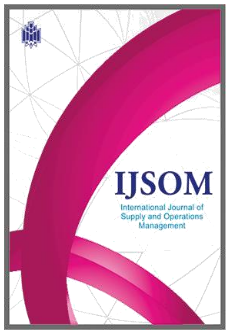
International Journal of Supply and Operations Management

IJSOM

February 2020, Volume 7, Issue 1, pp. 1-16
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com



A New Production-Inventory Planning Model for Joint Growing and Deteriorating Items

Hadi Mokhtari [a,*], Ali Salmasnia [b] and Javad Asadkhani [c]

[a] *Department of Industrial Engineering, University of Kashan, Kashan, Iran*
[b] *Department of Industrial Engineering, University of Qom, Qom, Iran*
[c] *Department of management, Faculty of Humanities, University of Kashan, Kashan, Iran*

Abstract
The production-inventory models are traditionally adopted for manufacturing systems. A relatively new area of production-inventory planning is related to livestock growing process. This research aims to propose a class of production-inventory model for new items titled growing items. In such a case, a rancher orders a quantity of newborn livestock like chicks or lambs, raises them to an appropriate weight during the growing period, slaughters them and then sells them to the meat market. The goal is to calculate the economic order quantity of the growing items at the start of a growing cycle, the optimal length of the growing cycle, and the optimal total profit. We, in this research, extend the previous work to the case of joint growing and deteriorating items, where livestock grow at growing period, and, additionally, the slaughtered livestock may be deteriorated during the sales period. Moreover, since some amount of slaughtered livestock is waste and should be disposed, a weight reduction factor is considered when livestock are slaughtered. The inventory models are constructed for this case, a heuristic solution algorithm is presented, a numerical example is discussed, and finally, sensitivity analysis is carried out to investigate the applicability of the problem.

Keywords: In Production-Inventory Systems; Growing Items; Deterioration; Livestock; Reduction Factor.

1. Introduction

In recent decades, the appropriate design and planning of production-inventory systems have played a great role in the performance of industrial systems. If inventories are not managed appropriately, they might become unreliable and costly. The important decisions in a production-inventory system are how much and when an industrial unit should order inventories. Since there are different types of inventories in any industrial sector, so far much research has been conducted on different types of inventories. Moreover, because of the inability of traditional production-inventory models to incorporate specific features of some product families, particular models have been introduced and studied for some specific types of goods. Till now, almost all of the production-inventory models have been designed for manufacturing units in the related literature. However, they can also be adapted and customized for other types of products like livestock. One of the major assumptions frequently used in the field is that the inventory items remain unchanged during the storage period. However, as discussed for the first time by Rezaei (2014), we can find another pattern of inventories, i.e., growing items such as livestock. The weight of the inventory of a regular good is usually constant when it is not consumed during the inventory cycle. For example, an automobile manufacturer stores 500 kilograms industrial paint in its raw material warehouse. As long as this manufacturer does not purchase extra paint, the weight of the inventory decreases if and only if existing paint is consumed by production line, and otherwise remains unchanged. However, in contrast to regular goods, the weight of inventory of growing items like livestock increases over the fattening period. For example, at the beginning of a growing cycle, a rancher company purchase 50 lambs with an overall weight of 300 kilograms, aiming to raise the lambs to an appropriate weight, then slaughter them and sell them as meat to market. Unlike regular products, the weight of the lambs may increase to 1000 kilograms after a few months, without purchasing extra lambs. This type of goods which is entitled growing items by Rezaei (2014) is a very new topic in the literature of inventory problems.

Corresponding author email address: mokhtari_ie@kashanu.ac.ir

To the best of our knowledge, there is no systematic research on this kind of issue in the area of production-inventory control. The work presented by Rezaei (2014) is the sole research on growing items. We will extend his work in the current research to joint growing and deteriorating items. In the work presented by Rezaei (2014), it is assumed that the slaughtered livestock remain unchanged during the sales period. However, slaughtered livestock might deteriorate after slaughtering process because the meat needs special holding conditions within the distribution stage in its market and the decay of inventory is therefore inevitable. One of the main features in production-inventory systems is associated with deteriorating items. Many physical goods and products deteriorate over time, and hence the deterioration of physical goods cannot be disregarded. The deterioration often leads to decreases in the usefulness of the items over time. In such cases, deterioration is defined as decay, damage, spoilage, evaporation or loss of the marginal value of goods. The examples are meat, vegetables, and fruits. Consequently, the inventory system in companies with deteriorating goods should be customized for the maintenance of perishable inventories. We can see a comprehensive review on the perishable literature till 2011 which is provided by Goyal and Giri (2001). Later, as an example, Balkhi (2011) discussed an inventory model for perishable products with time value of money. Bansal (2013) discussed an inventory problem for deteriorating products under inflation. Bhaula and Kumar (2014) considered two-parameter Weibull deterioration. Recently, Giri and Sharma, (2015) developed an inventory model for perishable products with shortages allowed. Li et al., (2015) developed a model with product deterioration and rework. Jaggi, et al., (2016) studied deteriorating items with two-warehouse inventory system. Moreover, Kouki et al., (2016) discussed an inventory system for perishable goods and employed a Markov process to formulate lead time. Teimoury and Kazemi (2016) studied a single deteriorating product with a constant rate within a two-stage supply chain. Janssen et al., (2016) reviewed major works on deteriorating items from 2012 to 2015. More recently, Chen (2017) considered joint optimization of pricing, replenishment and rework decisions for imperfect and deteriorating items. Last but not least, Bai et al., (2017) designed a sustainable supply chain with deteriorating items. The rest of the paper is organized as follows. In the next section, two mathematical models of the problem are presented. In Section 3, implementation and algorithms are discussed. Then, Section 4 presents and analyzes a numerical example to illustrate the problem and performs a sensitivity analysis. Finally, Section 5 concludes the paper and provides some research directions for the future.

2. Problem Definition and Modeling

As mentioned before, Rezaei (2014) introduced the first and sole research on production-inventory systems with growing items in his good work. In the current paper, we extend his work for a joint growing and deteriorating items. The traditional EOQ model is used as the basis of our production-inventory problem. In the basic EOQ, the aim is to determine the optimal order quantity, minimizing total costs, including setup, holding and ordering costs, so as to demand is satisfied. Two general mathematical models are presented based on feed intake function which can be used for every template of growing items in practice. In this problem, a rancher company buys a quantity of newborn livestock with the aim to raise them and then slaughters and sells them to the meat market. The main concern of rancher is to know (i) the quantity of newborn livestock to be purchased, and (ii) the optimal weight to slaughter them, so as to minimize the total costs including purchase cost of newborn livestock, setup cost of facilities in rancher company for a new growing cycle, feeding cost of growing livestock and holding cost of slaughtered livestock during the sales period, and simultaneously maximize the total revenue of slaughtered livestock. The total revenue is the sum of revenues obtained from sales of the total slaughtered livestock after the growing period. Assuming that the demand of the livestock is known, the optimal values of decision variables are the values which maximize the total profit of the rancher as follows.

$$\text{total profit} = \text{total revenue} - \text{total cost}$$

We are going to construct mathematical models to calculate the optimal values of decision variables, and the number of times the facilities should be set up to commence a new cycle of livestock fattening. Considering all the costs in the problem, the total profit is given as follows.

$$\text{total profit} = \text{total revenue} - \text{setup cost} - \text{total purchasing cost} - \text{total feeding cost} - \text{total holding cost}$$

At the beginning of the inventory cycle, a setup cost $\mathbf{K}$ will be incurred associated with the start of facilities operation. Assume $\mathbf{q}$ (as the first decision variable) denotes the quantities of newborn livestock the rancher purchases at the beginning of each growing cycle, with initial weight $\mathbf{w_0}$. Moreover, let $\mathbf{w_1}$ (as the second decision variable) shows the final weight of every livestock after fattening period which does not necessarily equal the net weight of slaughtered livestock meat. One of our extensions to Rezaei (2014)'s work is that he assumes the weight of livestock after slaughtering is certainly equal to the net weight of slaughtered livestock meat. However, in practice, some amount of slaughtered livestock is waste and should be disposed. To model this, we define the weight reduction factor of slaughtered livestock $\mathbf{r}$. That means for a given weight of livestock before slaughtering $\mathbf{w_1}$, the net amount of inventory after slaughtering will be just $(\mathbf{1-r})\,\mathbf{w_1}$. Considering the purchasing price per unit weight $\mathbf{p}$, the total purchasing cost is $\mathbf{pqw_0}$ and using selling price per unit weight $\mathbf{s}$, the total revenue would be $\mathbf{s(1-r)qw_1}$. During the growing cycle of newborn livestock, their feeding cost varies over time which is related to its feed intake function. Let

$\mathbf{F}(\mathbf{w})$ be a general function for feed intake as a function of current weight $\mathbf{w}$ representing the feeding rate of growing livestock when its current weight is $\mathbf{w}$. Therefore, considering $\mathbf{c_f}$ as the feeding cost per unit, the total feeding cost would be $\mathbf{c_f q} \int_{\mathbf{w_0}}^{\mathbf{w_1}} \mathbf{F}(\mathbf{w})\mathbf{dw}$, where the weight of the livestock at the end of inventory cycle is $\mathbf{w_1}$ (the slaughtered livestock). When the livestock are slaughtered, they should be held during the sales period. That means a holding cost is incurred within the sales period. The total holding cost, using $\mathbf{h}$ as the annual holding cost per weight unit and $\mathbf{Q}(\mathbf{t})$ as the total weight of inventory at time $\mathbf{t}$, can be calculated by $\mathbf{h} \int \mathbf{Q}(\mathbf{t})\mathbf{dt}$. In contrast to Rezaei's work, we consider the deteriorating case in $\mathbf{Q}(\mathbf{t})$ for slaughtered livestock. It is inevitable that an amount of meat inventory becomes perished during the sales period. Hence, considering the deteriorating case over sales period is a real-world assumption in a growing livestock inventory problem. So, we will investigate two different cases: (i) growing items, and (ii) joint growing and deteriorating items.

2.1. Notations

p	The purchasing price per unit weight
s	The selling price per unit weight
r	The weight reduction factor of slaughtered livestock per unit weight
θ	The deterioration rate after slaughtering per unit weight
t_1	The time of slaughtering (growing period)
t_2	The sales period
c_f	The feeding cost before slaughtering per unit weight per unit time
h	The holding cost after slaughtering per unit weight per unit time
$Q(t)$	The total weight of inventory at the time t
Q_0	The total weight of inventory at the time of purchasing
Q_1	The total weight of inventory at the time of slaughtering
D	The annual demand rate
w_t	The weight of unit livestock at the time t
w_0	The weight of unit livestock at the time of purchasing
w_1	The weight of unit livestock at the time of slaughtering (decision variable)
q	The number of ordered livestock at the beginning of the growing cycle (decision variable)

2.2. Growing Items

At this section, we consider, as the first model, a basic livestock inventory system where the items are just growing. Our model is different from the ones presented by Rezaei (2014) in two aspects: (i) we model the second decision variable by weight at time of slaughtering $\mathbf{w_1}$ instead of the time of slaughtering $\mathbf{t_1}$, and (ii) we define the weight reduction factor of slaughtered livestock. The behavior of the inventory system is shown in Figure 1. At the beginning of the cycle, the weight of inventory is calculated by multiplying the number of purchased livestock $\mathbf{q}$ by initial weight $\mathbf{w_0}$ as $\mathbf{qw_0}$, as depicted by Figure 1. A moment before the time of slaughtering which is the end of the growing cycle $\mathbf{t_1}$, the weight of individual livestock goes up to $\mathbf{w_1}$ and the total weight of the livestock reaches $\mathbf{qw_1}$. In contrast to Rezaei's model, the total weight of inventory after slaughtering reduces to $(\mathbf{1}-\mathbf{r})\mathbf{qw_1}$ due to waste in the slaughtering process. The slaughtered livestock are then gradually sold as meat at demand rate $\mathbf{D}$. At the end of the sales period $\mathbf{t_2}$, this cycle ends.
To achieve the optimal decision variables, the cost components and revenue are formulated in the sequel.

The setup cost per cycle is as follows.

$$SCC_I = K \tag{1}$$

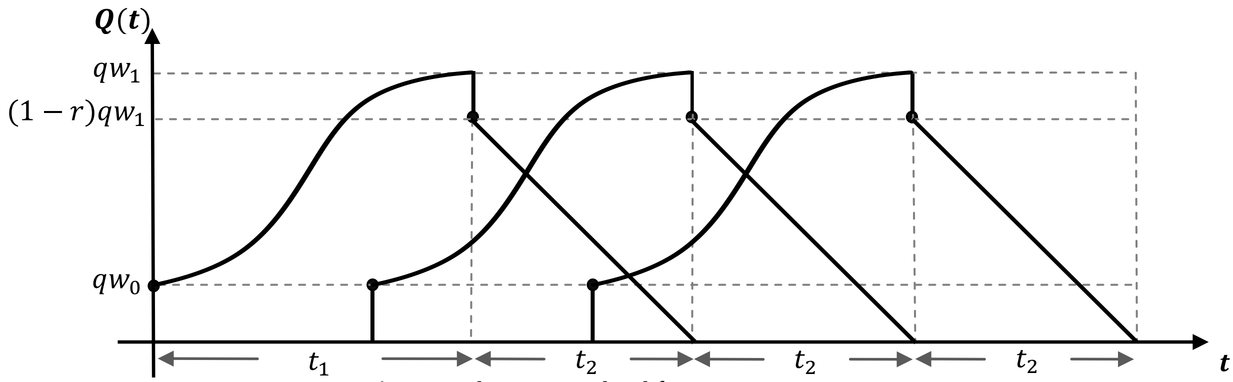


Figure 1. The inventory level for growing items

The purchasing cost per cycle, when we purchase q newborn livestock in each cycle and each one bears weight w_0, is obtained as:

$$PCC_I = pqw_0 \tag{2}$$

Where p represents price per unit weight.

The feeding cost per cycle for livestock before slaughtering is calculated by:

$$FCC_I = c_f q \int_{w_0}^{w_1} F(w)dw \tag{3}$$

Where $F(w)$ is a general feed intake function in terms of the weight of livestock.

The holding cost per cycle for slaughtered livestock meat is given as follows:

$$HCC_I = \ h\int_0^{t_2} Q(t)dt = h\frac{(1-r)qw_1t_2}{2} \tag{4}$$

The revenue per cycle achieved by selling total slaughtered livestock is gained as follows:

$$RCC_I = s(1-r)qw_1, \tag{5}$$

And the total profit per cycle can be computed by subtracting total revenue and sum of total costs by:

$$TPC_I(q,w) = s(1-r)qw_1 - K - pqw_0 - c_f q\int_{w_0}^{w_1} F(w)dw - h\frac{(1-r)qw_1t_2}{2} \tag{6}$$

Then the total profit per unit time is given by dividing $TPC_I(q,w)$ by t_2 as follows.

$$TPU_I(q,w_1) = \frac{TPC(q,w_1)}{t_2} = \frac{s(1-r)qw_1 - K - pqw_0 - c_f q\int_{w_0}^{w_1} F(w)dw - h\frac{(1-r)qw_1t_2}{2}}{t_2} \tag{7}$$

Since decision variables in the model are (i) order quantity for newborn livestock q and (ii) weight of livestock slaughtering w_1, we should remove all other dependent variables like t_2 in $TPU_I(q,w_1)$. For this purpose, knowing that t_2 is a time interval at which an inventory with the value of $(1-r)qw_1$ is sold under demand rate D, we can obtain t_2 as presented below.

t_{2} (8)

Substituting t_2 into $TPU_I(q,w_1)$, we reach final relation for $TPU_I(q,w_1)$ as a function of decision variables q and w_1 as follows:

$$TPU_I(q,w_1) = sD - \frac{KD}{(1-r)qw_1} - pD\frac{w_0}{w_1} - \frac{c_fD}{w_1}\int_{w_0}^{w_1} F(w)dw - h\frac{(1-r)qw_1}{2} \tag{9}$$

We will discuss a solution algorithm in Section 4.

2.3. Joint Growing and Deteriorating Items

In this section, we discuss, as the second model, an inventory system with joint growing and deteriorating items. The behavior of this inventory system is depicted in Figure 2. The growing cycle is completely same as the previous model in Section 3.2. The level of inventory is obtained by $\mathbf{qw_0}$ and final total weights reach $\mathbf{qw_1}$ at the end of the growing period, just a moment before slaughtering. After slaughtering, the total weight of inventory reduces to $(\mathbf{1-r})\mathbf{qw_1}$. The slaughtered livestock are then sold gradually. At this sales period, we consider deterioration with rate $\boldsymbol{\theta}$.

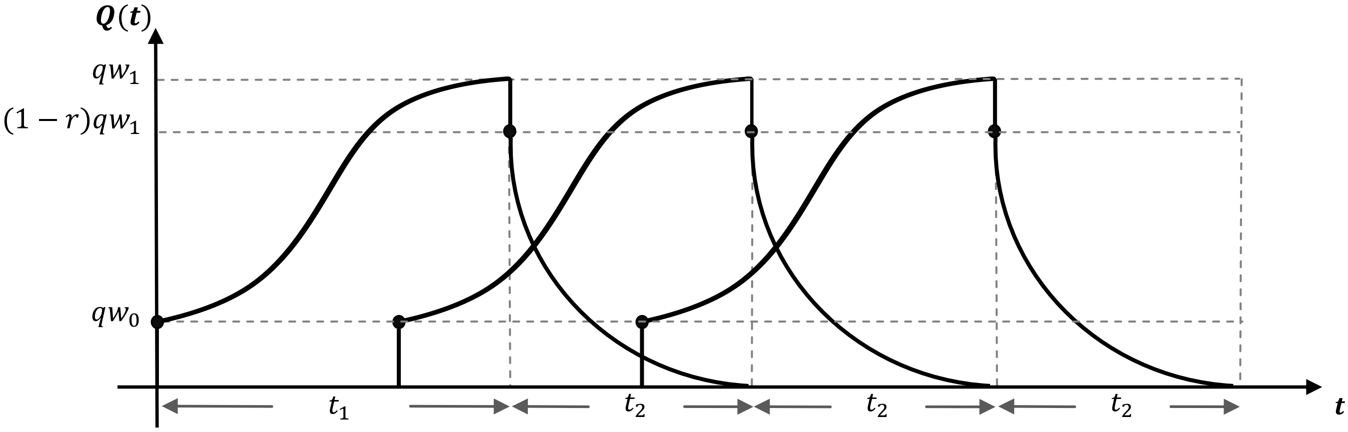


Figure 2: The inventory level for joint growing and deteriorating items

The setup, purchasing, and feeding costs are the same as the previous model as follows.

$SCC_{II} = K$

$PCC_{II} = pqw_0$

$FCC_{II} = c_f q \int_{w_0}^{w_1} F(w)dw$

However, the holding cost at sales period is completely different from the previous one. To compute the holding cost, the total weight of inventory at time interval $[t_1, t_1 + t_2]$ should be calculated first. To this end, we have:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -D \tag{10}$$

With boundary condition $Q(t_2) = 0$ we have:

$$Q(t) = \frac{D}{\theta}\left[e^{\theta(t_2 - t)} - 1\right] \tag{11}$$

In order to obtain t_2, we can use $Q(0) = (1-r)qw_1$ which results in:

$$t_2 = \frac{1}{\theta}\ln\left(\frac{D + \theta(1-r)qw_1}{D}\right) \tag{12}$$

Hence, we can write $Q(t)$ in terms of decision variables q and w_1 by substituting t_2 into the above equation as presented below.

$$Q(t) = \frac{D}{\theta}\left[e^{-\theta t} - 1\right] + (1-r)qw_1 e^{-\theta t} \tag{13}$$

Thus, the holding cost can be obtained as follows.

$$HCC_{II} = h\int_0^{t_2} Q(t)\,dt = h\left[\frac{(1-r)qw_1}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{D + \theta(1-r)qw_1}{D}\right)\right] \tag{14}$$

The total revenue per cycle will be:

$$TRC_{II} = s(1-r)qw_1 \tag{15}$$

And then the total profit per cycle is as follows.

$$TPC_{II} = s(1-r)qw_1 - K - pqw_0 - c_f q \int_{w_0}^{w_1} F(w)dw - h\left[\frac{(1-r)qw_1}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{D+\theta(1-r)qw_1}{D}\right)\right] \tag{16}$$

The total profit per unit time is

$$TPU_{II}(q,w_1) = \frac{TPC}{t_2} = \frac{s(1-r)qw_1 - K - pqw_0 - c_f q \int_{w_0}^{w_1} F(w)dw - h\left[\frac{(1-r)qw_1}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{D+\theta(1-r)qw_1}{D}\right)\right]}{\frac{1}{\theta}\ln\left[\frac{D+\theta(1-r)qw_1}{D}\right]} \tag{17}$$

3. Implementation and Algorithm

As mentioned before, a general function for the feed intake of livestock during growing period $F(w)$ is considered in two previous models, which can be used specifically for any type of livestock. To make it specific, appropriate corresponding functions should be replaced in these models and solved. There are several factors influencing the feed intake for different animals including:

- temperature and weather
- humidity and ventilation rates
- feed quality
- feed composition
- sex
- breed
- genetic

In the existing literature, several mathematical functions have been introduced to measure feed intake of livestock during the growth period. To keep generality, we investigate three major types of this function in our analysis, i.e., power feed intake, exponential feed intake, and quadratic feed intake.

3.1. Power Feed Intake

The power function is considered for livestock feed intake, in this section, as follows.

$$F(w) = F_0 w^{F_1} \tag{18}$$

Where F_0 and F_1 are adjusting parameters for this function. By considering the function, the feeding cost per cycle is given by

$$FCC = c_f q \int_{w_0}^{w_1} F(w)dw = c_f q \frac{F_0}{F_1+1}\left(w_1^{F_1+1} - w_0^{F_1+1}\right) \tag{19}$$

And then the total profit per cycle for the first model (growing items) is customized as

$$TPU_I(q,w_1) = sD - \frac{KD}{(1-r)qw_1} - pD\frac{w_0}{w_1} - \frac{c_f D}{w_1}\frac{F_0}{F_1+1}\left(w_1^{F_1+1} - w_0^{F_1+1}\right) - h\frac{(1-r)qw_1}{2} \tag{20}$$

To obtain the optimal values for q and w_1, we should solve the system of equations resulting from setting the partial derivative of $TPU_I(q,w_1)$ with respect to q and w_1 to zero, as follows:

$$\frac{\partial TPU_I(q,w_1)}{\partial q} = \frac{KD}{(1-r)q^2 w_1} - h\frac{(1-r)w_1}{2} = 0 \quad \Rightarrow \quad q = \frac{1}{w_1}\left[\frac{2KD}{h(1-r)}\right]^{\frac{1}{2}} \tag{21}$$

$$\frac{\partial TPU_I(q,w_1)}{\partial w_1} = \frac{D(K+pqw_0)}{(1-r)qw_1^2} + \frac{c_f D}{w_1^2}\left(\frac{F_0}{F_1+1}\left(w_1^{F_1+1} - w_0^{F_1+1}\right) - w_1 F(w_1)\right) - h\frac{(1-r)q}{2} = 0 \tag{22}$$

The same procedure is carried out for the second model (joint growing and deteriorating items), as follows:

$$TPU_{II}(q,w_1) == \frac{s(1-r)qw_1 - K - pqw_0 - c_f q\frac{F_0}{F_1+1}\left(w_1^{F_1+1} - w_0^{F_1+1}\right)}{\frac{1}{\theta}\ln\left[\frac{D+\theta(1-r)qw_1}{D}\right]} - \frac{h\left[\frac{(1-r)qw_1}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{D+\theta(1-r)qw_1}{D}\right)\right]}{\frac{1}{\theta}\ln\left[\frac{D+\theta(1-r)qw_1}{D}\right]} \tag{23}$$

The partial derivatives are as follows.

$$\frac{\partial TPU_{II}(q,w_1)}{\partial q} = \frac{-\theta\left\{pw_0 - h\left[\frac{w_1(r-1)}{\theta} - \frac{Dw_1(r-1)}{\theta\left(D-\theta qw_1(r-1)\right)}\right] + sw_1(r-1) - \frac{F_0c_f({w_0}^{F_1+1} - {w_1}^{F_1+1})}{F_1+1}\right\}}{\ln\left(\frac{D-\theta qw_1(r-1)}{D}\right)} - \frac{\theta^2 w_1(r-1)\left\{K - h\left[\frac{D\ln\left(\frac{D-\theta qw_1(r-1)}{D}\right)}{\theta^2} + \frac{qw_1(r-1)}{\theta}\right] + pqw_0 + qsw_1(r-1) - \frac{F_0c_fq({w_0}^{F_1+1} - {w_1}^{F_1+1})}{F_1+1}\right\}}{[D-\theta qw_1(r-1)]\ln\left[\frac{D-\theta qw_1(r-1)}{D}\right]^2} = 0 \tag{24}$$

$$\frac{\partial TPU_{II}(q,w_1)}{\partial w_1} = \frac{-\theta\left\{qs(r-1) - h\left[\frac{q(r-1)}{\theta} - \frac{Dq(r-1)}{\theta\left(D-\theta qw_1(r-1)\right)}\right] - F_0c_fq{w_1}^{F_1}\right\}}{\ln\left(\frac{D-\theta qw_1(r-1)}{D}\right)} - \frac{\theta^2 q(r-1)\left\{K - h\left[\frac{D\ln\left(\frac{D-\theta qw_1(r-1)}{D}\right)}{\theta^2} + \frac{qw_1(r-1)}{\theta}\right] + pqw_0 + qsw_1(r-1) - F_0c_fq({w_0}^{F_1+1} - {w_1}^{F_1+1})\right\}}{[D-\theta qw_1(r-1)]\ln\left[\frac{D-\theta qw_1(r-1)}{D}\right]^2} = 0 \tag{25}$$

3.2. Exponential Feed Intake

The exponential function is considered in this section for livestock feed intake as follows.

$$F(w) = A(1 - exp(-Bw)) \tag{26}$$

Where A and B are adjusting parameters for this function. By utilizing this function, the feeding cost per cycle is calculated as

$$FCC = c_f q\int_{w_0}^{w_1} F(w)dw = c_f q\left(A(w_1 - w_0) + \frac{A}{B}\left(exp(-Bw_1) - exp(-Bw_0)\right)\right) \tag{27}$$

Therefore, the total profit per cycle for the first model (growing items) is customized as

$$TPU_I(q,w_1) = sD - \frac{KD}{(1-r)qw_1} - pD\frac{w_0}{w_1} - \frac{c_f D}{w_1}\left(A(w_1 - w_0) + \frac{A}{B}\left(exp(-Bw_1) - exp(-Bw_0)\right)\right) - h\frac{(1-r)qw_1}{2} \tag{28}$$

By getting the partial derivatives, we reach a system of two equations with two unknown variables, q and w_1, as follows.

$$\frac{\partial TPU_I(q,w_1)}{\partial q} = \frac{KD}{(1-r)q^2 w_1} - h\frac{(1-r)w_1}{2} = 0 \quad \Rightarrow \quad q = \frac{1}{w_1}\left[\frac{2KD}{h(1-r)}\right]^{\frac{1}{2}} \tag{29}$$

$$\frac{\partial TPU_I(q,w_1)}{\partial w_1} = (hq(r-1))/2 + (Dpw_0)/w_1\hat{}2 - (Dc_f(A(w_0 - w_1) + (A(1/exp(Bw_0) - 1/exp(Bw_1)))/B))/w_1\hat{}2 + (DK)/((1-r)qw_1\hat{}2) - (Dc_f(A - A/exp(Bw_1)))/w_1 = 0 \tag{30}$$

The same procedure can be implemented for the second model (joint growing and deteriorating items), as follows:

$$TPU_{II}(q,w_1) = \frac{s(1-r)qw_1 - K - pqw_0 - c_f q\left(A(w_1 - w_0) + \frac{A}{B}\left(exp(-Bw_1) - exp(-Bw_0)\right)\right)}{\frac{1}{\theta}\ln\left[\frac{D+\theta(1-r)qw_1}{D}\right]} - \frac{h\left[\frac{(1-r)qw_1}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{D+\theta(1-r)qw_1}{D}\right)\right]}{\frac{1}{\theta}\ln\left[\frac{D+\theta(1-r)qw_1}{D}\right]} \tag{31}$$

with the system of partial derivatives as follows.

$$\frac{\partial TPU_{II}(q,w_1)}{\partial q} = \frac{-\theta\left\{pw_0 - c_f\left[A(w_0 - w_1) + \frac{A}{B}\left(\frac{1}{e^{Bw_0}} - \frac{1}{e^{Bw_1}}\right)\right] - h\left[\frac{w_1(r-1)}{\theta} - \frac{Dw_1(r-1)}{\theta\left(D - \theta qw_1(r-1)\right)}\right] + sw_1(r-1)\right\}}{\ln\left(\frac{D - \theta qw_1(r-1)}{D}\right)} - \{\theta^2 w_1(r-1)\{K - h\left[\frac{D\ln\left(D - \theta qw_1(r-1)\right)}{\theta^2} + \frac{qw_1(r-1)}{\theta}\right] + pqw_0 - c_f q\left(A(w_0 - w_1) + \frac{A}{B}\left(\frac{1}{e^{Bw_0}} - \frac{1}{e^{Bw_1}}\right)\right) + qsw_1(r-1)\}\}/([D - \theta qw_1(r-1)]\ln\left[\frac{D - \theta qw_1(r-1)}{D}\right]^2) = 0 \tag{32}$$

$$\frac{\partial TPU_{II}(q,w_1)}{\partial w_1} = \frac{-\theta\left\{qs(r-1) - h\left[\frac{q(r-1)}{\theta} - \frac{Dq(r-1)}{\theta\left(D - \theta qw_1(r-1)\right)}\right] - c_f q\left[A + \frac{A}{e^{Bw_1}}\right]\right\}}{\ln\left(\frac{D - \theta qw_1(r-1)}{D}\right)} - \{\theta^2 q(r-1)\left\{K - h\left[\frac{D\ln\left(\frac{D - \theta qw_1(r-1)}{D}\right)}{\theta^2} + \frac{qw_1(r-1)}{\theta}\right] + pqw_0 - c_f q\left(A(w_0 - w_1) + \frac{A}{B}\left(\frac{1}{e^{Bw_0}} - \frac{1}{e^{Bw_1}}\right)\right) + qsw_1(r-1)\right\}\} /([D - \theta qw_1(r-1)]\ln\left[\frac{D - \theta qw_1(r-1)}{D}\right]^2) = 0 \tag{33}$$

3.3. Quadratic Feed Intake

The quadratic function is considered in this section for livestock feed intake as follows.

$$F(w) = Aw^2 + Bw + C \tag{34}$$

Where A, B and C are adjusting parameters for the function. The feeding cost per cycle is calculated for this case as

$$FCC = c_f q \int_{w_0}^{w_1} F(w)dw = c_f q(A(w_1^3 - w_0^3)/3 + B(w_1^2 - w_0^2)/2 + C(w_1 - w_0)) \tag{35}$$

Thus, the total profit per cycle for the first model (growing items) is customized as.

$$TPU_I(q, w_1) = sD - \frac{KD}{(1-r)qw_1} - pD\frac{w_0}{w_1} - \frac{c_f D}{w_1}(A(w_1^3 - w_0^3)/3 + B(w_1^2 - w_0^2)/2 + C(w_1 - w_0)) - h\frac{(1-r)qw_1}{2} \tag{36}$$

By getting the partial derivatives, we reach a system of two equations with two unknown variables, q and w_1, as follows.

$$\frac{\partial TPU_I(q, w_1)}{\partial q} = \frac{KD}{(1-r)q^2 w_1} - h\frac{(1-r)w_1}{2} = 0 \Rightarrow q = \frac{1}{w_1}\left[\frac{2KD}{h(1-r)}\right]^{\frac{1}{2}} \tag{37}$$

$$\frac{\partial TPU_I(q, w_1)}{\partial w_1} = -(3hq\hat{}2 - 3hq\hat{}2r + 3BDc_f q)/(6q) - (D(2Ac_f q w_0\hat{}3 - 6pqw_0 - 6K/(1-r) + 3Bc_f q w_0\hat{}2 + 6Cc_f q w_0))/(6qw_1\hat{}2) - (2ADc_f w_1)/3 = 0 \tag{38}$$

The same procedure can be implemented for the second model (joint growing and deteriorating items), as follows:

$$TPU_{II}(q, w_1) = \frac{s(1-r)qw_1 - K - pqw_0 - c_f q(A(w_1^3 - w_0^3)/3 + B(w_1^2 - w_0^2)/2 + C(w_1 - w_0))}{\frac{1}{\theta}\ln\left[\frac{D + \theta(1-r)qw_1}{D}\right]} - \frac{h\left[\frac{(1-r)qw_1}{\theta} - \frac{D}{\theta^2}\ln\left(\frac{D + \theta(1-r)qw_1}{D}\right)\right]}{\frac{1}{\theta}\ln\left[\frac{D + \theta(1-r)qw_1}{D}\right]} \tag{39}$$

with system of partial derivatives as follows.

$$\frac{\partial TPU_{II}(q, w_1)}{\partial q} = \frac{-\theta\left\{pw_0 - c_f\left[\frac{A(w_0^3 - w_1^3)}{3} + \frac{B(w_0^2 - w_1^2)}{2} + C(w_0 - w_1)\right] - h\left[\frac{w_1(r-1)}{\theta} - \frac{Dw_1(r-1)}{\theta(D - \theta qw_1(r-1))}\right] + sw_1(r-1)\right\}}{\ln\left(\frac{D - \theta qw_1(r-1)}{D}\right)}$$
$$- \{\theta^2 w_1(r-1)\{K - h\left[\frac{D\ln(D - \theta qw_1(r-1))}{\theta^2} + \frac{qw_1(r-1)}{\theta}\right] + pqw_0 - c_f q\left[\frac{A(w_0^3 - w_1^3)}{3} + \frac{B(w_0^2 - w_1^2)}{2} + C(w_0 - w_1)\right] + qsw_1(r-1)\}\}/([D - \theta qw_1(r-1)]\ln\left[\frac{D - \theta qw_1(r-1)}{D}\right]^2) = 0 \tag{40}$$

$$\frac{\partial TPU_{II}(q, w_1)}{\partial w_1} = \frac{\left\{-\theta\left\{qs(r-1) - h\left[\frac{q(r-1)}{\theta} - \frac{Dq(r-1)}{\theta(D - \theta qw_1(r-1))}\right] - c_f q[Aw_1^2 + Bw_1 + C]\right\}\right\}}{\ln\left(\frac{D - \theta qw_1(r-1)}{D}\right)} -$$

$$\{\theta^2 q(r-1)\{K - h[\frac{D\ln\left(\frac{D-\theta q w_1(r-1)}{D}\right)}{\theta^2} + \frac{q w_1(r-1)}{\theta}] + pqw_0 - c_f q\left[\frac{A(w_0^3 - w_1^3)}{3} + \frac{B(w_0^2 - w_1^2)}{2} + C(w_0 - w_1)\right] + qsw_1(r-1)\}\}/([D - \theta q w_1(r-1)]\ln\left[\frac{D-\theta q w_1(r-1)}{D}\right]^2) = 0 \tag{41}$$

3.4. Solution Procedure

In this section, we attempt to find the most adequate approach for calculating the optimal decision variables in joint growing and deteriorating problems with all the functions of feed intake. As can be seen, equations 24, 25, 32, 33, 40, and 41 are complex and it is not possible to demonstrate the concavity of total profit functions. Therefore, optimal values cannot be obtained by the analytical methods. In addition, the number of ordered livestock at the beginning of the growing cycle q and the weight of unit livestock at the time of slaughtering w_1 are respectively an integer and positive continuous variables. Consequently, the proposed models are mixed-integer, non-concave, two-variable, and nonlinear problems which classify as complex and hard operational research programming. As a result, we are allowed to apply metaheuristic methods. Genetic algorithm (GA) is the most well-known metaheuristic which computes good (near optimal) values for decision variables for mixed-integer problems in an efficient way. In the next section, we present a combinatorial solution algorithm which is designed based on the priority of derivatives and when that is not feasible, GA is used.

3.5. Solution Algorithm

We use a simple algorithm to find the optimal value of decision variables (q^*, w_1^*) which is applicable to all types of feed intake functions discussed before. In this algorithm, we first try to solve the models by solving the system of derivative equations with respect to q and w_1, derived in the previous sub-sections. However, since the analysis of concavity for TPU function is and intractable and complex work, we use a simple genetic algorithm within our algorithm when the derivative equations cannot be solved during a maximum number of iterations j_{max}. To implement GA, we utilize the GA toolbox in MATLAB ('gatool') with all default settings.

Step 1: Start with $j = 0$ and the initial random value of $q_j = q_0$.

Step 2: By solving the derivative equation with respect to w_1, find the optimal value of w_{1_j}, for a given quantity of newborn livestock q_j. To this end, we can use a bisection root-finding method.

Step 3: Utilize the result in step 2 for w_{1_j}, and then find the optimal value of q_{j+1} by solving the derivative equation with respect to q. Use a bisection root-finding method.

Step 4: If the difference between q_j, q_{j+1} is sufficiently small (less than a small positive number like 0.001), set $q^* = q_{j+1}$ and $w_1^* = w_{1_j}$ as optimal solutions and stop. Otherwise, set $j = j + 1$.

Step 5: If $j < j_{max}$ (j_{max} denotes the maximum number of iterations) go back to step 2, otherwise, run gatool and stop.

4. Numerical Example

To investigate and illustrate the performance of the proposed models, we present and discuss a numerical example for lambs in this section. As mentioned before, we employ three types of feed intake function in this paper, i.e., power function $(F_0 w^{F_1})$, exponential function $\left(A(1 - exp(-Bw))\right)$ and quadratic function $(Aw^2 + Bw + C)$. Note that the feed intake unit is assumed as kilograms per day (KPD). For the functions, the adjusting parameters are considered, in this example, as follows:

$F_0 = 0.15,\ \ F_1 = 1.05$

$A = 3,\ \ B = 0.1$

$A = 0.01,\ \ B = 0.02,\ \ C = 0.1$

Considering these parameters, the feed curve used for this example is, therefore, as follows.

$0.15w^{1.05},\ \ 3(1 - exp(-0.1w)),\ \ 0.01w^2 + 0.02w + 0.1$

Moreover, Table 1 and Figure 3 depict the growth of feed consumption for lambs with three functions of feed intake (KPD) separately. It is notable the data presented are from 1 to 10 kg as some examples, while the range of weight can be extended more.

Table 1. The feed consumption by three types of feed intake function

Lamb weight (w)	Feed intake function $F(w)$		
	Power	Exponential	Quadratic
1	0.15	0.29	0.13
2	0.31	0.54	0.18
3	0.48	0.78	0.25
4	0.64	0.99	0.34
5	0.81	1.18	0.45
6	0.98	1.35	0.58
7	1.16	1.51	0.73
8	1.33	1.65	0.9
9	1.51	1.78	1.09
10	1.68	1.90	1.3

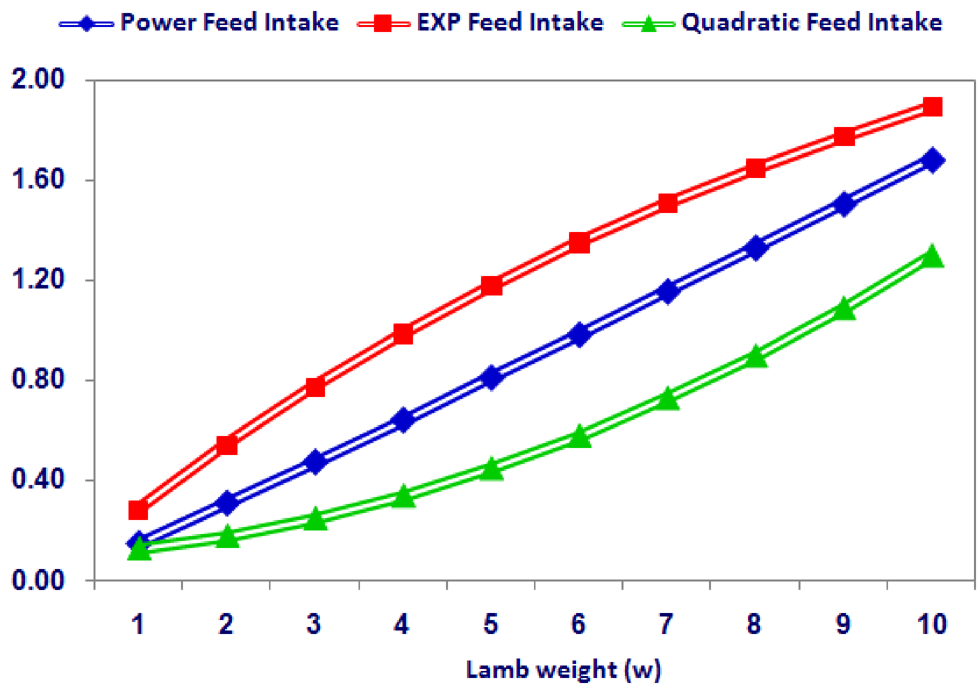


Figure 3. Behavior of feed consumption by three types of feed intake function

The other parameters are set as follows:

$p =$ 15 \$

$s =$ 12 \$

$r =$ 0.08

$\theta =$ 0.05

$c_f =$ 0.8 \$

$h =$ 0.35 \$

$D =$ 120000 kg/year

$w_0 =$ 3 kg

Using the system of derivative equations, the optimal results for three feed intake functions are obtained, as presented in Table 2.

Table 2. Results of implementations

Feed Intake Function	First model: growing items			Second model: joint growing and deteriorating items		
	q^*	w_1^*	$TPPU_I^*$	q^*	w_1^*	$TPPU_{II}^*$
Power	345	24.82	902429.057	476	24.90	983143.874
Exponential	822	68.23	992299.445	128	76.85	1092106.083
Quadratic	349	19.83	897490.865	401	19.85	977198.393

As an analysis of the first model, the results obtained reveal that the rancher should order 345, 822 and 349 lambs for power, exponential and quadratic feed intakes at the start of each growing cycle and slaughter them when they reach weight 24.82, 68.23 and 19.83, respectively. The net weight of lambs' meat according to the weight reduction factor $r = 0.08$ is, therefore, obtained as

- Power feed intake: $(1 - r)qw_1 = (1 - 0.08) \times 345 \times 24.82 = 8391.64$
- Exponential feed intake: $(1 - r)qw_1 = (1 - 0.08) \times 822 \times 68.23 = 54963.36$
- Quadtratic feed intake: $(1 - r)qw_1 = (1 - 0.08) \times 349 \times 19.83 = 6782.26$

Moreover, the sales period of slaughtered lambs t_2 is calculated as:

- Power feed intake: $(1 - r)qw_1/D = 8391.64/120000 = 0.070$ years ≈ 25.2 days
- Exponential feed intake: $(1 - r)qw_1/D = 54963.36/120000 = 0.458$ years ≈ 164.9 days
- Quadtratic feed intake: $(1 - r)qw_1/D = 6782.26/120000 = 0.032$ years ≈ 11.52 days

As further analysis of results, the numbers of growing cycles required per year to satisfy the annual demand are as follows:

- Power feed intake: $D/(1 - r)qw_1 = 120000/8391.64 = [14.29] = 15$ times per year
- Exponential feed intake: $D/(1 - r)qw_1 = 120000/54963.36 = [2.18] = 3$ times per year
- Quadtratic feed intake: $D/(1 - r)qw_1 = 120000/6782.26 = [17.69] = 18$ times per year

The annual profit of the rancher will be 902429.057, 992299.445 and 897490.865 for the three feed intake functions. Comparing them shows that the exponential feed intake is the most profitable case for the first model.

By analyzing the second model results, we find out that the rancher should order 476, 128 and 401 lambs for power, exponential, and quadratic feed intakes at the beginning of the growing cycle and fatten them to reach weight 24.90, 76.85 and 19.85, respectively, and then slaughter and sell them. The net weight of meat by the weight reduction factor $r = 0.08$ is thus as follows.

- Power feed intake: $(1 - r)qw_1 = (1 - 0.08) \times 476 \times 24.90 = 11615.35$
- Exponential feed intake: $(1 - r)qw_1 = (1 - 0.08) \times 128 \times 76.85 = 9640.06$
- Quadtratic feed intake: $(1 - r)qw_1 = (1 - 0.08) \times 401 \times 19.85 = 7800.65$

The sales period is calculated as:

- Power feed intake: $(1 - r)qw_1/D = 11615.35/120000 = 0.097$ years ≈ 34.85 days
- Exponential feed intake: $(1 - r)qw_1/D = 9640.06/120000 = 0.080$ years ≈ 28.92 days
- Quadtratic feed intake: $(1 - r)qw_1/D = 7800.65/120000 = 0.065$ years ≈ 23.40 days

Moreover, the numbers of growing cycles per year are as follows:

- Power feed intake: $D/(1 - r)qw_1 = 120000/11615.35 = [10.33] = 11$ times per year
- Exponential feed intake: $D/(1 - r)qw_1 = 120000/9640.06 = [12.45] = 13$ times per year
- Quadtratic feed intake: $D/(1 - r)qw_1 = 120000/7800.65 = [15.38] = 16$ times per year

As a result, the annual profit would be 983143.874, 1092106.083 and 977198.393 for the three feed intake functions which reveal that the exponential feed intake is the most profitable case for the second model.

In addition, a comparison between the first and second model indicates that the second one is more profitable in all types of feed intake function.

As a further investigation, Tables 3 shows a sensitivity analysis of the input parameters for both models. As shown by the results presented in Table 3, the sensitivity analysis of both models confirms that parameters s and D have a positive impact on TPU, while the parameters p, r, θ, c_f, h and w_0 have a negative impact on it. That means an increase in the former parameters (s and D) leads to increase in TPU, while an increase in the latter ones (p, r, θ, c_f, h and w_0) decrease TPU.

Table 3. Sensitivity analysis results (total profit per unit time)

Parameters	Change (%)	First model			Second model		
		Power	Exponential	Quadratic	Power	Exponential	Quadratic
p	−40	1125423.73	1175425.44	1087542.42	1164570.34	1297627.70	1157222.54
	−30	1078543.07	1145424.27	1036042.72	1107212.45	1255424.01	1104271.00
	−20	976354.58	1087423.72	964412.76	1064257.04	1172453.41	1052154.57
	−10	924755.12	1024126.78	923193.46	1004918.67	1115784.83	1004103.83
	+10	881804.76	975904.91	869203.73	957912.43	1058708.42	944987.51
	+20	8454290.71	934521.10	810412.24	914220.70	9845710.73	894512.71
	+30	8074120.22	904512.91	751201.04	874610.10	9441271.64	944987.51
	+40	7654124.04	854870.74	701452.15	848702.03	9054126.07	944987.51
s	−40	574525.62	648542.04	586541.65	601245.45	709547.65	605412.65
	−30	601542.03	704521.54	632514.43	686504.43	765425.03	679547.05
	−20	655414.70	792145.32	701241.56	742541.71	845652.74	732154.55
	−10	791124.35	900145.74	763278.14	836805.04	932467.41	829841.42
	+10	1055959.80	1122534.41	1028082.60	1125131.91	1261045.43	1118209.08
	+20	1120124.02	1240127.17	1187454.45	1242541.45	1387545.57	1287545.54
	+30	1185412.45	1298745.54	1274671.21	1325412.57	1465124.45	1398745.75
	+40	1265484.65	1381454.74	1387454.47	1398745.56	1564214.74	1487542.65
r	−40	1184542.85	1136521.55	995457.65	1065425.25	1242541.26	1045421.32
	−30	1112454.14	1095487.74	965456.45	1032124.45	1187545.57	1020321.02
	−20	1045424.54	1054215.65	935465.78	1012414.78	1144512.74	995421.54
	−10	935324.30	1018756.45	907441.57	984890.57	1119842.54	978088.37
	+10	882279.92	981240.17	883778.59	976884.24	1075746.80	969869.13
	+20	825415.54	959875.02	859874.32	955654.32	1045425.21	945784.02
	+30	765480.02	925468.32	826124.08	941265.30	9898650.06	913254.36
	+40	726548.45	885987.51	785456.81	928792.42	9587451.87	896885.74
θ	−40	10221031.63	11123214.54	9123541.03	1035412.14	12102124.32	1012425.58
	−30	985720.57	10754652.35	9054687.12	1012541.35	11705421.12	995412.25
	−20	956545.04	10454214.12	9021232.25	995412.52	11425412.32	985412.14
	−10	923828.94	1017985.00	898611.43	987844.14	1100120.51	977517.۳۲
	+10	893822.45	980758.57	895666.42	980427.81	1074521.00	973799.73
	+20	875412.87	955421.24	885468.21	978457.24	1045784.58	968754.24
	+30	854214.25	924125.38	876587.15	970245.14	1010245.45	956875.12
	+40	812541.41	896524.01	870214.47	965784.58	998542.21	950124.05

Parameters	Change (%)	First model			Second model		
		Power	Exponential	Quadratic	Power	Exponential	Quadratic
c_f	−40	1024521.42	1202421.78	1010124.14	1152457.21	1297874.52	1159874.07
	−30	996587.74	1154572.54	978542.01	1098754.78	1245245.47	1079854.42
	−20	965465.04	1098754.47	945754.45	1045987.45	1186898.65	1042541.54
	−10	933516.45	1032104.40	911332.89	1005124.64	1132103.83	991528.47
	+10	875175.58	964571.47	880655.06	959255.62	1068798.50	957890.42
	+20	845424.14	902451.12	856587.45	910021.04	1002454.50	915468.85
	+30	820124.54	856542.75	825468.87	865877.73	945784.50	875468.42
	+40	800254.84	800124.95	799869.90	815467.52	896587.50	820545.78
h	−40	942541.65	1028988.01	910254.70	996524.20	1160658.01	989895.45
	−30	935670.41	1021452.58	909758.05	990542.56	1140254.78	985425.01
	−20	925465.74	1010254.74	905425.45	989857.74	1119857.45	980241.12
	−10	903750.81	998792.54	899741.57	984516.90	1101987.10	978546.45
	+10	897685.04	986875.97	895371.13	980014.53	1087586.80	973981.25
	+20	895642.75	975845.74	891245.65	978987.04	1065778.74	970021.56
	+30	889857.24	970012.06	889568.47	974521.52	1049875.25	968975.75
	+40	882541.05	962014.45	882412.08	971234.83	1025464.03	960241.56
D	−40	423567.67	698754.01	523465.32	632604.65	510230.30	536541.88
	−30	577542.42	785452.78	624512.14	725468.75	624521.78	635421.65
	−20	652542.78	865874.65	745456.78	805421.54	798542.45	754251.75
	−10	811167.54	948935.78	835727.79	882593.09	977864.70	908936.00
	+10	1026228.79	1058764.48	985084.65	1079087.55	1217485.51	1071707.46
	+20	1203254.42	1165245.03	1123564.32	1202125.68	1432654.32	1232544.32
	+30	1368754.68	1266987.75	1265364.75	1326542.03	1603251.02	1387542.18
	+40	1536524.10	1367547.46	1398742.02	1454265.65	1854265.32	1512354.03
w_0	−40	1112568.45	1196536.87	1203524.57	1203254.32	1312042.02	1178542.87
	−30	1075421.41	1132540.21	1142560.41	1130204.45	1256442.21	1112402.45
	−20	1012154.05	1085640.45	978985.25	1086524.74	1197854.45	1065245.25
	−10	942480.54	1028969.94	922805.00	1013414.۹۳	1137896.42	1003335.77
	+10	884368.47	964798.14	869461.74	958631.95	1074201.61	945568.34
	+20	825201.65	905425.45	810021.54	890745.12	1020021.75	886598.45
	+30	765245.14	830201.25	752302.14	845625.47	865654.58	826598.08
	+40	723524.45	766568.74	700024.78	798542.65	810012.02	756524.91

6. Conclusions

This research proposed production-inventory models for joint growing and deteriorating items as an extension of the work presented by Rezaei (2014). A rancher buys a quantity of newborn livestock, raises them during a period, then slaughters them and sells them to the market. In addition to growing characteristics of livestock, the slaughtered livestock are deteriorated under a specific rate during the sales period. To meet the annual demand, rancher wants to know the optimal order quantity of newborn livestock and optimal weight they should reach before slaughtering, so as to maximize the total profit. To construct the production-inventory models for this problem, total profit was formulated for two cases separately, i.e., growing items, and (ii) joint growing and deteriorating items, via subtracting the total costs from the total revenue. The total costs include the total setup costs, the total purchasing costs, the total feeding costs, and the total holding cost, while the total revenue consists of the total sales of the slaughtered livestock. The previous model assumes that the weight of livestock after slaughtering certainly equals the weight of slaughtered livestock meat. However, in practice, some amount of slaughtered livestock is waste and should be disposed. To model this, we defined the weight reduction factor of slaughtered livestock r and used it in formulations. Moreover, three types of feed intake function, i.e., power feed intake, exponential feed intake, and quadratic feed intake, were discussed and formulated. A numerical example was also considered for managing growing and sales cycle of newborn lambs, where both models (growing items and joint growing and deteriorating items) were implemented for the three types of feed intake function (power, exponential, quadratic). The values of decision variables and resulting total profit were presented and discussed for all cases. Moreover, in order to assess the impact of the input parameters on the total profit, we performed a sensitivity analysis in terms of production cost p, selling price s, weight reduction factor r, deterioration rate θ, feeding cost c_f, holding cost h, annual demand D, and the initial weight of newborn livestock w_0. The results reveal that the parameters s and w_0 have the highest degree of positive and negative impacts on the total profit, respectively. As mentioned by Rezaei (2014), this class of growing items introduces a novel area for production-inventory problems. As further research, it can be extended via various aspects like backorder, lost sale, discount, pricing, dependent demand cases, etc. In addition, a future study can consider stochastic demand rate by using optical control theory which is more relevant to practical situations.

References

Bai, O., Chen, M., Xu, L. (2017) Revenue and promotional cost-sharing contract versus two-part tariff contract in coordinating sustainable supply chain systems with deteriorating items, *International Journal of Production Economics*, Vol. 187, pp. 85–101.

Balkhi, Z., (2011). Optimal economic ordering policy with deteriorating items under different supplier trade credits for finite horizon case. *International Journal of Production Economics*, Vol. 133, pp. 216–223.

Bansal, K.K., (2013). Inventory model for deteriorating items with the effect of inflation. *International Journal of Application and Innovation in Engineering and Management*, Vol. 2(5), pp. 143-150.

Bhaula, B., and Kumar, M.R., (2014). An economic order quantity model for weibull deteriorating items with stock dependent consumption rate and shortages under inflation. *International Journal of Computing and Technology*, Vol. 1(5), pp. 196-204.

Chen, T.-H., (2017). Optimizing pricing, replenishment and rework decision for imperfect and deteriorating items in a manufacturer-retailer channel, *International Journal of Production Economics*, Vol. 183, pp. 539–550.

Giri, B.C. Sharma, S. (2015) An integrated inventory model for a deteriorating item with allowable shortages and credit linked wholesale price, *Optimization Letters*, Vol. 9, pp. 1149-1175.

Goyal, S.K., Giri, B.C., (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research*, Vol. 134, pp. 1–16.

Jaggi, C.K. Eduardo Cárdenas-Barrón, L., Tiwari, S., Shafi, A.A., (2016) Two-warehouse inventory model for deteriorating items with imperfect quality under the conditions of permissible delay in payments, *Scientia Iranica*, In Press.

Janssen, L., Claus, T., Sauer, J. (2016). Literature review of deteriorating inventory models by key topics from 2012 to 2015, *International Journal of Production Economics*, Vol. 182, pp. 86–112.

Kouki, C., Zied Babai, M., Jemai, Z., Minner, S. (2016) A Coordinated Multi-Item Inventory System for Perishables with Random Lifetime, *International Journal of Production Economics*, In-Press.

Li, N., Chan, F.T.S., Chung, S.H., Tai, A.H. (2015). An EPQ model for deteriorating production system and items with rework, *Mathematical Problems in Engineering*, pp. 1-10.

Rezaei, J., (2014). Economic order quantity for growing items, *International Journal of Production Economics*, Vol. 155, pp. 109–113.

Teimoury, E., and Kazemi, S.M.M., (2016). An Integrated Pricing and Inventory Model for Deteriorating Products in a two stage supply chain under Replacement and Shortage, *Scientia Iranica*, In Press.