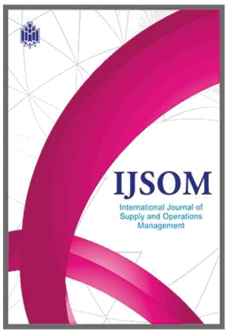
International Journal of Supply and Operations Management

IJSOM

August 2020, Volume 7, Issue 3, pp. 272-278
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com



On Single Machine Scheduling Problem with Distinct due Dates under Fuzzy Environment

H. A. Khalifa*

Mathematics Department, College of Science and Arts, Al- Badaya, Qassim University, Saudi Arabia

Abstract

This research article proposes a single machine scheduling problem subject to distinct due dates in fuzzy environment. The proposed problem is considered in fuzzy environment with an attempt to sequence of the $n-$jobs. The total penalty cost is considered as the composition of all the total earliness and tardiness cost. The aim of this research work is to minimize the total penalty cost. A method to minimize the total penalty cost due to earliness or lateness of job in fuzzy environment is proposed. A numerical example is illustrated to support the proposed method in this study.

Keywords: Single machine; Distinct due dates; Early/ lately machine problem; $(\alpha, \beta)-$ interval- valued fuzzy number; Signed distance ranking.

1. Introduction

Single machine, distinct due dates, early (lately) machine problem and fuzzy environment closely to the situation faced by "just in time" manufacture (Gupta and Rambha, 2011). Scheduling is defined as prescribing of when and where each operation necessary to manufacture the product to be performed. It aims to plane a sequence of work so as to the production can be systematically arranged towards the end of completion of all products by due dates. Senthilkumar and Narayanan (2010) introduced a survey on the classification of the literatures of single machine scheduling problem in three major categories, viz. Offline scheduling, online scheduling and miscellaneous scheduling. Baker and Scudder (1990) developed sequences with earliness and tardiness penalties under Just in Time production. Recently, the of earliness and tardiness penalties in scheduling models is necessary and inquiry (Liaw, 1999; Biskup and Feldmann, 2001; Belkaid et al. 2013; Belkaid et al. 2016; Che et al. 2017; Arlk, 2019; and Gerstl and Mosheiov, 2020).

Single machine scheduling problems bear complex computations and such problem analyzed important for a better understanding problem. Among those, related to earliness and tardiness. Single machine scheduling with single machine consists of single machine to process n jobs; this problem aims to schedule these n jobs on the single machine to minimize the given measure of performance. These jobs may be independent or dependent. Mohamad and Said in (2011) studied n-job single machine scheduling problem with common due date so as to minimize the sum of the total inventory and penalty costs. Belkaid et al. (2012) presented a branch and bound algorithm to minimize the makespan. They considered the identical parallel machines for consumable resources. Zhao et al. (2014) considered a model with job- dependent position effects for a single machine scheduling and due date assignment with set up time, which is proportional to the length of the processed jobs. Zhao et al. (2014) aims to minimize the sum of the scheduling criterion of the accepted jobs and total penalty of the rejected jobs for the single- machine scheduling and due date assignment problem. Ben-Yehoshua and Mosheiov (2016) proposed a single machine-scheduling problem with the objective to minimize total early work. Wang and Xu (2016) considered a single machine scheduling problem with a mandatory maintenance whose duration is workload- dependent aims to determine the start time of the maintenance and schedule all the jobs to the machine such that the maximum lateness is minimized through a proposed an approximation algorithm based on the classical earliest due date.

* Corresponding author email address: hamiden_2008@yahoo.com

Processing time of a job varies in different ways, may be due to factor environmental or the different work places. When a contractor takes the work form his/her department, he (she) calculates expenditure at the time of allotment. But due to enormous factors as non-available of labor, weather not favorable or sometimes abnormal conditions. In many scientific areas, such as system analysis and operators research, a model has to be setup-using data, which is only approximately known. Fuzzy sets theory, introduced by Zadeh (1965) makes this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Decision making in a fuzzy environment, developed by Bellman and Zadeh (1970) has an improvement and a great help in the management decision problems. Kaufmann and Gupta (1988) studied several fuzzy mathematical models with their applications to engineering and management sciences. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers by use of a fuzzification principle. Fuzzy linear constraints with fuzzy numbers have been studied by Dubois and Prade (1980). Recently, the study of earliness and tardiness penalties in scheduling is relatively of inquiry. Yazdani et al. (2017) studied a single- machine scheduling problem subject to several unavailability constraints. To solve the model, they considered a mathematical model and an enhanced variable neighborhood search approach. An enormous number of authors studied machine scheduling in uncertainty environment (Arlk and Toksarl, 2019; Ertem et al. 2019; Wu and Wang, 2020; and Yue et al. 2020). The job sequencing scheduling problem when processing time represented as fuzzy numbers is studied by Cahon and Lee (1992). Chong (1995), Ishibuchi et al. (1996) and Ishii and Tada (1995) fuzzified the scheduling problems by using a fixed due date. Jadhav and Bajaj (2012) proposed n - jobs to processed on single machine scheduling problem with fuzzy processing and fuzzy due dates so as to minimize the total penalty cost in the schedule. Dueness and Petrivic (1995) developed an approach under fuzzy flow as fuzzy in nature. They used the concepts of Average High Ranking Method. Toksari and Arık (2017) studied the single machine scheduling problem involving the position dependent fuzzy learning effect as well as fuzzy processing times. Sotskov and Egorova (2018) applied the stability approach to the single machine problem with uncertain durations of the given jobs to minimize the sum of the job completion times. Ponnalagu and Mounika (2018) considered the sequence performance measurements and job mean Koulamas and Kyparisis (2019) shown that the single machine scheduling problem with past- sequence- dependent setup times and either the minimum maximum lateness/ tardiness objective was solvable by an index priority rule followed by backward intentions of certain qualifying jobs.

The penalizing concept in both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. The using of earliness and tardiness in fuzzy environment give rise to a non-regular performance measures, because this is led to a new methodological in the design of the procedures of solutions.

In this article, the research objective is to consider the sequence of the jobs on a single machine with fuzzy due dates so as to minimize the total penalty cost. A method to minimize the total penalty fuzzy cost due to earliness or lateness of job in fuzzy environment is proposed.

The outlay of the paper is organized as follows: In the next Section, the basic concepts and results related to fuzzy numbers, and (α, β) interval-valued fuzzy numbers are introduced. Section 3 presents assumptions and notation needed in the paper. Section 4 formulate single machine scheduling problem with processing time represented as (α, β) interval valued fuzzy bi-objective criteria. Section 5 proposes solution procedure for solving the problem. Section 6 provides a numerical example to illustrate the efficiency of the solution procedure. Finally, some concluding remarks are reported in Section 7.

2. Preliminaries

This Section introduces some of basic concepts and results related to fuzzy numbers, and (α, β) interval valued fuzzy numbers.

Definition 1. A fuzzy set $\tilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be fuzzy numbers if its membership function

$\mu_{\tilde{\tilde{A}}}(x)\colon \mathbb{R} \rightarrow [0,1]$, have the following properties:

1. $\mu_{\tilde{\tilde{A}}}(x)$ is an upper semi- continuous membership function;
2. $\tilde{A}$ is convex fuzzy set, i.e., $\mu_{\tilde{\tilde{A}}}(\delta\, x + (1-\delta)\, y) \geq \min\{\mu_{\tilde{\tilde{A}}}(x), \mu_{\tilde{\tilde{A}}}(y)\}$ for all $x, y \in \mathbb{R}$; $0 \leq \delta \leq 1$;
3. $\tilde{A}$ is normal, i.e., $\exists\, x_0 \in \mathbb{R}$, for which $\mu_{\tilde{\tilde{A}}}(x_0) = 1$;
4. Supp $(\tilde{A}) = \{x \in \mathbb{R}\colon \mu_{\tilde{\tilde{A}}}(x) > 0\}$ is the support of $\tilde{A}$, and the closure $\mathrm{cl}(\mathrm{Supp}(\tilde{A}))$ is compact set.

Definition 2. (Chiang, 2001**).** If the membership function of the fuzzy set $\tilde{A}$ on $\mathbb{R}$ is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{\alpha\,(x-r)}{(s-r)}, & r < x \leq s, \\ \frac{\alpha\,(t-x)}{(t-s)}, & s \leq x < t, \\ 0, & otherwise, \end{cases}$$

Where $0 < \alpha \leq \alpha$ then $\tilde{A}$ is called a level α fuzzy number and it is denoted as $\tilde{A} = (r, s, t;\ \alpha)$.

Definition 3. (Chiang, 2001). An interval- valued fuzzy set $\tilde{A}$ on $\mathbb{R}$ is given by

$\tilde{A} \triangleq \{(x,\ [\ \mu_{\tilde{A}^-}(x), \mu_{\tilde{A}^+}(x)\ ]):x\in\mathbb{R}\}$, where $\mu_{\tilde{A}^-}(x), \mu_{\tilde{A}^+}(x) \in [0,\ 1]$, and

$\mu_{\tilde{A}^-}(x) \leq\ \mu_{\tilde{A}^+}(x)$; for all $x \in \mathbb{R}$ and is denoted as $\tilde{A} = [\tilde{A}^-, \tilde{A}^+]$. Let

$$\mu_{\tilde{A}^-}(x) = \begin{cases} \frac{\alpha\,(x-r)}{(s-r)}, & r < x \leq s, \\ \frac{\alpha\,(t-x)}{(t-s)}, & s \leq x < t, \\ 0, \text{otherwise} \end{cases}$$

Then $\tilde{A}^- = (r, s, t; \alpha)$.

Let

$$\mu_{\tilde{A}^+}(x) = \begin{cases} \frac{\beta\,(x-a)}{(s-a)}, & a < x \leq s, \\ \frac{\beta\,(b-x)}{(b-s)}, & s \leq x < c, \\ 0, \text{otherwise.} \end{cases}$$

Then $\tilde{A}^+ = (a, s, b;\ \beta)$.

It is clear that $0 < \alpha \leq \beta \leq 1$, and $a < r < s < t < b$. Then the interval-valued fuzzy set is defined as:

$\tilde{A} \triangleq \{(x,\ [\ \mu_{\tilde{A}^-}(x), \mu_{\tilde{A}^+}(x)\ ]): x \in \mathbb{R}\}$, is denoted as

$\tilde{A} = [(r, s, t; \alpha), (a, s, b; \beta)] = [\tilde{A}^-, \tilde{A}^+]$.

$\tilde{A}$ is called a level (α, β) interval-valued fuzzy number as shown in the following Fig 1.

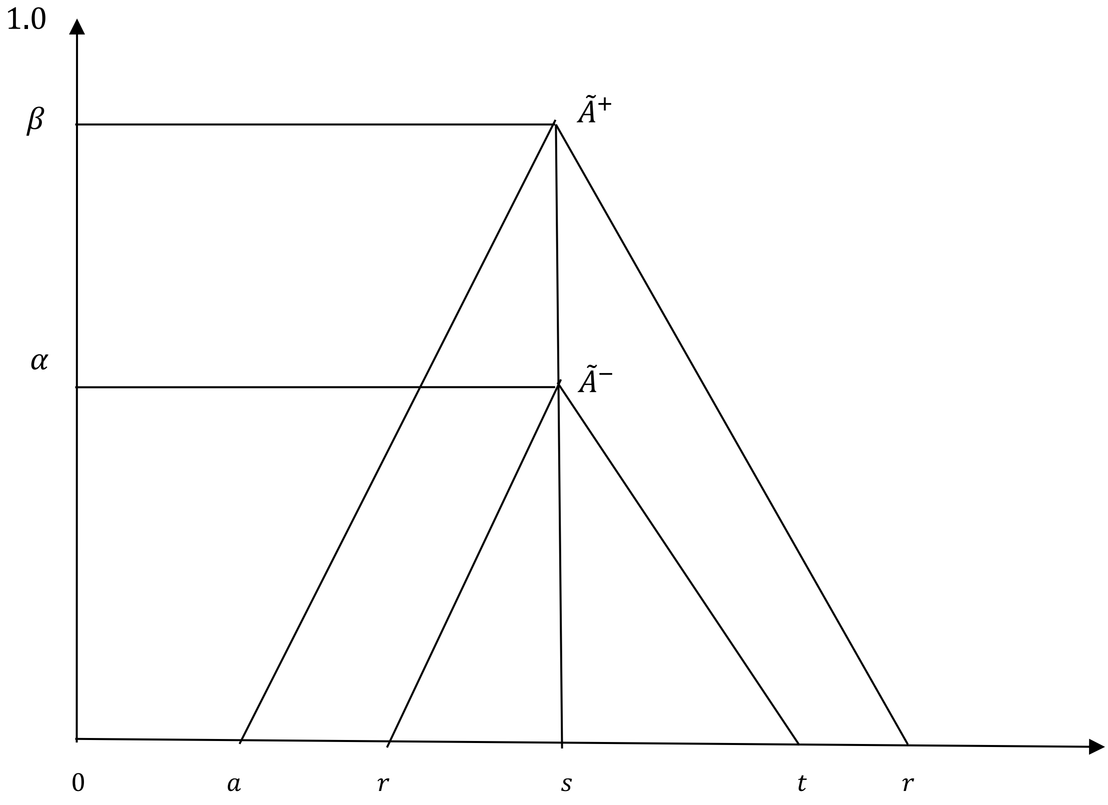


Figure 1. Level (α, β) interval- valued fuzzy number

Property 1. (Chiang, 2001).

Let, $F_{IVF}(\alpha,\beta) = \{[(r,s,t;\alpha),(a,s,b;\ \beta)]: for\ all\ \ a<r<s<t<b\}$,

$0<\alpha\leq\beta\leq 1$ be the family of (α,β) interval- valued fuzzy numbers.

Let,$\widetilde{P} = [(r,s,t;\alpha),(a,s,b;\beta)] \in F_{IVF}(\alpha,\beta)$, and$\widetilde{Q} = [(r_1,s_1,t_1;\alpha),(a_1,s_1,b_1;\beta)] \in F_{IVF}(\alpha,\beta)$. Then

1. $\widetilde{P}(+)\widetilde{Q} = [(r+r_1, s+s_1, t+t_1;\alpha),(a+a_1, s+s_1, b+b;\beta)]$,
2. $k\widetilde{P} = \begin{cases} [(kr,ks,kt;\alpha),(ka,ks,k\,b;\beta)], & k>0,\\ [(kt,ks,kr;\alpha),(kb,ks,k\,a;\beta)], & k<0,\\ [(0,0,0;\alpha),(0,0,0\,;\beta)], & k=0.\end{cases}$

Definition 4. (Chiang, 2001). Let $\widetilde{P} = [(r,s,t;\alpha),(a,s,b;\beta)] \in F_{IVF}(\alpha,\beta), 0<\alpha\leq\beta\leq 1$. The signed distance of $\tilde{P}$ from $\tilde{0}$ is given as

$$D_0(\widetilde{P},\tilde{0}) = \frac{1}{8}\left[6s+r+t+4a+4b+3(2s-a-b)\frac{\alpha}{\beta}\right].$$

Remark 1. $\widetilde{P} = [(a,a,a;\alpha),(a,a,a;\beta)]$, then $D_0(\widetilde{P},\tilde{0}) = 2a$.

Definition 5. (Chiang, 2001). Let $\widetilde{P},\widetilde{Q} \in F_{IVF}(\alpha,\beta)$, the ranking of level (α,β) interval- valued fuzzy numbers in $F_{IVF}(\alpha,\beta)$ using the distance function D_0 is defined as:

$\widetilde{Q} \prec \widetilde{P} \Leftrightarrow D_0(\widetilde{Q},\tilde{0}) < D_0(\widetilde{P},0)$,

$\widetilde{Q} \approx \widetilde{P} \Leftrightarrow D_0(\widetilde{Q},\tilde{0}) = D_0(\widetilde{P},0)$.

Property 2. (Chiang, 2001). Let $\widetilde{P} = [[(r,s,t;\alpha),(a,s,b;\beta)]]$ and

$\widetilde{Q} = [(r_1,s_1,t_1;\alpha),(a_1,s_1,b_1;\beta)]$ be (α,β) interval- valued fuzzy numbers in$F_{IVF}(\alpha,\beta)$. Then

1. $D_0(\widetilde{P}\oplus\widetilde{Q},\tilde{0}) = D_0(\widetilde{P},\tilde{0}) + D_0(\widetilde{Q},\tilde{0})$,
2. $D_0(k\,\widetilde{P},\tilde{0}) = k\,D_0(\widetilde{P},\tilde{0}), k>0$.

3. Assumptions and Notation

3.1. Assumptions

The machine-scheduling problem introduced in this paper requires the following assumptions:

1. All jobs are available for processing at time zero
2. The single machine can process at most one job at a time
3. No pre- emption is allowed

3.2. Notation

In the single machine scheduling, the following notation may be used

S: Schedule for the i job.

$P_i = [(r_i,s_i,t_i;\alpha),(a_i,s_i,b_i;\beta)]$: Processing time of job i on the machine under fuzzy environment.

D_i: Signed distance ranking of the processing time $[(r_i,s_i,t_i;\alpha),(a_i,s_i,b_i;\beta)]$,

$i = 1,2,\ldots,n$

d_i: Due date for the job i

c_i : Completion time for job i

T_i : $Max.(0, c_i - d_i)$

E_i : $Max.(0, d_i - dc_i)$

V_i : Slack time of job i

e_i: Penalty per unit time for the earliness of job i

l_i: Penalty per unit time for the tardiness of job i

4. Fuzzy processing time statement

Consider the following single machine scheduling problem. Assume there are a set of n jobs $(\mathrm{J}_1, \mathrm{J}_2, \ldots, \mathrm{J}_\mathrm{n})$ to be scheduled for processing on a single machine. Continuously, the machine is available from $\mathrm{t} = 0$ and process only one job at a time. Also, the jobs are continuously available from $\mathrm{t} = 0$ and requiring processing times $(\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_\mathrm{n})$ and due date $\mathrm{d}_\mathrm{i}(\mathrm{i} = 1, 2, \ldots, \mathrm{n})$ associated with each job are positive integers. The customer satisfaction decreases the completion time of job passes in between due date d_i and late date $\mathrm{d}_\mathrm{i}^{\circ}$ until vanishes in the delay case.

Mathematically, the problem can be formulated as follows

$$\mathrm{f(s)} = \min \sum |\mathrm{c}_\mathrm{i} - \mathrm{d}_\mathrm{i}| = \sum |\mathrm{E}_\mathrm{i} + \mathrm{T}_\mathrm{i}|.$$

5. Solution approach

In this section, the steps of the approach to determine the total penalty cost are:

Step 1: Calculate the signed distance ranking for the fuzzy processing time of all the jobs.

Step 2: Find the slack time of all the jobs $\mathrm{V}_\mathrm{i} = |D_i - d_i|$

Step 3: Arrange the jobs in increasing order of its slack time. If two jobs have the same slack time, then take the one with the lowest processing at the earlier position.

Step 4: Determine the total penalty cost of all jobs using the sequence in step3.

6. Numerical Example

Consider 7- jobs having fuzzy processing time, single machine and distinct due dates. Penalty cost for earliness is shown in the following Table 1:

Table 1. Penalty cost for earliness

Job	$\mathbf{P_i}$	$\mathbf{D_i}$	$\mathbf{d_i}$	$\mathbf{V_i}$	$\mathbf{e_i}$	$\mathbf{l_i}$
1	$[(0.5, 2, 5; 0.6), (0.25, 2, 7; 0.9)]$	5	8	3	2	3
2	$[(2, 3, 4; 0.6), (1, 3, 13; 0.9)]$	8	9	1	2	3
3	$[(4, 6, 12; 0.6), (2, 6, 16; 0.9)]$	14	9	6	2	3
4	$[(4, 5, 6; 0.6), (3, 5, 9; 0.9)]$	11	13	4	2	3
5	$[(2, 4, 64; 0.6), (1, 4, 11; 0.9)]$	9	11	2	2	3
6	$[(3, 5, 7; 0.6), (2, 5, 8; 0.9)]$	10	17	7	2	3
7	$[(5, 6, 13; 0.6), (4, 6, 17; 0.9)]$	15	11	5	2	3

7 jobs having(α, β)interval- valued fuzzy numbers due dates $[(r, s, t; \alpha), (\mathrm{a}, \mathrm{s}, \mathrm{b}; \beta)]$ which are converted into its average high ranking using the signed distance as in Definition 4 and referring to the solution approach in Section 5, the near optimal sequence is

$$\mathrm{S} = 2 > 5 > 1 > 4 > 7 > 3 > 6.$$

The total flow time of the system and the total optimized penalty cost are displayed in the following Table 2:

Table 2. Total flow time of the system and the total optimized penalty cost

Job	**Processing Time**	$\mathbf{d_i}$	$\mathbf{V_i}$	**Cost**
2	0-8	8	1	3*2
5	8- 17	9	2	1*3
1	17-22	9	3	6*3
4	22-33	13	4	4*3
7	33-48	11	5	2*3
3	48-82	17	6	7*3
6	82- 92	11	7	5*3

It is clear from table2 that the flow time of the system and the total optimized penalty cost due date to earliness / tardiness of the job is equal to $81

7. Concluding Remarks

In this paper, a single machine scheduling problem with distinct due dates has studied. We determined the optimal scheduling which attempted to minimize a cost function consisting of the earliness and tardiness costs with penalties under fuzzy environment. The criteria to calculate the optimal schedule according to the signed distance of fuzzy numbers has investigated. Moreover, it is determined that the proposed technique facilitates to determine the optimal solution for small systems, satisfactory to the decision maker (DM), and help the DM to determine the best schedule for a given set of jobs that is to control penalty cost.

In real life situations, the uncertainty may be handled by considering the intuitionistic fuzzy set. While the proposed work did not consider the intuitionistic fuzzy set. This may be a limitation of the present study. For future research scope, there are several research directions to work. The better one is to introduce the concept of intuitionistic fuzzy set to the present work.

Acknowledgment

The author would like to thank the Editor-In–Chief and the two anonymous referees for their various suggestions and helpful comments that have led to the improved in the quality and clarity version of the paper.

Conflicts and Interest

The author declares no conflict of interest.

References

Arlk, O. A., (2019). Dissatisfaction levels of earliness and tardiness durations by relaxing common due date on single machine scheduling problems. Journal of Multidisciplinary Modeling and Optimization, Vol. 2(1), pp. 1-15.

Arlk, O. A., and Toksarl, M. D., (2019). Fuzzy parallel machine scheduling problem under fuzzy job deterioration and learning effects with fuzzy processing times. In Advanced Fuzzy Logic Approaches in Engineering Science, pp. 49-67, IGI Global.

Baker, K. S., and Scudder, G. D., (1990). Sequencing with earliness and tardiness penalties. A Review- Operations Research, Vol. 38, pp. 22-36.

Belkaid, F., Maliki, F., Boudahri, F., and Sari, Z. (2012). A branch and bound algorithm to minimize makespan on identical parallel machines with consumable resources. In Advances in mechanical and electronic engineering (pp. 217-221). Springer, Berlin, Heidelberg.

Belkaid, F., Sari, Z., and Yalaoui, F., (2013). A hybrid genetic algorithm for parallel machine scheduling problem with consumable resources. International Conference of Control, Decision and Information Technologies, pp. 143- 148.

Belkaid, F., Yalaoui, F., and Sari, Z., (2016). An efficient approach for the reentrant parallel machine scheduling problem with consumable resources constraints. International Journal of Information Systems and Supply Chain Management, Vol. 9(3), pp. 1-25.

Ben-Yehoshua, Y., and Mosheiov, G. (2016). A single machine scheduling problem to minimize total early work. Computers & Operations Research, Vol. 73, pp. 115-118.

Bellmann, R., and Zadeh,L., (1970). Decision making in a fuzzy environment. Management Science, Vol.17, pp. 141-164.

Biskup, D., and Feldmann, M., (2001). Benchmarks for scheduling on a single machine against restrictive and unrestrictive common due dates. Computers& Operations Research, Vol. 28(8), pp. 787-801.

Cahon, C. S. Mc., and Lee, E. S., (1992). Fuzzy job sequencing for a job shop. European Journal of Operations Research, Vol. 62, pp.294-301.

Che, A., Wu, X., Peng, J., and Yan, P., (2017). Energy- efficient bi- objective single machine scheduling with power-down mechanism. Computers& Operations Research, Vol. 28, pp. 172-183.

Chiang, J., (2001). Fuzzy linear programming based on statistical confidence interval and interval- valued fuzzy set. European Journal of Operational Research, Vol. 129, pp. 65-86.

Cong, T. C. E., (1995). Optimal due date determining and sequence on $n-$ jobs on a single machine. Journal of Operations Research Society, Vol. 35(5), pp. 425-433.

Dubois, D., and Prade, H. (1980). Fuzzy sets and systems: theory and applications, Academic Press, New York.

Duenes, A., and Petrivic, A. A new Approach to multi- objective Single Machine Scheduling Problem Under Fuzziness, CTAC School of Maths and Information Sciences, Coventry, U. K., 1995.

Ertem, M., Ozcelik, F., and Sarac, T., (2019). Single machine scheduling problem with stochastic sequence- dependent setup times. International Journal of Production Research, Vol. 57 (10), pp. 3273-3289.

Gerstl, E., and Mosheiov, G., (2020). Single machine scheduling to maximize the number of on- time jobs with generalized due- dates. Journal of Scheduling, Vol. 23, pp. 289-299.

Gupta, S., and Rambha, M.M.G.I., (2011). Single machine scheduling with distinct due dates under fuzzy environment. International Journal of Enterprise Computing and Business Systems, Vol. 1(2), pp. 1-9.

Ishii, H., and Tada, M., (1995). Single machine scheduling with fuzzy precedence relation. European Journal of Operations Research, Vol. 87, pp. 284-288.

Ishubuchi, H. T.M., and Lee, K. H. (1996). Formulation of fuzzy flow shop scheduling with fuzzy processing time. Proceedings of IEEE International Conference of Fuzzy Systems, pp. 199- 205.

Jadhav, V. S., and Bajaj V. H., (2012). Single machine scheduling problem under processing time and fuzzy due dates. International Journal of Computer Engineering Science, Vol. 2(5), pp. 12-19.

Kaufmann, A., and Gupta, M.M. (1988). Fuzzy Mathematical Models in Engineering and Management Science, Elsevier Science Publishing Company INC, New York,

Koulamas, C., Kyparisis, G. J., (2019). New results for single- machine scheduling with past- sequence- dependent setup times and due date- related objectives. European Journal of Operational Research, Vol. 278, pp. 149-159.

Liaw, C. F., (1999). A branch- and- bound algorithm for the single machine earliness and tardiness-scheduling problem. Computers& Operations Research, Vol. 26(7), pp. 679-693.

Mohamad, N. H., and Said, F., (2011). Solving single machine scheduling problem with common due date. Business Management Dynamics, Vol. 1(4), pp. 63-72.

Ponnalagu, K., and Mounika, P., (2018). A study on sequencing problem in two different environments. International Journal of Mathematics and its Applications, Vol. 6(1- D), pp. 835-839.

Senthil kumar, P., and Narayanan, S., (2010). Literature review of single machine scheduling problem with uniform parallel machines, Intelligent Information Management, Vol. 2, pp.457-474.

Stoskov, Y. N., and Egorova, N. G. (2018). Single machine scheduling problem with interval processing times and total completion time objective. Algorithms, Vol. 11(5), pp. 47-66.

Toksari, M. D., and Arık, O. A. (2017). Single machine scheduling problems under position dependent fuzzy learning effect with fuzzy processing times. Journal of Manufacturing Systems, Vol. 45, pp. 159-179.

Wang, T., and Xu, D., (2015). Single machine scheduling with workload- dependent maintenance duration to minimize maximum lateness. Mathematical Problems in Engineering, pp. 1-5

Wu, H., and Wang, B., (2020). Single machine scheduling problem with fuzzy time delays and mixed precedence constraints. Journal of Intelligent & Fuzzy Systems, (preprint), pp. 1-14.

Yazdani, M., Khalili, S. M., Babagolzadeh, M., and Jolai, F., (2017). A single- machine-scheduling problem with multiple unavailability constraints. A mathematical model and an enhanced variable neighborhood search approach. Journal of Computational Design and Engineering, Vol. 4(1), pp. 46-59.

Yue, F., Song, S., Jia, P., Wu, G., and Zhao, H. (2020). Robust single machine scheduling problem with uncertain job due dates for industrial mass production. Journal of Systems Engineering and Electronics, Vol. 31(2), pp. 350- 358.

Zadeh, L. A. (1965). Fuzzy sets. Information Control, Vol. 8(3), pp. 338-353.