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An Application of Cooperative Grey Games to Post-Disaster Housing Problem

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Abstract
This paper shows that cooperative grey game theory can help us to establish a fair cost share between private organizations for supporting the temporary housing problem by using facility location games under uncertainty. Temporary accommodation might be a method that ought to get started before a tragedy happens, as a preventative pre-planning. In spite of being temporary constructions, the housing buildings are one of the most essential parts to produce in emergency situations, to contribute to the reconstruction and to recover better. Our study is based on a default earthquake in Izmir of western Turkey. A number of tents are being built in three cities near Izmir. Two companies are selected to distribute the tents in a fair way between the three cities. For this purpose, we use cooperative grey game theory to help us define a fair cost allocation between private organizations for supporting the housing problem by using facility location games under uncertainty.

Keywords: Temporary housing; Earthquake; Cooperative grey games; Grey numbers; Facility location situations.

1. Introduction

Izmir is one of the largest cities in Turkey's with a population of around 4 million, which has the second biggest seaport after Istanbul. Izmir is a vibrant city where half of its population is below the age of thirty. The town hosts tens of thousands of university students, scientist, artists, business leaders and lecturers. It's an apace growing town on the Central Aegean coast of Turkey. Izmir survived as a giant town throughout its history of 5000 years and has been oftentimes restored beneath political science and earth science influences. The city has been greatly littered with some disasters like earthquakes, fires, epidemics, etc. Therefore, several edifices that might reflect historical background of the town did not survive until these days, present remains are generally few and known only by experts and neighboring people (Report of Radius Project August 2001).

The map in Figure 1, shows seismic zones in Turkey and Izmir city. It divided into five zones according to the magnitude of earthquakes, ranging from the most dangerous to the least dangerous. According to statistics, about 44% of the population of Turkey lives in the most dangerous zones (Kilci et al., 2015). Communities suffer after the disasters, and their impact are tragic. From the destruction of infrastructure to the unfolding of sickness, natural disasters will devastate entire countries long. Tsunamis, earthquakes and typhoons do not just make a disturbance on land; they additionally disrupt people's lives in both densely inhabited cities and remote villages. In this study, we try to help those people who have lost their houses and became homeless. Uncertainty in the likelihood of disasters arises from a number of sources. These uncertainties, we can deal into it by cooperative interval game and interval solutions which solutions are interval Centre-of gravity of the Imputation-Set value, shortly denoted by ICIS-value, Interval Egalitarian Non-Separable Contribution's value, shortly denoted by IENSC-value and therefore the interval equal division solution, shortly denoted by IED-solution (Usta et al., 2018). Our main goal is to extend of these solutions by using grey uncertainty.

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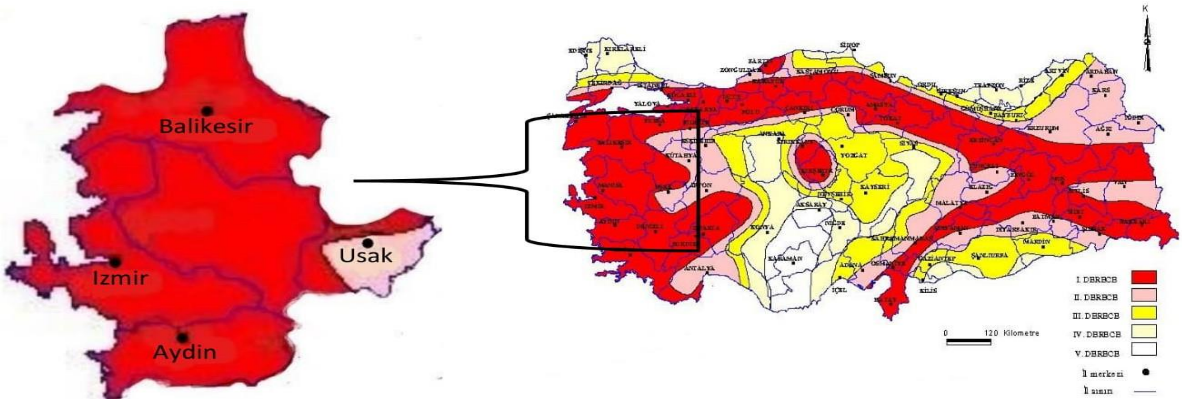

Figure 1. Seismic zones map of Turkey and Izmir and Cities of tents (https://www.deprem.afad.gov.tr/deprem-bolgeleri-haritasi?lang=en)

In this paper, we focus on an application on a facility location situation and related games with grey data. Each facility is made to please the players during a facility location situation of affairs. Here, the matter is to attenuate the overall cost. This cost consists of each of the player's distance and also the construction of every facility (Usta et al., 2018). The paper is organized as follows: We provide some basic notions and resolution concepts from grey numbers and cooperative grey games in Section 2. Our cooperative facility location game based on the cooperative grey game model created after an earthquake as a natural disaster is given in Section 3. In Section 4, we tend to give some interpretations relating to our solutions. Finally, Section 5 ends this paper with a conclusion and an outlook to future studies.

2. Basic Concepts

In this section, in order to supply the readers with all the required background to follow this paper, we formally provide some basics from cooperative grey games and connected grey solution ideas. A facility location scenario that is critical to construct our model is additionally given.

2.1. Grey numbers system

Grey theory (Deng, 1982), originally developed by Professor Deng in 1982, has become a very effective method of solving uncertainty problems under discrete data and incomplete information. Grey theory has now been applied to various areas such as forecasting, system control, computer graphics and decision-making. Here, we give some basic definitions regarding relevant mathematical background of grey numbers in grey theory:

A grey number takes an unknown distribution between fixed lower and upper bounds, denoted as $\otimes \in [\underline{a}, \overline{a}]$, where $\underline{a}$ and $\overline{a}$ are respectively, the lower and upper bounds for $\otimes$.

Let $\otimes_1 \in [\underline{a}, \overline{a}]$, $\otimes_2 \in [\underline{b}, \overline{b}]$ and α is a positive real number, then

1. $\otimes_1 \in [\underline{a}, \overline{a}] + \otimes_2 \in [\underline{b}, \overline{b}] \Leftrightarrow \otimes_1 + \otimes_2 \in [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$
2. The scalar multiplication of α and $\otimes$ is defined as follows:
$$\alpha \otimes \in [\alpha \underline{a}, \alpha \overline{a}]$$
We denote by $\mathcal{G}(\mathbb{R})$ the set of interval grey numbers in R. Let $\otimes_1, \otimes_2 \in \mathcal{G}(\mathbb{R})$ with $\otimes_1 \in [\underline{a}, \overline{a}]$, $\otimes_2 \in [\underline{b}, \overline{b}]$, $|\otimes_1| = \underline{a} - \overline{a}$ and $\alpha \in \mathbb{R}_+$. Then by parts 1 and 2 we see that $\mathcal{G}(\mathbb{R})$ has a cone structure.
3. In general, the difference of $\otimes_1$ and $\otimes_2$ is defined as follows:
$$\otimes_1 \ominus \otimes_2 = \otimes_1 + (-\otimes_2) \in [\underline{a} - \underline{b}, \overline{a} - \overline{b}]$$
Different from the above subtraction we use a partial subtraction operator.

We define $\otimes_1 \ominus \otimes_2$, only if $|\underline{a} - \overline{a}| \geq |\underline{b} - \overline{b}|$, by $\otimes_1 - \otimes_2 = [\underline{a} - \underline{b}, \overline{a} - \overline{b}]$. (Alparslan Gok et al., 2009).

2.2. Cooperative Grey Games

In this section, we introduce the notion of cooperative grey games. A cooperative grey game is an ordered pair $\langle N, w' \rangle$ with the player set $N = \{1, \dots, n\}$ in which $w' = \otimes : 2^N \to \mathcal{G}(\mathbb{R})$ is the grey payoff characteristic function such that $w'(\emptyset) = \otimes_\emptyset \in [0,0]$, grey payoff function $w'(S) = \otimes_S \in [\underline{A_S}, \overline{A_S}]$ refers to the valuing area of the grey expectation benefit which is belonged to a coalition $S \in 2^N$, where $\underline{A_S}$ and $\overline{A_S}$ represent the maximum and minimum possible profits of the coalition S. So, a cooperative grey game can be considered as a classical cooperative game with grey profits $\otimes$. Grey solutions are useful to solve reward/cost sharing problems with grey data using cooperative grey games as a tool. Building blocks for grey solutions are grey payoff vectors, i.e. vectors whose components belong to $\mathcal{G}(\mathbb{R})$. We denote by $\mathcal{G}(\mathbb{R})^N$ the set of all such grey payoff vectors. We denote by $\mathcal{GG}^N$ the family of all cooperative grey games.

The following example illustrates a grey game.

Example 2.1. (Grey glove game) Let $N = \{1, \ldots, n\}$ be the set of players consisting of two disjoint subsets L and R. The members of L possess each one left-hand glove, the members of R one right-hand glove. A single glove is worth nothing, a right-left pair of gloves is worth between 10 and 20 Euros. In case $L = \{1, 2\}$ and $R = \{3\}$, this situation can be modelled as a three-person grey game, where the coalitions formed by players 1 and 3, players 2 and 3, and the grand coalition obtain an element of the worth $[10, 20]$. The worth gained in other cases is $[0, 0]$, i.e. $\otimes_{13} = w'(1, 3) = \otimes_{23} = w'(2, 3) = \otimes_N = w'(N) \in [10, 20]$ and $\otimes_S = w'(S) \in [0, 0]$, otherwise (Palanci et al., 2015b, Alparslan Gök et al. 2014).

Example 2.2. Consider a production situation with 3 departments involved in the working process of a raw material. Each department assures one stage of processing and there is a hierarchy between them: the material is processed at stage i only after its processes in stages $1, \ldots, i-1$. At any stage $i, i = 1, 2, 3$, there is a fixed cost necessary to process the material. However, the cost at stage 2 may increase with an additional amount, for example due to a machinery accident and related maintenance. Suppose that the cost at stages 1 and 3 are 7 and 12, respectively, whereas the cost of stage 2 is in between 5 and 10. The uncertainty due to department 2 affects the departments that are not its superiors. This situation is modelled as the cooperative grey $\langle N, w' \rangle$ with $N = \{1, \ldots, n\}$ and $\otimes_1 = w'(1) = \otimes_{13} = w'(1, 3) \in [7 + 5, 7 + 10] = [12, 17]$, $\otimes_{123} = w'(1, 2, 3) \in [7 + 5 + 12, 7 + 10 + 12] = [24, 29]$ and $\otimes_S = w'(S) \in [0, 0]$ in any other case (Palanci et al., 2015b).

2.3. Grey solutions

Let us recall the definition of the grey solution that is critical during this study (Yilmaz et al., 2018, Palanci et al., 2015a, Palanci et al., 2017b).

2.3.1. Grey Shapley value

Now, we introduce some theoretical notions from the theory of cooperative grey games. For $w, w_1, w_2 \in IG^N$ and $w', w_1', w_2' \in \mathcal{G}G^N$ we say that $w_1' \in w_1 \leq w_2' \in w_2$ if $w_1'(S) \leq w_2(S)$, where $w_1'(S) \in w_1(S)$ and $w_2'(S) \in w_2(S)$, for each $S \in 2^N$. For $w_1', w_2' \in \mathcal{G}G^N$ and $\alpha \in \mathbb{R}_+$ we define $\langle N, w_1' + w_2' \rangle$ and $\langle N, \alpha w' \rangle$ by $(w_1' + w_2')(S) = w_1'(S) + w_2'(S)$ and $(\alpha w')(S) = \alpha w'(S)$ for each $S \in 2^N$. So, we conclude that $\mathcal{G}G^N$ endowed with "$\leq$" has a cone structure with respect to addition and multiplication with non-negative scalars above. For $w_1', w_2' \in \mathcal{G}G^N$ where $w_1' \in w_1, w_2' \in w_2$ with $|w_1(S)| \geq |w_2(S)|$ for each $S \in 2^N$, $\langle N, w_1' - w_2' \rangle$ is defined by $(w_1' - w_2')(S) = w_1'(S) - w_2'(S) \in w_1(S) - w_2(S)$. We call a game $\langle N, w' \rangle$ grey size monotonic if $\langle N, |w| \rangle$ is monotonic, i.e. $|w|(S) \leq |w|(T)$ for all $S, T \in 2^N$ with $S \subset T$. For further use we denote by $SM\mathcal{G}G^N$ the class of grey size monotonic games with player set N. The grey marginal operators and the grey Shapley value are defined on $SM\mathcal{G}G^N$. Denote by $\prod(N)$ the set of permutations $\sigma : N \to N$ of N. The grey marginal operator $m^\sigma : SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ corresponding to σ, associates with each $w' \in SM\mathcal{G}G^N$ the grey marginal vector $m^\sigma(w')$ of w' with respect to σ defined by

$$m_i^\sigma(w') := w'(P^\sigma(i) \cup \{i\}) - w'\left(P^\sigma(i)\right) \in [\underline{A_{P^\sigma(i) \cup \{i\}}} - \underline{A_{P^\sigma(i)}}, \overline{A_{P^\sigma(i) \cup \{i\}}} - \overline{A_{P^\sigma(i)}}], \text{ for each } i \in N,$$

where $P^\sigma(i) = \{r \in N \mid \sigma^{-1}(r) < \sigma^{-1}(i)\}$, and $\sigma^{-1}(i)$ denotes the entrance number of player i. For grey size monotonic games $\langle N, w' \rangle$, $w'(T) - w'(S) \in w(T) - w(S)$ is defined for all $S, T \in 2^N$ with $S \subset T$ since $|w(T)| = |w|(T) \geq |w|(S) = |w(S)|$. We notice that for each $w' \in SM\mathcal{G}G^N$ the grey marginal vectors $m^\sigma(w')$ are defined for each $\sigma \in \prod(N)$, because the monotonicity of $|w|$ implies $\overline{A_{S \cup \{i\}}} - \underline{A_{S \cup \{i\}}} \geq \overline{A_S} - \underline{A_S}$, which can be rewritten as $\overline{A_{S \cup \{i\}}} - \overline{A_S} \geq \underline{A_{S \cup \{i\}}} - \underline{A_S}$. So, $w'(S \cup \{i\}) - w'(S) \in w(S \cup \{i\}) - w(S)$ is defined for each $S \subset N$ and $i \notin S$. Next, we notice that all the grey marginal vectors of a grey size monotonic game are efficient grey payoff vectors. The grey Shapley value $\Phi' : SM\mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ is defined by

$$\Phi'(w') := \frac{1}{n!} \sum_{\sigma \in \prod(N)} m^\sigma(w') \in \left[\frac{1}{n!} \sum_{\sigma \in \prod(N)} m^\sigma(\underline{A}), \frac{1}{n!} \sum_{\sigma \in \prod(N)} m^\sigma(\overline{A})\right], \text{ for each } w' \in SM\mathcal{G}G^N. \quad (1)$$

We can write the last equation as follows:

$$\Phi_i'(w') := \frac{1}{n!} \sum_{\sigma \in \prod(N)} \left[w'(P^\sigma(i) \cup \{i\}) - w'\left(P^\sigma(i)\right)\right] \in [\frac{1}{n!} \sum_{\sigma \in \prod(N)} \underline{A_{P^\sigma(i) \cup \{i\}}} - \underline{A_{P^\sigma(i)}}, \frac{1}{n!} \sum_{\sigma \in \prod(N)} \overline{A_{P^\sigma(i) \cup \{i\}}} - \overline{A_{P^\sigma(i)}}] \quad (2)$$

The following example illustrates the calculation of the grey Shapley value.

Example 2.3. (Palanci et al., 2015b) Let $\langle N, w' \rangle$, be a cooperative grey game with $N = \{1, \ldots, n\}$ and $\otimes_1 = w'(1) = \otimes_{13} = w'(1, 3) \in [7, 7]$, $\otimes_{12} = w'(12) \in [12, 17]$, $\otimes_N = w'(123) \in [24, 29]$ and $\otimes_S = w'(S) \in [0, 0]$ otherwise. Then the grey marginal vectors are given in the following table, where $\sigma : N \to N$ is identified with $(\sigma(1), \sigma(2), \sigma(3))$. Firstly, for $\sigma_1 = (1,2,3)$, we calculate the grey marginal vectors. Then,

$m_1^{\sigma_1}(w') = w'(1) \in [7, 7]$,

$m_2^{\sigma_1}(w') = w'(12) - w'(1) \in [12, 17] - [7, 7] = [5, 10]$,

$m_3^{\sigma_1}(w') = w'(123) - w'(12) \in [24, 29] - [12, 17] = [12, 12]$.
The others can be calculated similarly, which is shown in Table 1.

Table 1. Grey marginal vectors of the cooperative grey game

σ	$m_1^{\sigma}(w')$	$m_2^{\sigma}(w')$	$m_3^{\sigma}(w')$
$\sigma_1 = (1,2,3)$	$m_1^{\sigma_1}(w') \in [7,7]$	$m_2^{\sigma_1}(w') \in [5,10]$	$m_3^{\sigma_1}(w') \in [12,12]$
$\sigma_2 = (1,3,2)$	$m_1^{\sigma_2}(w') \in [7,7]$	$m_2^{\sigma_2}(w') \in [17,22]$	$m_3^{\sigma_2}(w') \in [0,0]$
$\sigma_3 = (2,1,3)$	$m_1^{\sigma_3}(w') \in [12,17]$	$m_2^{\sigma_3}(w') \in [0,0]$	$m_3^{\sigma_3}(w') \in [12,12]$
$\sigma_4 = (2,3,1)$	$m_1^{\sigma_4}(w') \in [24,29]$	$m_2^{\sigma_4}(w') \in [0,0]$	$m_3^{\sigma_4}(w') \in [0,0]$
$\sigma_5 = (3,1,2)$	$m_1^{\sigma_5}(w') \in [7,7]$	$m_2^{\sigma_5}(w') \in [17,22]$	$m_3^{\sigma_5}(w') \in [0,0]$
$\sigma_6 = (3,2,1)$	$m_1^{\sigma_6}(w') \in [24,29]$	$m_2^{\sigma_6}(w') \in [0,0]$	$m_3^{\sigma_6}(w') \in [0,0]$

The average of the six grey marginal vectors is the grey Shapley value of this game which can be shown as:
$\Phi_i'(w') \in \left(\left[\frac{27}{2}, 16\right], \left[\frac{13}{2}, 9\right], [4,4]\right)$

2.3.2. The grey Banzhaf value

The grey Banzhaf value $\beta: SMGG^N \to \mathcal{G}(\mathbb{R})^N$, $\forall\, w' \in SMGG^N$ is defined as

$$\beta(w') = \frac{1}{2^{|N|-1}} \sum_{i \in S} [w'(S) - w'(S\backslash\{i\})] \tag{3}$$

2.3.3. The $\mathcal{G}CIS$ -value

The CIS-value (Driessen and Funaki, 1991) assigns to every player its individual worth, and distributes the remainder of the worth of the grand coalition N equally among all players (van den Brink and Funaki, 2009).
The grey CIS-value assings every player to its individual grey worth, and distributes the remainder of the grey worth of the grand coalition N equally among all players (Yilmaz et al, 2018). The $\mathcal{G}CIS$-value $\mathcal{G}CIS: SMGG^N \to \mathcal{G}(\mathbb{R})^N$ is defined by

$$\mathcal{G}CIS_i(w') = w'(\{i\}) + \frac{1}{|N|}\left[w'(N) - \sum_{j \in N} w'(\{j\})\right] \tag{4}$$

2.3.4. The $\mathcal{G}ENSC$-value

The grey $ENSC$-value ($\mathcal{G}ENSC$ -value) assigns to every game w' the $\mathcal{G}CIS$-value of its dual game, i.e.

$$\mathcal{G}ENSC_i(w') = \mathcal{G}CIS_i(w'^*) = \frac{1}{|N|}\left[w'(N) - \sum_{j \in N} w'(N\{j\})\right] - w'(N\backslash\{i\}) \tag{5}$$

The $\mathcal{G}ENSC$-value assigns to every player in a game its grey marginal contribution to the "grand coalition" and distributes the remainder equally among the players (Yilmaz et al, 2018; van den Brink and Funaki, 2009).

2.3.5. The $\mathcal{G}ED$-solution

The grey ED-solution ($\mathcal{G}ED$ -solution) $\mathcal{G}ED: \mathcal{G}G^N \to \mathcal{G}(\mathbb{R})^N$ is given by

$$\mathcal{G}ED_i(w') = \frac{w'(N)}{|N|}, \text{ for all } i \in N \tag{6}$$

For further details see Yilmaz et al, 2018 and van den Brink and Funaki, 2009.

3. Facility Location Situations

In this section, we inspired from cite the facility paper (Usta et al., 2018). In a facility location game, a set $\mathcal{A}$ of agents (also known as cities, clients, or demand points), a set $\mathcal{F}$ of facilities, a facility opening cost f_i for every facility $i \in \mathcal{F}$, and a distance d_{ij} between every pair (i,j) of points in $\mathcal{A} \cup \mathcal{F}$ indicating the cost of connecting j to i are given. We assume that the distances come from a metric space; i.e., they are symmetric and obey the triangle inequality. For a set $S \subseteq \mathcal{A}$ of agents, the cost of this set is defined as the minimum cost of opening a set of facilities and connecting every agent in S to an open facility. More precisely, the cost function c is defined by Mallozzi (2011) as follows:

$$c(S) = \min_{\mathcal{F}^* \subseteq \mathcal{F}} \left\{\sum_{i \in \mathcal{F}^*} f_i + \sum_{j \in S} \min_{i \in \mathcal{F}^*} d_{ij}\right\} \tag{7}$$

Facility location games are studied by Nisan et al. (2007) in the literature. Further, facility location interval games are introduced by Palanci et al. (2017a). In this study, we introduce the facility location grey games. In a facility location grey game, a set $\mathcal{A}$ of agents (also known as cities, clients, or demand points), a set $\mathcal{F}$ of facilities, a grey facility opening cost $f_i{}'$ for every facility $i \in \mathcal{F}$ and a distance d_{ij} between every pair (i,j) of points in $\mathcal{A} \cup \mathcal{F}$ indicating the rey cost of connecting j to i are given. Here, $f_i{}' \in \left[\underline{f_i{}'}, \overline{f_i{}'}\right]$, $d_{ij}' \in \left[\underline{d_{ij}'}, \overline{d_{ij}'}\right] \in \mathcal{G}(\mathbb{R})$. The distances are supposed to come from a metric space. So, these distances are symmetric and satisfy the triangle inequality. For a set $S \subseteq \mathcal{A}$ of agents, the grey cost of this set is defined as the minimum grey cost of opening a set of facilities and connecting every agent in S to an open facility. More precisely, the grey cost function w' is defined by

$$w'(S) \in \left[\min_{\mathcal{F}^*\subseteq\mathcal{F}}\left\{\sum_{i\in\mathcal{F}^*}\underline{f_i}+\sum_{j\in S}\min_{i\in\mathcal{F}^*}\underline{d_{ij}}\right\},\min_{\mathcal{F}^*\subseteq\mathcal{F}}\left\{\sum_{i\in\mathcal{F}^*}\overline{f_i}+\sum_{j\in S}\min_{i\in\mathcal{F}^*}\overline{d_{ij}}\right\}\right] \in \mathcal{G}(\mathbb{R}) \tag{8}$$

A facility location game has two aims: the primary one is to outline a applicable position for the ability consistent with some given facility location rule; the second one deals instead with the problem of how to deal out the total cost among the members of the coalition, where the dispersion of total cost is worked with cooperative game theoretic solution ideas, corresponding to grey solutions. Now, we present an application from a facility location situation and related game with grey data, where we inspired by Palanci et al. (2017a).

Example 3.1. Figure 2 shows a facility location game with 3 cities Player 1 (City 1), Player 2 (City 2), Player 3 (City 3) in Turkey and 2 hospitals$\{f_1, f_2\}$.
The costs of each coalitions are calculated by using (8) as follows:

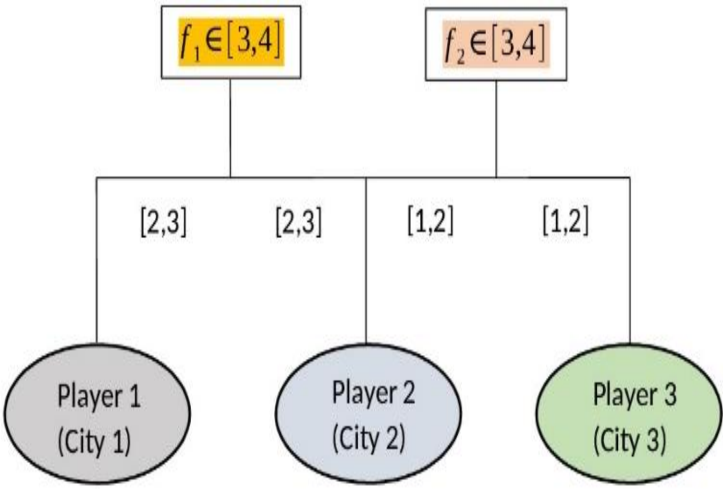

Figure 2. An application of a facility location game with grey data.

$w'(\{1\}) \in [5,7]$; $w'(\{2\}) \in [4,6]$; $w'(\{3\}) \in [4,6]$;
$w'(\{1,2\}) \in [7,10]$; $w'(\{2,3\}) \in [5,8]$; $w'(\{1,3\}) \in [9,13]$;
$w'(\{1,2,3\}) \in [10,15]$.

The grey marginal vectors are given in the following table:

Table 2. Grey marginal vectors

σ	$m_1^\sigma(w')$	$m_2^\sigma(w')$	$m_3^\sigma(w')$
$\sigma_1 = (1,2,3)$	$m_1^{\sigma_1}(w') \in [5,7]$	$m_2^{\sigma_1}(w') \in [2,3]$	$m_3^{\sigma_1}(w') \in [3,5]$
$\sigma_2 = (1,3,2)$	$m_1^{\sigma_2}(w') \in [5,7]$	$m_2^{\sigma_2}(w') \in [1,2]$	$m_3^{\sigma_2}(w') \in [4,6]$
$\sigma_3 = (2,1,3)$	$m_1^{\sigma_3}(w') \in [3,4]$	$m_2^{\sigma_3}(w') \in [4,6]$	$m_3^{\sigma_3}(w') \in [3,5]$
$\sigma_4 = (2,3,1)$	$m_1^{\sigma_4}(w') \in [5,7]$	$m_2^{\sigma_4}(w') \in [4,6]$	$m_3^{\sigma_4}(w') \in [1,2]$
$\sigma_5 = (3,1,2)$	$m_1^{\sigma_5}(w') \in [5,7]$	$m_2^{\sigma_5}(w') \in [1,2]$	$m_3^{\sigma_5}(w') \in [4,6]$
$\sigma_6 = (3,2,1)$	$m_1^{\sigma_6}(w') \in [5,7]$	$m_2^{\sigma_6}(w') \in [1,2]$	$m_3^{\sigma_6}(w') \in [4,6]$

The average of the six grey marginal vectors is the grey Shapley value of this game which can be shown as:
$\Phi'(w') \in \left(\left[\frac{14}{3},\frac{13}{2}\right],\left[\frac{13}{6},\frac{7}{2}\right],\left[\frac{19}{6},5\right]\right)$.
Now, Let us look at how the grey Banzhaf value for this game. For player 1, we have:
$\beta_1(w') \in \frac{1}{2^2}\sum_{1\in S}[w'(S) - w'(S\setminus\{1\})]$

$$\in \frac{1}{4}[w'(\{1\}) + w'(\{1,2\}) + w'(\{1,2,3\}) + w'(\{1,3\}) - w'(\{2\}) - w'(\{3\}) - w'(\{2,3\})] \in \left[\frac{9}{2},\frac{25}{4}\right].$$

The grey Banzhaf values of other players can be examined similarly as follows:

$\beta_2(w') \in \left[2, \frac{13}{4}\right], \beta_3(w') \in \left[3, \frac{19}{4}\right].$
At that rate, the grey Banzhaf value is
$\beta(w') \in \left(\left[\frac{9}{2}, \frac{25}{4}\right], \left[2, \frac{13}{4}\right], \left[3, \frac{19}{4}\right]\right)$

Now, we want to calculate $\mathcal{G}CIS$-value, $\mathcal{G}ENSC$-value and $\mathcal{G}ED$-solution. We calculate the $\mathcal{G}CIS$-value of our game as follows:
$\mathcal{G}CIS_1(w') \in w'(\{1\}) + \frac{1}{3}\left[w'(\{1,2,3\}) - \sum_{j \in N} w'(\{j\})\right]$
$\mathcal{G}CIS_1(w') \in w'(\{1\}) + \frac{1}{3}[w'(\{1,2,3\}) - (w'(\{1\}) + w'(\{2\}) + w'(\{3\}))] = \left[4, \frac{17}{3}\right].$
$\mathcal{G}CIS_2(w') \in w'(\{2\}) + \frac{1}{3}[w'(\{1,2,3\}) - (w'(\{1\}) + w'(\{2\}) + w'(\{3\}))] = \left[3, \frac{14}{3}\right].$
$\mathcal{G}CIS_3(w') \in w'(\{3\}) + \frac{1}{3}[w'(\{1,2,3\}) - (w'(\{1\}) + w'(\{2\}) + w'(\{3\}))] = \left[3, \frac{14}{3}\right].$

Then, the $\mathcal{G}CIS$-value is obtained by
$\mathcal{G}CIS(w') \in \left(\left[4, \frac{17}{3}\right], \left[3, \frac{14}{3}\right], \left[3, \frac{14}{3}\right]\right).$

We calculate the $\mathcal{G}ENSC$-value of our game as follows:
$\mathcal{G}ENSC_1(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \sum_{j \in N} w'(N \backslash \{j\})\right] - w'(N \backslash \{1\})$
$\mathcal{G}ENSC_1(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \left(w'(\{1,2\}) + w'(\{1,3\}) + w'(\{2,3\})\right)\right] - w'(\{2,3\}) = \left[\frac{16}{3}, \frac{22}{3}\right].$
$\mathcal{G}ENSC_2(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \left(w'(\{1,2\}) + w'(\{1,3\}) + w'(\{2,3\})\right)\right] - w'(\{1,3\}) = \left[\frac{4}{3}, \frac{7}{3}\right].$
$\mathcal{G}ENSC_3(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \left(w'(\{1,2\}) + w'(\{1,3\}) + w'(\{2,3\})\right)\right] - w'(\{1,2\}) = \left[\frac{10}{3}, \frac{16}{3}\right].$

Then, the $\mathcal{G}ENSC$-value is obtained by
$\mathcal{G}ENSC(w') \in \left(\left[\frac{16}{3}, \frac{22}{3}\right], \left[\frac{4}{3}, \frac{7}{3}\right], \left[\frac{10}{3}, \frac{16}{3}\right]\right).$

Finally, we calculate the $\mathcal{G}ED$-solution of our game as follows:
$\mathcal{G}ED_1(w') = \mathcal{G}ED_2(w') = \mathcal{G}ED_3(w') \in \frac{w'(\{1,2,3\})}{3} = \left[\frac{10}{3}, \frac{15}{3}\right].$

So, we have
$\mathcal{G}ED(w') \in \left(\left[\frac{10}{3}, \frac{15}{3}\right], \left[\frac{10}{3}, \frac{15}{3}\right], \left[\frac{10}{3}, \frac{15}{3}\right]\right).$

Table 3 illustrates the results of this application.

Table 3. The grey solutions of our model

Grey Solutions	Player 1	Player 2	Player 3
Grey Shapley value	$\in \left[\frac{14}{3}, \frac{13}{2}\right]$	$\in \left[\frac{13}{6}, \frac{7}{2}\right]$	$\in \left[\frac{19}{6}, 5\right]$
Grey Banzhaf value	$\in \left[\frac{9}{2}, \frac{25}{4}\right]$	$\in \left[2, \frac{13}{4}\right]$	$\in \left[3, \frac{19}{4}\right]$
$\mathcal{G}CIS$-value	$\in \left[4, \frac{17}{3}\right]$	$\in \left[3, \frac{13}{3}\right]$	$\in \left[3, \frac{14}{3}\right]$
$\mathcal{G}ENSC$-value	$\in \left[\frac{16}{3}, \frac{22}{3}\right]$	$\in \left[\frac{4}{3}, \frac{7}{3}\right]$	$\in \left[\frac{10}{3}, \frac{16}{3}\right]$
$\mathcal{G}ED$-value	$\in \left[\frac{10}{3}, \frac{15}{3}\right]$	$\in \left[\frac{10}{3}, \frac{15}{3}\right]$	$\in \left[\frac{10}{3}, \frac{15}{3}\right]$

4. Case Study: Tent City Development After The earthquake In Izmir

The application in this section is inspired by Usta et al. (2018). After the severe Earthquake, a large emergency sheltering and temporary sheltering demand occurred, since the results are very huge. If the disaster is huge, the requirement for housing may be the same that thousands of dwellings were required urgently. In this period the Turkish government began to evaluate the rehabilitation of the districts and to make post-disaster housing (permanent housing). However, it had been clear that everyone could not be met in a single region as requested, therefore the government began a study for finding suitable districts for building post-disaster housing settlements. It took some time to resolve of these problems (Ozden, 2005).

Our case study relies on attainable facility location after an earthquake in Izmir, Turkey. Take into account that there is an earthquake in Izmir and after the earthquake, nearly 14000 tents are distributed. Three tent cities are established in Aydin, Usak and Balikesir which are near Izmir. There are nearly 8000 tents in the hands of the Kizilay that is the beneficiary of Turkey. The distribution of the approximately remaining 6000 tents is undertaken by one local and one foreign company. The cost of bringing services to the people living in the tent cities belongs to these companies. Almost 50 percent of the 6000 tents are built in Aydin, almost 35 percent of the 6000 tents are built in Usak, and the approximately rest of the tents are built in Balikesir. Three kinds of tent types are distributed (Table 4). In Aydin, one tent is between 500 and 700 Turkish Liras (TL) and is for 8 or 10 persons. In Usak, one tent is between 850 and 1050 TL and is for 15 or 17 persons. In Balikesir, one tent is between 650 and 850 TL and is for 10 or 12 persons.

Table 4. The costs of building tent cities and some properties.

Tent city no	Tent city name	Property of tent	Number of tents established by companies	Total
1	Aydin	1 tent=[500,700] TL and for [8,10] persons	[3000,3200] (by local company)	[1500000,2240000] TL for [24000,32000] persons
2	Usak	1 tent=[850,1050] TL and for [15,17] persons	[500,700] (by local company) [1600,1800] (by foreign company)	[6375000,22495000] TL for [7500,11900] persons (by local company) [20400000,32130000] TL for [24000,30600] persons (by foreign company)
3	Balikesir	1 tent=[650,850] TL and for [10,12] persons	[900,1100] (by foreign company)	[585000,935000] TL for [9000,13200] persons (by foreign company)

Additionally, the delivery services for facility location problems should be given, as well. In our case study, the service cost per person is between 50 and 70 TL. In Table 5, the costs of delivery services of companies are given.

Table 5. The costs of bringing services of companies

Tent city no	Tent city name	The costs of bringing services of local company	The costs of bringing services of foreign company
1	Aydin	[1200000,2240000] TL for [3000,3200] tents	-
2	Usak	[375000,833000] TL for [500,700] tents	[1200000,2142000] TL for [1600,1800] tents
3	Balikesir	-	[450000,924000] TL for [900,1100] tents

Figure 3 shows a facility location game with 3 cities (Aydin (Player 1), Usak (Player 2) and Balikesir (Player 3)) in Turkey and 2 companies.

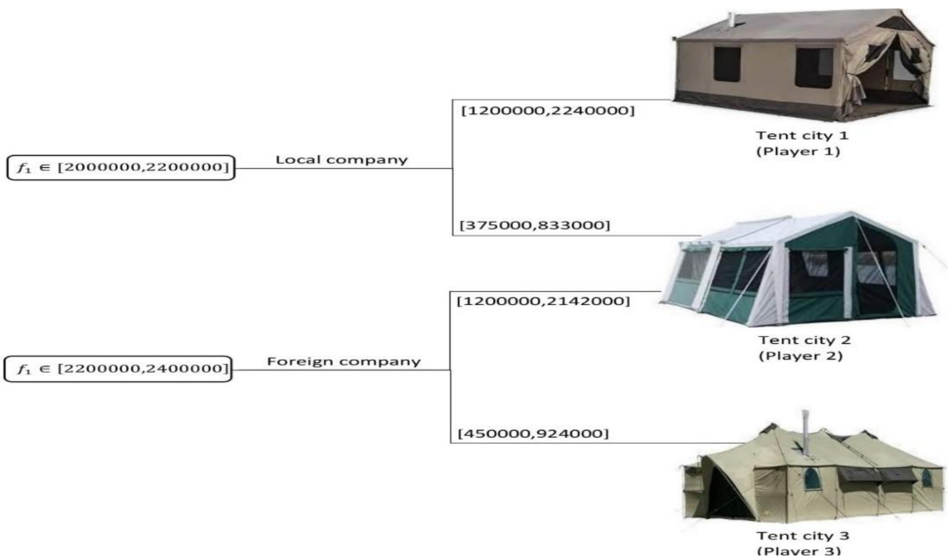

Figure 3. The illustration of our case study.

The costs for each coalition are calculated by using (8) as follows:
$w'(\{1\}) \in [3200000, 4440000]$,
$w'(\{2\}) \in [2375000, 3033000]$,
$w'(\{3\}) \in [2650000, 3324000]$,
$w'(\{1, 2\}) \in [3575000, 5273000]$,
$w'(\{1, 3\}) \in [5850000, 7764000]$,
$w'(\{2, 3\}) \in [3850000, 5466000]$,
$w'(\{1, 2, 3\}) \in [6225000, 8597000]$.

Table 6 shows the marginal vectors of our model, where $\sigma : N \to N$ consists of three components with the order $(\sigma\ (1), \sigma\ (2), \sigma\ (3))$.

Table 6. The marginal vectors of our model

σ	$m_1^{\sigma}(w')$	$m_2^{\sigma}(w')$	$m_3^{\sigma}(w')$
$\sigma_1 = (1,2,3)$	$m_1^{\sigma_1}(w') \in [3200000,4440000]$	$m_2^{\sigma_1}(w') \in [375000,833000]$	$m_3^{\sigma_1}(w') \in [2650000,3324000]$
$\sigma_2 = (1,3,2)$	$m_1^{\sigma_2}(w') \in [3200000.4440000]$	$m_2^{\sigma_2}(w') \in [375000,833000]$	$m_3^{\sigma_2}(w') \in [2650000,7320000]$
$\sigma_3 = (2,1,3)$	$m_1^{\sigma_3}(w') \in [1200000,2240000]$	$m_2^{\sigma_3}(w') \in [2375000,3033000]$	$m_3^{\sigma_3}(w') \in [2650000,3324000]$
$\sigma_4 = (2,3,1)$	$m_1^{\sigma_4}(w') \in [2375000,3131000]$	$m_2^{\sigma_4}(w') \in [2375000,3033000]$	$m_3^{\sigma_4}(w') \in [1475000,2433000]$
$\sigma_5 = (3,1,2)$	$m_1^{\sigma_5}(w') \in [3200000,4440000]$	$m_2^{\sigma_5}(w') \in [375000,833000]$	$m_3^{\sigma_5}(w') \in [2650000,3324000]$
$\sigma_6 = (3,2,1)$	$m_1^{\sigma_6}(w') \in [2375000,2298000]$	$m_2^{\sigma_6}(w') \in [1200,2142000]$	$m_3^{\sigma_6}(w') \in [2650000,3324000]$

The average of the six marginal vectors is the grey Shapley value of this game which can be calculated as:
$\Phi'(w') \in ([2591666.67, 3498166.67], [1179166.67, 1784500], [2454166.67, 3841500])$

Now, Let us look at how the grey Banzhaf value for this game. For player 1, we have:

$$\beta_1(w') \in \frac{1}{2^2}\sum_{1\in S}[w'(S) - w'(S\backslash\{1\})]$$

$$\in \frac{1}{4}[w'(\{1\}) + w'(\{1, 2\}) + w'(\{1, 2, 3\}) + w'(\{1, 3\}) - w'(\{2\}) - w'(\{3\}) - w'(\{2, 3\})] \in [2493750, 3562750].$$

The grey Banzhaf values of other players can be examined similarly as follows:
$\beta_2(w') \in [1081250, 1710250], \beta_3(w') \in [2356250, 3101250]$.

At that rate, the grey Banzhaf value is
$\beta(w') \in ([2493750, 3562750], [1081250, 1710250], [2356250, 3101250])$

Now, we want to calculate $\mathcal{G}CIS$-value, $\mathcal{G}ENSC$-value and $\mathcal{G}ED$-solution. We calculate the $\mathcal{G}CIS$-value of our game as follows:

$$\mathcal{G}CIS_1(w') \in\ w'(\{1\}) + \frac{1}{3}\left[w'(\{1, 2, 3\}) - \sum_{j\in N} w'(\{j\})\right]$$

$$\mathcal{G}CIS_1(w') \in\ w'(\{1\}) + \frac{1}{3}[w'(\{1, 2, 3\}) - (w'(\{1\}) + w'(\{2\}) + w'(\{3\}))] = [2533330, 3720000].$$

$$\mathcal{G}CIS_2(w') \in\ w'(\{2\}) + \frac{1}{3}[w'(\{1, 2, 3\}) - (w'(\{1\}) + w'(\{2\}) + w'(\{3\}))] = [1708330, 2313000].$$

$$\mathcal{G}CIS_3(w') \in\ w'(\{3\}) + \frac{1}{3}[w'(\{1, 2, 3\}) - (w'(\{1\}) + w'(\{2\}) + w'(\{3\}))] = [1983330, 2604000].$$

Then, the $\mathcal{G}CIS$-value is obtained by
$\mathcal{G}CIS(w') \in ([2533330, 3720000], [1708330, 2313000], [1983330, 2604000])$.

We calculate the $\mathcal{G}ENSC$-value of our game as follows:

$$\mathcal{G}ENSC_1(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \sum_{j\in N} w'(N\backslash\{j\})\right] - w'(N\backslash\{1\})$$
$$\mathcal{G}ENSC_1(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \left(w'(\{1,2\}) + w'(\{1,3\}) + w'(\{2,3\})\right)\right] - w'(\{2,3\}) = [2650000, 3567333].$$
$$\mathcal{G}ENSC_2(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \left(w'(\{1,2\}) + w'(\{1,3\}) + w'(\{2,3\})\right)\right] - w'(\{1,3\}) = [650000, 1269333].$$
$$\mathcal{G}ENSC_3(w') \in \frac{1}{3}\left[w'(\{1,2,3\}) + \left(w'(\{1,2\}) + w'(\{1,3\}) + w'(\{2,3\})\right)\right] - w'(\{1,2\}) = [2925000, 3760333].$$

Then, the $\mathcal{G}ENSC$-value is obtained by
$\mathcal{G}ENSC(w') \in ([2650000, 3567333], [650000, 1269333], [2925000, 3760333])$.

Finally, we calculate the $\mathcal{G}ED$-solution of our game as follows:
$$\mathcal{G}ED_1(w') = \mathcal{G}ED_2(w') = \mathcal{G}ED_3(w') \in \frac{w'(\{1,2,3\})}{3} = [2075000, 2865666.66].$$

So, we have
$\mathcal{G}ED(w') \in ([2075000, 2865666.66], [2075000, 2865666.66], [2075000, 2865666.66])$.

Table 7 illustrates the results of this application.

Table 7. The grey solutions of our model.

Grey Solutions	Player 1	Player 2	Player 3
Grey Shapley value	$\in$ [2591666.67,3498166.67]	$\in$ [1179166.67,1784500]	$\in$ [2454166.67,3841500]
Grey Banzhaf value	$\in$ [2493750,3562750]	$\in$ [1081250,1710250]	$\in$ [2356250,3101250]
$\mathcal{G}CIS$-value	$\in$ [2533330,3720000]	$\in$ [1708330,2313000]	$\in$ [1983330,2604000]
$\mathcal{G}ENSC$-value	$\in$ [2650000,3567333]	$\in$ [650000,1269333]	$\in$ [2925000,3760333]
$\mathcal{G}ED$-value	$\in$ [2075000,2865666.66]	$\in$ [2075000,2865666.66]	$\in$ [2075000,2865666.66]

These values will be utilized in completely different application areas corresponding to Operations Research, economic and management situations.

5. Research Literature

In this section, some literature leading to grey system theory and the theory of cooperative games is provided in historical order. Shapley (1953) introduced the Shapley value which is one of the common solution concepts in cooperative game theory. Deng (1982) studied the stability and stabilization of a grey system whose state matrix is triangular. Alparslan Gök et al. (2009) introduced the class of convex interval games and extended classical results regarding the characterizations of convex games and the properties of solution concepts to the interval setting. Branzei et al. (2010) briefly presented the state-of-the-art of the cooperative interval games. They discussed how the model of cooperative interval games extends the cooperative game theory literature, and reviewed its existing and potential applications in economic and Operations Research situations with interval data. Palanci et al. (2015b) characterized the Shapley value by using the grey numbers. Palanci et al. (2017b) introduced some set-valued solution concepts using grey payoffs, namely, the grey core, the grey dominance core and the grey stable sets for cooperative grey games. Yılmaz et al. (2018) considered some grey division rules, called the grey equal surplus sharing solutions.

6. Conclusion and Outlook

The catastrophic events have expanded dramatically, especially in Turkey, creating outstanding harms to the various infrastructures within the country. Many buildings have suffered damage and became uninhabitable, triggering a high range of vagrants who inhabitants became homeless. Re-housing of homeless is one of the main jobs of reconstruction programs after disasters. Reconstruction works often last long, and through that time, it is essential to provide affected individuals with the minimum conditions to live with self-esteem, privacy, and protection. The rapid pace of change and a paucity of information after disasters make it inherently difficult to specify action items. Under these conditions of uncertainty, we use the cooperative grey games. The basic aim of the theory is to study ways to sustain cooperation between players under these conditions. The recommendations of this paper identify several key questions and challenges, including the need for efforts to improve the quality and quantity of data in facility location situations, the need to continue efforts to develop models of post-earthquake relief of addressing the temporary housing issues after a disaster, how the total costs can be allocated among players in a fair way. Through this study, we handle with a housing problem after the earthquake in Izmir. Structured on the situation study, we constructed the cooperative facility location game with three cities of the camping. We have shown some concepts of grey solutions like grey Shapley value, grey Banzhaf value, $\mathcal{G}CSI$-value, $\mathcal{G}ENSC$-value and $\mathcal{G}ED$-value. If we compare our results obtained from grey numbers with the results obtained from other methods, we see that our results give insight for interval solutions too. Because to obtain a grey solution, we have to calculate the interval solutions too. Since real world models include uncertainty, it is interesting to work with grey numbers instead of crisp solutions. Focused suggestions can be developed for future work of catastrophe management to solve problems such as minimization, preparedness, recovery, health issues and re-housing of homeless people it allows victims to have a private and secure place to return to their normal life, to until reconstruction of permanent housing.

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