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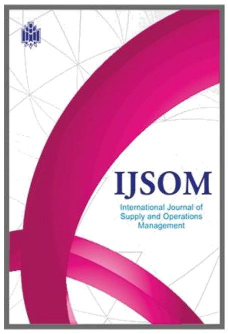

A Multi-attribute Approach to Simultaneous Determination of Preventive Replacement Times and Order Quantity of Spare Parts

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Abstract
One of the most important activities in preventive maintenance is replacement of spare parts prior to failure. The aim of this paper is to propose an approach which determines jointly the preventive replacement interval and the spare parts inventory by considering different criteria and interacting with decision makers. In this approach, preventive replacement intervals, determined by experts of production and maintenance, are ranked by the analytical hierarchy process (AHP). Criteria such as cost per unit of time, availability, remaining lifetime, and reliability are used. Then, a mixed integer nonlinear multi-objective model is presented that simultaneously specifies the period of preventive replacement and the required number of spare parts. This model considers the mentioned criteria and the inventory control costs of spare parts as different objective functions. Since the solution of the problem depends on the decision maker's strategy, it needs to interact with the decision-makers, and consequently the proposed model could be solved using the goal programming approach. The applicability of the proposed approach is illustrated by two numerical examples. The effect of key parameters on the optimal decisions is investigated for the examples.

Keywords: Preventive replacement; Spare parts; Analytical hierarchy process (AHP); Goal programming.

1. Introduction

Each system may fail during operational processes, and maintenance is often needed to keep a system in a condition where it can perform its function (Fallahnezhad and Pourgharibshahi, 2017). Maintenance operations play a major role in production systems since they result in a number of desired business outcomes including higher product quality, lower production line-off costs, reliable delivery of products and services, and improved safety. Thus, as a competency, maintenance management affects not only profitability of business but also quality of products and services (Zeng, 1997; Alsyouf, 2007; Najari et al., 2018).

The effective implementation of maintenance requirements is guaranteed by spare parts provision. In many classical maintenance policies it is ideally assumed that spare parts are always available in maintenance practice (Sharma and Yadava, 2011). However, excess inventory may impose additional costs whereas shortage of spare parts may increase the risk of system shutdown and lead to production losses (Au-Yong et al., 2016). So, it is important to balance the inventory and shortage costs by making the optimal decisions about the maintenance spare parts inventory. Extensive reviews of the maintenance spare parts inventory management models have been provided by Basten & Van Houtum, 2014. Practically, management of spare parts and maintenance are mutually influenced by each other; therefore, coordination of the planning of spare parts inventory and maintenance is important (Wang, 2011). This coordination has been considered separately or sequentially in many researches (Cheng and Tsao, 2010, de Almeida et al., 2015). However, Sarker and Haque, (2000) demonstrated that joint optimization is superior to separately or sequentially policies.

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This paper also focuses on the timely supply of spare parts and feasible preventive replacement times which are necessary to perform successful preventive maintenance by considering the interactions among decision makers of the production and maintenance departments. In our proposed approach, feasible preventive replacement intervals are the times in which the production line or equipment is shut down and also is available to perform the maintenance operations. These intervals are evaluated with respect to different criteria by the analytical hierarchy process (AHP). Then, a multi-objective is developed that simultaneously determines the preventive replacement times and the quantity of spare parts over a planning horizon that is solved by goal programming approach.

The rest of this paper is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 defines the problem under study and presents the details of the solution procedure. It also presents the parameters and formulation of model. Section 4 provides the numerical experiments to illustrate and analyze the performance of the procedure. Finally, concluding remarks are given in Section 5.

2. Literature review

The literature on the preventive replacement identifies four general types of single-criterion approaches: (1) age replacement presented by Barlow and Proschan (1965) and developed by Chien et al. (2010); (2) periodic replacement developed by Boland and Proschan (1982) and used by Wang and Zhang (2006); (3) block replacement presented by Barlow and Proschan (Nakagawa, 2005) developed by Huang et al (2008), and finally (4) replacement based on the number of failures proposed by Makabe and Marimura (1965) and developed by Chien (2009).

Given the close tie between the planning of maintenance and production, the problem of maintenance planning has become a multi-attribute decision making (MADM) environment (Chareonsuk, 1997). Gopalaswamy et al. (1993) investigated the problem of replacing the transportation fleet by applying multi-criteria decision making (MCDM) techniques. They considered a number of criteria such as cost rate, availability, and reliability to determine a preventive replacement strategy. Triantaphyllou et al. (1997) introduced several effective criteria for maintenance decision making and discussed their significance. Chareonsuk et al. (1997) proposed a strategy for preventive maintenance of parts and sub-systems using PROMETHEE II and considering some criteria such as cost per unit of time and reliability. Cavalcante (2008) dealt with the problem of determining a replacement strategy under the condition of uncertainty or in case of incomplete failure data. Yoo et al. (2001) proposed an optimization model of joint optimization policy of replacement and inventory using renewal theory. Jiang and Ji (2002) developed a multi-criteria model along with a utility function in order to compute the optimal period of preventive replacement according to cost, reliability and life of parts. Cavalcante and de Almeida (2007) investigated the problem of preventive replacement of parts in case of lacking adequate data about equipment failure. They used Bayesian analysis to handle the problem of insufficient data and uncertainty. Ilgin and Tunali (2007) proposed a combinational approach to genetic algorithm and simulation that identifies the joint optimization policies of preventive maintenance and spare provisioning of a manufacturing system operating in the automotive sector. Panagiotidou (2014) studied the joint maintenance and spare parts ordering problem with multiple identical items subject to silent failures, including minor and major failure, based on two ordering policies containing periodic and continuous review policies. Jiang et al. (2015) considered the joint optimization models of block replacement and periodic review policies of deteriorating inventory for a multi-unit system. These optimization models find the preventive replacement interval and the maximum inventory level to minimize the expected system total cost rate. Shi et al. (2016) developed a general simulation model which allows a closer-to-reality analysis of joint decisions on preventive maintenance and spare parts inventory for a fleet of equipment by both block- and age-based preventive maintenance policies with increasing failure rates. Ba et al. (2016) proposed an optimization model of joint preventive maintenance and spare parts inventory which determines a cost-effective production plan and a maintenance policy. Keizer et al. (2016) formulated an optimization model of joint condition-based maintenance and spares planning for multi-component systems by Markov Decision Process. Yang and Kang (2017) proposed a simulation optimization approach by using Monte Carlo simulation that optimizes jointly block preventive replacement and spare parts inventory for multi-component systems. Zahedi-Hosseini et al. (2017) developed several simulation models to optimize the joint spare parts inventory and inspection interval based on the delay time concept for a complex system. Zahedi-Hosseini et al. (2018) presented a model of simultaneous inspection and spare parts inventory policy for maintaining machines in a parallel system.

Dekker (1996) described four types of maintenance objectives which involve ensuring: system function, system life, safety, and human well-being. Quan et al. (2007) formulated the problem of scheduling maintenance operations as a multi-objective problem and applied an evolutionary algorithm to tackle the model. An interactive approach was proposed by Bashiri et al. (2011) based on linear assignment which uses quantitative and qualitative criteria to rank maintenance strategies. In addition, Nosoohi and Hejazi (2011) developed a multi-objective model to simultaneously calculate the quantity of spare parts and preventive replacement times over a planning horizon; their proposed model has been solved by the ε-constraint method.

In these researches, the optimal replacement interval and order quantity of spare parts were obtained in order to reach the maximum level of inventory. Usually, the optimal replacement intervals are obtained from mathematical models which are sometimes complex and not practical. Because, in the modeling process, it is not considered that replacement of spare parts is done when the production line is shut down or equipment is off. These downtimes are usually holiday times and allowance ones determined by industry managers based on market demand of products and competence conditions. Therefore, replacement intervals of spare parts in particular for preventive maintenance activities should be determined as interactively so that intervals are practical. Since the optimal policy of ordering spare parts depends on their consumption at replacement interval and optimal replacement interval obtained from models is infeasible, this policy of ordering spare parts is naturally inefficient in practice.

In most previous studies, reliability and anticipated cost per unit of time are used to identify the preventive replacement policies while other criteria including equipment accessibility and remaining life of spare parts are important in decisions making about preventive maintenance. However, previous researches failed to take such criteria into account. Holding and supplying costs of spare parts are important criteria from the perspective of decision makers. Moreover, warehouse service level in replacing pieces (either corrective or preventive maintenance operations) significantly affects preventive replacement intervals. This issue is ignored in the prior investigations except in Nosoohi and Hejazi (2011). Preventive replacement intervals identified by Nosoohi and Hejazi (2011) and Jiang et al. (2015) are not practical since the decision makers are not involved in determining replacement intervals in order to implement these operations. In this paper, we take advantage of the criteria by Jiang et al. (2015), excluding the average cycle time of replacement which is modified according to the assumptions and conditions of the study. In the previous researches, the formula of the criteria was created according to the continuous solution space. Therefore, the obtained results were not applicable in practice and should be adjusted according to the manufacturing conditions. Since the maintenance unit performs its operations on a production line or equipment in discrete time intervals, the solution space should be considered discrete. Some of the formula intended for the criteria with respect to the discrete solution space must be mathematically reconstructed and then used in the calculations.

In practice, industries determine the production policies based on factors such as on-time reaction to demand market and do not stop their production line(s) to perform preventive maintenance activities including preventive replacement of spare parts. The replacement time intervals obtained from the above-mentioned researches are not efficient because the researches fail to consider practical time intervals. However, replacement time intervals should be determined so that maintenance and production departments have identical viewpoints on these times. This paper develops a methodology which provides the optimal order quantity of spare parts as well as the feasible preventive replacement intervals. This method is a combined methodology of the Analytic Hierarchy Process (AHP) and multi-objective modeling which considers different decision-making criteria in production and maintenance units within a company. The main advantage of the proposed methodology is to consider diverse goals that are helpful in making good decisions. Hence, a lexicographic goal programming approach with consecutive transformations is used to solve a multi-objective model. As a result, the coordination between production and maintenance departments is able to communicate more effectively and faulty parts are replaced when equipment shuts down.

In this paper, a combination of the AHP and goal programming has been used for analyzing the problem under study. When the AHP method is applied, instead of using experts' opinion to accomplish the pairwise comparisons of alternatives based on criteria, mathematical relations are used and alternatives are compared with each other without the involvement of expert opinion. The result is a more precise comparison of the criteria and the achievement of more compatible comparisons in decision making. In order to do this, comparisons of time intervals based on some criteria such as the remaining lifetime are done by innovative relations. The use of GP modeling considers the final decision makers in problem analysis and guarantees the applicability of the solution provided by the model in practice.

3. The problem and specification of solution procedure

Each subsystem or any component encounters a potential failure or a failure that it needs to be replaced with a new one. Replacement costs include shut down, purchases, human resources costs, etc. Furthermore, the issue of supplying spare parts is not separate from the preventive replacement problem, and decisions should be proportional to the number of replacements (both preventive and corrective). It cannot be said with certainty whether the component or subsystem will fail, or its conditions change in terms of performance, and what costs will be imposed with this unexpected downtime. If a component / subsystem fails in the age $x < t_p$, it will be replaced by a new one.

The inventory control policy in this paper is continuous review (s, S). In this policy, an order up-to-S is placed according to the reorder points.
In the problem, the life or preventive interval t_p and spare parts inventory are decision variables. The assumptions of the problem under study are as follows:

- Part lifetimes are stochastic. End-of-life parts are preferred; however, replacement must precede failure.
- Equipment only fails due to faulty parts.
- In case of failure, the part is replaced and there is no room for partial repair.
- The planning horizon is limited.
- Preventive replacement is performed after operational time of t_p; if a failure does not occur prior to that point, preventive maintenance is performed; otherwise corrective maintenance is taken.
- After replacing the part, the system goes back to its initial state.

- The replacement takes time; thus, operations should be stopped when the replacement is being performed.
- The replacement cycle time is defined as the summation of the mean replacement time (MTTR) and operational times of each part. Indeed, a cycle is a time period between two successive replacements.
- The spare parts can be ordered and provided during the planning horizon.
- The holding cost of spare parts during the planning horizon is considered in the computations.
- The inventory control policy of spare parts is continuous review.
- The initial inventory and safety stock of spare parts is set to zero.
- The consumption rate of spare parts in the planning horizon is constant.
- Shortage of spare parts is not allowed.

Preventative maintenance tasks such as replacement are often planned to be carried out during equipment downtimes. This influences the usage of spare parts; additionally, as a result of inventory control policies, spare parts may be unavailable at the planned time interval. Therefore, this paper focuses on optimizing jointly preventive replacement times and ordering quantities of spare parts by considering various decision-making criteria as well as managerial goals.

This procedure is composed of three general phases demonstrated in Figure 1.

Phase 1: Determine the criteria and intervals of preventive replacement (the minimum number of intervals is 10).
Phase 2: Rank the preventive replacement intervals by the AHP proposed by Saaty (1980).
Phase 3: Build the multi-objective model and solve it using goal programming.

3.1. Criteria and intervals of preventive replacement

This subsection describes the criteria of preventive replacement in detail and then demonstrates how preventive replacement intervals are determined.

3.1.1. Planning criteria

To plan preventive replacement, previous studies mostly applied these four criteria: cost per unit of time, availability, remaining lifetime, and reliability. Each criterion is briefly explained below.

a) Cost per unit of time

Suppose t_p is the time at which preventive replacement occurs based on age and t_f represents the time of corrective replacement; then, C_{p} is the cost of a preventive replacement and C_{f} is cost of a corrective replacement such that it includes all the costs incurred by unexpected failure and replacement. The average cost per time unit is calculated via Equation (1) (Quan et al, 2007):

$$C(t_p) = \frac{C_f F(t_p) + C_p R(t_p)}{(t_p + MTTR_p)R(t_p) + \left(E(t_f|t_f < t_p) + MTTR_f\right)F(t_p)} \tag{1}$$

where the numerator yields the expected cost in each cycle and the denominator is the length of the expected replacement cycle. In this regard, $F(t_p)$ and $R(t) = 1 - F(t_p)$ are cumulative distribution of failure and reliability function, respectively. Besides, $MTTR_p$ and $MTTR_f$ are time of preventive and corrective replacement, respectively; finally, $E(t_f|t_f < t_p)$ is the expected conditional time between two corrective failures.

b) Availability

Availability is the percentage of cycle time in which the system is available. It is equal to the proportion of the expected time between two failures to the sum of the expected time between two failures and the expected time of replacement/repair. The criterion can be calculated as Equation (2). (Triantaphyllou et al., 1997):

$$A(t_p) = \left[1 + \frac{MTTR_f F(t_p) + MTTR_p R(t_p)}{t_p R(t_p) + E(t_f|t_f < t_p)F(t_p)}\right]^{-1} \tag{2}$$

Higher $A(t_p)$ values are preferred. The aforementioned equation is borrowed from Gopalaswamy et al. (1993), where the denominator of the relation is considered as the average time between two failures whereas here the time intervals are determined beforehand. Thus, during the intervals, replacement is carried out in only two cases: one preventive replacement operation or another corrective replacement in case of unpredicted failure.

c) Remaining lifetime

Let μ and $m(t)$ denote the average lifetime and average remaining lifetime of a replaced part, respectively (Nosoohi and Hejazi, 2013). This criterion is given by Equation (3).

$$m(t_p) = E(t - t_p|t > t_p) = \frac{1}{R(t)}\int_{t_p}^{\infty} tf(t)\,dt - t_p = \frac{1}{R(t_p)}\int_{t_p}^{\infty} R(t)dt \tag{3}$$

For the age replacement strategy with one period of t_p, after preventive maintenance, the replaced part remaining age is equal to an average of remaining lifetime $m(t_p)$ and before the replacement, the part's average lifetime equals μ.

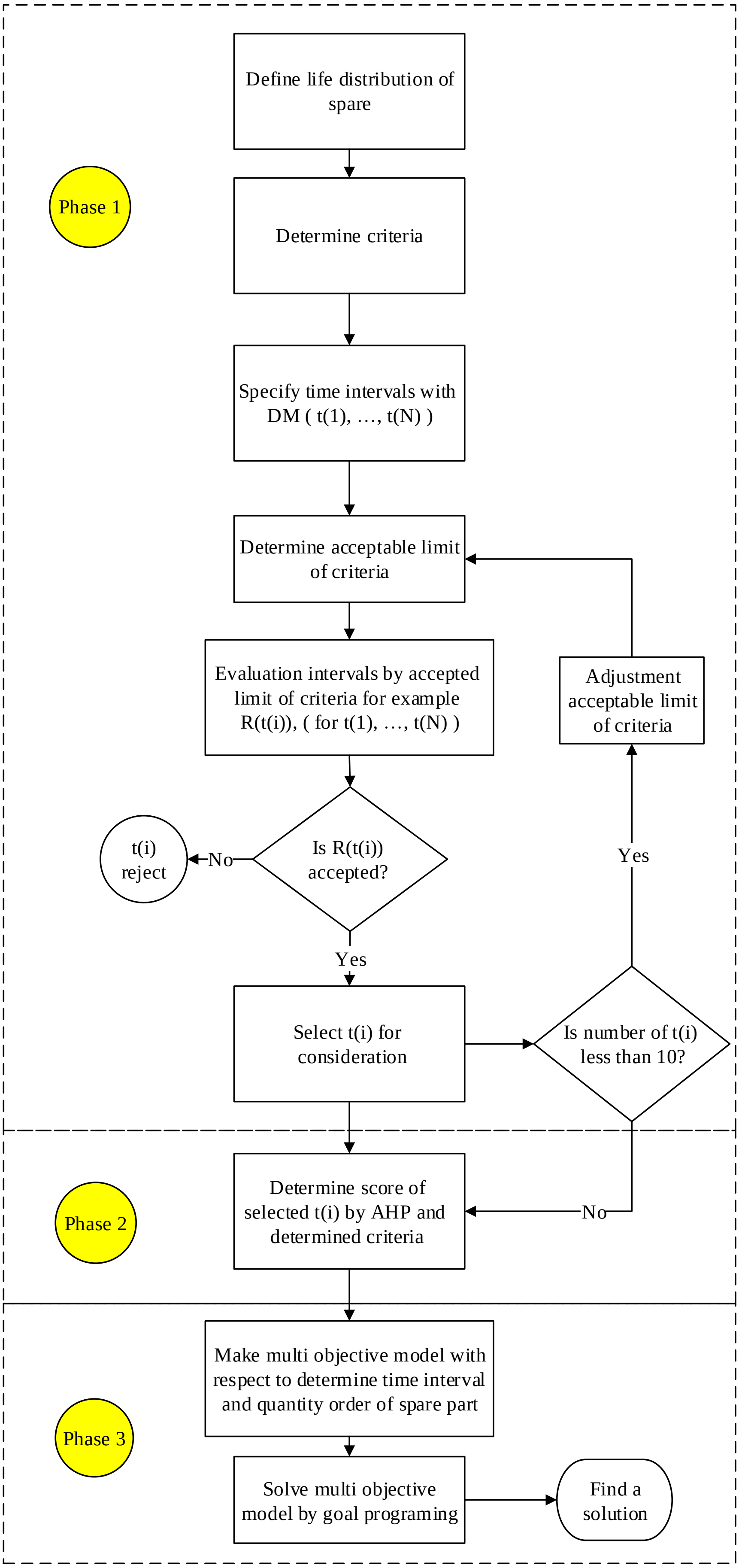

Figure 1. The proposed Procedure

d) Reliability

Reliability is defined as the probability of a part or subsystem to operate under a given condition and in a given time. It is computed by Equation (4)

$$R(t_p) = 1 - F(t_p) \tag{4}$$

3.1.2. Preventive replacement intervals

In order to define preventive replacement intervals, opinions of decision makers and experts such as maintenance and production manager are collected. To this end, at first, part lifetime distribution must be estimated. According to this distribution, the intervals are evaluated with regard to the different selected criteria. Thereafter, based on the defined upper or lower bounds of the criteria, the number of the determined intervals is reduced.

3.2. Ranking time intervals

In this paper, the AHP is applied to rank candidate intervals of preventive replacement. The AHP, as a powerful decision-making tool, is based on three principles, namely construction of a hierarchy, priority setting, and logical consistency. In the AHP Method, decision-makers (DMs) have the alternative to make a more comprehensive conceptual comparison based on different reciprocally exclusive multivariate criteria (Hasannia Kolaee and Torabi, 2017). The steps of this technique are described below.

Step 1: Build the hierarchical structure of the problem

The first step is to create a graphical view of the problem in which the objective, criteria and alternatives are depicted. Figure 2 shows the hierarchical structure of the problem.

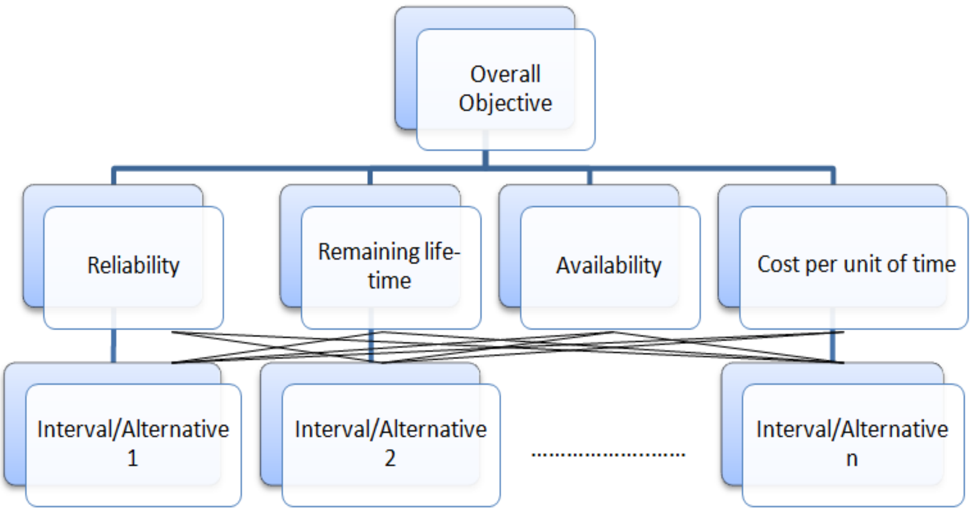

Figure 2. The hierarchical structure of the problem.

Step 2: Determining relative weights of alternatives with respect to the criteria

Relative weights of replacement time intervals (alternatives) are obtained through pairwise comparisons of alternatives with respect to each criterion. Each pairwise comparison matrix might be consistent or inconsistent; if the matrix is consistent, the weight computation is straightforward and it is done by normalizing the elements of each column. A pairwise comparison matrix sometimes becomes inconsistent once it is formed based on expert opinions. However, in this paper, such opinions are not applied. Instead, the value of each interval for each criterion is computed and the resulting intervals are compared to each other. Accordingly, the comparison matrix computed for this problem is consistent and weights are extracted from the matrix by utilizing the arithmetic average of normalized column vectors. The pairwise comparisons matrixes related to preventive replacement intervals with respect to cost per unit of time, availability, remaining life-time, and reliability are presented in Tables 1-4, respectively. For computing these matrixes, negativity or positivity of the criterion should be taken into account (Arunraj and Maiti, 2010).

Table 1. The pairwise comparisons matrix-cost per unit of time

With respect to $C(t_p)$	t_1	t_2	...	t_n
t_1	*1*	$1\Big/\frac{C(t_1)}{C(t_2)}$	...	$1\Big/\frac{C(t_1)}{C(t_n)}$
t_2	$1\Big/\frac{C(t_2)}{C(t_1)}$	*1*	...	$1\Big/\frac{C(t_2)}{C(t_n)}$
. . .	. . .	. . .	. . .	. . .
t_n	$1\Big/\frac{C(t_n)}{C(t_1)}$	$1\Big/\frac{C(t_n)}{C(t_2)}$	...	*1*

Table 2. The pairwise comparisons matrix-availability

With respect to $A(t_p)$	t_1	t_2	...	t_n
t_1	*1*	$\frac{A(t_1)}{A(t_2)}$	...	$\frac{A(t_1)}{A(t_n)}$
t_2	$\frac{A(t_2)}{A(t_n)}$	*1*	...	$\frac{A(t_2)}{A(t_n)}$
. . .	. . .	. . .	. . .	. . .
t_n	$\frac{A(t_n)}{A(t_1)}$	$\frac{A(t_n)}{A(t_2)}$	...	*1*

Table 3. The pairwise comparisons matrix-remaining life-time

With respect to $m(t_p)$	t_1	t_2	...	t_n
t_1	*1*	$\frac{\lvert m(t_1)\rvert}{\lvert m(t_1)-m(t_2)\rvert+\lvert m(t_1)\rvert}$	...	$\frac{\lvert m(t_1)\rvert}{\lvert m(t_1)-m(t_n)\rvert+\lvert m(t_1)\rvert}$
t_2	$\frac{\lvert m(t_1)-m(t_2)\rvert}{\lvert m(t_1)\rvert}+1$	*1*	...	$\frac{\lvert m(t_2)\rvert}{\lvert m(t_2)-m(t_n)\rvert+\lvert m(t_2)\rvert}$
. . .	. . .	. . .	. . .	. . .
t_n	$\frac{\lvert m(t_1)-m(t_n)\rvert}{\lvert m(t_1)\rvert}+1$	$\frac{\lvert m(t_2)-m(t_n)\rvert}{\lvert m(t_2)\rvert}+1$	...	*1*

Table 4. The pairwise comparison matrix-reliability

With respect to $\mathrm{R}(\mathrm{t_p})$	$\mathrm{t_1}$	$\mathrm{t_2}$	...	$\mathrm{t_n}$
$\mathrm{t_1}$	1	$\frac{\mathrm{R(t_1)}}{\mathrm{R(t_2)}}$	...	$\frac{\mathrm{R(t_1)}}{\mathrm{R(t_n)}}$
$\mathrm{t_2}$	$\frac{\mathrm{R(t_2)}}{\mathrm{R(t_1)}}$	1	...	$\frac{\mathrm{R(t_2)}}{\mathrm{R(t_n)}}$
. . .	. . .	. . .	. . .	. . .
$\mathrm{t_n}$	$\frac{\mathrm{R(t_n)}}{\mathrm{R(t_1)}}$	$\frac{\mathrm{R(t_n)}}{\mathrm{R(t_2)}}$	...	1

Since the criterion of remaining lifetime $m(\mathrm{t_p})$ takes negative and positive values, the comparison is not performed like that of other criteria. Furthermore, because the value of $m(\mathrm{t_p})$ is better for larger intervals, the preference ratio of each interval relative to its former time interval is used for evaluation where $\frac{|m(\mathrm{t_i})-m(\mathrm{t_{i+1}})|}{|m(\mathrm{t_i})|}+1$ represents the ratio of improvement from interval i to interval $i+1$.

After constructing the pairwise comparison matrixes, the weights or values of each preventive replacement interval for each criterion are obtained by using Equation (5). The matrix shown in Table 5 is constructed using the obtained weights. Assume a_{ij} denotes the elements of each matrix, the weight of each preventive replacement interval with respect to each criterion is calculated by Equation (5) where "i" is the number of alternatives/intervals and "j" is the number of criteria. So w_{ij} is the weight of the "ith" interval with respect to the "jth" criterion.

$$w_{ij} = \frac{\sum_{j=1}^{4}\left(\frac{a_{ij}}{\sum_{i=1}^{n} a_{i1}}\right)}{n} \quad \text{For } i,j = 1,\dots,n \tag{5}$$

Afterward, the final weight of each preventive replacement interval according to all criteria is given by Equation (6).

$$w_i = \sum_{j=1}^{4} w_j \times w_{ij} \;; \quad i = 1,..,n \tag{6}$$

Thus, w_i denotes the weight of each interval with respect to all criteria. Note that w_j values are equal and $\sum w_j = 1$.

Table 5. The evaluation matrix of preventive replacement intervals or replacement alternatives with respect to the criteria

Criteria weights	$\mathrm{w_1}$	$\mathrm{w_2}$	$\mathrm{w_3}$	$\mathrm{w_4}$
Criteria / *Intervals/alternatives*	$\mathrm{C(t_p)}$	$\mathrm{A(t_p)}$	$\mathrm{m(t_p)}$	$\mathrm{R(t_p)}$
$\mathrm{t_1}$	$\mathrm{w_{11}}$	$\mathrm{w_{12}}$	$\mathrm{w_{13}}$	$\mathrm{w_{14}}$
$\mathrm{t_2}$	$\mathrm{w_{21}}$	$\mathrm{w_{22}}$	$\mathrm{w_{23}}$	$\mathrm{w_{24}}$
. . .	. . .	. . .	. . .	. . .
$\mathrm{t_n}$	$\mathrm{w_{n1}}$	$\mathrm{w_{n2}}$	$\mathrm{w_{n3}}$	$\mathrm{w_{n4}}$

3.3. Model formulation

Prior to describing the model, the parameters and decision variables are introduced

3.3.1. Decision variables and parameters

Once the weights (scores) of all preventive replacement time intervals are computed, they are applied and a multi-objective model is put forward to simultaneously determine preventive replacement policies and order quantity of spare parts. Decision variables and parameters are expressed as follows:

Decision variables

x_i	Binary variable, represents selecting or not selecting i^{th} interval
Q	Order quantity of spare parts

Parameters of model

$\mathrm{t_i}$	Preventive replacement time intervals determined by decision makers
a_{ij}	Coefficients of decision variable x_i in the objective functions (the value of the i^{th} replacement interval for the j^{th} criterion)
w_i	Final score of preventive replacement intervals (the AHP output)

c_o	Cost of each order
c_h	Holding cost of a unit of spare part during planning horizon
c_b	Purchasing cost of a unit of spare part
A	Average order during planning horizon
B	Available budget for purchasing spare parts during planning horizon
t_p	Preventive replacement interval
t_f	Corrective replacement time
$MTTR_p$	Average time of preventive replacement
$MTTR_f$	Average time of preventive correction
$f(t)$	Probability density function of part lifetime t
$F(t)$	Cumulative distribution of part lifetime t
$F(\mathrm{t_p})$	Probability of failure prior to preventive replacement
$R(\mathrm{t_p})$	Reliability (probability of no failure) before preventive replacement
$E(\mathrm{t_f}\|\mathrm{t_f} < \mathrm{t_p})$	Expected time between two corrective replacements provided that they are performed before preventive replacement
T	Planning horizon
$E(c)$	Average time of a replacement cycle
g_i	Goal related to the ith objective
b_i	the determined bound of the ith constraint
n	Number of pre-determined replacement time intervals for selection

3.3.2. The proposed multi-objective model

In most prior studies on preventive replacement, cost objectives are the main concern in maintenance planning. However, in practical situations and especially for more interaction with decision makers, other objectives also need to be considered. In this subsection, a multi-objective model is proposed with considering other objectives and constraints. The model is given by Equations (7) to (15).

$$Minf_1 = \sum_{i=1}^{n} a_{i1}x_i \tag{7}$$

$$Maxf_2 = \sum_{i=1}^{n} a_{i2}x_i \tag{8}$$

$$Minf_3 = \sum_{i=1}^{n} a_{i3}x_i \tag{9}$$

$$Maxf_4 = \sum_{i=1}^{n} a_{i4}x_i \tag{10}$$

$$Maxf_5 = \sum_{i=1}^{n} w_i x_i \tag{11}$$

$$Minf_6 = \frac{A}{Q}c_o + c_h\frac{Q}{2} \tag{12}$$

$$Subjectto$$

$$\frac{c_b \mathrm{T}}{\mathrm{E(c)}} \leq B, \tag{13}$$

$$\sum_{i=1}^{n} x_i = 1, \tag{14}$$

$$Q \geq {}^{T}/_{\sum_{i=1}^{n} x_i t_i}, \tag{15}$$
$x_i = 0 \; or \; 1\,, \quad \forall i$
$Q \in \mathbb{Z} \cup 0$

where $\frac{\mathrm{T}}{\mathrm{E(c)}}$ is the average order of spare parts during the planning horizon and $\mathrm{E(c)}$ is the average time of a replacement cycle which is calculated by Equation (16):

$$E(c) = \left[t_p + MTTR_p\right] R(t_p) + \left[E\left(t_f \middle| t_f < t_p\right) + MTTR_f\right] F(t_p) \tag{16}$$

where $\mathrm{t_p}$ represents preventive replacement interval which is equal to the time intervals determined for preventive replacement that is $\mathrm{t_p} = \sum_{i=1}^{n} x_i t_i$. At the end, one of these intervals is selected.

Equations (7) through (10) pertain to the objectives of maintenance management and attempt to optimize criteria of cost per unit of time, availability, remaining life-time and reliability, respectively. Equation (11) maximizes the overall utility of the AHP while Equation (12) minimizes the inventory costs of spare parts; that is, the summation of ordering and holding costs. In this objective, it is assumed that the consumption rate of a spare part is constant during the planning horizon. Equation (13) enforces the limitation of available budget for purchasing spare parts during the planning horizon. Equation (14) guarantees the selection of an interval among determined intervals for preventive replacement. Finally, Equation (15) ensures that order size at each time exceeds the number of preventive replacements in the planning horizon.

3.3.3. Solving the multi-objective model

One of important techniques of solving the multi-objectives decision making (MODM) problems is the goal programming (GP) which finds a set of satisfying solutions. When decision makers (DMs) set a possible target value or aspiration level, GP is applied. In fact, the aim of GP is to minimize the deviations between the achievement of goals (or targets) and their aspiration levels. The GP model is formulated to obtain all parameters via an interactive process to achieve the predefined aspiration level (Tamiz et al., 1998).

In the short term, priority of production unit is the continuous and no-delay production whilst maintenance unit always endeavors to maintain the production line ready to produce. Because of the available incomprehensive information and the conflicts of interest in this kind of decision environment, making a reliable mathematical representation of the DMs' preferences is almost impossible. The GP provides a good framework for achieving a set of goals as closely as possible. Therefore, the multi-objective problem in this paper is re-modeled by the lexicographic goal programming (LGP) and solved by an iterative approach. In the lexicographic goal programming, deviational variables are minimized in a lexicographic sense and assigned to a number of priority levels. Tamiz et al. (1998) stated that “lexicographic goal programming is defined as sequential minimization of each priority whilst maintaining the minimal values reached by all higher priority level minimization”. The following steps are used to solve the multi-objective model:

Step 1: Converting the multi-objective model to a lexicographic goal programming model.
Subsequent to the conversion, the model can be expressed as:

$$Min\, D = \{w_1 h_1(d^+, d^-), w_2 h_2(d^+, d^-),\; w_3 h_3(d^+, d^-)\} \tag{17}$$

$$\sum_{i=1}^{n} a_{i1} x_i + d_1^- - d_1^+ = g_1 \tag{18}$$

$$\sum_{i=1}^{n} a_{i2} x_i + d_2^- - d_2^+ = g_2 \tag{19}$$

$$\sum_{i=1}^{n} a_{i3} x_i + d_3^- - d_3^+ = g_3 \tag{20}$$

$$\sum_{i=1}^{n} a_{i4} x_i + d_4^- - d_4^+ = g_4 \tag{21}$$

$$\sum_{i=1}^{n} w_i x_i + d_5^- - d_5^+ = g_5 \tag{22}$$

$$\frac{A}{Q} c_o + c_h \frac{Q}{2} + d_6^- - d_6^+ = g_6 \tag{23}$$

$$c_b A + d_7^- - d_7^+ = B \tag{24}$$

$$\sum_{i=1}^{n} x_i + d_8^- - d_8^+ = 1 \tag{25}$$

$$Q \times \sum_{i=1}^{n} x_i t_i - T + d_9^- - d_9^+ = 0 \tag{26}$$

$$d_i^- \times d_i^+ = 0, \qquad \forall i \tag{27}$$

$$x_i = 0 \ or \ 1 \ \ \forall i$$

$$d_i^+, d_i^- \geq 0 \ , \forall \ i$$

$$Q \in Z \cup 0$$

w_1, w_2, and w_3 represent the ranking weights regarding objectives preferences; here, the preferences are considered as $w_1 \gg w_2 \gg w_3$. This means that the objectives with higher priority are given stronger preferences. In other words, initially, objectives of the first priority are optimized followed by the objectives of lower order. d_i^+ and d_i^- denote the negative and positive deviation variables of the i^{th} goal.

$\mathrm{h}_1(\mathrm{d}^+, \mathrm{d}^-)$ represents the deviation function from the goals with the first priority which is related to constraints of the model and examines the feasibility of the solution space. When the function value equals zero, objective functions of the next priority will be solved. Analogously, $\mathrm{h}_2(\mathrm{d}^+, \mathrm{d}^-)$ denotes the deviation function from objectives of the second priority which include Equations (22) and (23). The equations are assigned to the second priority since this study tries to simultaneously optimize inventory and maintenance functions and the two functions cover all the objectives of inventory and maintenance. Thus, by jointly optimizing the two objectives, the aforementioned goals can be achieved. And finally, $\mathrm{h}_3(\mathrm{d}^+, \mathrm{d}^-)$ represents the deviation function from objectives of the third priority which are to be minimized. These objectives include Equations (18), (19), (20), and (21), which are related to the objectives pursued by the maintenance department. After simultaneously improving the overall utility of the objectives and inventory, at this level, a solution is sought that improves these objectives without deteriorating previous values.

Step 2: Solving the lexicographic goal programming by an iterative approach.

The LGP model is then solved by using an iterative approach (Romero, 2001). The output of the problem is a preventive replacement time interval $\mathrm{t_p}$ and order quantity of the spare part Q such that deviations from the inventory and maintenance objectives are optimized at the same time. Since Q is the order quantity, the time interval of each order is calculated by the ratio ($\frac{Q}{\frac{T}{E(c)}}$).

4. Numerical results and sensitivity analysis

In this section, two examples are investigated. In each example, consider a part in a system with the following specifications.

C_o	\$3000 in each order
C_h	\$1000 for a part in a year
c_b	\$8000
B	\$1000000
$\mathrm{MTTR_p}$	0.009 month
$\mathrm{MTTR_f}$	0.022 month
C_p	\$30000 for each preventive replacement
C_f	\$50000 for each corrective replacement
T	One year
n	10 intervals

Part lifetime distribution is the difference between two examples such that a Weibull distribution with parameters β and α and an exponential distribution with a failure rate λ are part lifetime distribution functions at examples 1 and 2, respectively.

It is assumed that the preventive replacement intervals, presented in Table 6, are known and provided by the decision maker(s).

Table 6. Time intervals of preventive replacement/replacement alternatives for two examples

Row	1	2	3	4	5	6	7	8	9	10
Replacement interval (month)	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5

Example 1. The random variable of part lifetime is a Weibull distribution with parameters $\beta = 2$ and $\alpha = 1$. Therefore,

$$f(t) \qquad \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} \quad , \qquad \alpha > 0, \beta > 0\,, t \geq 0$$
$$F(t) \qquad 1 - e^{-\alpha t^{\beta}}, \qquad \alpha > 0, \beta > 0\,, t \geq 0$$
$$F(\mathrm{t_p}) \qquad 1 - e^{-\alpha t_p{}^{\beta}}, \qquad \alpha > 0, \beta > 0\,, t \geq 0$$
$$R(\mathrm{t_p}) \qquad e^{-\alpha t_p{}^{\beta}}, \qquad \alpha > 0, \beta > 0\,, t \geq 0$$

Example 2. The random variable of part lifetime is an exponential distribution with a failure rate of twice in a month (λ= 2). Therefore**,**

$$f(t) \qquad \lambda\, \mathrm{e}^{-\lambda \mathrm{t}}\,, \qquad t \geq 0$$
$$F(t) \qquad 1 - \mathrm{e}^{-\lambda \mathrm{t}}\,, \qquad t \geq 0$$
$$F(\mathrm{t_p}) \qquad 1 - \mathrm{e}^{-\lambda \mathrm{t_p}}$$
$$R(\mathrm{t_p}) \qquad \mathrm{e}^{-\lambda \mathrm{t_p}}$$

Numerical results of examples 1 and 2 are shown as follows. To determine the optimal policy, the proposed procedure is applied.

Step 1. Equations (1) through (4) are rewritten and analyzed under the hypothesis of distribution function for part lifetime. At the end, all four criteria are used to evaluate time intervals for preventive replacement.

Step 2. After selecting the intervals and determining the criteria, the intervals are evaluated using the AHP and the results are reported in Table 7.

The weights of criteria are assumed to be equal; therefore, $\mathrm{w}_i = 0.25 \quad \forall i$ for Weibull distribution and $\mathrm{w}_i = 0.33 \quad \forall i$ for Exponential distribution. Using Equation (6), the final score of each interval is computed as shown in Table 8.

Step 3. After computing the total scores of time intervals using the AHP, the multi-objective model is constructed. The coefficients $a_{\mathrm{i}1}$, $a_{\mathrm{i}2}$, $a_{\mathrm{i}3}$ and $a_{\mathrm{i}4}$ for each interval are computed via Equation (1), (2), (3), and (4), respectively. The values of the considered goals are shown in Table 9.

Then, the goal programming model is developed. Cost-related objective functions, including replacement cost for each unit of time and inventory control cost are different in terms of scale from other objective functions. While the latter objectives are already without dimensions, cost-related objective functions must be dimensionless. This is also true for coefficients of budget constraint.

The model is solved by a consecutive transformation approach using the GAMS 22.1, solver BARON. The values of objectives regarding maintenance and inventory control of spare parts, for replacement time interval $\mathrm{t_p} = 0.5$ can be observed in Table 10 for the two examples.

Exploring the effect of changing several model parameters on the current solution is useful. To perform sensitivity analysis on goal programming parameters, three types of changes are to be taken into consideration: switching priorities, weight or importance of objectives in each priority, and values of objective goals.

By switching priorities and modifying the weights of objectives in each priority, no change is seen in the solution of the model. However, the goal values of the objective function determined by decision makers are of importance and vital parameters to specify the solution. This result showed that decision maker's aims and goals affect strategies for preventive replacement and ordering spare parts. This can also be managed intuitively.

By changing the values of goals g_1, g_2, g_3, g_4 and g_5, no change occurs in the solution. So, the model is not sensitive to changes in these values. As seen in Table 11 and Table 12, the replacement time interval and the numbers of spare parts begin to change by increasing values of the sixth goal. Therefore, the solution is sensitive to the sixth objective which requires decision-maker discretion.

Table 7. The calculated scores for time intervals of preventive replacement for the two examples

Type of lifetime distribution	t_i	$C(T)$	$A(T)$	$M(T)$	$R(T)$
Weibull distribution	t_1	0.036689	0.097966	0.058195	0.109474
	t_2	0.066217	0.099609	0.069047	0.108656
	t_3	0.091221	0.100105	0.080669	0.107306
	t_4	0.106685	0.100304	0.091597	0.105445
	t_5	0.115151	0.10038	0.100907	0.103099
	t_6	0.118587	0.100394	0.108483	0.100302
	t_7	0.118947	0.100376	0.114763	0.097095
	t_8	0.117656	0.100338	0.120289	0.093521
	t_9	0.115603	0.100291	0.125483	0.08963
	t_{10}	0.113244	0.100237	0.130566	0.085472
Exponential distribution	t_1	0.033270	0.089426	0.100000	0.150545
	t_2	0.054129	0.096566	0.100000	0.136219
	t_3	0.074072	0.099198	0.100000	0.123256
	t_4	0.089361	0.100561	0.100000	0.111526
	t_5	0.102215	0.101391	0.100000	0.100913
	t_6	0.113120	0.101948	0.100000	0.091310
	t_7	0.122447	0.102345	0.100000	0.082621
	t_8	0.130476	0.102642	0.100000	0.074758
	t_9	0.137429	0.102871	0.100000	0.067644
	t_{10}	0.143480	0.103052	0.100000	0.061207

Table 8. Final score of replacement time intervals for the two examples

Type of lifetime distribution	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
Weibull distribution	0.075581	0.085882	0.094826	0.101008	0.104884	0.106942	0.107795	0.107951	0.107752	0.10738
Exponential distribution	0.0911	0.0956	0.0988	0.1005	0.1015	0.1021	0.1025	0.1026	0.10265	0.10258

Table 9. The values of goals related to the objectives for the two examples

Type of lifetime distribution	g_1	g_2	g_3	g_4	g_5	g_6
Weibull distribution	150000($)	1	-1	0.7	0.2	15000($)
Exponential distribution	150000($)	1	0	0.7	0.2	15000($)

Table 10. Values of objective functions with respect to maintenance and inventory for the two examples

Type of lifetime distribution	t_p	Q	$C(T)$	$A(T)$	$M(T)$	$R(T)$	Final score
Weibull distribution	0.5	24	38354	0.99	-0.50	0.78	0.107
Exponential distribution	0.5	24	127949	0.9483	0.10	0.3679	0.1026

Table 11. Solution changes in response to changes in values of the 6th objective in example 1

Solution g_6	t_p	Q	$C(T)$	$A(T)$	$M(T)$	$R(T)$	Final score of time interval
2000	0.5	24	38354	0.99	-0.5	0.78	0.107
15000	0.5	24	38354	0.99	-0.5	0.78	0.107
20000	0.45	29	37572	0.99	-0.45	0.82	0.108
25000	0.45	42	37572	0.99	0-0.45	0.82	0.108
30000	0.5	30	38354	0.99	-0.5	0.78	0.107
50000	0.5	51	38354	0.99	-0.5	0.78	0.107

Table 12. Solution changes in response to changes in values of the 6th objective in example 2

Solution g_6	t_p	Q	C(T)	A(T)	R(T)	TC(Q)	Final score of time interval
10000	0.5	24	127949	0.9483	0.3679	16501	0.1026
12000	0.5	24	127949	0.9483	0.3679	16501	0.1026
16000	0.5	24	127949	0.9483	0.3679	16501	0.1026
18000	0.45	27	133582	0.9467	0.4066	17754	0.1026
20000	0.45	33	133582	0.9467	0.4066	19981	0.1026
22000	0.35	36	149927	0.9418	0.4966	21742	0.1025
24000	0.3	40	162288	0.9382	0.5488	23743	0.1021
30000	0.25	53	179604	0.9331	0.6065	29721.5	0.1015

Furthermore, it is assumed the number of preventive replacement intervals is increased to 15. In fact, times 0.55, 0.6, 0.65, 0.7 and 0.75 were added to the times presented in Table 6. It is presumed that these intervals are known and provided by the decision maker(s) for example 2. The obtained results are presented in Table 13.

Table 13. Values of objective functions with respect to maintenance and inventory for example 2 with 15 intervals

t_p	Q	$C(T)$	$A(T)$	$M(T)$	$R(T)$	Final score
0.7	20	13953	0.95	0.10	0.247	0.063

It is clear from Table 13 that the preventive replacement interval is 0.7 months and the order quantity of spare parts is 20.

5. Conclusion

In this paper, the problems of preventive maintenance and inventory control of spare parts were solved simultaneously. The feasible replacement time intervals were gained from decision makers and experts of the production unit to implement the preventive maintenance. These time intervals were evaluated through the AHP by using the pre-determined criteria such as cost per unit of time, availability, remaining lifetime, and reliability. These criteria were presented by a mathematical analysis. After the time intervals were ranked, a multi-objective model was created, which was re-modeled by goal programming. The model validity was established by solving two examples and performing sensitivity analysis. By evaluating sensitivity of the model to the parameters of goal programming, it was concluded that the model is sensitive to the defined magnitude of objective goals and the solution relies on the objectives which decision makers aim to achieve.

The presented procedure solved the problem of replacing a part or a subsystem in equipment or a system; this procedure can be extended to the situation of multiunit systems. The joint optimization of dependent units in parallel systems might also be investigated. In addition, this procedure can be generalized to comprise group replacement or other types of replacement (block, failure-based, etc.). The authors believe that the proposed procedure could be applied in other maintenance problems such as determining sequence of maintenance actions while objectives of other units are considered. The procedure needs to be implemented in realistic industrial situations to be validated.

References

Alsyouf I. (2007). The role of maintenance in improving companies, productivity and profitability, *International Journal of Production Economics*, Vol. 105, pp. 70-78.

Arunraj N.S. and Maiti J. (2010). Risk-based maintenance policy selection using AHP and goal programming, *Safety science*, Vol.48, pp.238-247.

Au-Yong Ch. P., Ali A.Sh. and Ahmad F. (2016). Enhancing building maintenance cost performance with proper management of spare parts, *Journal of Quality in Maintenance Engineering*, Vol. 22(1), pp. 51-61.

Ba K., Dellagi S., Rezg N. and Erray W. (2016). Joint optimization of preventive maintenance and spare parts inventory for an optimal production plan with consideration of CO 2 emission, *Reliability Engineering & System Safety*, Vol. 149, pp. 172-186.

Barlow R. E. and Proschan F. (1965). *Mathematical theory of reliability*, Wiley, New York.

Bashiri M., Badri H. and Hejazi T.H. (2011), Selecting optimum maintenance strategy by fuzzy interactive linear assignment method, *Applied Mathematical Modelling*, Vol. 35, pp. 152–164.

Basten R. and Van Houtum, G. (2014). System-oriented inventory models for spare parts, *Surveys in operations research and management science,* Vol. 19(1), pp. 34-55.

Boland P.J. and Proschan F. (1982). Periodic replacement with increasing minimal repair costs at failure, *Operation research*, Vol.30, pp.1183-1189.

Cavalcante C.A.V. and de Almeida A.T. (2007). A multi-criteria decision-adding model using PROMETHEE III for preventive maintenance planning under uncertain conditions, *Journal of Quality in maintenance engineering*, Vol. 13(4), pp.385-397.

Cavalcante C.A.V. (2008). A preventive decision model based on multicriteria method PROMETHEE II integrated with Bayesian approach, *IMA Journal of management mathematics*, Vol.21(4), pp.333-348.

Chareonsuk C., Nagarura N. and Tabucanona M.T. (1997). A multicriteria approach to the selection of preventive maintenance intervals, *International Journal of Production Economics*, Vol.49, pp.55-46.

Cheng Y.H. and Tsao H.L. (2010). Rolling stock maintenance strategy selection, spares parts' estimation, and replacements' interval calculation, *International Journal of Production Economics,* Vol. 128, pp. 404-412.

Chien Y.H. (2009). A number dependent replacement policy for a system with continuous preventive maintenance and random lead times, *Applied mathematical modeling*, Vol. 33, pp.1708-1718.

Chien Y.H., Chang C. and Sheu S.H. (2010). Optimal age-replacement model with age-dependent type of failure and random lead time based on cumulative repair-cost limit policy, *Journal of Annals of Operation Research*, Vol. 181(1), pp.723-744.

De Almeida A.T., Ferreira R.J.P. and Cavalcante C.A.V. (2015). A review of the use of multicriteria and multi-objective models in maintenance and reliability, *IMA Journal of Management Mathematics*, Vol. 26, pp. 249-271.

Dekker R. (1996). Applications of maintenance optimization models: a review and analysis, *Reliability engineering and system safety*, Vol. 51, pp.229-240.

Fallahnezhad M.S. and Pourgharibshahi M. (2017). Determining an optimal maintenance policy for three state machine replacement problem using dynamic programming, *International Journal of Supply and Operations Management*, Vol. 4 (2), pp. 180-192.

Gopalaswamy V., Rice J. A. and Miller F. G. (1993). Transit vehicle component maintenance policy via multiple criteria decision making methods, *Journal of the Operational Research Society*, Vol.44, pp.37–50.

Hasannia Kolaee M. and Torabi S. A. (2017). Supplier and Carrier Selection and Order Allocation by Considering Disruptions with AHP and Multi-objective Mathematical Programming, *International Journal of Supply and Operations Management,* Vol. 4(4), pp. 359-369.

Huang R.H., Meng L., Xi L. and Liu C.R. (2008). Modeling and analyzing a joint optimization policy of block-replacement and spare inventory with random-lead time, *IEEE Transaction reliability*, Vol. 57(1), pp. 113-124.

Ilgin MA, Tunali S. (2007). Joint optimization of spare parts inventory and maintenance policies using genetic algorithms, International journal advanced manufacturing technology, Vol. 34, pp. 594–604.

Jiang R. and Ji P. (2002). Age replacement policy: a multi attribute value model, *Reliability engineering and system safety*, Vol.76, pp.311-318.

Jiang Y., Chen M. and Zhou D. (2015). Joint optimization of preventive maintenance and inventory policies for multi-unit systems subject to deteriorating spare part inventory, *Journal of Manufacturing Systems*, Vol. 35, pp. 191–205.

Keizer M. C. A. O., Teunter R. H. and Veldman J. (2017). Joint condition-based maintenance and inventory optimization for systems with multiple components, European Journal of Operational Research, Vol. 257(1), 209-222.

Makabe H. and Marimura H. (1965). Some considerations on preventive maintenance policies with numerical analysis, *The Operations Research Society of Japan*, Vol. 7(4), pp. 154-171.

Nakagawa T. (2005). *Maintenance theory of reliability*, Springer, USA.

Nosoohi I. and Hejazi S.R. (2011). A multi-objective approach to simultaneous determination of spare part numbers and preventive replacement times, *Applied Mathematical Modelling*, Vol.35(3), pp.1157-1166.

Najari F., Sobhanallahi M.A. and Mohammadi M. (2018). Improving maintenance strategy by physical asset management considering the use of MFOP instead of MTBF in petrochemical, *International Journal of Supply and Operations Management*, Vol. 5(4), pp. 319-337.

Panagiotidou S. (2014). Joint optimization of spare parts ordering and maintenance policies for multiple identical items subject to silent failures, *European Journal of Operational Research*, Vol. 235, pp. 300–314.

Quan G., Greenwood G.W., Liu D. and Hu S. (2007). Searching for multiobjective preventive maintenance schedules: combining preferences with evolutionary algorithms, *European Journal of Operational Research*, Vol.177, pp.1969–1984.

Romero C. (2001). Extended lexicographic goal programming: a unifying approach, *Omega*, Vol. 29(1), pp. 63-71.

Saaty T.L. (1980). *The Analytic Hierarchy Process*, McGraw-Hill International, New York, USA.

Sarker R. and Haque A. (2000). Optimization of maintenance and spare provisioning policy using simulation, *Applied Mathematical Modelling*, Vol. 24, pp. 751-760.

Sharma A. and Yadava G.S. (2011). A literature review and future perspectives on maintenance optimization, *Journal of Quality in Maintenance Engineering*, Vol. 17, pp. 5-25.

Shi Y., Xiang Y., Jin T. and Li Y. (2016). Joint planning for spare parts inventory and preventive maintenance in a multi-echelon network, *International Journal of Inventory Research*, Vol. 3(3), pp. 263-281.

Tamiz M, Jones D. and Romero C. (1998). Goal programming for decision making: An overview of the current state-of-the-art. *European Journal of Operational Research*, Vol. 111, pp. 569-581.

Triantaphyllou E., Ovalerchuk B., Mann L. and Knapp G.M. (1997). Determining the most important criteria in maintenance decision making, *Journal of quality maintenance engineering*, Vol. 3(1), pp.16-28.

Wang G.J. and Zhang Y.L. (2006). Optimal periodic preventive replacement policy assuming Geometric process repair, *IEEE Transactions on reliability*, Vol. 55(1), pp. 118-122.

Wang W. (2011). A joint spare part and maintenance inspection optimisation model using the delay time concept, *Reliability Engineering and Systems Safety*, Vol. 96, pp. 1535-1541.

Yang R. and Kang J. (2017). A joint optimal policy of block preventive replacement and spare part inventory, *International Journal system assurance engineering management*, Vol. 8(4), pp. 740-746.

Yoo Y.K., Kim K.J. and Seo J. (2001). Optimal joint spare stocking and block replacement policy, *International journal advanced manufacturing technology*, Vol. 18, pp. 906–909.

Zahedi-Hosseini F., Scarf Ph. and Syntetos A. (2017). Joint optimisation of inspection maintenance and spare parts provisioning: a comparative study of inventory policies using simulation and survey data, *Reliability Engineering & Systems Safety*, Vol 168, pp. 306-316.

Zahedi-Hosseini F., Scarf Ph. And Syntetos A. (2018). Joint maintenance-inventory optimisation of parallel production systems, *Journal of Manufacturing Systems*, Vol. 48, pp. 73–86.

Zeng S. W. (1997). Discussion on maintenance strategy, policy and corresponding maintenance systems in manufacturing, *Reliability Engineering & Systems Safety*, Vol. 55, pp. 151-162.