

## A Novel Cell Layout Problem with Reliability and Stochastic Failures

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### Abstract

The facility layout design and Cell Formation (CF) problems are the important sectors in designing a cellular manufacturing system (CMS). These problems are interrelated and simultaneous consideration of them is essential for a successful design of CMS. In this paper, a new non-linear mixed integer programming model is presented to solve the integrated cell formation and inter/intra cell layouts in continuous space. The proposed approach incorporated machine reliability with a stochastic time between failures. Some important factors such as stochastic process time, part demand, cell size, variable process routing, and both inter-cell and intra-cell layout are considered in proposed model. The objective is to minimize the total inter/intra cell transportation cost and total breakdown cost. The proposed model is then linearized to reduce computation time and an exact solver by using GAMS is proposed to tackle the computational complexity of the developed model. Results indicate the efficiency and the application of proposed model in the area of CMS conceptually.

**Keywords:** Cellular manufacturing system; Cell formation; Cell layout; Machine reliability.

### 1. Introduction

Facility design is still a hot topic in the area of production and manufacturing systems (Tompkins, 2003). About 20% to 50% of the total manufacturing cost is ascribed to material handling. The initial reasons for developing cellular manufacturing systems (CMS) is to minimize material transferring movements (Golmohammadi et al., 2018). Cellular manufacturing systems (CMS) based on Group Technology is an approach applying the benefits of both flexible and mass production approaches. CMS attempts to assign machines and parts to produce cells on the basis of the their similarities in production process, design and geometrical characteristics (Sakhaii et al., 2014; Tajdin et al., 2017).

Application of CMS yields numerous advantages including reduced material transferring cost, setup time, delivery time, lot-size and work-in-process (WIP) (Aghajani-Delavar et al., 2015; Alfa et al., 1992). It can also cause better supervisory control and improvement in productivity (Aghajani-Delavar et al., 2015). One of the important steps in designing a CMS is the cell formation (CF) problem having been widely considered in the scientific works (Alfa et al., 1992). It contains two fundamental issues: machine-cell formation and part-family formation for minimization of some objectives such as inter/intra-cell movements. In this regard, exceptional elements (EEs) which are common in CMS manufacturing environments are recognized as the major obstacle in cell forming and cell scheduling processes (Mak et al., 2000). An EE is a product that needs to be processed in more than one cells and it causes inter-cell transferring of materials. Therefore, a common objective in the CF problem is minimization of the inter-cell material handling cost and material flow (Bayram & Şahin, 2016).

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Facility layout is also a key element in designing a CMS which considers the layout of machines within the cells (Intra-cell layout), and Layout of cells on the shop floor (Inter-cell layout). An efficient facility layout can reduce material handling cost, work-in-process, and throughput rate (Mohammadi & Forghani, 2016). A competent layout not only improves the performance of the system but also minimizes around 50 % of the production costs on average (Wu et al., 2007). The layout problem in CMS haven't paid much attention, since most of the relevant research only investigate the CF problem (Chang, 2013).

As reported from the literature, most approach in the CF and facility layout problems (Jolai et al., 2012), for simplicity, usually aim to minimize the number of intra-cell movements or inter-cell movements or both (Forghani et al., 2015). Although in order to minimize the real cost of transporting, the exact information about facility layout design considering the notion of distance must be considered.

Furthermore, those approaches that seek to minimize the material handling cost usually consider unrealistic assumptions such as fixed cells and machines locations in the layout problem (Golmohammadi et al., 2018). As a consequent, the resulting layout may be inefficient. Also, in most of the previous studies line formed locations were the only consideration for locating the machines in manufacturing cell space (Golmohammadi et al., 2018).

Recently, some studies incorporated reliability into the CF problem. Logenderan et al. (1997) showed that reliability is an important design factor in the CMS. Chung et al. (2011) introduced an efficient TS algorithm for solving the CF problem considering machine reliability and alternative process routings. Das et al. (2007) proposed a model for incorporating reliability considerations into the CF problem. When machine breakdowns occur, each machine type is replaced with another similar copy. In their approach, maximization of the system reliability and minimization of the total costs are considered as two objective functions. Arkat et al. (2012) used chance-constrained programming (CCP) in cell formation problem considering machine reliability. Their proposed model minimizes the total CMS costs, that consists of both inter/ intra-cellular material handling costs and machine breakdown costs. Alhourani (2016) considered alternative process routings and machine reliability together for solving the problem, which can help in realistic of process routing selection for parts. Rafiee et al. (2011) introduced an integrated CF and inventory lot sizing problem for cost minimization in CMS. Furthermore, machine breakdowns and the process deterioration were considered in their approach to make the model more realistic and practical.

More recently, Golmohammadi et al., (2020) proposed a bi-objective CMS to simultaneously minimize the total breakdown costs and the process time in a fuzzy environment. They also used a novel hybrid metaheuristic algorithm based on the red deer and Keshtel algorithms. Karampour et al., (2020) proposed a green CMS to consider the green emissions of production systems with a vendor managed inventory contract. At last but not least, Zhang et al., (2020) applied a risk assessment method with a fuzzy method to model this system.

To alleviate the disadvantages of previous works and make this problem more practical, this study proposes a new nonlinear mathematical model for CF employing the rectilinear distance concept to determine layout in a continuous space. In order to improve the accuracy of the inter /intra-cell layouts, the material handling cost is obtained based on the actual location of machines and cells on the shop floor considering dimensions of machines and aisle widths. Having employed the benefits of cellular layout consideration, target function accurately calculates the inter/ intra-cell relocation costs for parts considering machine reliability with stochastic operation time. Cells and machines should not be overlapped in an accurate cellular layout.

It goes without saying that the main difference of this paper with aforementioned literature especially Golmohammadi et al., (2020), is the use of an exponential distribution for the failures.

This paper is outlined as follows. Section 2, addresses the proposed problem along with main assumptions and formulation. Section 3, a numerical example and computational analyses on the proposed model are explained. Finally, discussion and some suggestions to improve this study are investigated in Section 4.

## 2. Problem description and modeling

Here, the main structure of proposed problem and its modeling approach is illustrated in details.

### 2.1 Model assumption

- Demand of parts are definite.
- The Operation sequences of each part are determined.
- A part may have variable process routings.
- Time between failures follows exponential distribution.
- Machine breakdown cost is assumed to be known in advance and is on the basis of its repair, install/uninstall costs.
- The inter-cell transportation cost is on the basis of the unit of distance and it remains the same over time.
- The number of cells is known.
- The maximum capacity of cells is determined.
- There is only one from each type of machine.

### 2.2 Sets

$r = \{1, 2, \dots, R_i\}$	Set of process routs of parts;
$i = \{1, 2, \dots, n\}$	Set of parts;
$m, m' = \{1, 2, \dots, m\}$	Set of machines;
$j = \{1, 2, \dots, J_{ir}\}$	Set of operation $j$ of selected rout $r$ for part number $i$
$c, c' = \{1, 2, \dots, C\}$	Set of cells;

### 2.3 Model Parameters

$C$	Number of considered cells;
$P_i$	Demand of part $i$ ;
$R_i$	Number of process routs for part $i$
$UC$	The maximum number of machines in each cell;
$J_{ir}$	Number of operations for part $i$ under process rout $r$ ;
$C_i^{inter}$	The inter-cell transportation cost for transporting part $i$ for per unit distance;
$C_i^{intra}$	The intra-cell transportation cost for transporting part $i$ for per unit distance;
$F$	Vertical length of the shop
$E$	Horizontal length of the shop
$A_{cc'}, B_{cc'}$	Binary random variables
$N$	Positive big number
$\lambda_m$	failure rate for machine $m$ that follows a exponential distribution
$\alpha$	pre-specified confidence level
$\beta_m$	the breakdown unit cost for machine $m$

### 2.4 Decision variables

$Z_{ir} = \begin{cases} 1 & \text{If process rout } r \text{ is selected for part } i \\ 0 & \text{Otherwise} \end{cases}$	
$V_{mc} = \begin{cases} 1 & \text{If machine } m \text{ is allocated to cell } c \\ 0 & \text{Otherwise} \end{cases}$	
$x_m$	Horizontal component of machine $m$
$y_m$	Vertical component of machine $m$
$p_c^1$	Left side horizontal component of cell $c$
$p_c^2$	Right side horizontal component of cell $c$
$q_c^1$	Bottom side vertical component of cell $c$
$q_c^2$	Top side vertical component of cell $c$

### 2.5 Reliability consideration with stochastic operation time

machine breakdown is one of the most important matter influencing the performance of the CMS, that results in many manufacturing problems like longer production period and higher production costs. Machines in most of the previous studies are assumed reliable without any breakdown. Since machine reliability has a probabilistic nature, in this study

assumed that machine reliability follows an exponential distribution with a probabilistic failure rate ( $\lambda_m$ ). Also, machine breakdown cost is assumed to be known in advance and is on the basis of its repair and install/uninstall costs. Here, in order to calculate the total cost of breakdowns, it is assumed that  $\lambda_m$  follows a normal distribution. Considering  $\beta_m$  as the breakdown unit cost for machine  $m$ , and  $\zeta_{irj}$  as the number of breakdowns occurring for  $j$ th machine under process rout  $r$  for  $i$ th part, the total breakdown cost (TBC) is calculated as follows:

$$TBC = \sum_{i=1}^n \sum_{r=1}^{R_i} \sum_{j=1}^{J_{ir}} Z_{ir} \zeta_{irj} \beta_{M_{ir}^j} \tag{1}$$

Eq. (1) is the total breakdown cost for all parts of CMS. Since time between failures has an exponential distribution, the number of breakdowns for each machine follows a Poisson distribution. Due to the probabilistic nature of the time between machine breakdowns, calculating the exact amount of  $\zeta_{irj}$  is impossible. Hence, based on the concept of CCP, the stochastic variable  $\zeta_{irj}$  in equation (1) can be replaced by  $\varphi_{irj}$  as a new deterministic variable in Eq. (2), and a chance constraint according to Eq. (3) will be added to the model to ensure that the number of breakdowns never exceeds  $\varphi_{irj}$  in at least  $\alpha$  of time.

$$\Pr(\zeta_{irj} = \varphi_{irj}, \lambda) = \frac{e^{-\lambda} \times \lambda^\varphi}{\varphi!} \quad \varphi = 0,1,2, \dots \tag{2}$$

$$\Pr(\zeta_{irj} \leq \varphi_{irj}) \geq \alpha \tag{3}$$

Where  $\varphi_{irj}$  is an integer variable and  $\alpha$  is the pre-specified confidence level. If  $\lambda$  follows a continuous distribution, then Eq. (2) can be replaced as follows:

$$\Pr(\zeta_{irj} = \varphi_{irj}, \lambda) = \int_0^\infty \frac{e^{-\lambda} \cdot \lambda^\varphi}{\varphi!} f(\lambda) d\lambda \tag{4}$$

In this research it is assumed that  $\lambda_m$  follows a normal distribution, so the above equation can be considered as follows:

$$\Pr(\zeta = \varphi, \lambda) = \int_0^\infty \frac{e^{-\lambda} \cdot \lambda^\varphi}{\varphi!} \times \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\lambda-\mu)^2}{2\sigma^2}} d\lambda \tag{5}$$

Considering  $T_{i(M_{ir}^j)}$  as the process time of machine  $j$  for part  $i$  under process rout  $r$ , and  $\lambda_{M_{ir}^j}$  as the failure rate of machine  $j$  for part  $i$  under process route  $r$ , the number of breakdowns in total process time of  $T_{i(M_{ir}^j)}$  follows a Poisson distribution with following parameter:

$$\lambda'_{M_{ir}^j} = \lambda_{M_{ir}^j} \times T_{i(M_{ir}^j)} \tag{6}$$

If  $T_{i(M_{ir}^j)}$  follows a normal distribution with expected value of  $\mu_{M_{ir}^j}$  and standard deviation of  $\sigma_{M_{ir}^j}$ , then  $\lambda'_{M_{ir}^j}$  has a normal distribution with expected value of  $\lambda_{M_{ir}^j} \times \mu_{M_{ir}^j}$  and standard deviation of  $\lambda_{M_{ir}^j} \times \sigma_{M_{ir}^j}$ . Considering  $P_i$  as the production volume of part  $i$ , these values can be calculated as follows:

$$\mu'_{M_{ir}^j} = \lambda_{M_{ir}^j} \times P_i \times \mu_{M_{ir}^j} \tag{7}$$

$$\sigma'_{M_{ir}^j} = \lambda_{M_{ir}^j} \times P_i \times \sigma_{M_{ir}^j} \tag{8}$$

Considering  $\lambda'_{M_{ir}^j}$ , the equation (5) is equivalent to the following equation.

$$Pr(\zeta_{irj} = \varphi_{irj}) = \int_0^\infty \frac{(\lambda'_{M_{ir}^j})^{\varphi_{irj}} \times \exp(-\lambda'_{M_{ir}^j})}{\varphi_{irj}!} \times \frac{1}{\sigma'_{M_{ir}^j} \times \sqrt{2\pi}} e^{-\frac{(\lambda'_{M_{ir}^j} - \mu'_{M_{ir}^j})^2}{2\sigma'^2_{M_{ir}^j}}} d\lambda' \quad (9)$$

Considering Eq. (3), the above constraint can be rewritten as follows:

$$\sum_{\omega=0}^{\varphi_{irj}} \int_0^\infty \frac{(\lambda'_{M_{ir}^j})^\omega \times \exp(-\lambda'_{M_{ir}^j})}{\omega!} \times \frac{1}{\sigma'_{M_{ir}^j} \times \sqrt{2\pi}} e^{-\frac{(\lambda'_{M_{ir}^j} - \mu'_{M_{ir}^j})^2}{2\sigma'^2_{M_{ir}^j}}} d\lambda' \geq \alpha \quad (10)$$

On the other hand, as the total breakdown cost given in Eq. (1) is a nonlinear one, we define the following equation to simplify the term of the problem:

$$W_{irj} = Z_{ir} \times \varphi_{irj} \quad (11)$$

Where  $\varphi_{irj}$  is an integer variable and  $Z_{ir}$  is a binary variable. Hence,  $W_{irj}$  is an integer variable. The newly defined equation is equivalent to the following equation:

$$W_{irj} \geq \varphi_{irj} - (1 - Z_{ir})N \quad (12)$$

In which  $N$  is a large positive number. If  $Z_{ir}$  takes 1, this constraint becomes  $W_{irj} \geq \varphi_{irj}$ , and due to the minimization form of the objective function,  $W_{irj}$  takes  $\varphi_{irj}$ . Similarly, if  $\zeta_{ij}$  is 0, the constraint becomes  $W_{irj} \geq -N$  and again, because of the minimization of the objective function,  $W_{irj}$  takes 0.

### 2.6 Mathematical formulation

Regarding the aforementioned suppositions for the proposed model, the final formulation is presented as follows:

$$\begin{aligned} \text{Min } TC = & \sum_{i=1}^n \sum_{r=1}^{R_i} \sum_{j=1}^{J_{ir}-1} \sum_{c=1}^c \sum_{c'=1}^c C_i^{inter} Z_{ir} P_i V_{M_{ir}^j, c} V_{M_{ir}^{j+1}, c'} \left( |x_{M_{ir}^j} - x_{M_{ir}^{j+1}}| + |y_{M_{ir}^j} - y_{M_{ir}^{j+1}}| \right) \\ & + \sum_{i=1}^n \sum_{r=1}^{R_i} \sum_{j=1}^{J_{ir}-1} \sum_{c=1}^c C_i^{intra} Z_{ir} P_i V_{M_{ir}^j, c} V_{M_{ir}^{j+1}, c} \left( |x_{M_{ir}^j} - x_{M_{ir}^{j+1}}| + |y_{M_{ir}^j} - y_{M_{ir}^{j+1}}| \right) \\ & + \sum_{i=1}^n \sum_{r=1}^{R_i} \sum_{j=1}^{J_{ir}} W_{irj} \beta_{M_{ir}^j} \end{aligned} \quad (14)$$

Subject to:

$$\sum_{c=1}^c V_{mc} = 1 \quad \forall m = 1, 2, \dots, M \quad (15)$$

$$\sum_{r=1}^{R_i} Z_{ir} = 1 \quad \forall i = 1, 2, \dots, n \quad (16)$$

$$\sum_{m=1}^M V_{mc} \leq UC \quad \forall c = 1, 2, \dots, C \quad (17)$$

$$|x_m - x_{m'}| + |y_m - y_{m'}| \geq 1 \tag{18}$$

$$\begin{cases} y_m \geq q_c^1 + N(1 - V_{mc}) \\ y_m \leq q_c^2 + N(1 - V_{mc}) \\ x_m \geq p_c^1 + N(1 - V_{mc}) \\ x_m \leq p_c^2 + N(1 - V_{mc}) \end{cases} \quad \forall m, c \tag{19}$$

$$\begin{cases} p_c^1 \geq 0 \\ q_c^1 \geq 0 \\ p_c^2 \leq E \\ q_c^2 \leq F \end{cases} \quad \forall c \tag{20}$$

$$\begin{cases} p_c^1 - p_{c'}^2 + NA_{cc'} + NB_{cc'} \geq 0 \\ p_{c'}^2 - p_c^1 - NA_{cc'} - N(1 - B_{cc'}) \leq 0 \\ q_c^1 - q_{c'}^2 + N(1 - A_{cc'}) + NB_{cc'} \geq 0 \\ q_{c'}^2 - q_c^1 - N(1 - A_{cc'}) - N(1 - B_{cc'}) \leq 0 \end{cases} \quad 0 \leq c < c' \leq C \tag{21}$$

$$W_{irj} \geq \varphi_{irj} - (1 - Z_{ir})N \quad \forall i, r, j \tag{22}$$

$$\sum_{\omega=0}^{\varphi_{irj}} \int_0^{\infty} \frac{(\lambda'_{M_{ir}^j})^\omega \cdot \exp(-\lambda'_{M_{ir}^j})}{\omega!} \times \frac{1}{\sigma'_{M_{ir}^j} \times \sqrt{2\pi}} e^{-\frac{(\lambda'_{M_{ir}^j} - \mu'_{M_{ir}^j})^2}{2\sigma'^2_{M_{ir}^j}}} d\lambda' \quad \forall i, r, j \tag{23}$$

$$\geq \alpha$$

$$Z_{ir}, V_{mc} \in \{0,1\} \tag{24}$$

$$x_m, y_m, p_c^1, p_c^2, q_c^1, q_c^2 \geq 0 \text{ and integer}$$

The objective in the presented model seeks to minimizing the total transportation cost regarding both inter/intra cell movements and total breakdown cost. The first set of constraints (Eq. (15)) ensures that each machine is allocated only to one cell. The second set of constraints (Eq. (16)) indicates that only one process rout is assigned to each part. The third set of constraints (Eq. (17)) ensures that the number of machines located in each cell don't exceed UC. The forth set of constraints (Eq. (18)) leads to no-overlap between the machines. This means that regarding the dimensions of machines (1×1), this relationship ensures that the machines are not overlapped. Constraints (19) ensure that each machine is relocating in space of its corresponding cell. The set of relationship (20) causes relocating of each cell in space of a job shop and the set of constraints (21) ensures that cells are not overlapped. Constraints (22) and (23) are related to reliability consideration in CMS which are explained before in Section 2.5. Finally the nature of the decision variables are represented in the set of Constraints (24).

### 2.7 Linearization of the Model

Here, these terms are linearized to improve the efficiency of the model. To linearize these terms, a common procedure is employed having been used in many papers such as (Golmohammadi et al., 2019; Hadian et al., 2020; Liu; 2020). The following variables have been used to convert the binary variables in the objective function to a linear ones:

$$R_{irjcc'} = Z_{ir} \times V_{M_{ir}^j} \times V_{M_{ir}^{j+1}c'} \tag{25}$$

$$S_{irjc} = Z_{ir} \times V_{M_{ir}^j} \times V_{M_{ir}^{j+1}c} \tag{26}$$

$R_{irjcc'}$  is equal to 1 when the rout  $r$  is selected for component  $i$  and operations  $j$  and  $(j + 1)$  are processed in cell  $c$  and  $c'$ , consequently.  $S_{irjc}$  is equal to 1 when rout  $r$  is selected for part  $i$  and operations  $j$  and  $(j + 1)$  are both processed in cell  $c$ . Considering the new variables in the objective function, the following constraints have been added to the model.

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c'} - R_{irjcc'} \leq 2 \quad \forall i, r, j, c, c' \quad (27)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c'} - 3R_{irjcc'} \geq 0 \quad \forall i, r, j, c, c' \quad (28)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c} - S_{irjc} \leq 2 \quad \forall i, r, j, c \quad (29)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c} - 3S_{irjc} \geq 0 \quad \forall i, r, j, c \quad (30)$$

Also, for the linearization of absolute terms in objective function, non-negative variables  $x_{ii'h}^+$ ,  $x_{ii'h}^-$ ,  $y_{ii'h}^+$  and  $y_{ii'h}^-$  are presented as follows:

$$x_{M_{ir}^{jj+1}}^+ = \begin{cases} (x_{M_{ir}^j} - x_{M_{ir}^{j+1}}) & \text{if } x_{M_{ir}^j} - x_{M_{ir}^{j+1}} > 0 \\ 0 & \text{Otherwise} \end{cases} \quad (31)$$

$$x_{M_{ir}^{jj+1}}^- = \begin{cases} (x_{M_{ir}^{j+1}} - x_{M_{ir}^j}) & \text{if } x_{M_{ir}^j} - x_{M_{ir}^{j+1}} \leq 0 \\ 0 & \text{Otherwise} \end{cases} \quad (32)$$

$$y_{M_{ir}^{jj+1}}^+ = \begin{cases} (y_{M_{ir}^j} - y_{M_{ir}^{j+1}}) & \text{if } y_{M_{ir}^j} - y_{M_{ir}^{j+1}} > 0 \\ 0 & \text{Otherwise} \end{cases} \quad (33)$$

$$y_{M_{ir}^{jj+1}}^- = \begin{cases} (y_{M_{ir}^{j+1}} - y_{M_{ir}^j}) & \text{if } y_{M_{ir}^j} - y_{M_{ir}^{j+1}} \leq 0 \\ 0 & \text{Otherwise} \end{cases} \quad (34)$$

This linearization is carried out by rewriting the absolute term in objective function as follows:

$$|x_{M_{ir}^j} - x_{M_{ir}^{j+1}}| = x_{M_{ir}^{jj+1}}^+ + x_{M_{ir}^{jj+1}}^- \quad (35)$$

$$|y_{M_{ir}^j} - y_{M_{ir}^{j+1}}| = y_{M_{ir}^{jj+1}}^+ + y_{M_{ir}^{jj+1}}^- \quad (36)$$

where the following constraints must be considered in the main model:

$$x_{M_{ir}^j} - x_{M_{ir}^{j+1}} = x_{M_{ir}^{jj+1}}^+ - x_{M_{ir}^{jj+1}}^- \quad (37)$$

$$y_{M_{ir}^j} - y_{M_{ir}^{j+1}} = y_{M_{ir}^{jj+1}}^+ - y_{M_{ir}^{jj+1}}^- \quad (38)$$

Now the transformed mixed integer linear programming formulation is presented as follows:

$$\text{Min } TC = \sum_{i=1}^n \sum_{r=1}^{R_i} \sum_{j=1}^{J_{ir}-1} \sum_{c=1}^C \sum_{c'=1}^C C_i^{\text{inter}} P_i R_{irjcc'} (x_{M_{ir}^{jj+1}}^+ + x_{M_{ir}^{jj+1}}^- + y_{M_{ir}^{jj+1}}^+ + y_{M_{ir}^{jj+1}}^-) \quad (39)$$

$$+ \sum_{i=1}^n \sum_{r=1}^{R_i} \sum_{j=1}^{J_{ir}-1} \sum_{c=1}^C C_i^{\text{intra}} P_i S_{irjc} (x_{M_{ir}^{jj+1}}^+ + x_{M_{ir}^{jj+1}}^- + y_{M_{ir}^{jj+1}}^+ + y_{M_{ir}^{jj+1}}^-)$$

Subject to: (40)

$$\sum_{r=1}^{R_i} Z_{ir} = 1 \quad \forall i = 1, 2, \dots, n \quad (41)$$

$$\sum_{c=1}^C V_{mc} = 1 \quad \forall m = 1, 2, \dots, M \quad (42)$$

$$\sum_{m=1}^M V_{mc} \leq UC \quad \forall c = 1, 2, \dots, C \quad (43)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c'} - R_{irjcc'} \leq 2 \quad \forall i, r, j, c, c' \quad (44)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c'} - 3R_{irjcc'} \geq 0 \quad \forall i, r, j, c, c' \quad (45)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c} - S_{irjc} \leq 2 \quad \forall i, r, j, c \quad (46)$$

$$Z_{ir} + V_{M_{ir}^j c} + V_{M_{ir}^{j+1} c} - 3S_{irjc} \geq 0 \quad \forall i, r, j, c \quad (47)$$

$$\begin{cases} x_{M_{ir}^j} - x_{M_{ir}^{j+1}} = x_{M_{ir}^{j,j+1}}^+ - x_{M_{ir}^{j,j+1}}^- \\ y_{M_{ir}^j} - y_{M_{ir}^{j+1}} = y_{M_{ir}^{j,j+1}}^+ - y_{M_{ir}^{j,j+1}}^- \end{cases} \quad \forall m, m' \quad (48)$$

$$\begin{cases} x_{M_{ir}^j} - x_{M_{ir}^{j+1}} + NA_{j,j+1} + NB_{j,j+1} \geq 1 \\ x_{M_{ir}^{j+1}} - x_{M_{ir}^j} - NA_{j,j+1} - N(1 - B_{j,j+1}) \geq 1 \\ y_{M_{ir}^j} - y_{M_{ir}^{j+1}} + N(1 - A_{j,j+1}) + NB_{j,j+1} \geq 1 \\ y_{M_{ir}^{j+1}} - y_{M_{ir}^j} - N(1 - A_{j,j+1}) - N(1 - B_{j,j+1}) \geq 1 \end{cases} \quad \forall 1 \leq m < m \leq M \quad (49)$$

$$\begin{cases} y_m \geq q_c^1 + N(1 - V_{mc}) \\ y_m \leq q_c^2 + N(1 - V_{mc}) \\ x_m \geq p_c^1 + N(1 - V_{mc}) \\ x_m \leq p_c^2 + N(1 - V_{mc}) \end{cases} \quad \forall m, c \quad (50)$$

$$\begin{cases} p_c^1 \geq 0 \\ q_c^1 \geq 0 \\ p_c^2 \leq E \\ q_c^2 \leq F \end{cases} \quad \forall c \quad (51)$$

$$\begin{cases} p_c^1 - p_c^2 + NA_{cc'} + NB_{cc'} \geq 0 \\ p_c^2 - p_c^1 - NA_{cc'} - N(1 - B_{cc'}) \leq 0 \\ q_c^1 - q_c^2 + N(1 - A_{cc'}) + NB_{cc'} \geq 0 \\ q_c^2 - q_c^1 - N(1 - A_{cc'}) - N(1 - B_{cc'}) \leq 0 \end{cases} \quad 0 \leq c < c' \leq C \quad (52)$$

$$W_{irj} \geq \varphi_{irj} - (1 - Z_{ir})N \quad \forall i, r, j \quad (53)$$

$$\sum_{\omega=0}^{\varphi_{irj}} \int_0^{\infty} \frac{(\lambda'_{M_{ir}^j})^\omega \cdot \exp(-\lambda'_{M_{ir}^j})}{\omega!} \times \frac{1}{\sigma'_{M_{ir}^j} \times \sqrt{2\pi}} e^{-\frac{(\lambda'_{M_{ir}^j} - \mu'_{M_{ir}^j})^2}{2\sigma'^2_{M_{ir}^j}}} d\lambda' \geq \alpha \quad \forall i, r, j \quad (54)$$

$$Z_{ir}, V_{mc}, R_{irjcc'}, S_{irjc} \in \{0,1\} \quad (55)$$

$$x_m, y_m, p_c^1, p_c^2, q_c^1, q_c^2 \geq 0 \text{ and integer}$$

### 3. Computational experiments

Here, a numerical example to validate the proposed formulation is provided. This example includes 8 parts and 5 machines in which all the presented hypotheses in the “model assumption section” are valid. Our goal is to determine machine locations and cell allocation. The part-machine matrix and all the input parameters for the model are depicted in Fig.1. and Tables 1 and 2 respectively:

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$	$p_7$	$p_8$
$M_1$	0	0	1	3	0	1	2	0
$M_2$	3	2	0	0	0	2	1	3
$M_3$	2	0	2	0	2	3	0	0
$M_4$	0	1	0	2	3	0	3	1
$M_5$	1	3	3	0	1	0	0	2

Figure. 1. machine-part matrix of the problem sized 5×8

Table 1 :Considered parameters of the problem

$m$	$n$	$C_{intra}^j$	$C_{intra}^j$	$C$	$UC$	$E$	$F$
5	8	10	1	2	3	4	4

Table 2: Demand of parts

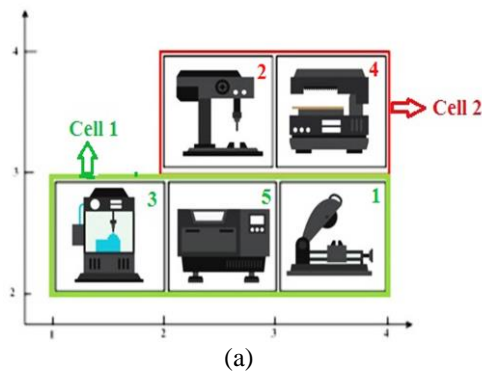
PN	1	2	3	4	5	6	7	8
De	177	194	143	191	234	341	231	189
PN: Part Number								
De: Demand								



The model of the problem is coded and solved employing GAMS software and run on a Pentium 4 personal computer (PC). The results are obtained after lunching the program for two hours. The parts, machines, and locations assignments to cells and also layouts for machines and cells are illustrated in Table 3 and Fig. 2. respectively. For example, in cell 2, machines 2, 4 are assigned. The results are depicted by Fig. 1 as well.

**Table 3:** Output information related to assign machines and parts to each cell using GAMS

Parts assigned to		Machine in	
Cell 1	Cell 2	Cell 1	Cell 2
1,2,3,5,8	4,6,7	1,3,5	2,4



**Figure. 2.** Layout for Machines and Cells

#### 4. Conclusions

A non-linear mixed integer programming model was applied in this paper to consider the simultaneous cell formation and inter/intra cell layouts in the continuous space. The purpose of the model was to determine concurrently the formation of cells and the intra and inter-cellular layout in a way that the total transportation cost of parts, and total breakdown cost were minimized. The excellent advantage of the proposed model was to calculate the material handling costs realistically. It was computed based on the actual location of machines and cells within the shop floor considering dimensions of equal-sized facilities. Both intra and inter-cellular material handling were considered to calculate the transportation cost based on the distance traveled based on the center-to-center interval between machines through a rectilinear distance. In the proposed model cells were configured in flexible shapes considering cell capacity in planning horizon. Some other important factors such as variable process routing are also considered in proposed model. Furthermore, the total breakdown cost was determined considering stochastic operation time. The problem was then simplified by linearizing the nonlinear constraints to enhance the efficiency of the problem.

Future directions, incorporating scheduling problem and some production data such as setup times and inventory cost makes the model become more realistic (Fathollahi-Fard et al. 2020; Rasay et al. 2020). Since the proposed model is complex and NP-hard, efficient heuristics like Lagrangian relaxation (Fathollahi-Fard et al. 2020; Hadian 2020), red deer algorithm (Fathollahi-Fard et al., 2020; Liu; 2020; Fathollahi-Fard et al., 2020) and social engineering optimizer (Fathollahi-Fard et al. 2020; Nezhadroshan et al. 2020).

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