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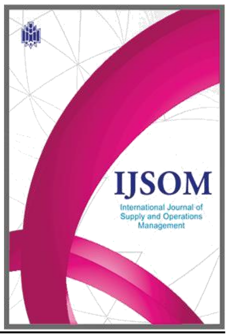

Two-Echelon Supply Chain Model for Deteriorating Items in an Imperfect Production System with Advertisement and Stock Dependent Demand under Trade Credit

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Abstract
This article presents a two-echelon supply chain model for deteriorating items, consisting of a single manufacturer and a single retailer, where the customer's demand to the retailer depends on advertisement and the displayed stock level of the retailer. Due to the imperfect production system, the manufacturer produces a certain quantity of defective items with the perfect products. The manufacturer inspects all the products immediately after production and sells the ideal quality items to the retailer. To entice the retailer to purchase more products, the manufacturer offers the retailer a trade-credit policy so that the retailer can get a chance to settle his account before the payment for the products. We have developed a cost function of this model. Numerical examples have been presented to clarify the applicability of this model and the sensitivity analysis with respect to different parameters involved with the model has been performed to study the effect of the parameter change on the decision variables.

Keywords: Supply chain; Deterioration; Imperfect production; Advertisement; Stock dependent demand, Trade-credit.

1. Introduction

A common phenomenon, observed in almost any manufacturing organization is the imperfect production system. As a natural consequence, as production is proceeded in the factory, we find out that a portion of items produced is of imperfect quality. So, researchers in this era focus their attention on developing supply chain models with the imperfect production system. Chiu, Gong, and Wee (2004) presented an inventory model with imperfect production, where they assumed that the imperfect items produced are of two main categories, i.e. repairable and non-repairable. The repairable defective items are reworked after the termination of the regular production. Panda, Kar, Maity, and Maiti (2008) derived an imperfect production inventory model under budget and shortage constraints. Sana (2010) developed a production lot-size model by assuming that the production system may shift to an out-of-control state at any random time and produce a certain portion of defective products. Furthermore, Manna, Dey, andMondal (2014) presented a three-echelon supply chain model consisting of a single supplier, a single manufacturer, and a single retailer. In this model, they considered the product reliability and the change of the defective items. Khalilpourazari and Pasandideh (2016) developed a multi-product economic production quantity model in an imperfect production system, where the non-conforming products produce at a random rate. In this model, they aimed to minimize the total inventory costs as well as the total warehouse space to keep the products. Khalilpourazari and Pasandideh (2018) also presented a multi-objective economic order quantity model, where the supply batches are inspected immediately after receiving the products and the total batch is rejected if it is found that the products are below standard. In the same year, Khalilpourazari, Pasandideh, and Ghodratnama (2018) studied another model which considers the defective products. They used two metaheuristic algorithms, namely Whale Optimization and Water Cycle Algorithms to solve the model.

The main objective of any business is to increase the number of customers and expand itself as the only way to gain profit is the selling of the products. To fulfil this aim, the supply chain players offer various policies to their customers

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so as to entice them, among which the most significant is the trade credit policy. Under this policy, the customers do not need to pay the vendor immediately after purchasing, but after a certain time period- known as the trade credit period. This makes an economic sense to the customers as they get an opportunity to settle their account in this time period and consequently, are encouraged to buy more products. Numerous researchers have studied supply chain model under the trade credit policy(e.g. Annadurai & Uthayakumar, 2013; Jaggi, Tiwari, & Goel,,2017; Karuppuchamy Annadurai ,2013; Kumar, Chauhan, & Kumar,2012; Liao, Huang, & Chung, ,2012; Mahata, 2012, Mahata & De, 2016; Min, Zhou, & Zhao, 2010;Shah, Shah, & Shah, 2013;Soni,2013; Sundara Rajan, & Uthayakumar, 2017). In practice, almost every supply chain inventory system faces the problem of product deterioration as most of the physical goods may lose their utility partially or totally over time. Researchers have shown interest in developing supply chain models considering the deterioration of the products. Pal, Bhunia, and Mukherjee (2006) presented a lot-size model considering the constant rate of product deterioration. Sarkar (2013) presented a two-echelon supply chain model, taking the rate of deterioration as probabilistic. Saha and Chakrabarti (2017) developed a supply chain inventory model assuming the constant rate of product deterioration. In this model, they assumed that the cycle time is uncertain and described this as a triangular fuzzy number. Saha and Chakrabarti (2018a) developed a two-echelon supply chain model for deteriorating products which compose of a single manufacturer and a single retailer. Saha and Chakrabarti (2018b) determined an economic production quantity model by taking a constant rate of deterioration and probabilistic demand into account. Tiwari, Jaggi, Gupta, and Cárdenas-Barrón (2018) developed a pricing model for the deteriorating products considering the capacity of the warehouse for displaying the products is limited. Maihami, Karimi, and Ghomi (2017) formulated a manufacturer-retailer supply chain model for deteriorating items taking into consideration the rate of deterioration and the demand for the product as probabilistic in nature.

Besides the deterioration of the products, another important factor related to the selling rate of any product is the advertisement and the stock level dependency of the customers' demand. In reality, it is observed that customers are attracted to any product by watching the advertisement in various media as well as by checking the displayed stock of the products in the showrooms. Li, Wang, and Yan (2013) considered the dependency of the demand rate on the advertisement in the two-echelon supply chain model (a single vendor and a single retailer). Palanivel and Uthayakumar (2014) studied a production inventory model assuming demand rate as a deterministic function of selling price and advertisement cost. Pal et al. (2006) developed a lot-size model for the deteriorating items, where they considered the dependency of the demand rate on both the advertisement of the product on different media and on the displayed stock level of the items.

2. Notations and Assumptions

The following notations have been used to develop the model:

$I_{1m}(t)$: the inventory level of the manufacturer at any time t, $0 \leq t \leq t_2$.
$I_{1r}(t)$: the inventory level of the retailer at any time t, $0 \leq t \leq T$.
θ: deterioration rate of the item.
z: fraction of defective items produced per unit time.
P_m: manufacturer's production rate per unit time.
D_r : the demand rate of the retailer to the manufacturer.
D_c : the demand rate of the customer to the retailer.
A_m: setup cost of the manufacturer.
A_r: the ordering cost of the retailer.
C_p: the production cost of the manufacturer per unit item per unit time.
h_m: holding cost per unit per unit time of the manufacturer.
C_m: the inspection cost of the manufacturer per unit item.
h_r: holding cost per unit item per unit time of the retailer.
d_m: deterioration cost per unit item per unit time of the manufacturer.
d_r: deterioration cost per unit item per unit time of the retailer.
A_c: advertisement cost for the items of the manufacturer.
S_m: selling price per unit of the manufacturer.
S_r: selling price per unit of the retailer.
t_1: the production period of the manufacturer.
t_2: the cycle length of the manufacturer as well as the procurement period of the retailer.
T: cycle length of the system.
M: length of credit period offered by the manufacturer to the retailer.
I_e : interest earned by the retailer per dollar per unit time.
I_c: interest payable by the retailer per dollar per unit time.
I_p: lost opportunity cost per dollar per unit time of the manufacturer for offering trade credit period to the retailer.

The model is based on the following assumptions:

The customer's demand to the retailer depends on the inventory level of the retailer and the advertisement of the product, i.e. $D_c = {A_c}^{\gamma} + bI_{1r}(t)$, where γ and b are real constants, $0 < \gamma, b < 1$.

The unit production cost $C_p = C_r + \frac{a_1}{{p_m}^{\delta_1}} + a_2{p_m}^{\delta_2} + a_3{p_m}^{\delta_3}$, where C_r is the material cost, $\frac{a_1}{{p_m}^{\delta_1}}$ is the labour cost, $a_2{p_m}^{\delta_2}$ is the environment protection cost, and $a_3{p_m}^{\delta_3}$ is the maintenance cost of the machinery system, where a_1, a_2, a_3 and δ_1, δ_2, δ_3 are real constants, $0 \leq \delta_1$, $\delta_2, \delta_3 \leq 1$.

The manufacturer gives the advertisement to different media to promote his band.

The retailer earns interest from the sale revenue during the credit period at a rate of I_e. At the end of the credit period, the retailer pays the purchasing cost to the manufacturer and incurs a capital opportunity cost at a rate of I_c for the items still left in stock.
The manufacturer incurs a lost opportunity cost at a rate I_p for offering the retailer the trade credit policy.

3. Problem definition

In this model, we have developed a two-echelon supply chain model for deteriorating items which comprises a single manufacturer and a single retailer. Here, it is assumed that the production system is imperfect and produces a certain proportion of defective products with the perfect products. The manufacturer inspects all the products immediately after production and sells the perfect quality items to the retailer. In the literature, we found that some authors considered the change of the defective products (Chiu et al., 2004; Pal & Mahapatra, 2017; Roy, Sana, & Chaudhuri, 2014; Sarkar, Cárdenas-Barrón, Sarkar, & Singgih, 2014). But, there are some products, the defects of which cannot be removed by reworking. For example, for pharmaceutical products, the manufacturer should take the responsibility for the quality of the products so that the patients may not fall at risk due to inadequate safety, quality, or efficacy (on Specifications for Pharmaceutical Preparations & others, 2014). For such products, it is not expected that the manufacturer does any repairing work of the non-conforming products to remove the defects (on Specifications for Pharmaceutical Preparations & others, 2014). For such cases, it is wise for the manufacturer to get salvage value by selling these items. For this purpose, we have considered the salvage value of the defective products instead of reworking these products.

In practice, it is usually noticed that the customers are attracted to any products following the displayed stock level of the products. If the retailer has a sufficient stock of the products, it is convenient for the customers to select the products of their choices. Again, customers are also encouraged to buy any product by watching the advertisement on the different media. So, in this model, we have considered the demand rate for the product as the stock and advertisement dependent.

Moreover, in this model, we have considered the trade credit policy as a coordination strategy. To encourage the retailer to purchase more, the manufacturer offers him a trade credit period so that he can get an opportunity to settle his account. Here we have discussed two cases depending on the length of the trade credit period and derived the cost function of the model for both cases. We have provided numerical examples to illustrate the applicability of the proposed model and presented the sensitivity analysis to know the effect of parameter change on the optimum decision variables.

The differences between this proposed model and the relevant existing models are shown in Table1. It is clear from Table1 that none of the authors has developed a two-echelon supply chain model with the combination of stock level and advertisement dependent demand, variable production cost, salvage value, and trade credit policy. Hence, we have developed a model to determine the optimal total cost considering all these issues.

4. Mathematical model

In this section, we have developed the mathematical models for the manufacturer as well as the retailer. A pictorial representation has been shown in Figure1.

4.1. Manufacturer's model

The manufacturer starts producing the items at time $t = 0$ and continues up to time $t = t_1$. The production rate of the manufacturer for the good quality items is $(1 - z)p_m$. The inventory level depleted due to both demand and deterioration that reaches to zero level at time $t = t_2$. The differential equations describing the inventory level of the manufacturer are

$$\frac{dI_{1m}(t)}{dt}+\theta I_{1m}(t)=\begin{cases}(1-z)p_m-D_r, & 0\le t\le t_1\\ -D_r, & t_1\le t\le t_2\end{cases} \quad \dots\dots\dots(1)$$

With $I_{1m}(0)=0$ and $I_{1m}(t_2)=0$.

Solving (1) we have,

$$I_{1m}(t)=\begin{cases}\dfrac{\{(1-z)p_m-D_r\}}{\theta}\left(1-e^{-\theta t}\right), & 0\le t\le t_1\\ \dfrac{D_r}{\theta}\left(e^{\theta(t_2-t)}-1\right), & t_1\le t\le t_2\end{cases} \quad \dots\dots\dots(2)$$

From the continuity condition of $I_{1m}(t)$ at $t=t_1$ we have,

$$\frac{\{(1-z)p_m-D_r\}}{\theta}\left(1-e^{-\theta t_1}\right)=\frac{D_r}{\theta}\left(e^{\theta(t_2-t_1)}-1\right) \quad \dots\dots\dots(3)$$

Now, holding cost of the manufacturer,

$$=h_m\left[\int_0^{t_1}I_{1m}(t)dt+\int_{t_1}^{t_2}I_{1m}(t)dt\right]$$

$$=h_m\left[\frac{\{(1-z)p_m-D_r\}}{\theta}\int_0^{t_1}\left(1-e^{-\theta t}\right)dt+\frac{D_r}{\theta}\int_{t_1}^{t_2}\left(e^{\theta(t_2-t)}-1\right)dt\right]$$

$$=\frac{h_m}{\theta}\left[\{(1-z)p_m-D_r\}t_1-\frac{\{(1-z)p_m-D_r\}}{\theta}\left(1-e^{-\theta t_1}\right)+\frac{D_r}{\theta}\left(e^{\theta(t_2-t_1)}-1\right)-D_r(t_2-t_1)\right]$$

$$=\frac{h_m}{\theta}\left[(1-z)p_mt_1-D_rt_2\right], \qquad \text{(using (3))}.$$

Deterioration cost,

$$=d_m\left[\int_0^{t_1}\theta I_{1m}(t)dt+\int_{t_1}^{t_2}\theta I_{1m}(t)dt\right]$$

$$=d_m\left[(1-z)p_mt_1-D_rt_2\right]$$

Production cost $=C_pp_mt_1$.

Inspection cost $=C_mp_mt_1$.

Lost opportunity cost per unit time for offering trade-credit period M to the retailer,
$=S_mD_rI_pM$

Salvage value of the defective products $=S'_mzp_mt_1$.

Therefore, total cost of the manufacturer per unit time –

$$TC_m=\frac{1}{T}\left\{A_m+A_c+C_pp_mt_1+\frac{(h_{m+d_m\theta})}{\theta}\{(1-z)p_mt_1-D_rt_2\}-\ S'_mzp_mt_1\right\}+S_mD_rI_pM \quad \dots\dots\dots(4)$$

4.2. Retailer’s Model

The retailer purchases the product from the manufacturer at a rate of D_r and sells these to the customers at a rate of $D_c={A_c}^{\gamma}+bI_{1r}(t)$. At time $t=t_2$, he stops purchasing the items and then the inventory level depleted due to both demand and deterioration, and reaches to the zero level at time $t=T$. The retailer’s inventory level is described by the following differential equations –

$$\frac{dI_{1r}(t)}{dt}+\theta I_{1r}(t)=\begin{cases}D_r-\left({A_c}^{\gamma}+bI_{1r}(t)\right), & 0\le t\le t_2\\ -\left({A_c}^{\gamma}+bI_{1r}(t)\right), & t_2\le t\le T\end{cases} \quad \dots\dots\dots(5)$$

With boundary conditions, $I_{1r}(0)=0$ and $I_{1r}(T)=0$.

Solving equation (5) we have,

$$I_{1r}(t)=\begin{cases}\dfrac{(D_r-{A_c}^{\gamma})}{(\theta+b)}\left(1-e^{-(\theta+b)t}\right), & 0\le t\le t_2\\ \dfrac{{A_c}^{\gamma}}{(\theta+b)}\left(e^{(\theta+b)(T-t)}-1\right), & t_2\le t\le T\end{cases} \quad \dots\dots\dots(6)$$

From the continuity condition of $I_{1r}(t)$ at $t = t_2$ we have,

$$\frac{(D_r - A_c{}^{\gamma})}{(\theta + b)}\left(1 - e^{-(\theta+b)\,t_2}\right) = \frac{A_c{}^{\gamma}}{(\theta + b)}\left(e^{(\theta+b)(T-\,t_2)} - 1\right) \dots\dots\dots (7)$$

Now, holding cost of the retailer,

$$= h_r\left[\int_0^{t_2} I_{1r}(t)dt + \int_{t_2}^{T} I_{1r}(t)dt\right]$$
$$= h_r\left[\frac{(D_r - A_c{}^{\gamma})}{(\theta + b)}\int_0^{t_2}\left(1 - e^{-(\theta+b)\,t}\right)dt + \frac{A_c{}^{\gamma}}{(\theta + b)}\int_{t_2}^{T}\left(e^{(\theta+b)(T-t)} - 1\right)dt\right]$$
$$= \frac{h_r}{(\theta + b)}\left[(D_r - A_c{}^{\gamma})\left\{t_2 + \frac{1}{(\theta + b)}\left(e^{-(\theta+b)\,t_2} - 1\right)\right\} + A_c{}^{\gamma}\left\{-\frac{1}{(\theta + b)}\left(1 - e^{(\theta+b)(T-\,t_2)}\right) - (T - t_2)\right\}\right]$$
$$= \frac{h_r}{(\theta + b)}\left[(D_r - A_c{}^{\gamma})t_2 - \frac{(D_r - A_c{}^{\gamma})}{(\theta + b)}\left(1 - e^{-(\theta+b)\,t_2}\right) + \frac{A_c{}^{\gamma}}{(\theta + b)}\left(e^{(\theta+b)(T-\,t_2)} - 1\right) - A_c{}^{\gamma}(T - t_2)\right]$$
$$= \frac{h_r}{(\theta+b)}(D_r t_2 - A_c{}^{\gamma}T), \quad \text{(Using equation (7))}$$

Deterioration cost of the retailer,

$$= d_r\left[\int_0^{t_2} \theta I_{1r}(t)dt + \int_{t_2}^{T} \theta I_{1r}(t)dt\right]$$
$$= \frac{\theta d_r}{(\theta + b)}(D_r t_2 - A_c{}^{\gamma}T)$$

Purchasing cost $= S_m D_r t_2$

Now, we consider the two major cases based on the length of the credit period.

Case-1: When $T < M$.

In this case, the retailer can sell all the items before the due date of the delayed period. Therefore, the retailer does not need to pay any opportunity cost to the manufacturer, but he can earn interest at a rate of I_e from the sales revenue. This situation has been depicted in Figure2.

The interest earned by the retailer form the sales revenue,

$$= \frac{S_r I_e}{T}\left[\int_0^{T}\left(A_c{}^{\gamma} + bI_{1r}(t)\right)tdt + (M - T)\int_0^{T}\left(A_c{}^{\gamma} + bI_{1r}(t)\right)dt\right]$$
$$= \frac{S_r I_e}{T}\left[\frac{A_c{}^{\gamma}T^2}{2} + b\left\{\int_0^{t_2} I_{1r}(t)tdt + \int_{t_2}^{T} I_{1r}(t)tdt\right\} + (M - T)\left\{A_c{}^{\gamma}T + b\int_0^{t_2} I_{1r}(t)dt + b\int_{t_2}^{T} I_{1r}(t)dt\right\}\right]$$

$$= \frac{S_r I_e}{T}\left[\frac{A_c{}^{\gamma}T^2}{2} + b\left\{\frac{(D_r - A_c{}^{\gamma})}{(\theta + b)}\int_0^{t_2}\left(1 - e^{-(\theta+b)t}\right)tdt + \frac{A_c{}^{\gamma}}{(\theta + b)}\int_{t_2}^{T}\left(e^{(\theta+b)(T-t)} - 1\right)tdt\right\}\right.$$
$$\left. + (M - T)\left\{A_c{}^{\gamma}T + b\frac{(D_r - A_c{}^{\gamma})}{(\theta + b)}\int_0^{t_2}\left(1 - e^{-(\theta+b)t}\right)dt + b\frac{A_c{}^{\gamma}}{(\theta + b)}\int_{t_2}^{T}\left(e^{(\theta+b)(T-t)} - 1\right)dt\right\}\right]$$

$$= \frac{S_r I_e}{T}\left[\frac{A_c{}^{\gamma}T^2}{2} + \frac{b}{(\theta + b)}\left\{(D_r - A_c{}^{\gamma})\left(\frac{t_2{}^2}{2} + \frac{1}{(\theta + b)}\left(t_2 + \frac{1}{(\theta + b)}\right)e^{-(\theta+b)t_2} - \frac{1}{(\theta + b)^2}\right)\right.\right.$$
$$\left. + A_c{}^{\gamma}\left(-\frac{1}{(\theta + b)}\left(T + \frac{1}{\theta + b}\right) - \frac{T^2}{2} + \frac{1}{(\theta + b)}\left(t_2 + \frac{1}{(\theta + b)}\right)e^{(\theta+b)(T-t_2)} + \frac{t_2{}^2}{2}\right)\right\}$$
$$+ (M - T)\left\{A_c{}^{\gamma}T\right.$$
$$+ \frac{b}{\theta + b}\left\{(D_r - A_c{}^{\gamma})t_2 - \frac{(D_r - A_c{}^{\gamma})}{(\theta + b)}\left(1 - e^{-(\theta+b)t_2}\right) + \frac{A_c{}^{\gamma}}{(\theta + b)}\left(e^{(\theta+b)(T-t_2)} - 1\right)\right.$$
$$\left.\left.\left. - A_c{}^{\gamma}(T - t_2)\right\}\right\}\right]$$

$=\frac{S_rI_e}{T}\Big[\frac{A_c{}^{\gamma}T^2}{2}+\frac{b}{(\theta+b)}\Big\{(D_r-A_c{}^{\gamma})\Big(\frac{t_2{}^2}{2}+\frac{1}{(\theta+b)}\Big(t_2+\frac{1}{(\theta+b)}\Big)(1-(\theta+b)t_2-\frac{1}{(\theta+b)^2}\Big)+A_c{}^{\gamma}\Big(-\frac{1}{(\theta+b)}\Big(T+\frac{1}{\theta+b}\Big)-\frac{T^2}{2}+\frac{1}{(\theta+b)}\Big(t_2+\frac{1}{(\theta+b)}\Big)\big(1+(\theta+b)(T-t_2)\big)+\frac{t_2{}^2}{2}\Big)\Big\}+(M-T)\Big\{A_c{}^{\gamma}T+\frac{b}{\theta+b}(D_rt_2-A_c{}^{\gamma}T)\Big\}\Big]$ (Neglecting the higher power of θ and using (7))

$$=\frac{S_rI_e}{T}\left[\frac{A_c{}^{\gamma}T^2}{2}+\frac{b}{(\theta+b)}\left\{A_c{}^{\gamma}\left(t_2-\frac{T}{2}\right)T-\frac{D_rt_2{}^2}{2}\right\}+(M-T)\left\{A_c{}^{\gamma}T+\frac{b}{\theta+b}(D_rt_2-A_c{}^{\gamma}T)\right\}\right]$$
$$=S_rI_e\left[A_c{}^{\gamma}\left(M-\frac{T}{2}\right)+\frac{b}{(\theta+b)}\left\{A_c{}^{\gamma}\left(\frac{T}{2}+t_2-M\right)+\frac{D_r}{T}\left(M-T-\frac{t_2}{2}\right)t_2\right\}\right]$$

Case-2: When $T \geq M$.

In this case, the trade credit period ends before the retailer's cycle time. Here, the retailer can earn interest in the trade credit period, but after that, he has to pay interest to the manufacturer for the unsold products. We have described this situation in figure -3.

Interest charged

$$=\frac{S_mI_c}{T}\left[\int_M^T(A_c{}^{\gamma}+bI_{1r}(t))(T-t)dt\right]$$
$$=\frac{S_mI_c}{T}\left[A_c{}^{\gamma}\int_M^T(T-t)dt+b\int_M^T\frac{A_c{}^{\gamma}}{(\theta+b)}\left(e^{(\theta+b)(T-t)}-1\right)(T-t)dt\right]$$
$$=\frac{S_mI_c}{T}\left[\frac{A_c{}^{\gamma}(T-M)^2}{2}+\frac{bA_c{}^{\gamma}}{(\theta+b)}\left\{\frac{1}{(\theta+b)^2}-\frac{T^2}{2}+(T-M)\left(\frac{1}{(\theta+b)}e^{(\theta+b)(T-M)}+M\right)-\frac{1}{(\theta+b)^2}e^{(\theta+b)(T-M)}+\frac{M^2}{2}\right\}\right]$$
$$=\frac{S_mI_c}{T}\left[\frac{A_c{}^{\gamma}(T-M)^2}{2}+\frac{bA_c{}^{\gamma}}{(\theta+b)}\left\{\frac{1}{(\theta+b)^2}-\frac{T^2}{2}+(T-M)\left(\frac{1}{(\theta+b)}\big(1+(\theta+b)(T-M)\big)+M\right)-\frac{1}{(\theta+b)^2}\big(1+(\theta+b)(T-M)\big)+\frac{M^2}{2}\right\}\right]$$
$$=\frac{S_mI_c}{T}\left[\frac{A_c{}^{\gamma}(T-M)^2}{2}+\frac{bA_c{}^{\gamma}}{2(\theta+b)}(T-M)^2\right]$$
$$=\frac{S_mI_cA_c{}^{\gamma}}{2T}\left(1+\frac{b}{(\theta+b)}\right)(T-M)^2$$

Interest earned

$$=\frac{S_rI_e}{T}\int_0^M(A_c{}^{\gamma}+bI_{1r}(t))tdt$$
$$=\frac{S_rI_e}{T}\left[\frac{A_c{}^{\gamma}M^2}{2}+b\left\{\int_0^{t_2}I_{1r}(t)tdt+\int_{t_2}^{M}I_{1r}(t)tdt\right\}\right]$$
$$=\frac{S_rI_e}{T}\left[\frac{A_c{}^{\gamma}M^2}{2}+b\left\{\frac{(D_r-A_c{}^{\gamma})}{(\theta+b)}\int_0^{t_2}\left(1-e^{-(\theta+b)t}\right)tdt+\frac{A_c{}^{\gamma}}{(\theta+b)}\int_{t_2}^{M}\left(e^{(\theta+b)(T-t)}-1\right)tdt\right\}\right]$$

$$=\frac{S_rI_e}{T}\left[\frac{A_c{}^{\gamma}M^2}{2}+\frac{b(D_r-A_c{}^{\gamma})}{(\theta+b)}\left\{\frac{t_2{}^2}{2}+\frac{1}{(\theta+b)}\left(t_2+\frac{1}{(\theta+b)}\right)e^{-(\theta+b)t_2}-\frac{1}{(\theta+b)^2}\right\}+\frac{bA_c{}^{\gamma}}{(\theta+b)}\left\{-\frac{1}{(\theta+b)}\left(M+\frac{1}{(\theta+b)}\right)e^{(\theta+b)(T-M)}-\frac{M^2}{2}+\frac{1}{(\theta+b)}\left(t_2+\frac{1}{(\theta+b)}\right)e^{(\theta+b)(T-t_2)}+\frac{t_2{}^2}{2}\right\}\right]$$

$=\frac{S_rI_e}{T}\Big[\frac{A_c{}^{\gamma}M^2}{2}+\frac{b(D_r-A_c{}^{\gamma})}{(\theta+b)}\Big\{\frac{t_2{}^2}{2}+\frac{1}{(\theta+b)}\Big(t_2+\frac{1}{(\theta+b)}\Big)(1-(\theta+b)t_2)-\frac{1}{(\theta+b)^2}\Big\}+\frac{bA_c{}^{\gamma}}{(\theta+b)}\Big\{-\frac{1}{(\theta+b)}\Big(M+\frac{1}{(\theta+b)}\Big)\big(1+(\theta+b)(T-M)\big)-\frac{M^2}{2}+\frac{1}{(\theta+b)}\Big(t_2+\frac{1}{(\theta+b)}\Big)\big(1+(\theta+b)(T-t_2)\big)+\frac{t_2{}^2}{2}\Big\}\Big]$ (Neglecting the higher power of θ)

$$=\frac{S_rI_e}{T}\left[\frac{A_c{}^{\gamma}M^2}{2}+\frac{b}{(\theta+b)}\left\{A_c{}^{\gamma}\left(\frac{M^2}{2}-MT+t_2T\right)-\frac{D_rt_2{}^2}{2}\right\}\right]$$

Therefore, total cost of the retailer is given by

$$TC_r = \begin{cases} TC_{r1}, & if\ T < M \\ TC_{r2}, & if\ T \geq M \end{cases} \ldots\ldots\ldots (8)$$

Where,

$$TC_{r1} = \frac{1}{T}\left\{A_r + \frac{(h_r + \theta d_r)}{(\theta + b)}(D_r t_2 - A_c{}^{\gamma}T) + S_m D_r t_2\right\} - S_r I_e \left\{A_c{}^{\gamma}\left(M - \frac{T}{2}\right) + \frac{b}{(\theta + b)}\left\{A_c{}^{\gamma}\left(\frac{T}{2} + t_2 - M\right) + \frac{D_r}{T}\left(M - T - \frac{t_2}{2}\right)t_2\right\}\right\} \ldots\ldots\ldots (9)$$

And,

$$TC_{r2} = \frac{1}{T}\left\{A_r + \frac{(h_r + \theta d_r)}{(\theta + b)}(D_r t_2 - A_c{}^{\gamma}T) + S_m D_r t_2\right\} + \frac{S_m I_c A_c{}^{\gamma}}{2T}\left(1 + \frac{b}{(\theta + b)}\right)(T - M)^2 - \frac{S_r I_e}{T}\left\{\frac{A_c{}^{\gamma}M^2}{2} + \frac{b}{(\theta + b)}\left\{A_c{}^{\gamma}\left(\frac{M^2}{2} - MT + t_2 T\right) - \frac{D_r t_2{}^2}{2}\right\}\right\} \ldots\ldots\ldots (10)$$

Hence, the total cost of the supply chain system

$$TC = \begin{cases} TC_1 & if\ T < M \\ TC_2 & if\ T \geq M \end{cases} \ldots\ldots\ldots (11)$$

Where, $TC_1 = TC_m + TC_{r1}$

And, $TC_2 = TC_m + TC_{r2}$

Therefore,

$$TC_1 = \frac{1}{T}\left\{A_m + A_c + A_r + C_p p_m t_1 + \frac{(h_{m+d_m\theta})}{\theta}\{(1 - z)p_m t_1 - D_r t_2\} + \frac{(h_r + \theta d_r)}{(\theta + b)}(D_r t_2 - A_c{}^{\gamma}T) + S_m D_r t_2 - S'_m z p_m t_1\right\} + S_m D_r I_p M - S_r I_e \left\{A_c{}^{\gamma}\left(M - \frac{T}{2}\right) + \frac{b}{(\theta + b)}\left\{A_c{}^{\gamma}\left(\frac{T}{2} + t_2 - M\right) + \frac{D_r}{T}\left(M - T - \frac{t_2}{2}\right)t_2\right\}\right\} \ldots\ldots\ldots (12)$$

$$TC_2 = \frac{1}{T}\left\{A_m + A_c + A_r + C_p p_m t_1 + \frac{(h_{m+d_m\theta})}{\theta}\{(1 - z)p_m t_1 - D_r t_2\} + \frac{(h_r + \theta d_r)}{(\theta + b)}(D_r t_2 - A_c{}^{\gamma}T) + S_m D_r t_2 - S'_m z p_m t_1 + \frac{S_m I_c A_c{}^{\gamma}}{2}\left(1 + \frac{b}{(\theta + b)}\right)(T - M)^2 - S_r I_e \left\{\frac{A_c{}^{\gamma}M^2}{2} + \frac{b}{(\theta + b)}\left\{A_c{}^{\gamma}\left(\frac{M^2}{2} - MT + t_2 T\right) - \frac{D_r t_2{}^2}{2}\right\}\right\}\right\} + S_m D_r I_p M \ldots\ldots\ldots (13)$$

Now, let us take $t_1 = \gamma_1 T$ and $t_2 = \gamma_2 T$, then the equations (12) and (13) can be rewritten as

$$TC_1 = \frac{1}{T}(A_m + A_c + A_r) + C_p p_m \gamma_1 + \frac{(h_m + d_m\theta)}{\theta}\{(1 - z)p_m\gamma_1 - D_r\gamma_2\} + \frac{(h_r + \theta d_r)}{(\theta + b)}(D_r\gamma_2 - A_c{}^{\gamma}) + S_m D_r \gamma_2 - S'_m z p_m \gamma_1 + S_m D_r I_p M - S_r I_e \left\{A_c{}^{\gamma}\left(M - \frac{T}{2}\right) + \frac{b}{(\theta + b)}\left\{A_c{}^{\gamma}\left(\frac{T}{2} + \gamma_2 T - M\right) + D_r\left(M - T - \frac{\gamma_2 T}{2}\right)\gamma_2\right\}\right\} \ldots\ldots\ldots (14)$$

$$TC_2 = \frac{1}{T}\left[A_m + A_c + A_r + \frac{S_m I_c A_c{}^{\gamma}}{2}\left(1 + \frac{b}{(\theta + b)}\right)(T - M)^2 - S_r I_e \left\{\frac{A_c{}^{\gamma}M^2}{2} + \frac{b}{(\theta + b)}\left\{A_c{}^{\gamma}\left(\frac{M^2}{2} - MT + \gamma_2 T^2\right) - \frac{D_r(\gamma_2 T)^2}{2}\right\}\right\}\right] + C_p p_m \gamma_1 + \frac{(h_{m+d_m\theta})}{\theta}\{(1 - z)p_m\gamma_1 - D_r\gamma_2\} + \frac{(h_r + \theta d_r)}{(\theta + b)}(D_r\gamma_2 - A_c{}^{\gamma}) + S_m D_r\gamma_2 - S'_m z p_m \gamma_1 + S_m D_r I_p M \ldots\ldots\ldots (15)$$

Lemma-1: The cost function TC_1 is convex with respect to T.

Differentiating (14) partially with respect to T we have,

$$\frac{\partial(TC_1)}{\partial T} = -\frac{1}{T^2}(A_m + A_c + A_r) + \frac{S_r I_e A_c{}^{\gamma}}{2} + \frac{b}{(\theta+b)}\left\{A_c{}^{\gamma}\left(\gamma_2 + \frac{1}{2}\right) - D_r\left(1 + \frac{\gamma_2}{2}\right)\gamma_2\right\}$$

$$\frac{\partial^2(TC_1)}{\partial T^2} = \frac{2}{T^3}(A_m + A_c + A_r) > 0,$$

Therefore, TC_1 is convex with respect to T. ■

Therefore, we can get the optimum value of T, say T^*, which minimizes the total cost TC_1 from the equation

$$\frac{\partial(TC_1)}{\partial T} = -\frac{1}{T^2}(A_m + A_c + A_r) + \frac{S_r I_e A_c{}^{\gamma}}{2} + \frac{b}{(\theta+b)}\left\{A_c{}^{\gamma}\left(\gamma_2 + \frac{1}{2}\right) - D_r\left(1 + \frac{\gamma_2}{2}\right)\gamma_2\right\} = 0 \ldots\ldots\ldots (16)$$

Again, the necessary condition for the optimum value of T, which minimizes the total cost TC_2 is $\frac{\partial(TC_2)}{\partial T} = 0$.

Now, $\frac{\partial(TC_2)}{\partial T} = 0$ gives,

$$\frac{\partial(TC_2)}{\partial T} = -\frac{1}{T^2}(A_m + A_c + A_r) - \frac{S_m I_c A_c{}^{\gamma}}{T^2}\left(1 + \frac{b}{(\theta+b)}\right)\{(T-M)^2 - T(T-M)\}$$
$$+ \frac{S_r I_e}{T^2}\left\{\frac{A_c{}^{\gamma} M^2}{2} + \frac{b}{(\theta+b)}\left(A_c{}^{\gamma}\left(\frac{M^2}{2} - \gamma_2 T^2\right) + \frac{D_r(\gamma_2 T)^2}{2}\right)\right\} = 0 \ldots\ldots\ldots (17)$$

We can get the optimum value of T, say T^{**}, that minimizes TC_2 from the equation (17), provided that the sufficiency condition must be satisfied, i.e.

$$\frac{\partial^2(TC_2)}{\partial T^2} > 0$$

i.e. $\frac{\partial^2(TC_2)}{\partial T^2} = \frac{2}{T^3}(A_m + A_c + A_r) + \frac{S_m I_c A_c{}^{\gamma}}{T^3}\left(1 + \frac{b}{(\theta+b)}\right)M^2 - \frac{S_r I_e}{T^3}\left\{A_c{}^{\gamma} M^2\left(1 + \frac{b}{(\theta+b)}\right)\right\} > 0$

5. Numerical Examples

Example1. We have considered the following numerical values of the variables to illustrate the applicability of the proposed model.

$A_m = 500, A_r = 350, A_c = 50, h_m = 1.5, h_r = 2, d_m = 3, d_r = 7, p_m = 55,$
$D_r = 26, b = 0.06, z = 0.08, \theta = 0.1, \gamma = 0.7, S_m = 45, S_r = 60, S'_m = 25,$
$I_c = 0.1, I_p = 0.08, I_e = 0.2, \gamma_1 = 0.4, \gamma_2 = 0.7, C_r = 3, a_1 = 35,$
$a_2 = 0.04, a_3 = 0.12, \delta_1 = 0.7, \delta_2 = 1, \delta_3 = 0.5.$

We have taken $M = 4$ for Case-1 and $M = 2$ for Case-2.
The optimum results are shown in Table2 and Table3, respectively.

Example-2: Here, we have taken the following values of the variables in the appropriate units.
$A_m = 300, A_r = 150, A_c = 60, h_m = 3.5, h_r = 4, d_m = 5, d_r = 8, p_m = 65,$
$D_r = 25, b = 0.06, z = 0.07, \theta = 0.08, \gamma = 0.6, S_m = 50, S_r = 65, S'_m = 30,$
$I_c = 0.2, I_p = 0.08, I_e = 0.3, \gamma_1 = 0.5, \gamma_2 = 0.8, C_r = 4, a_1 = 40,$
$a_2 = 0.05, a_3 = 0.15, \delta_1 = 0.6, \delta_2 = 0.08, \delta_3 = 0.6.$

We have taken $M = 5$ for Case-1 and $M = 1$ for Case-2.
The optimum results are shown in Table4 and Table5, respectively.

6. Sensitivity Analysis

In this section, we have studied the effect of changes of the parameters on the optimum decision variables (taking the values of the parameters from example1)

6.1. Observations

From the Tables 6- 10, it is observed that:

i) The cycle times and the total costs for both the cases increase with the increase in the setup cost of the manufacturer as well as with the increase in the setup costs of the retailer (See Table6 and Table7).
ii) With the increase in the cost of advertisement, the cycle times and the total costs for both the cases decrease (See Table8).
iii) From the demand function of the customer, it is seen that the demand rate increases with the increase in the parameter b. So, as the Table9 shows, we can say that with the increase in the parameter b i.e., with the increase in the customers' demand, the cycle length and the total costs for both cases increase.
iv) With the increase in the interest rate earned by the retailer (I_e), the cycle length for both cases decreases. The decrease in the TC_1 and TC_2 is obvious in this case (from Table10).

7. Conclusion

This article presents a two-echelon supply chain model for deteriorating items which includes a single manufacturer and a single retailer. In this model, we have assumed that due to the imperfect production system of the manufacturer, it produces a certain quantity of imperfect products with the perfect products. The manufacturer does not rework the defective products. Instead, he sells these products at the reduced price and earns the salvage value. Moreover, to encourage the retailer to purchase more, the manufacturer offers him a trade credit period so that the retailer can settle his account before the payment for the products. Again, in practice, it is observed that the customers are motivated to purchase more by watching the advertisement of the product as well as by checking the displayed stock level of the retailer. If the retailer has an adequate amount of the stock, the customers can choose the products of their choices, and consequently, the demand rate for the product increases. Here, we have discussed two cases depending on the length of the trade credit period offered by the manufacturer to the retailer and derived the cost functions in both cases. The applicability of this model has been verified with the help of numerical examples and the sensitivity analysis has been presented to see the effect of the parameter change on the optimum decision variables.

This model can be extended to a three-echelon supply chain model by introducing one supply chain player, such as a supplier or a wholesaler, and considering two-level of credit financing. Furthermore, in this article, we have considered the rate of deterioration and the defective rate of the products during manufacturing as constant although these factors may not be constant in practice. So, we can consider the rate of deterioration and the defective rate of the product as fuzzy or stochastic in nature to extend this model.

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