively from infinite production and assure that each of them makes a positive profit. And the last restriction corresponds that the system's per unit selling price is higher than expected per unit cost.

4. Centralized supply chain model

Conceptually here only one decision-maker is involved to maximize the system profit and the wholesale price charged by the manufacturer to the retailer could be viewed as a transfer of internal revenue. The whole supply chain's expected profit is given by

$$\begin{aligned}\Pi_c(Q,Q_m,p,\eta) &= pE[\min\{X,Q,yQ_m\}] + v_rE[(\min\{Q,yQ_m\}-x)^+] - gE[(x-\min\{Q,yQ_m\})^+ \\ &\quad - c_rE[\min\{Q,yQ_m\}] + v_mE[(yQ_m-Q)^+] - c_mQ_m \\ &= (p+g-v_r)E[\min\{X,Q,yQ_m\}] - (c_r-v_r+v_m)E[\min\{Q,yQ_m\}] \\ &\quad -(c_m-v_m\bar{y})Q_m - \eta - g\bar{x}\end{aligned} \tag{1}$$

We can rewrite above profit function as follows:

$$\begin{aligned}\Pi_c(Q,Q_m,p,\eta) &= (p+g-v_r) \\ &\times\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx\right.\right. \\ &+\left.\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\right)g(y)dy \\ &+\left.\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx+\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u}Qf(x)dx\right)g(y)dy\right\} - (c_r \\ &- v_r+v_m)\times\left\{\int_a^{\frac{Q}{Q_m}}yQ_mg(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\right\}-(c_m-v_m\bar{y})Q_m-\eta-g\bar{x}\end{aligned} \tag{2}$$

The following theorem characterizes the centralized benchmark model's optimal decisions.

Theorem 1. In the centralized model, the entire system's objective function is jointly concave in Q, Q_m, η and p, and the optimal order quantity Q^c, production decision Q_m^c, retail price $p^c$and CSR investment η^csatisfy the following equations:

$$Q = (\alpha-\beta p+k\sqrt{\eta}) + F^{-1}\left(\frac{c_r-v_r+v_m}{p+g-v_r}\right) \tag{3}$$

$$(p+g-v_r)\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy \;-(c_r-v_m+v_r)\int_a^{\frac{Q}{Q_m}}yg(y)dy = (c_m-v_m\bar{y}) \tag{4}$$

$$\begin{aligned}&\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx+\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u}(yQ_m)f(x)dx\right)g(y)dy\right. \\ &\quad\left.+\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}(\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx+\int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u}Qf(x)dx\right)g(y)dy\right\} \\ &= \beta(p+g-v_r)\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\right)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})}f(x)dx\right)g(y)dy\right\}\end{aligned} \tag{5}$$

And

$$\frac{K}{2\sqrt{\eta}}(p+g-v_r)\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\right)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\right)()g(y)dy\right\}=1 \tag{6}$$

where $F^{-1}(v)=inf\{u:F(u)=v\}$. From equation (3), we observe that the order quantity Q of the final product is affected by both exogenous and endogenous parameters related to the final product as well as the raw materials except the distribution of the demand, which is quite natural. After putting the optimal values of the decisions variables in (2) we get the channel profit $\Pi_c(Q^c,Q_m^c,p^c,y^c)$. Although, due to the complexity, we can't find the closed-form solution from $\Pi_c(Q^c,Q_m^c,p^c,\eta^c)$, with the help of Theorem 1, we can show that CSR investment increases the order quantity which results in a higher expected channel profit.

4.1 Centralized model without CSR

We now present the optimal decisions and maximal expected profit of the centralized decision model without CSR for comparison purpose. In this scenario, the expected profit function can be described as

$$\Pi_{c0}(Q,Q_m,p)=(p+g-v_r)\times\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx+\int_{yQ_m-(\alpha-\beta p)}^{u}(yQ_m)f(x)dx\right)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx+\int_{Q-(\alpha-\beta p)}^{u}Qf(x)dx\right)g(y)dy\right\}-(c_r-v_r+v_m)\times\left\{\int_a^{\frac{Q}{Q_m}}yQ_mg(y)dy+\int_{\frac{Q}{Q_m}}^{b}Qg(y)dy\right\}-(c_m-v_m\bar{y})Q_m-g\bar{x} \tag{7}$$

The following theorem characterizes the centralized benchmark model's optimal decisions.

Theorem 2. The whole supply chain's profit function Π_{c0} in the centralized model without CSR is jointly concave in Q,Q_m and p, and the optimal order quantity Q^{c0}, production decision Q_m^{c0} and retail price p^{c0} are the solutions of the following equations:

$$Q=(\alpha-\beta p)+F^{-1}\left(\frac{c_r-v_r+v_m}{p+g-v_r}\right) \tag{8}$$

$$(p+g-v_r)\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p)}^{u} yf(x)dxg(y)dy-(c_r-v_m+v_r)\int_a^{\frac{Q}{Q_m}}yg(y)dy=(c_m-v_m\bar{y}) \tag{9}$$

$$\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx+\int_{yQ_m-(\alpha-\beta p)}^{u}(yQ_m)f(x)dx\right)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p)}(\alpha-\beta p+x)f(x)dx+\int_{Q-(\alpha-\beta p)}^{u}Qf(x)dx\right)g(y)dy=\beta(p+g-v_r)\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p)}f(x)dx\right)g(y)dy+\int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p)}f(x)dx\right)g(y)dy\right\} \tag{10}$$

The entire system's expected profit functions given in (2) and (7), with and without CSR investment, respectively are too complicated to compare its properties. The optimal decision variables are implicitly calculated through functional equations. In Section 7, a numerical example is taken to compare the results.

4.2 Centralized model with CSR in the presence of a secondary resource

In this segment, we investigate the same model in the presence of a secondary resource for the manufacturer to mitigate his random production yield. Let c_m' be the purchase cost of an item from the secondary resource. Under this circumstance, the expected profit of the entire supply chain is described as

$$\Pi_{cs}(Q, Q_m, p, \eta) = (p + g - c_r - v_m)Q - (p + g - v_r) \times \int_l^{Q-(\alpha+\sqrt{\eta}k-\beta p)} F(x)dx$$
$$-(c_m' - v_m) \times Q_m \int_a^{\frac{Q}{Q_m}} g dy - Q_m(cm - v\bar{y}) - g_r\bar{X} - \eta. \quad (11)$$

The entire chain's optimum decisions satisfy the followings equations:

$$(p + g - c_r - v_m) - (p + g - v_r)F(Q) = (c_m' - v_m)G\left(\frac{Q}{Q_m}\right) \quad (12)$$

$$(c_m' - v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (cm - v\bar{y}) \quad (13)$$

$$Q - \int_l^{Q-(\alpha+\sqrt{\eta}k-\beta p)} F(x)dx = (p + g - v_r)\beta \int_l^{Q-(\alpha+\sqrt{\eta}k-\beta p)} f(x)dx \quad (14)$$

$$(p + g - v_r)\frac{K}{2\sqrt{\eta}}\int_l^{Q-(\alpha+\sqrt{\eta}k-\beta p)} f(x)dx = 1 \quad (15)$$

It is difficult to get analytical solution of the model. Most of the literatures have shown that the secondary resource has a positive impact on the supply chain (Lee and Whang, 2002). It is noted that the accessibility of the secondary resource is helpful to the supply chain in order to obtain a higher profit while production is suffering from random yield. The presence of the secondary market provides the manufacturer with more ways to overcome the production uncertainty and increases the supply chain's performance effectively. Moreover, the double marginalization in the decentralized supply chain is decreases in the presence of a secondary resource.

5. Decentralized model with price only contract

Although the integrated model provides the most system potency, it is far from the real business situation. In reality, supply chain entities are freelance decision makers and that they select the most effective decisions to maximize their individual profits. We currently think about a decentralized system wherever there's a price-only contract among the supply chain entities. The method flow is as follows. The manufacturer simultaneously decides its wholesale price w_m and CSR investment η first. Then, with the knowledge of demand uncertainty and wholesale price offered by the manufacturer, the retailer determines to buy Q units from the manufacturer and the manufacturer decides to produce Q_m units. The amount $min\{Q, yQ_m\}$ is shipped by the manufacturer to the retailer. Lastly, the market demand x occurs and the retailer trades the quantity $min\{X, Q, yQ_m\}$ to the end-customers. We consider a Nash sequence where the manufacturer is the first decision maker, and the system is solved through backward substitution. Therefore, the retailer first determines his optimal decisions. For, given $Q_m$and η, the retailer's profit function $\Pi_r(Q, p)$can be derived as follows:

$$\Pi_r(Q, p) = pE[\min\{X, Q, yQ_m\}] + v_rE[(\min\{Q, yQ_m\} - x)^+] - gE[(x - \min\{Q, yQ_m\})^+ - (w + c_r)E[\min\{Q, yQ_m\}]$$
$$= (p + g - v_r)E[\min\{X, Q, yQ_m\}] - (w + c_r - v_r)E[\min\{Q, yQ_m\}] - g\bar{x} \quad (16)$$

We can rewrite above profit function as given below:

$$\Pi_r(Q,p) = (p+g-v_r) \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x) f(x)dx + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x) f(x)dx + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Qf(x)dx \right) g(y)dy \right\} - (w + c_r - v_r) \left\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy \right\} - g\bar{x} \quad (17)$$

The retailer's optimal ordering and pricing decisions are provided in the following theorem.

Theorem 3. The retailer's objective function in the decentralized setting under wholesale price-only contract is concave in Q and p. The optimal order quantity Q and retail price p can be obtained from the following equations:

$$Q = (\alpha-\beta p+k\sqrt{\eta}) + F^{-1}\left(\frac{w+c_r-v_r}{p+g-v_r}\right) \quad (18)$$

$$\int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x) f(x)dx + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x) f(x)dx + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Qf(x)dx \right) g(y)dy = \beta(p+g-v_r) \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \right) g(y)dy \right\} \quad (19)$$

Comparing the retailer's optimal order quantity with that of the centralized system, we find that the retailer orders less. The manufacturer's self-interest bears the double marginalization impact by pricing higher than the production cost, and this is the only way by which he can gain a positive expected profit. Underneath this contract, the manufacturer doesn't have any incentive to cut back his wholesale value because the retailer takes order decisions with no assurance to the manufacturer.

After investigating the retailer's problem and getting the optimum decisions (Q^d, p^d), we tend to derive the manufacturer's expected profit function $\Pi_m(Q_m,\eta)$as follows:

$$\Pi_m(Q_m,\eta) = wE[\min\{Q, yQ_m\}] + v_m E[(yQ_m - Q)^+] - c_m Q_m - \eta$$
$$= (w - v_m)E[\min\{Q, yQ_m\}] - (c_m - v_m\bar{y})Q_m - \eta$$

which can be written as

$$\Pi_m(Q_m,\eta) = (w-v_m) \left\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy \right\} - (c_m - v_m\bar{y})Q_m - \eta \quad (20)$$

The following theorem characterizes the manufacturer's optimal production amount and CSR investment in the decentralized model under wholesale price-only contract.

Theorem 4. The objective function $\Pi_m(Q_m,\eta)$ of the manufacturer is concave in both Q_m and η and the optimal input amount Q_m satisfies the following equation

$$(w - v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (c_m - v_m\bar{y}) \tag{21}$$

The CSR investment η in the manufacturer's objective function is in linear form with a negative sign and there is no other term related to η. Therefore, it is optimal for the manufacturer not to invest in CSR.

Under the price-only contract, the manufacturer does not have any influence on the retailer's order quantity. The retailer also takes his decision without any promise to the manufacturer. The manufacturer takes the risk of his production uncertainties alone. In this situation, we consider that the wholesale price is negotiated based on the bargaining powers of the manufacturer and the retailer, keeping the gross margin higher than a desired level of acceptance. Now, we compare the above benchmark models in the following theorem.

Theorem 5. Both the order and production quantities in the decentralized model are strictly less than their counterparts in the centralized model. A lower-order amount results in a lower expected supply chain profit in the decentralized model.

The above Theorem is a generalization of the finding for the two-level supply chain that can be a step back model (Spengler, 1950), demonstrating that, in the decentralized environment, the total network channel output is not reached to its maximum profit even if all chain participants maximize their respective incomes. In the decentralized scenario where the decision power is distributed across the various chain participants, there is a possible deviation from the optimal decisions achieved under the centralized model. In order to align each member's decision with the entire channel, contract mechanisms come into play to prevent sub-optimization by removing members' rivalry without affecting the structure of the supply chain and the decision making powers of its members.

6. Coordination contract

Contract mechanism is a technique to attain coordination by modifying each player's expected profit exploitation in terms of trade parameters like valuation, order amount, quality among the players which also enhance the profit of the whole supply chain. A decentralized supply chain is claimed to be coordinated when it attains an equivalent potency as the centralized scenario in terms of profit. We currently utilize an appropriate contract so as to coordinate our decentralized model.

6.1 Revenue sharing contract

We first analyze the case where the manufacturer decides production quantity, CSR investment, and unit wholesale price which is less than its unit production cost. Depending on the manufacturer's decisions, the retailer decides its retail price and order quantity. At the end of the marketing season, the retailer keeps a fraction φ of his total revenue and returns $(1-\varphi)$ proportion to the manufacturer so as to compensate it's reduced wholesale price. Under this contract, for given retail price and order quantity, the expected profit of the manufacturer can be described as

$$\begin{aligned}\Pi_m^{ors}(Q_m,\eta) &= (1-\phi)pE[\min\{X,Q,yQ_m\}] + wE[\min\{Q,yQ_m\}] + v_mE[(yQ_m-Q)^+] - c_mQ_m - \phi\eta \\ &= (1-\phi)pE[\min\{X,Q,yQ_m\}] - (w-v_m)E[\min\{Q,yQ_m\}] - (c_m - v_m\bar{y})Q_m - \eta\end{aligned} \tag{22}$$

We can rewrite above profit function as given below:

$$\Pi_m^{ors}(Q_m,\eta) = (1-\phi)p \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x) f(x)dx + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x) f(x)dx + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Q f(x)dx \right) g(y)dy \right\} - (w - v_m) \times \left\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Q g(y)dy \right\} - (c_m - v_m\bar{y})Q_m - \eta \tag{23}$$

The optimal production lot size and CSR investment of the manufacturer satisfy the following equations:

$$(1-\phi)\times p\int_a^{\frac{Q}{Q_m}}\int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy \ - (w-v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (c_m - v_m\bar{y}) \tag{24}$$

And

$$\frac{K}{2\sqrt{\eta}}(1-\phi)p\left\{\int_a^{\frac{Q}{Q_m}}\left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\right)g(y)dy + \int_{\frac{Q}{Q_m}}^{b}\left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx\right)g(y)dy\right\} = 1 \tag{25}$$

For channel coordination, we take $\eta^{ors} = \eta^c$, which gives $\phi p - v_r + g = 0$. The expected profit function of the retailer under this contract is given by

$$\begin{aligned}\Pi_{rc}(Q,p) &= \phi p E[\min\{X,Q,yQ_m\}] + v_r E[(\min\{Q,yQ_m\}-x)^+] - gE[(x-\min\{Q,yQ_m\})^+] \\ &\quad -(w+c_r)E[\min\{Q,yQ_m\}] \\ &= (\phi p - v_r + g)E[\min\{X,Q,yQ_m\}] - (w+c_r-v_r)E[\min\{Q,yQ_m\}] - g\bar{x}\end{aligned} \tag{26}$$

If the relation $\eta^{ors} = \eta^c$ is satisfied then the expected profit of the retailer becomes negative. Hence the retailer would not agree to sign up the standard revenue-sharing contract.

Theorem 6. The standard revenue-sharing contract fails to coordinate the supply chain.

6.2 Revenue-sharing with cost sharing contract

Now, we intercommunicate the case where the manufacturer offers a cost-sharing contract, in addition to the revenue sharing policy with the retailer. During a cost sharing contract, the retailer is actuated to share the CSR investment of the manufacturer. Using this contract, the manufacturer influences the retailer to share the CSR investment more, which successively enhances customer demand. Under this setting, the retailer's expected profit is given by

$$\begin{aligned}\Pi_{rc}(Q,p) &= \phi p E[\min\{X,Q,yQ_m\}] + v_r E[(\min\{Q,yQ_m\}-x)^+] - gE[(x-\min\{Q,yQ_m\})^+] \\ &\quad -(w+c_r)E[\min\{Q,yQ_m\}] - \phi\eta \\ &= (\phi p - v_r + g)E[\min\{X,Q,yQ_m\}] - (w+c_r-v_r)E[\min\{Q,yQ_m\}] - g\bar{x} - \phi\eta\end{aligned} \tag{27}$$

An equal representation of the expected profit of the retailer is given by

$$\Pi_{rc}(Q,p) = (\phi p - v_r + g) \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Qf(x)dx \right) g(y)dy \right\} - (w + c_r - v_r) \times \left\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy \right\} - g\bar{x} - \eta\phi \quad (28)$$

The following theorem characterizes the optimum order quantity Q and retail price P of the retailer under the revenue-sharing and cost sharing agreement.

Theorem 7: In the decentralized model under revenue sharing with a cost sharing agreement, the expected profit function $\Pi_{rc}(Q,p)$ of the retailer is concave in Q and p and the optimal order quantity Q^* and retail price p^* can be obtained from the following equations:

$$Q = (\alpha - \beta p + k\sqrt{\eta}) + F^{-1}\left(\frac{w+c_r-v_r}{\phi p+g-v_r}\right) \quad (29)$$

$$\phi \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m) f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha - \beta p + k\sqrt{\eta} + x) f(x)dx + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Qf(x)dx \right) g(y)dy \right\} = \beta(\phi p + g - v_r) \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \right) g(y)dy \right\} \quad (30)$$

From Theorem 7 we notice that the retailer's optimal order quantity is an increasing function of the retail price p and a decreasing function of its purchasing cost w_m and treating cost c_r.
Now, taking into account the retailer's optimum responses, we determine the manufacturer's optimal decisions. His expected profit function is given by

$$\Pi_{mc}(Q_m,\eta) = (1-\phi)pE[\min\{X,Q,yQ_m\}] + wE[\min\{Q,yQ_m\}] + v_m E[(yQ_m - Q)^+] - c_m Q_m - (1-\phi)\eta$$

$$= (1-\phi)pE[\min\{X,Q,yQ_m\}] - (w - v_m)E[\min\{Q,yQ_m\}] - (c_m - v_m\bar{y})Q_m - (1-\phi)\eta \quad (31)$$

We extract an alternative representation of the manufacturer's profit function as given below:

$$\Pi_{mc}(Q_m,\eta) = (1-\phi)p \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx + \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} (yQ_m)f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} (\alpha-\beta p+k\sqrt{\eta}+x)f(x)dx + \int_{Q-(\alpha-\beta p+k\sqrt{\eta})}^{u} Qf(x)dx \right) g(y)dy \right\} - (w - v_m)\left\{ \int_a^{\frac{Q}{Q_m}} yQ_m g(y)dy + \int_{\frac{Q}{Q_m}}^{b} Qg(y)dy \right\} - (c_m - v_m\bar{y})Q_m - (1-\phi)\eta \quad (32)$$

The following theorem characterizes the manufacturer's optimal production amount and CSR investment in the decentralized setting under the proposed composite contract.

Theorem 8. The objective function of the manufacturer is concave in both Q_m and η and the optimal input amount Q_m^* and CSR expenditure η^* satisfy the following equations:

$$(1-\phi)p \int_a^{\frac{Q}{Q_m}} \int_{yQ_m-(\alpha-\beta p+k\sqrt{\eta})}^{u} yf(x)dxg(y)dy - (w-v_m)\int_a^{\frac{Q}{Q_m}} yg(y)dy = (c_m - v_m\bar{y}) \quad (33)$$

And

$$\frac{K}{2\sqrt{\eta}}(1-\phi)p \times \left\{ \int_a^{\frac{Q}{Q_m}} \left(\int_l^{yQ_m-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \right) g(y)dy + \int_{\frac{Q}{Q_m}}^{b} \left(\int_l^{Q-(\alpha-\beta p+k\sqrt{\eta})} f(x)dx \right) g(y)dy \right\} = (1-\phi) \quad (34)$$

In order to find the conditions for a win-win outcome, we now characterize the participation problem of the chain members. In the following theorem, we get the circumstances under which the contract coordinates the supply chain:

Theorem 9. Under the voluntary compliance, the revenue sharing with cost-sharing contract with the wholesale price of the manufacturer

$$w_m = \phi \times v_m - (1-\phi)(c_r - v_r) \quad (35)$$

achieves the channel coordination.

If the supply chain is coordinated then it follows that the optimal decisions of both the retailer and the manufacturer are the same as the optimal decisions in the centralized benchmark setting, i.e. $Q^* = Q^c, Q_m^* = Q_m^c, \eta^* = \eta^c$ and $p^* = p^c$.

Now, our attention is focused on the issue of an individual firm's participation. A situation can occur where a participant of the chain becomes worse by signing the proposed contract. Clearly, the manufacturer wants to receive higher compensation (higher value of φ) from the retailer to reduce his wholesale price. On the other hand, the retailer wants to share CSR investment cost as small as possible (lower value of φ) to incentive the manufacturer to invest in CSR activities to increase CSR sensitive customer demand. Therefore, the manufacturer wants to increase the share φ but the retailer wants to decrease it. Hence, a question arises - how to determine the suitable contract parameters (ϕ, w_m) under which supply chain entities are motivated to engage in the proposed coordinating contract mechanism. We will find the answer to this question from the numerical experiment.

7. Numerical analysis

In this section, a numerical example is used to analyze the effect of various parameters on the optimal decision variables and the expected profit of the whole supply chain as well as the individual profits of the channel members. The parameter-values are:

$\bar{X} = 50; \bar{z} = 0.7143; \sigma_z = 0.1; \sigma_y = 0.05; \bar{y} = 0.8; \sigma_x = \frac{50}{\sqrt{3}}; \alpha = 500; \beta = 10; c_m = 5; c_r = 2.5; g = 1.5; v_r = 4; v_m = 4.5.$

In Table 1, the optimal decision variables and expected profits for the SC's centralized decisions with CSR, the SC's centralized decisions without CSR, the SC's centralized decisions with a secondary resource, the decentralized decisions under wholesale price-only contract and SC coordination under profit and cost sharing contract are compared. Table 1 shows that the optimal retail price $p^c = 32.52$ with CSR is greater than the optimal retail price $p^0 = 31.10$ without CSR. Also, the data in Table 1 reveal that the expected profit Π_c=5735.50 is much higher than Π_0=5400.33. It is possible to explain that Π_c is more than Π_0 as follows. The SC with CSR has extra reward demand (S) relative to the SC without CSR and hence has higher expected sales (EPS in Table 1). As the extra revenue arising from additional rewarded demand and the higher selling price ($p^c > p^0$) covers the CSR investment η, the expected profit of the SC with CSR is higher than the expected profit of the SC without CSR.

Table 1. The optimal decisions and profits of three scenarios

Variables	Centralized model with CSR	Centralized model without CSR	Centralized model with secondary resource	Decentralized model under revenue sharing cost sharing	Decentralized model under wholesale price
Q	293.036	278.490	287.191	293.036	186.786
Q_m	370.851	350.648	387.987	370.851	253.023
p	32.524	31.102	32.59	32.524	36.482
η	355.225		354.871	355.225	0.010
EPS	251.299	237.111	251.024	251.299	173.312
np	31.110	31.102	31.18	31.110	36.481
w				3.27	18
Π_d	5735.50	5400.33	5717.7	5735.50	4901.30
Π_r				3401.94	2740.20
Π_m				2333.56	2161.10

It is noted that, for the supply chain with CSR, the selling price is greater than that of the supply chain without CSR. At the first glance, it appears that relatively high optimum retail price for the SC with CSR does not help the consumer. However, the CSR activities support the stakeholders including the consumer. For example, Indian company P&G spends a fraction of revenue from all its products for girl's education which might give the chance of a healthier and happier life for the girls to meet their own needs. After all, there are also extensive benefits for the community as a whole. An educated woman has the skills, knowledge, and self-confidence to be a better citizen, parent, and employee. In this way, the CSR activity is beneficial to the customers. Here, the manufacturer's CSR activity helps both the members of the SC and other stakeholders. In other terms, the manufacturer's CSR operation accomplishes "win-win" for the supply chain and the community.

For the decentralized decision under revenue and cost sharing contract, Q, Q_m, η, and p are calculated by Eqs. (29), (30), (33), (34), respectively. The wholesale price w_m is determined following Theorem 9 as $w_m = \phi \times v_m - (1 - \phi)(c_r - v_r)$. Table 1 displays the computational outcome for the decentralized scenario under the composite contract. Comparing the results in the second column with those in the fifth column in Table 1, we observe that all decision variables and predicted incomes are the same. Such findings suggest that the composite contract can achieve channel coordination, as described in Theorem 9. Making a comparison of the results in the fifth column with those in the sixth column in Table 1 we observe that SC's order quantities and CSR's commitment under a composite contract agreement are greater than those under whole-sale price only contract. The expected profits of the supply chain and its members are grater under the proposed composite contract than those under whole-sale price only contract. Based on the criteria in the empirical case, we consider that if the requirements in Theorem 9 are fulfilled, i.e. the supply chain is coordinated, then in turn, for $\phi = 0.59$ all supply chain participants receive the same amount of additional profit due to revenue-sharing and cost sharing contract relative to the decentralized model with whole-sale price contract only.

Table 2. The effects of k on the decision variables and profit functions

k	Π_d	Π_r	Π_m	Q	Q_m	p	η	EPS	np
1.1	5575.72	3306.65	2269.07	286.107	361.225	31.846	180.889	244.536	31.106
1.3	5648.38	3349.98	2298.40	289.259	365.604	32.154	259.042	247.612	31.108
1.5	5735.50	3401.94	2333.56	293.036	370.851	32.524	355.225	251.299	31.110
1.7	5838.29	3463.24	2375.05	297.489	377.041	32.960	472.199	255.649	31.113
1.9	5958.27	3534.78	2423.48	302.683	384.262	33.469	613.496	260.725	31.116
2.1	6097.29	3617.69	2479.60	308.696	392.624	34.058	783.641	266.606	31.119
2.3	6257.64	3713.30	2544.33	315.625	402.263	34.738	988.445	273.387	31.122

Now, we investigate the effects of CSR-sensitive coefficient k on the expected profits and optimal decisions. To investigate the impact of k under the composite contract, we take ϕ = 0.59. Optimal decision variables and estimated profits for various k are mentioned in Table 2. It is evident from the results shown in Table 2 that all the decision variables, including order quantity and retail price, increase w.r.t. k. Also the SC's expected profit, each member's expected benefit, and CSR investment increase w.r.t. k. However, the increased rate of CSR investment with k tends to be higher than the expected profit amount. Table 2 provides an interesting comparison of the CSR contribution level. Apparently, the ratio of CSR investment to the profit of the supply chain and the ratio of CSR investment to the profit of the manufacturer increase w.r.t. k. This result implies that if the reward demand arising from CSR investment increases, the manufacturer wants to increase its CSR investment. In Table 2, it is found out that the increased rate of *np* (31.10-31.12) is very low with k but the CSR investment rises with k at a high rate. A higher value of k is therefore advantageous not only for the SC but also for other SC owners, including customers. A higher value of k is one of the strongest ways to boost the entire channel's revenue and consumers' welfare. The significance of k indicates that when handling the SC with CSR, we strive to choose acceptable CSR practices and boost the income by improving CSR investment. Similar findings can be created on other parameters such as c_m' and σ_y, etc. through a sensitivity analysis.

Most of the researches have shown that there is a beneficial allocation influence of a secondary market for the supply chain. In our numerical analysis, we have quite similar observations. Remember that getting access to the secondary market as an emergency resource is advantageous to the supply chain (Lee and Whang, 2002) to achieve a higher expected profit when producing less. From the numerical example, we see that the profit in the centralized model with a secondary resource is greater than the profit in the centralized model without a secondary resource. The presence of the secondary market offers more alternatives which ultimately increase the supply chain's efficiency. In the presence of a secondary market, the double marginalization effect in the decentralized supply chain is decreased, and the amount moved to the retailer is increased.

Table 3. The effect of c_m' on the decision variables and profit functions

c_m'	Π_d	Q	Q_m	p	η	EPS	np
10	5738	287.772	367.594	32.5625	355.978	250.367	31.1407
15	5723.47	287.356	382.083	32.585	355.185	250.955	31.1697
20	5717.7	287.191	387.987	32.5939	354.871	251.024	31.1803
25	5714.61	287.102	391.194	32.5987	354.702	251.029	31.1857
30	5712.68	287.047	393.208	32.6017	354.596	251.022	31.1891
35	5711.36	287.009	394.591	32.6038	354.524	251.013	31.1914

It is a little surprising when we find that there may be a situation when a secondary resource may not be favorable for the supply chain. It seems that the manufacturer does not need to invest that much for production to satisfy the same amount Q in the presence of a secondary resource which decreases his production investment as well as the risk of salvaging, compare to the absence of a secondary resource. However, the manufacturer produces less under the expectation that he will purchase from the secondary resource while producing less. But when the purchasing cost of the secondary resource increases, the performance of the supply chain decreases as shown in Table 3. The presence of the secondary resource with high purchasing cost indeed generates more fear on the manufacturer's mind that compels him to increase the amount of his production decision, under the force compliance. The effect on the retailer's decision, when the purchasing cost of the secondary resource is varying, is negligible. Consequently, the expected sales of the retailer almost remain unchanged. Therefore, the risk of salvaging of the final product at the manufacturer level increases as the purchasing cost of the secondary resource increases which leaves a negative impact on the supply chain performance.

Finally, we explore how the production yield uncertainty impacts on the supply chain under the proposed composite contract. Table 4 shows the influence of yield uncertainty on the decisions of the supply chain. For comparison purpose, six different values of yield variance are utilized in Table 4. It is shown that the supply chain benefit rises as the yield uncertainty decreases. This supports the fact that a lower risk leads to the efficiency of the supply chain. Not unexpectedly, the difference between order quantity and planed production amount decreases as the yield uncertainty decreases, i.e., the manufacturer plans to use less input to generate the very same output and satisfy the demand.

Table 4. The effect of σ_y on the decision variables and profit functions

σ_y	Π_d	Π_r	Π_m	Q	Q_m	P	H	EPS	np
.01	5763.05	3407.94	2355.11	293.527	359.564	32.484	357.795	252.206	31.065
.03	5751.30	3404.67	2346.63	293.315	363.465	32.500	356.567	251.773	31.084
.05	5735.50	3401.94	2333.56	293.036	370.851	32.524	355.225	251.299	31.110
.07	5717.18	3399.21	2317.97	292.712	379.991	32.551	353.758	250.779	31.141
.09	5696.64	3396.32	2300.32	292.349	390.457	32.582	352.147	250.208	31.175
.11	5673.86	3393.15	2280.71	291.946	402.103	32.617	350.372	249.576	31.213

We also find that the optimal CSR investment increases as the yield uncertainty decreases. This shows that yield uncertainty may encourage the manufacturer to increase the investment in CSR for a higher rewarded demand rather than his other investment, (production investment). When the manufacturer invests more in CSR, it creates more demand and the efficiency of the supply chain is increased in general.

8. Conclusion

In this article, we analyze the two-level supply chain coordination problem consisting of a manufacturer and a retailer where only the manufacturer invests in CSR activities. The issue discussed in this article is an extension of the traditional price setting newsvendor model and the existing socio responsible supply chain coordination problem with demand uncertainty and exogenous retail price. After modeling and addressing SC's centralized decision issue without CSR or with CSR in the presence or absence of a secondary resource, we have solved the problem of SC coordination under the revenue-sharing and cost sharing contract. Because of the difficulty of the problem, we have addressed the problem in the case of linear price-dependent demand, *i.e.*, $d(p) = a - bp$ as taken by Zhao and Yin (2018). Our work is firmly linked to Zhao and Yin (2018) who assumed that stochastic demand follows uniform distribution and the demand is linear in price. We extend their work by taking arbitrary distribution of demand function and random yield in production. Furthermore, the coordination effect of the modified revenue sharing contract in Zhao and Yin (2018), where only one member exhibits CSR is modified by revenue and cost sharing contract, where both members exhibit CSR.

The following conclusions are drawn from our theoretical analysis and empirical study: First, the SC's estimated benefit with CSR is persuaded to be greater than the SC without CSR. Second, the traditional revenue sharing contract is unable to coordinate the SC under the Nash system, but the proposed composite contract can coordinate the SC. Third, in particular, the secondary resource is detected to have a positive effect on supply chain performance, but we also find a situation where the presence of secondary resource might not be beneficial for a supply chain. Ultimately, the SC's expected income, each member's expected benefit, and stakeholder welfare (CSR investment) increase with k. The above results suggest that raising k is crucial for enhancing the SC's profit and its stakeholders' welfare while handling the SC with CSR.

For a socio responsible two-echelon supply chain faced with price and CSR-dependent random demand, we have concentrated on the coordination issue of the supply chain. Although the revenue sharing and cost sharing contract guarantees that SC's target is met, there are some limitations of this research work. We have assumed that the demand is linear in retail price which can be relaxed. The expansion of this work includes exploring the problems of SC coordination with dual networks for the manufacturer-retailer SC, in the sense of CSR. Extending the revenue sharing and cost sharing contract into a multi-period supply chain is another avenue for future research.

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