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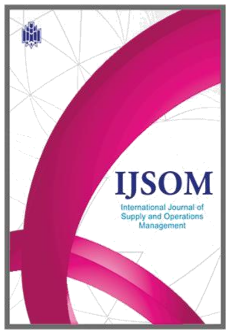

The Stackelberg Game Model for Vendor-managed Inventory Systems: A Wholesale Price versus a Two-part Tariff Contract

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Abstract

This paper investigates the issue of replenishment coordination for a two-echelon supply chain with one manufacturing vendor and multi retailers under vendor-managed inventory mode of operation. The demand is influenced by the retail price and market scale generally recognized as a Cobb- Douglas demand function in the available studies. The authors developed a Stackelberg game theoretic model for examining vendor-managed inventory supply chain performance in terms of combined profit. The aim was to find the wholesale and retail price, the replenishment interval of the retailers, and the fraction backlogging so as to the combined profit of supply chain being optimized. The problem was formulated as NLP models while two different contracts including wholesale price and two-part tariff were also taken into consideration. Using some benchmark data, the authors compared such contracts and illustrated how they can optimize the performance of VMI-type supply chains. Results of the numerical experimental study show that the wholesale price is more beneficial to all supply chain members under specific conditions. Sensitivity analyses of the model's parameters are also performed to assess the impacts of changes on the performance of supply chain.

Keywords: Vendor managed inventory; Game theory; Wholesale price; Two-part tariff; Supply chain**.**

1. Introduction

According to Mattson (2000), a supply chain is a physical network in which different entities of material, cash and information are transferred amongst the players. Supply chain management is defined as an integrative approach dealing with the planning and control of materials and information from suppliers to end customers (Monczaka et al., 2007; Jones and Riley, 1985). Today, various coordination strategies have caught attention encouraging all members of supply chain to coordinate their linked decisions such as inventory replenishment in order to optimize the system-wide performance. Thus, the emphasis on supply-chain coordination has increased in recent years (e.g., see Arntzen et al., 1995; Lee and Billington, 1992; Lee et al., 1997; Tayur et al., 1999), and a bunch of coordination strategies including but not limited to quick response (QR), continuous partnerships, collaborative planning, forecasting and replenishment (CPFR), advanced continuous replenishments, and vendor-managed inventory (VMI) have been discussed in the literature (Disney and Towill, 2002; Simchi-Levi et al., 2007). VMI as one of the best identified strategies in the area of vendor-buyer coordination was introduced in the mid 80's following the partnership between Wal-Mart and Procter and Gamble (Vigtil, 2007). In a VMI-type supply chain, vendor (supplier) takes broader responsibility for inventory management that leads to taking care of buyer (customer)'s inventory too. The benefits of VMI are successfully recognized by many famous companies such as Wal-Mart, Fruit of the Loom, Shell Chemical, Kmart, Dillard Department Stores, J.C. Penney, Lucent Technologies, Electrolux Italia, Dell, HP, ST Microelectronics, Target, Walgreens, Barnes & Nobel, Eckerd, and Procter & Gamble in several studies (Vigtil A., 2007; Holmstro J., 1998;TowillD.R, 2003; Hu W., 2008; Gurenius P, Wicander J. 2007; Tyana J, Wee H. M. 2003; Ronen D, et al. 1987; Miller D. M. 1987; Sadeghi J, et al. 2014; Chen X, et al. 2012; Pasandideh S. H. R, et al. 2011; Hariga M. A, Al-Ahmari A. 2013; Yu Y, et al. 2013; Shu J, et al. 2012; Tu-Chen L. 2013). Utilization of VMI is occasionally aligned with the employment of well-known contracts like sales rebate contract (Vigtil, 2007), incentive contract (Achabal D. D, et al. 2000), and explicitly incorporated contract (Jones & Riley 1985). Cachon (2003) provided a detailed-

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investigation into different contract types established in simple or complex supply chains.

The purpose of this research was to consider VMI supply chain with one manufacturing vendor and multi retailers that purchase one unified product from the manufacturer following a deterministic demand. For this reason, the authors formulated the replenishment decisions using a Stackelberg game theoretic model consisting of non-linear mixed integer programming models at two levels. Additionally, two different types of partnership protocols including wholesale price contract and two-part tariff contract were applied as the type of agreement between manufacturer and its retailers. All members of supply chain were seeking to reach the equilibrium condition that optimizes their benefits. The objective was to determine appropriate values of wholesale price, retail price, backlogging percentage, and the common replenishment cycle so that the total combined supply chain would maximize. Besides, the performance of supply chain was examined and compared once wholesale price and two-part tariff were respectively assumed as the contract type amongst the SC members. Numerical examples are provided for demonstrating the application of the proposed methodology while evaluating the behavior of supply chain members with wholesale price and two-part tariff contracts. This research is most probably one of the pioneer works that investigates VMI-type supply chain with different contracts in a game-theoretic context.

We organize the remainder of this paper as follows: Section 2 reviews the related works. In Sections 3, we describe the VMI supply chain problem and its relevant definitions for variables and parameters. Section 4 formulates the MINLP problem using Stackelberg game model for both types of contracts discussed. Besides, some examples alongside sensitivity analyses are reported in the same section. Finally, conclusion and recommendation for future research are in Section 5.

2. Literature review

There has been an incremental growth of researches on vendor managed inventory (VMI) especially from 1998 to 2014. VMI is mainly considered a coordination scheme in which the vendor takes the responsibility of overall inventory management. Based on this strategy, all members of supply chain are seeking to integrate their linked decisions so as to optimize the operations and find better benefits. The related research literature can be divided into two parts: those VMI models in which at least on type of contractual agreement is reflected on and discussed in the paper, and those papers that have applied a game-theoretic approach so as to optimize the decisions and reach equilibrium. According to Tsay (1999), the supply chain contract is “a coordination mechanism that provides incentives to all of its members so that the decentralized supply chain behaves nearly or exactly the same as the integrated one”. Cachon (2003) provided a detailed investigation on different contract types established in a typical supply chain. He studied a number of contracts including buy backs, revenue sharing, quantity flexibility and sales-rebate to coordinate the newsvendor supply chain model. Moreover, he showed how these contracts can behave differently. Some explorations inside the literature show that most of the VMI-type supply chains discuss wholesale price, backup, and consignment agreements. However, in the remaining ones, no exact agreement is explicitly presented in the body of research. For instance, Darwish and Odah (2009) examined a single-vendor multi-retailer supply chain operating under VMI mode when a contractual agreement explicitly included in an upper bound was also placed on each retailer's inventory level. It was assumed this upper bound can protect retailers against exceeding their inventories. By imposing a penalty cost on vendor for conditions that inventories exceed the upper bound, the objective of the model was to find the optimal parameters that can minimize the total cost of supply chain. Sue-Ann et al. (2012) discussed and developed an evolutionary algorithm using Particle Swarm Optimization (PSO) and a hybrid of Genetic Algorithm (GA) and Artificial Immune System (GA–AIS) to optimize the operational parameters of a two-echelon VMI supply chain with single vendor multiple buyers. They considered a revenue share ratio in their model to maximize and evaluate the channel profits generated by using the two mentioned algorithms. Nachiappan and Jawaher (2007) modeled VMI problem in the context of a two-echelon supply chain with single vendor multiple buyers where a revenue-sharing contract is included. A nonlinear integer programming problem (NIP) which is incorporated in a GA based heuristic model is proposed to solve this problem for finding the optimal sales quantities. Wong and Leung (2009) investigated the effect of a sales rebate contract on a two-echelon supply chain with a single supplier serving multiple retailers in a VMI partnership. In their paper, two different configurations of retailers were also compared: independent retailers with a demand function sensitive only to their own price and competing retailers with a demand function depending on all retailers’ prices. Furthermore, the effect of learning and forgetting functions in the vendor’s production process under a VMI with consignment agreement was evaluated by Zanoni et al. (2012). They evaluated learning effects on replenishment decisions (size and time) in a VMI supply chain with a single vendor and a single-buyer at each level. A more recent study by Hariga et al. (2013) considered a VMI supply chain composed of a single vendor and multiple retailers under storage contractual contract. They addressed synchronization of both vendor and buyer’s ordering cycles so as to minimize the total inventory costs over the entire supply chain.

With respect to the various contract types reflected in VMI-supply chains, the most popular models that are usually used to solve VMI problems include analytical, operation research algorithms such as nonlinear programming, systems dynamics, game theory models such as Stackelberg or Nash, meta-heuristics such as ant-colony optimization, and so on (Akhbari et al., 2013). Here, we mainly focus on and review background of the game models being used to solve VMI problems. Tu Chen (2013) developed a Stackelberg game model to solve a VMI problem with revenue-sharing contract. They assessed the applicability of problem in a case study involving a regional seafood supplier and a local store of a large national retail chain. Yu et al. (2013) introduced a Stackelberg game model to firstly select and form the surrounding retail chain. Then, they provided a hybrid algorithm combining dynamic programming (DP), genetic algorithm (GA) and analytical methods so as to solve the developed game model. Comparing two different strategies of retailer selection and non-selection, they found the manufacturer's profit is positively influenced by his optimal decision about retailer selection. Yu et al. (2009a) studied a VMI system which consists of one vendor serving multiple retailers, and observed how the vendor can take advantage of the information that he obtained from his multiple retailers for maximizing his profit by using a Stackelberg game where the VMI partnership has been implemented. Yu et al. (2009b) also provided a Stackelberg game model to optimize the supply chain profits through the determination of wholesale price, advertising investment, replenishment cycles, and backorder quantities. They considered a supply chain system composed of one manufacturing vendor and multiple retailers geographically dispersed and independent of each other. More papers which involved utilization of Stackelberg game models in the area of VMI supply chain can be accessible in the researches by Almehdawe and Mantin (2010)[40] andChen et al. (2010).

Reviewing the related works shows although the dominant Stackelberg game model has been used by many researchers, there are some other kinds of game models that a few researchers tend to apply them in order to solve VMI problems. For instance, Yu and Huang (2010) modeled the VMI problem as a dual Nash game model with two sub-games. The first game was between the retailers' chain and the next one was between the manufacturer and its retailers. The objective was to optimize individual profits of the manufacturer and the retailers. In another study, Kim and Park (2010) assumed a two- echelon supply chain with the objective of maximizing revenues of both manufacturer and retailer. They developed a differential game model on top of a system dynamics model in order to compare and evaluate different pricing strategies of retailers including constant, linear, and non-linear.

Summarizing the review presented above, it could be said that the papers were mainly focused on the optimization of VMI problems with various solution models, either game-centric or non-game. Although different contract types have been used in studying supply chain models, as discussed, the literature is limited with regard to the utilization of various contract types under VMI partnership. Indeed, there is only a few types of contracts, mainly the inventory-related ones, which have been addressed by researchers till now. Thus, this study tries to fill this gap by examining and comparing Stackelberg game models' performance for two types of candidate contracts including the wholesale price and two-part tariff under a VMI agreement. Sensitivity analyses also help researchers and practitioners to have a better understanding of the benefits of using these contracts for more real-life problems.

3. Problem setup

We consider a two-echelon supply chain including a manufacturing vendor and multiple retailers with a VMI partnership contract. The manufacturing vendor supplies the finished products on a wholesale price to the retailers who sell the products to their end customers on a possible different retail price. Based on Hu (2008), in a wholesale price contract, the manufacturer charges the retailer C_p per unit purchased. We assume that each retailer faces a demand which is elastic to the retailer price and can be affected by the market scale. The end customer's demand is assumed to be characterized by following a downward convex function which is recognized as Cobb- Douglas in the relevant literature (Yu Y, et al. 2013; Yu Y, et al. 2009a; Yu Y, et al. 2009b; and Almehdawe E, Mantin B. 2010).

$D_i(P_i) = K_i p_i^{-e_i}$(1)

where K_i is the market scale, $e_i > 1$the demand elasticity, and p_i the i[th] retailer price.

Since the VMI system is in place, the overall inventory management is under the manufacturer's responsibility. Therefore, each retailer is motivated to share his demand and inventory data with the manufacturer while the manufacturer also undertakes taking control of the retailer's inventory as well as his own inventory. In this condition, each retailer pays .dollar/unit/time to manufacturer as a cost of inventory. By the way, .is practically being determined through the negotiations between manufacturing vendor and his retailers. In this model, the manufacturing vendor makes the replenishment decisions for the retailers in a common replenishment cycle (C) so as to save inventory-related costs (Darwish M. A, Odah O. M. 2010; Yu Y, et al. 2013; Yu Y, et al. 2009a; Yu Y, et al. 2009b; and Viswanathan S,Piplani R. 2001) and dispatches a quantity of product that may be fixed or variable. In fact, the replenishment orders could be only distributed in identical time intervals (for example, every two months). In addition, the entire batch of finished products is delivered simultaneously, i.e. the retailer should wait for the manufacturer to produce the exact order quantity and then, he will be able to take over the batch and pay. The manufacturer's production capacity (U) is fixed and limited. The manufacturer incurs both manufacturing/production and transpirations costs respectively identified as C_m and $\phi_i$in our model. Besides, the manufacturer encounters the entire inventory holding costs including H_{b_i} at retailers side as well as H_p and H_r at his side.

In short, the remaining specifications and assumptions of our VMI supply chain in which the manufacturing vendor and its retailers interact with each other are described as follows:

(1) There is only one product in the model.

(2) The mutual agreement is based on wholesale price contract. (3) The holding cost at retailer's side shall be higher than that of manufacturer's.

(4) No discount is available.

(5) The backorder cost per unit product shall be higher than holding cost per unit product.

(6) Both parties are interested to have a long-term partnership so as to control their ordering and inventory costs reasonably and efficiently.

(7) Practically, the wholesale price (w) will be less than retail price within the entire time span of manufacturer-retailer agreement.

(8) The retailers do not compete against each other.

(9) Shortages are allowed and compensated with backorder costs.

To model the discussed supply chain problem with VMI, we first define the following notations:

Indices:
m: total number of retailers
i: index of retailers, i = 1, 2,. . . ,m
Parameters:
U: production capacity of manufacturer (unit/time)
C_m: production/manufacturing cost of the product ($/unit)
$D_i(p_i)$: demand of retailer i per unit time which is a function of P_i
e_i: price elasticity of retailer i's demand rate (unit/time)
A_i: fixed franchise fee per replenishment order in two-part tariff contract ($/order)
H_{b_i}: holding cost of the product at retailer i's side paid by manufacturing vendor($/unit/time)
H_p: holding cost of the product at the manufacturer's side ($/unit/time)
K_i: a constant in the demand function $D_i(P_i)$ of retailer's i representing his market scale
L_{b_i}: backorder/shortage cost of retailer i for one unit ($/unit/time)
R: production/manufacturing rate of the manufacturer, which is a known constant (unit/time)
S_{b_i}: fixed replenishment's cost of the product for retailer i paid by manufacturing vendor ($/order)
S_p: fixed production setup cost of the finished product at the manufacturer's side ($/setup)
ϕ_i: direct transportation cost for shipping one unit product from the manufacturer to retailer i ($/unit)
ξ_i: inventory cost of one unit product paid to manufacturing vendor by retailer i($/unit/time)
$\mathrm{TDC_p}$: total direct cost for finished product at manufacture's side ($/time)
$\mathrm{TIC_p}$: total indirect cost for finished product at manufacture's side ($/time)
π_i: retailer's profit ($/time)
π_p: manufacturer's profit ($/time)
π_t: total profit ($/time)
x: binary variable indicating whether the total retailers' demand is less than the production capacity $(x = 1)$, and $(x = 0)$otherwise
***Decision variable of retailer** i, i = 1, 2,. . . ,m*
p_i: retail price charged by the retailer$i$($/unit)
Decision variable of manufacturing vendor
b_i: fraction of backlogging time in a cycle of retailer i (backlogging percentage)
C: common replenishment cycle of the product
C_p: wholesale price of the finished product set by the manufacturing vendor ($/unit)

4. Mathematical model

Since we described all problem notifications in the previous section, we present the mathematical formulation of the problem in this section.

4.1 Proposition 1: wholesale price contract

Mathematical model of the problem:

In order to setup the mathematical model of the problem, we firstly need to calculate the profit (revenue mines cost) of both manufacturer and retailers using the notations defined above.

Retailer's payoff function:

For each finished product unit sold, retailer i gains a revenue of $p_i D_i(p_i)$while its cost consists of two components: purchasing cost of $C_p D_i(p_i)$ and inventory cost which is asked by manufacturer, $\xi_i D_i(p_i)$. Then, the net profit or payoff function for retailer i can be given by:

$$\pi_i = p_i D_i(p_i) - C_p D_i(p_i) - \xi_i D_i(p_i) = \left(p_i - C_p - \xi_i\right) D_i(p_i) \tag{2}$$

Manufacturer's payoff function:

Likewise, the total revenue for the manufacturer derives from the selling of finished products to its retailers at wholesale price i.e. $C_p \sum_{i=1}^{m} D_i(p_i)$ as well as the revenue from the inventory costs paid by all retailers equal to $\xi_i \sum_{i=1}^{m} D_i(P_i)$.

The manufacturer's cost is acquired by summation of total direct and indirect costs. The total direct cost consists of two components including production/manufacturing cost and transportation cost, mathematically formulated as:

$$TDC_p = \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) \tag{3}$$

The total indirect cost for the manufacturer mainly consists of two components: inventory management costs at retailers' sides and inventory management cost at manufacturer's side. The inventory costs at each retailer's side are the fixed inventory costs, variable inventory costs, and back-ordering costs equal to:$\frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i){b_i}^2 C^2}{2} L_{b_i}\right]$. Similarly, the inventory cost at manufacturer's side is the sum of setup cost and inventory holding cost. Setup costs are incurred if the whole retailers' demand is less than manufacturer's production capacity ($x = 1$); otherwise, the production process will be continuous without any production setup cost ($x = 0$). The inventory management cost of manufacturer per unit time can be formulated as: $\frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right]$. Now, the total indirect cost of manufacturer denoted by $TIC_p$is expressed by the following formula:

$$TIC_p = \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i){b_i}^2 C^2}{2} L_{b_i}\right] + \frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right] \tag{4}$$

The manufacture profit, π_p, is the total revenue acquired from all of the retailers minus the total costs (direct and indirect) faced by the manufacturer which are already described. The following equation can determine the net profit or payoff function of manufacturer:

$$\pi_p = C_p \sum_{i=1}^{m} D_i(p_i) + \xi_i \sum_{i=1}^{m} D_i(p_i) - TDC_p - TIC_p$$

$$\pi_p = \sum_{i=1}^{m} D_i(p_i)\left(C_p + \xi_i\right) - \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) - \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i){b_i}^2 C^2}{2} L_{b_i}\right] - \frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right] \tag{5}$$

Stackelberg game model for wholesale price contract

In the previous section, we formulated the payoff functions of manufacturer and its retailers as defined in Eqs. (2) and (5). With the assumption that the manufacturer is the leader, in this section, we can formulate the VMI supply chain as a Stackelberg game model. The sequence of decisions is as follows: In the first step, the manufacturer determines the quantities of its decision variables noted as common replenishment cycle of the product (C), fraction of backlogging time (b_i), the wholesale price of the finished product (C_p), and the binary variable of x so as to maximize the net profit or payoff function. Then, the retailers identified as followers in the game model determine the retail price in an attempt to maximize their net profits based on the given values of manufacture. This process continues reciprocally until the manufacture cannot increase his net profit. In this condition, both manufacturer and retailers reach the state of *Stackelberg equilibrium* in which their net profits are maximized.

Considering the above description, two optimization problems should be resolved to obtain the optimum and exact values. The manufacturer solves the following non-linear mixed-integer programming model (MINLP), denoted by "Opt1":

Opt1:

$$\text{Max}\pi_p = \sum_{i=1}^{m} D_i(p_i)\left(C_p + \xi_i\right) - \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) - \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i){b_i}^2 C^2}{2} L_{b_i}\right] - \frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right],\ i = 1, 2, \dots, m$$

Subject to $\sum_{i=1}^{m} D_i(p_i) \leq U$ (6)

$U - \sum_{i=1}^{m} D_i(p_i) \leq x.M$

$0 \leq b_i \leq 1$

$C \geq 0$

$C_p \geq 0$

$x = [0,1]$

The first constraint guarantees that the retailers' demand is always less than the manufacturer's production capacity. The second one determines $x = 1$when there is redundancy in manufacturer's capacity. The fraction of backlogging that cannot exceed the market demand is shown in the third constraint. The remaining constraints are self-explanatory.

The objective function of each retailer is being formulated as follows (Opt2):

Opt2:

$\text{Max}\pi_i = (p_i - C_p - \xi_i)D_i(p_i),\ i = 1,2,\dots,m$

Subject to $p_i - C_p - \xi_i \geq 0$ (7)

$p_i \geq 0$

The first constraint emphasizes the quantity of retail price which pessimistically equals the sum of wholesale price and inventory cost paid to the manufacturer. Next, the constraint represents a basic rule. To solve the model and find out the optimal decisions, we use the *Karush– Kuhn–Tucker (KKT)* conditions as formerly utilized by some other researchers in (Darwish & Odah 2010; Almehdawe & Mantin 2010; and Yu & Huang 2010).

In this regard, two conditions need to be added to the model as follows:

$p_i - C_m - \xi_i \geq 0 \ \perp \ r_i \geq 0,$ (8)

$(1 - e_i)K_i p_i^{-e_i} + \ e_i(C_p + \xi_i)K_i p_i^{-(e_i+1)} + r_i = 0 \ \perp p_i \geq 0$

In the following KKT conditions, r_i represents the dual variable of each retailer's constraint which is assumed to be a non-negative variable in our mathematical modeling. The symbol of $\perp$ specifies the perpendicular relationship between retailers KKT conditions. Now, we supplement the leader optimization problem by adding the KKT conditions to it while we also add the M as a big number or penalty for the feasible disruption from the above mentioned conditions.

Furthermore, as it is clear in the following obtained optimization problem (*Opt*), we relax the binary constraint for x and consider a penalty to the objective function denoted by $Mx(1 - x)$to prevent the deviation of x from 0 or 1. All of the mentioned changes on the model will result in obtaining the below non-linear problem (NLP) identified as Opt:
Opt:

$\text{Max}\pi_p = \sum_{i=1}^{m} D_i(p_i)\left(C_p + \xi_i\right) - \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) - \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i) {b_i}^2 C^2}{2} L_{b_i}\right] - \frac{1}{c}\left[xS_p + \sum_{i=1}^{m} \frac{D_i(p_i)C^2}{2R} H_p\right] - \sum_{i=1}^{m} M r_i(p_i - C_p - \xi_i) - \sum_{i=1}^{m} M p_i[(1 - e_i)D_i(p_i) + K_i p_i^{-(e_i+1)}(C_p + \xi_i)e_i + r_i] - Mx(1 - x),$
$i = 1,2,\dots,m$

Subject to $\sum_{i=1}^{m} D_i(p_i) \leq U$ (9)

$U - \sum_{i=1}^{m} D_i(p_i) \leq x.M$

$0 \leq b_i \leq 1$

$C, C_p, x \ \geq 0$

$(1 - e_i)D_i(p_i) + K_i p_i^{-(e_i+1)}(C_p + \xi_i)e_i + r_i = 0$

4.2 Numerical example

In order to evaluate the benefits of Stackelberg game model for a VMI-type supply chain with wholesale price contract, we conduct some numerical examples in this section. We construct the NLP model in AIMMS[1] v3.12. This tool, introduced by Paragon Decision Technology in 1993, is one of the well-known and advanced mathematical software for the areas of operation research and optimization modeling that supports multiple world-class solvers such as KNITRO, CPLEX, BARON, GUROBI, and CONOPT (www.aimms.com, 2012).The model is solved on a Core i5, 2.4 GHz and 4GB RAM using the KNITRO 8.0 solver of AIMMS. For your reference, KNITRO is a software package for solving large-scale continuous smooth optimization problems especially nonlinear programming ones with or without complementary constrains (www.aimms.com, 2012).

In the first example, we consider a supply chain with a single manufacturer and three retailers who follow the base parameters denoted above. In addition, we consider 10 variations on the base settings to conduct the sensitivity analysis accordingly. We also use some other solvers including CONOPT 3.14 and IPOPT 3.10.1 so as to find their effectiveness compared to KNITRO 8.0. The insights from this example are discussed below.

Table 1. Optimal results and sensitivity analysis for Stackelberg equilibrium: the wholesale price contract

	Manufacturer						Retailer					
Settings	C	C_p	x	b_1	b_2	b_3	p_1	p_2	p_3	D_1	D_2	D_3
Base[a]	0.71	588	0.0	0.027	0.027	0.027	1592	2588	2090	22.6	73.2	33.7
1 $C_m = 320$	1.44	1199	1.0	0.013	0.013	0.013	3220	5233	4227	7.3	29.3	12.6
2 $H_p = 1.5$	0.74	588	0.0	0.026	0.026	0.026	1591	2585	2088	22.6	73.3	33.8
3 U=400	0.69	427	1.0	0.028	0.028	0.028	1162	1888	1525	37.4	110.2	52.4
4 $K_1 = 1.5 \times 10^6$	0.91	620	1.0	0.021	0.021	0.021	1678	2727	2203	10.4	68.3	31.3
5 $e_1 = 1.2$	0.68	815	1.0	0.028	0.028	0.028	4947	3573	2886	110.6	48.1	21.5
6 $\xi_i = 12$	0.85	587	1.0	0.023	0.023	0.023	1606	2610	2108	22.3	73.3	33.3
7 $H_{b_i} = 6$	0.59	589	0.0	0.048	0.048	0.048	1593	2589	2091	22.6	73.1	33.7
8 $L_{b_i} = 20$	0.86	592	1.0	0.18	0.18	0.18	1604	2606	2105	22.3	72.5	33.4
9 $S_p = 300$	0.85	586	0.0	0.023	0.023	0.023	1586	2577	2081	22.7	73.6	33.9
10 $\phi_i = 9$	0.74	611	0.0	0.026	0.026	0.026	1654	2687	2170	21.3	69.7	32.0
Solver Conopt 3.14	0.85	593	1.0	0.023	0.023	0.023	1606	2610	2108	22.3	72.3	33.3
Solver IPOPT 3.10.1	0.71	588	0.0	0.027	0.027	0.027	1592	2588	2090	22.6	73.2	33.7

[1]Advanced Integrated Multidimensional Modeling Software

Table 1. Continued

Profit/Payoff π_1	Profit/Payoff π_2	Profit/Payoff π_3	Profit/Payoff π_p	Profit/Payoff π_t	Solving time(Sec)	Memory Used(Mb)
22478	145617	50339	55725	274159	0.03	24.2
14732	117885	37981	43206	213804	0.05	38.6
22493	145665	50361	55744	274632	0.00	39.2
27157	160055	57101	53921	298234	0.00	39.2
10891	143343	49293	50840	254367	1.88	39.3
456119	132188	44246	118565	751118	0.09	42.7
22363	145241	50165	55688	273457	0.09	26.6
22470	145589	50326	55648	274033	0.09	26.6
22383	145309	50197	55725	273614	0.06	29.4
22536	145802	50424	55635	274397	0.03	29.4
21976	143980	49585	54969	270510	0.03	23.9
22363	145241	50165	55688	273457	0.00	68.3
22478	145617	50339	55725.4	274159.4	0.05	68.3

[a] $K_1 = 3\times10^6,\ K_2 = 2\times10^6,\ K_1 = 1.5\times10^6,\ e_1 = 1.6,\ e_2 = 1.3, e_3 = 1.4,\ C_m = 160,\ \xi_i = 9, \phi_i = 3,\ H_{b_i} = 4,$ $L_{b_i} = 200, S_{b_i}$=40, $H_p = 3, S_p = 100, U$=200

With the increase of production cost (C_m) to 320, the wholesale price increases while retailers also increase their selling prices. In this situation, the market demand of all retailers strictly decreases that leads to about 25% reduction in manufacture's profit while he needs to adjust the production line to operate with lower production capacity. When the holding cost at manufacturer's side (H_p) decreases, with no change in market demands, all the retailer's and manufacture's profits smoothly increase as shown in setting 2. With more production capacity of manufacturer, we face a situation in which both wholesale and retail prices are decreased. In this case, the retailers profit more, but the manufacturer's profit is decreased. This can have different reasons such as the increase of setup costs since the capacity is not totally used up. Besides, the relevant inventory costs of retailers increase because the order quantities are increased with the same holding cost paid by the manufacturer. To analyze the influence of market scale on the problem, we decrease the market scale by 50% from 3000000 denoted in the base example to 1500000 in setting 4. In this condition, the first retailer's demand (D_1) will decrease from 22.6 to 10.4. Then, he increases the retail price (p_1) to 1678 to compensate the demand reduction in the market. The decision is followed by the mutual decision of manufacturer to increase the wholesale price accordingly. These decisions lead to increase in the profits of the manufacturer and all relating retailers. It should be pointed out that the other retailers' demands are sensitive to the change on the first market scale since they are indirectly affected by the manufacturer's control. In the next setting, the price reduction of elasticity by 25% (from 1.6 to 1.2) causes the manufacturer's and the first retailer's profits to be increased by about 1900% and 112%, respectively.

When the inventory cost of one unit product paid to the manufacturer is increased by about 33%, the profit of manufacturer strangely decreases. This is due to the increase in retail price of the first retailer which subsequently gains less revenue.

Setting 7 represents the effect of change on holding cost for the retailer's inventory. In our case, when the relevant cost (H_{b_i}) increases from 4 to 6, the replenishment cycle will be shorten (from 0.71 to 0.59), i.e. the manufacturer prefers to distribute the products in shorter schedules and do not keep the inventory on his side. With no major change in the wholesale price, since the holding cost paid by the manufacturer increases, the manufacturer's profit slightly decreases. On the other hand, the profit generated by all retailers will also decrease.

In the next scenario, the backorder cost paid by the manufacturer to all retailers has decreased. This leads to a huge decrease in backlogging fractions that via its comparison with the base example, we can find the difference about 6 times more (from 2.7% to 18%). With the increase of production setup cost (S_p) from 100 to 300, the wholesale price and retail prices for all retailers partially decrease. So we can see the manufacturer's profit has decreased up to 55635. It is also shown in Table 1 that when $\phi_i$increases, all of the profits either for the manufacturer or its retailers decrease.

This is true because it will directly impact the manufacturing costs of manufacturer in which he will also prefer to increase the wholesale price leading to decrease in the retailers' demands. As a result, the retailers' profits are decreased.

In summary, from the ten discussed settings in Table 1, in six settings, the payoff functions for both the manufacturer and the retailers change in the same direction, i.e. when the manufacture's profit increases/decreases, the retailers' profits also increase/decrease. In the remaining settings, they behave differently. As mentioned earlier, we solve the model via CONOPT 3.14[2] and IPOPT 3.10.1[3] solvers. As shown in Table 1, using the first solver can increase the profits of all the players while the second one works appositely.

4.3 Proposition 2: two-part tariff (TPT) contract

In the previous model, we assumed a VMI-type supply chain with a wholesale price contract. Now, we consider constructing the mentioned system on the basis of a two-part tariff agreement. As mentioned earlier, this type of contract was discussed in some previous studies, e.g. by Wang et al. (2009), Chen et al. (2012), Ma et al. (2013), and Oliveira et al. (2013). In this kind of contract, the retailer makes a fixed lump sum payment to the supplier that we have assumed it as a fixed franchise fee per order in this paper (A_i) (Lariviere, 1999)

In the other definition available in literature, the standard two-part contract, consisting of a fixed or lump sum franchise fee and a wholesale price, usually implies the wholesale price is set equal to the manufacturer's marginal cost and the fixed fee is used to capture all of the retailer's excess (monopoly) rents (Goering 2012).

We construct the payoff functions of both retailers and the manufacturer as follows:

Retailer's payoff function:

The payoff function of retailer will be kept the same as in the previous contract, only he additionally encounters a fixed or lump sum franchise fee per replenishment order (A). Therefore, the payoff function for retailer i can be given by:

$$\pi_i = p_i D_i(p_i) - C_p D_i(p_i) - \xi_i D_i(p_i) - \frac{1}{C} A_i = \left(p_i - C_p - \xi_i\right) D_i(p_i) - \frac{1}{C} A_i \qquad (10)$$

Manufacturer's payoff function:

Similarly to the manufacturer's payoff functions described by the formulas of (3) to (5), the TPT agreement necessitates taking the revenue of all orders' franchise fees into consideration. Egri (2008) believes that this is to compensate the manufacturer for his fixed setup cost. The relevant formula for the manufacturer's payoff can be denoted as below:

$$\pi_p = C_p \sum_{i=1}^{m} D_i(p_i) + \xi_i \sum_{i=1}^{m} D_i(p_i) + \frac{1}{C} \sum_{i=1}^{m} A_i - TDC_p - TIC_p$$

$$\pi_p = \sum_{i=1}^{m} D_i(p_i)\left(C_p + \xi_i\right) + \frac{1}{C}\sum_{i=1}^{m} A_i - \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) - \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i) {b_i}^2 C^2}{2} L_{b_i}\right] - \frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right] \qquad (11)$$

Stackelberg game model for TPT contract

The formulations of Stackelberg model for TPT agreement is the same as in the previous section for the wholesale price agreement except those necessary modifications which will change both MINLP model and the relevant KKT conditions. In this condition, the resulting MINLP model once the mentioned changes on both objective function and its constraints are applied is as follows:

Opt1:

$$\text{Max}\pi_p = \sum_{i=1}^{m} D_i(p_i)\left(C_p + \xi_i\right) + \frac{1}{C}\sum_{i=1}^{m} A_i - \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) - \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i) {b_i}^2 C^2}{2} L_{b_i}\right] - \frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right],\ i = 1, 2, \dots, m$$

$$\begin{aligned} \text{Subject to}\quad & \sum_{i=1}^{m} D_i(p_i) \le U \\ & U - \sum_{i=1}^{m} D_i(p_i) \le x.M \\ & 0 \le b_i \le 1 \\ & C \ge 0 \\ & C_p \ge 0 \end{aligned} \qquad (12)$$

[2]CONOPT is a FORTRAN-based Generalized Reduced Gradient (GRG) algorithm specifically designed for large nonlinear programming problems.
[3]IPOPT (Interior Point OPTimizer) is a software package for large-scale nonlinear optimization. As an open-source solver, it is designed to find (local) solutions for mathematical optimization problems, https://projects.coin-or.org/ipopt.

$x = [0, 1]$

Opt2:

$$\text{Max}\pi_i = \left(p_i - C_p - \xi_i\right) D_i(p_i) - \frac{1}{C} A_i, i = 1, 2, \dots, m$$

$$\text{Subject to } \; p_i - C_p - \xi_i - A_i \geq 0 \qquad (13)$$
$$p_i \geq 0$$
$$p_i - A_i \geq 0$$

Where the first constraint represents the possible value for the retail price which shall be bigger than or equal to the summation of wholesale price, inventory cost paid to manufacturer and the franchise fee for each order. Two later constraints are showing some basic conditions. Now, we again derive the KKT conditions for each of the retailers. These conditions for retailer i can be outlined as below:

$$p_i - C_m - \xi_i - A_i \geq 0 \;\perp\; r_i \geq 0, (13)$$

$$(1 - e_i) K_i p_i^{-e_i} + \; e_i\left(C_p + \xi_i + A_i / C\right) K_i p_i^{-(e_i+1)} + r_i = 0 \;\perp\; p_i \geq 0$$

Therefore, the resulting NLP model with all its complementarity conditions can be denoted as below:

Opt:

$$\text{Max}\pi_p = \sum_{i=1}^{m} D_i(p_i)\left(C_p + \xi_i\right) + \frac{1}{C}\sum_{i=1}^{m} A_i - \sum_{i=1}^{m} D_i(p_i)\left(C_m + \phi_i\right) - \frac{1}{c}\left[\sum_{i=1}^{m} S_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i)(1-b_i)^2 C^2}{2} H_{b_i} + \sum_{i=1}^{m} \frac{D_i(p_i) {b_i}^2 C^2}{2} L_{b_i}\right] - \frac{1}{c}\left[x S_p + \sum_{i=1}^{m} \frac{D_i(p_i) C^2}{2R} H_p\right] - \sum_{i=1}^{m} M r_i\left(p_i - C_p - \xi_i - A_i\right) - \sum_{i=1}^{m} M p_i\left[(1 - e_i) D_i(p_i) + K_i p_i^{-(e_i+1)}\left(C_p + \xi_i + A_i / C\right) e_i + r_i\right] - M x (1 - x), i = 1, 2, \dots, m$$

$$\text{Subject to } \; \sum_{i=1}^{m} D_i(p_i) \leq U \qquad (14)$$
$$U - \sum_{i=1}^{m} D_i(p_i) \leq x.M$$
$$0 \leq b_i \leq 1$$
$$C, C_p, x \geq 0$$
$$(1 - e_i) D_i(p_i) + K_i p_i^{-(e_i+1)}\left(C_p + \xi_i + A_i / C\right) e_i + r_i = 0$$

4.4 Numerical example

In this example, we consider a supply chain with a single manufacturer and two retailers with a two-part tariff (TPT) contract. We also consider 11 variations on the base settings to conduct the sensitivity analysis. The base example is solved using CONOPT 3.14 and IPOPT 3.10.1 solvers to let us compare the obtained result with that of the default solver of KNITRO 8.0. (See Table 2).

Table 2. Optimal results and sensitivity analysis for Stackelberg equilibrium: two-part tariff contract

	Settings	Base[a]	1 $C_m = 100$	2 $H_p = 6$	3 U=400	4 $K_1 = 2.5 \times 10^6$	5 $e_2 = 1.1$	6 $\xi_i = 5$	7 $H_{b_i} = 3$	8 $L_{b_i} = 600$	9 $S_p = 50$	10 $\phi_i = 20$	11 $A_i = 50$	Solver Conopt 3.14	Solver IPOPT 3.10.1
Manufacturer	C	1.80	1.71	1.61	1.81	1.83	1.77	1.80	2.35	1.8	1.74	1.77	3.81	1.80	1.80
	C_p	659	506	653	507	652	1103	661	651	659	659	715	714	659	659
	x	1.0	1.0	0.0	1.0	1.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	b_1	0.011	0.012	0.012	0.011	0.011	0.011	0.011	0.004	0.005	0.011	0.011	0.005	0.011	0.011
	b_2	0.011	0.012	0.012	0.011	0.011	0.011	0.011	0.004	0.005	0.011	0.011	0.005	0.011	0.011

Table 2. Continueed

Retailer				Profit/Payoff				Solving time(Sec)	Memory Used(Mb)
p_1	p_2	D_1	D_2	π_1	π_2	π_p	π_t		
2910	2351	94.2	47.7	211382	80307	71464	363153	1.52	48.9
2250	1817	131.6	68.4	228559	89124	80230	397913	0.17	46.4
2888	2333	95.1	48.2	211933	80585	71341	363859	0.05	46.8
2250	1817	131.6	68.4	228517	89102	70375	387994	55.77	46.8
2875	2322	79.7	48.5	176792	80697	63564	321053	0.03	46.8
4835	12272	48.7	79.5	181349	886892	121473	1189714	0.49	46.8
2910	2351	94.2	47.7	211382	80307	71464	363153	0.05	46.8
2868	2316	96.0	48.7	212200	80722	71895	364817	0.00	26.5
2911	2351	94.2	47.7	211371	80301	71460	363132	0.00	26.5
2909	2350	94.2	47.7	211419	80326	71520	363265	0.95	26.5
3155	2548	84.8	42.6	206288	77736	69460	353484	0.05	46.8
3179	2568	84.0	42.1	206430	77804	69585	353819	0.20	46.8
2910	2351	94.2	47.7	211382	80307	71464	363153	0.02	46.8
2910	2351	94.2	47.7	211382	80307	71464	363153	0.05	46.6

[a] $K_1 = 3 \times 10^6$,$K_2 = 2.5 \times 10^6 e_1 = 1.3$,$e_2 = 1.4 C_m = 150$, $\xi_i = 7$, $\phi_i = 5$, $H_{b_i} = 6$, $L_{b_i} = 300$, S_{b_i}=50, $H_p = 3$, $S_p = 150$, U=200, A_i=10

Once the production cost (C_m) decreases by 33%, both profits of the manufacturer and its retailers have increased. This is because of the lower wholesale and retail price which result in more demands. Setting 2 represents the effect of change on holding cost at manufacturer's side (H_p) while it increases by 100%. This makes the manufacturer to experience fewer profits while its retailers can purchase the finished products with lower price and acquire somehow more profits. Once the fixed capacity of manufacture (U) is doubled as denoted in Setting 3, only the retailers' profits increases. Since the wholesale price decreases in this case, we face lower profit for the manufacturer. With the decrease of holding cost paid by the manufacturer for the retailer's inventory, the manufacturer decides to transmit and replenish the products to retailers with more delays (Replenishment cycle (C) has increased by about 30%).This also leads to have the minimum backlogging fractions which is three times less than the base data. We have also examined the behavior of VMI model once we change the franchise fee per replenishment order (A) in Setting 11. In this scenario, we increased the franchise fee up to five times. So the following results are obtained: *(i)* the replenishment cycle has increased more about 112%; *(ii)* retailer price for both retailers increases. Thus, they face with less demand rates and consequently lower profits; *(iii)* the manufacturer's profit decreases as he sells the products with higher prices. Reduction of demand elasticity for the second retailer from 1.4 to 1.1 has a huge effect on both wholesale price and retail price. Besides, the combined profit of supply chain has increased about four times. Moreover, application of the other two solvers including CONOPT 3.14 and IPOPT 3.10.1 has no effect on the obtained results for the base example (Table 2).

Subsequently, we compare and contrast the above obtained results of TPT contract with the wholesale price contract that is previously discussed. Illustrative results are summarized in Table 3.

Table 3. Optimal results of VMI supply chain for wholesale price versus two-part tariff contract

	Contract type	wholesale price	two-part tariff (TPT)	Diff (%)
Manufacturer	C	0.48	1.80	+275
	C_p	612	659	+8
	x	0.0	1.0	N/A
	b_1	0.04	0.011	-73
	b_2	0.04	0.011	-73
Retailer	p_1	2681	2910	+9
	p_2	2165	2351	+9
	D_1	104.8	94.2	-10
	D_2	53.5	47.7	-11
Profit/Payoff	π_1	216115	211382	-2
	π_2	82716	80307	-3
	π_p	72815	71464	-2
	π_t	371646	363153	-2
Solving time(Sec)		0.06	1.52	+2433
Memory Used(Mb)		73.9	48.9	-34

The comparison of wholesale price and TPT contracts as mentioned in Table 3 provides the following outcomes: *(i)* the SC players in the two-part tariff pursue longer replenishment cycles. Indeed, retailers prefer not to pay franchise fee time-to-time; *(ii)* the wholesale price in the TPT contract is bigger than that wholesale price. This is why the retailers in wholesale price receive much more demands from their own markets; *(iii)* with the same quantity for backorder cost (L_{b_i}), the TPT contract is facing the backlogging percentage that is about three times less than the wholesale price contract; *(iv)* in short, the VMI supply chain in the wholesale price scheme can generate higher individual and combined profits than when there is a TPT contract in place. The third column shows the difference (%) between the obtained results of the two contracts with (+) or (-) signs.

Below we also draw some curves and graphs to demonstrate the behavior of supply chain once the contract is shifting from wholesale price to TPT. In the left side of Fig 1., we illustrate and compare the behavior of market demands for both types of contracts while the market scale varies from the base setting of $K_1 = 3 \times 10^6$ and $K_2 = 2.5 \times 10^6$ respectively by 10, 30, and 50% increase. As the market scale increases, the retailers' demands are also increasing. For 10 and 30%, the curve's slope is positively high, but it seems to have a smooth increase afterwards. The presented curve at the right side also shows the retail prices. In opposite to the demand graph, the retail prices for the TPT contract are bigger than the same variables in the wholesale price contract.

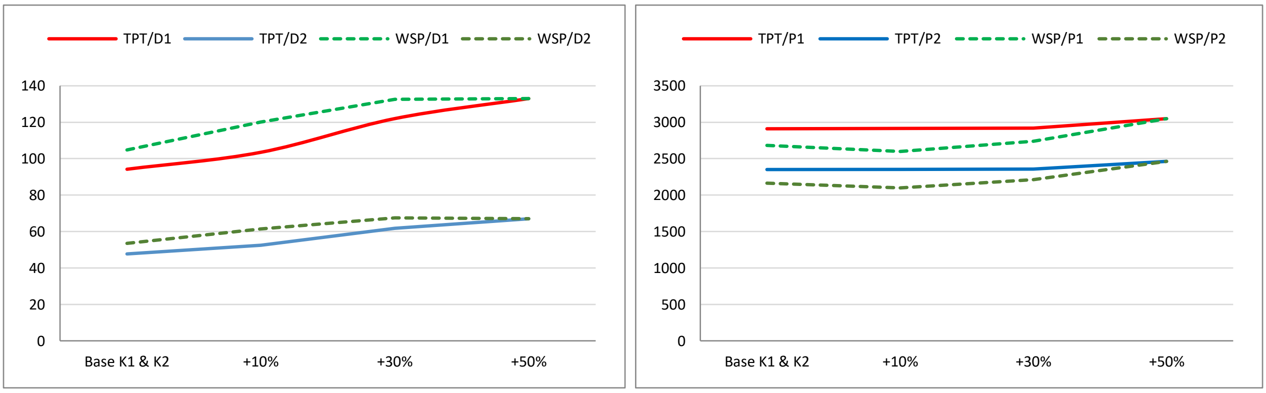

Figure 1. The influence of change in market scale on market demands (right) and retail prices (left)

Figure 2 illustrates how the market scale impacts on the individual profits of manufacturer and retailers and the combined profit of supply chain. As the base market scales (K_1andK_1) increases, the individual and combined profits of supply chain have increased (Figure 3). The combined profits for the wholesale price agreement is always much bigger than those of the TPT contract. This condition is true for individual profits only in the base setting, 10%, and 30%. Once we let for 50% increase in market scale, we can see the TPT contract provides more profits than the wholesale price (Figure 4).

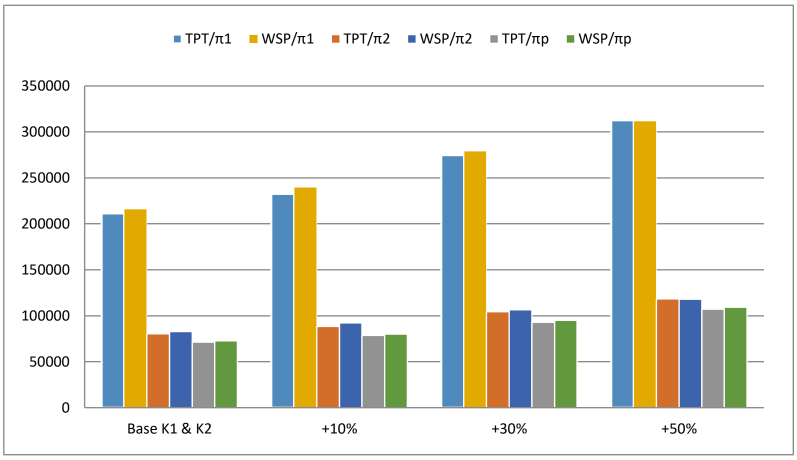

Figure 2. The influence of change in market scale on individual profits of the SC members

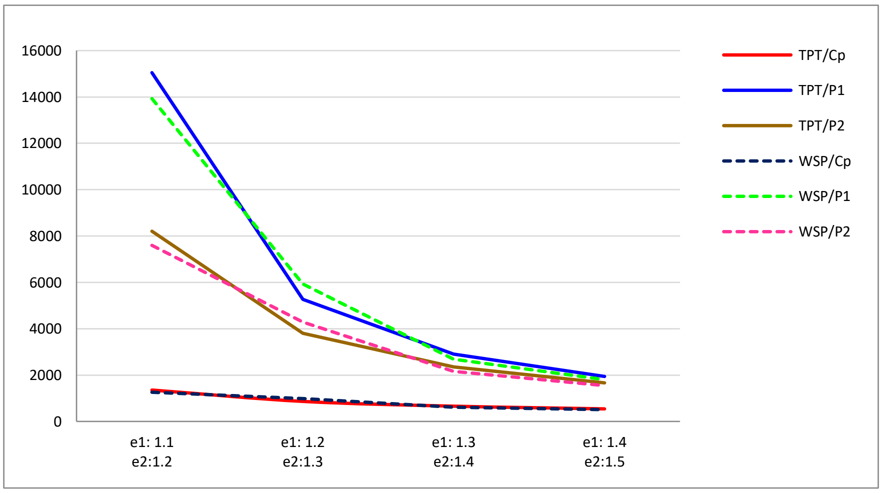

Figure 3. Change in demand elasticity and its effects on wholesale and retail prices

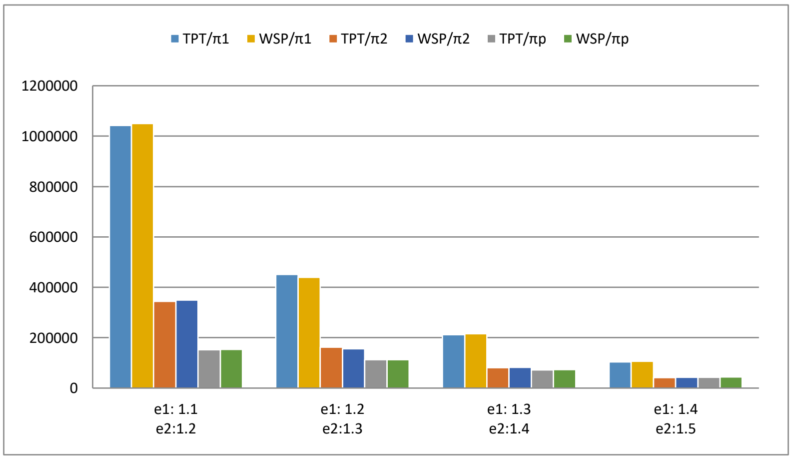

Figure 4. Change in demand elasticity and its effects on individual profits of SC members

5. Conclusion and recommendation for future research

This is one of the first papers that examines and compares the performance of a two-echelon supply chain under VMI partnership with two different contract types. The manufacturing vendor decides on the replenishment cycles, wholesale price and backlogging percentage for all retailers. Each retailer also decides on the product's retail price. Under these conditions, we formulated the problems as non-linear integer-programming (NIP) models and proposed the Stackelberg game to solve them. The two contract types include wholesale price and two-part tariff (TPT) contracts which are formulated in AIMMS software and solved by KNITRO solver. Some numerical examples are presented to demonstrate the performance of each contract. We have further compared both individual and combined profits of these supply chains. In addition, we carried out a few extensive sensitivity analyses to investigate the impact of key parameters on the problem decision variables and on both parties' profits. It is clear that in most of the cases, this is the wholesale price contract which provides better results over the time.

For future research in this area, we recommend the following directions:

(a) It would be interesting to study VMI systems with multi-manufacturers and multi-products that make the models much more complex.
(b) Other optimization and meta-heuristic algorithms such as particle swarm, ant colony, genetic algorithm, and soft computing technologies along with their comparative studies can be utilized.
(c) In addition to Stakelberg game, other possible game models shall be taken into consideration.
(d) Other VMI contract types mentioned in this paper such as buy-back, revenue-sharing, consignment, quantity flexibility, and backup agreement can be examined. Besides, in contrast to our assumption that all retailers have an identical contract with manufacturer, the condition that each retailer follows unique contract can be also studied.
(e) Another avenue for future research is to consider the cases when randomness and fuzziness in introduced into the demand pattern and production rates.
(f) The same research can be conducted when some parameters like manufacturer's production rate or retailers' demands are subject to learning effects.

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