How to Estimate the Supplier Fill Rate When the Supply Order and the Supply Lead-time Are Uncertain?

Slim Harbi *,a, Mohamed Bahroun b, Hanen Bouchriha a

*a OASIS Laboratory (ENIT-Tunis), National Engineering School of Carthage, University of Tunis El Manar, Tunisia
*b ACS Laboratory, National Engineering School of Tunis, University of Tunis El Manar, Tunisia

Abstract
Modern retail supply chains are more and more exposed to risks and uncertainties. Supply risks such as the uncertainty of the supplier fill rate (SFR) directly affect the performance of a retail supply chain. The purpose of this paper is to investigate the supply uncertainty, where the order size and the supply lead-time are considered as decision variables. We aim at developing a more realistic approach to predicting the SFR. Reviewing the relevant literature was the first step taken. We pointed out that while the scientific research on supply risk is growing, the literature lacks an accurate support tool that can predict the SFR. Then, a case study was conducted in order to have a comprehensive view of the real context of SFR parameters. Accordingly, we propose a new approach to predicting the SFR using the bivariate normal distribution. We illustrate the proposed approach using a real case study in Tunisia.

Keywords: Modern retail supply chain; Supply risk; Bivariate distribution; Supplier fill rate.

1. Introduction and statement of the problem
Supply chain managers are becoming increasingly aware of the importance of managing supply chain risks effectively (Gualandris and Kalchschmidt 2015) (Hamdi et al 2018). In the real supply chain environment, retailers need to protect themselves from uncertainties in demand and supply. Demand and supply chain planning is very complex (Hübner et al. 2013). According to Schmitt 2008, the study of supply risk and uncertainty is a growing field. While uncertain demand has been exhaustively explored, the impacts of supply uncertainties are not as well studied. Within the retail supply chain, many inventory control systems are used through the application of ERP. Most of these systems consider the supplied quantities equal to ordered quantities. However, in practice, suppliers fail to deliver the needs in terms of ordered quantities and/or lead-time (Gurnani et al. 2013). When the SFR is high, retailers can achieve a given service level to end-consumers while holding less inventory. Most retailers are concerned with the low SFR because not only it contributes to lost sales but also it allows consumers to switch to competitors. (Gurnani et al. 2013) and (Nagarajan and Shechter 2013) studied the ordering decisions of procurement professionals including supplier service level. They found that procurement professionals increase orders for an unreliable supplier. (Aastrup and Kotzab 2009) examined out-of-stock (OOS) challenges in the independent grocery sector. They revealed that the major part of OOS situations in the independent grocery sector originates directly from the store ordering practices and SFR. The researchers call for future work to explore more realistic procurement contexts in order to understand how retail ordering works in practice (Hopp et al. 2009) and (Gurnani et al. 2013).

This research is based on a case study in the modern retail supply chain in Tunisia. In this section, we describe the general structure and processes of the supply chain. Then, we present the statement of the problem, the scope and the purpose of this research.

Corresponding author email address: slim.harbi@enicarthege.rnu.tn
The considered retail supply chain is composed of hundreds of suppliers, a retailer-owned warehouse center (WC), and 90 stores around the country with multiple formats. Each store carries thousands of items. Stocking volume levels vary according to the size of the store and its geographic location. Suppliers replenish some products such as fresh goods directly to the stores. While most of the items are replenished through the WC, the demand in the WC is fulfilled by shipments from the suppliers (Figure 1). In our study, we focus on the items delivered via the WC.

![Supply chain configuration](image)

**Figure 1.** Supply chain configuration

We consider a three-echelon supply chain consisting of one supplier, one WC, and multiples stores. As shown in Figure 2, to control the WC inventory, the manager uses a replenishment policy similar to the standard periodic review base-stock policy \((T, S)\) with random demand and random lead-time. \(T\) and \(S\) denote review period and base-stock level respectively.

![Replenishment policy](image)

**Figure 2.** Replenishment policy

The order-up-to-level \(S\) is fixed to achieve a desired service level to stores and end-customers. \(S\) is obtained using the following formula:

\[
S = (T + L) \cdot D + \sqrt{(T + L) \cdot \sigma_D^2 + \sigma_L^2} \tag{1}
\]

where, \(L\) : Average supply lead-time; \(D\) : Average aggregate (all stores) demand; \(z\) : Value to meet a desired service level; \(\sigma_D^2\) : Standard deviation of the aggregate demand; \(\sigma_L^2\) : Standard deviation of the supply lead-time.

In the real supply chain environment, safety stock is needed to protect against variability. It is commonly known that supplier lead times have a direct impact on the retailer’s safety stock. In our case, the WC’s manager readjusts \(S\) every period based on historical results, demand forecast and professional experience. The manager proceeds according to the following process:

- Every \(T_j\) period, the inventory \(I_j\) at the WC is checked and compared with the order-up-to-level \(S\).
- If \(I_j < S\), a replenishment quantity \(Q_j\) is determined by \(Q_j = S - I_j\). Therefore, for each period \(T_j\), the order quantity \(Q_j\) and the supply lead-time \(L_j\) are decided in order to minimize the out of stock risk at the WC that may occur during the cycle period.
- \(L_j\) is estimated based on the inventory \(I_j\) to avoid out of stock during the supply lead-time, where \(L_j < T_j\).
- If the delivery lead-time exceeds \(L_j\), the order \(Q_j\) will be cancelled and another order will be sent to the supplier with other parameters (size and lead-time).

The fill rate is the fraction of demand that can be immediately fulfilled from the inventory on hand (Zipkin, 2000). We define the WC global fill rate (GFR) as the fraction of aggregate demand from all stores that is immediately fulfilled from the WC’s on hand inventory. In our case study, the GFR was less than 80 percent at the WC. Consequently, an item that was out of stock at the WC was even more likely to be out of stock at many stores (those without storage backroom). This situation which had a negative impact on customer loyalty and long-term profitability was attributed to many risks within the supply chain including the WC ordering process and random SFR.
Many suppliers were unable to fulfil 100% of the order on time. The average percentage of items delivered on time compared to the quantities ordered was approximately 65%. As in many supply chains, the SFR is unknown to the retailer and changes over time. That is why the prediction of the SFR by the managers when sending an order is needed.

We observed that the manager increases order size for an unreliable supplier based on his historical SFR. By managing the supply risks, the SFR may increase and at the same time, the WC in-stock percentage will increase. Moreover, the improved SFR may reduce the amount of safety stock at the WC and stores. Therefore, it is important to analyze the interaction between the WC’s replenishment decision (order size and lead-time) and the response of the supplier (supply order and supply lead-time).

Based on the case study, we noted that the SFR randomness was due to the supplier “random yield risk”, the “supplier capacity risk”, the “lead-time variability”, and the “order quality variability”. Clearly, this situation indicated that there was a need to improve the WC replenishment practices taking into account the SFR. Through the real case data analysis, we note that we encounter a supplier lead-time/order dependency problem. The WC’s replenishment decision depends on the expected supplier’s lead-time, whereas the order fulfillment and SFR depend on the replenishment decision (order size and lead-time). In this type of setting, the variability of the order pattern combined with the variability of the lead-time pattern all have an impact on the SFR.

This paper discusses previous research on the supply risk and the diverse parameters and formulation of SFR. Then, it highlights what makes the relationship between SFR and retailer order size (demand) and supply lead-time, in this specific context, different from the previously explored formulations of SFR. Therefore, in such a complex supply chain, statistical models are needed in order to predict uncertain events. Hence, analysis of dependence variables is often required. In recent years, interest in multivariate problems concerning uncertain events has increased. The present work studies the bivariate distribution extension in such a supply chain. In fact, we investigate the relevance of the normal two-dimensional distribution to predict the SFR in this specific situation.

The remainder of this paper is organized as follows. Section 2 reviews the literature and highlights the research gap. Section 3 presents our proposal for a new approach to predicting the SFR. In section 4, we discuss some of our findings through the practical application of the methodology using a real data computational example. Finally, we conclude in section 5 with some perspectives about future research.

## 2. Review of the literature

In this section, a summary of the main literature on supply risk and uncertainty is provided. The literature on supply risk has been growing over the last decade (Gualandris and Kalchschmidt 2015). According to (Zsidisin, 2003), supply risk can be defined as the uncertainty associated with suppliers’ activities and obligations. It can be divided into two types: disruption risk (supplier is either available or not) and operational risk. The existing research on the operational supply risk focuses on unreliable suppliers. As shown in Table 1, supply risk and uncertainty is often modeled using random yields, random SFR or supply service level (SSL) and supply lead-time variability.

<table>
<thead>
<tr>
<th>Supply risk</th>
<th>Papers</th>
</tr>
</thead>
</table>

a) Random supply yield

Random supply yield resides in the flow of products from suppliers to the company when it is not on time or of the required quality and quantity (Bahroun and Harbi 2015). Several factors are linked to random supply yield such as supply lead-time, the production capacity and the product quality that become unpredictable. (Yano and Lee 1995) present five basic approaches to dealing with supply yield: Bernoulli process; stochastically proportional yield; stochastic yield proportional to order quantity; random capacity; and general model that specifies the probability of each output for each order quantity. (Keren 2009) shows how stochastic supply yield impacts supply chain coordination. In practice, retailers do not know their suppliers’ yield distributions and must instead develop forecasts or beliefs about them. (Du et al 2018)

---

*Int J Supply Oper Manage (IJSOM), Vol.5, No.3*
studied a supply chain composed of a loss-averse supplier with yield randomness and a loss-averse retailer with demand uncertainty.

b) Random SFR (or SSL)

Most inventory models assumed that the quantity received is the same as the quantity ordered. However, as mentioned in (Priyan and Uthayakumar 2015), in practice the quantity received may not match the quantity ordered due to worker’s strike, rejection during inspection, damage during transportation, human errors in counting, etc. Accordingly, managers often must make decisions under uncertain quantity received circumstances. In this study, they investigate the continuous review inventory model with shortages including the case where the quantity received is uncertain, in which the lead time, lost sales rate, and order processing cost are decision variables.

The study of service level is as old as the theory of inventory itself. The most popular and frequently used definitions can be found in (Tempelmeier 2000). There are two types of service levels. Type-1 service level, denoted by \( \alpha \), is an event-based measurement, which describes the proportion of cycles in which no stock out occurs. Type-2 service level, denoted by \( \beta \), is a quantity-based measurement that not only describes the probability of a stock-out, but also provides an average expected number of backorders or loss for every demand period. The type-2 service level is often called fill rate or item fill rate. \( \beta \) service level is typically considered a more relevant measure of service level compared to \( \alpha \). The type-1 service level can be modeled using relatively simpler expressions and hence appears widely in the inventory literature whereas the type-2 service level is less commonly used in research due to the complex form of backorder/loss quantity calculation, which makes it hard to model it. The item fill rate, sometimes referred to as volume fill rate or unit fill rate (Guijarro et al., 2012), is different to the order fill rate, which applies to the proportion of fulfilled customer orders that may consist of multiple products (Larsen and Thorstenson, 2014). (Disney et al. 2015) investigated the fill rate as an inventory service metric and proposed a new calculation that ensures the target fill rate is achieved without excessive inventory investments.

Previous research on unreliable suppliers studied the distribution of supplier’s service level (SFR). According to (Chen et al. 2010), retailers may track changes in the SFR informally, as in the case of a buyer’s attitude toward a particular supplier, or formally through the use of automated software and supplier scorecards. They study how a retailer’s orders change as it receives information and updates its beliefs about a SFR. (Yang et al. 2012) investigated a related model in which a SFR is private information. (Burke et al. 2009) found that an increased SFR can increase orders for a supplier. For the single supplier case, an increased SFR decreases a retailer’s orders. The retailer decreases its order quantity if it is unlikely to receive a smaller quantity than that requested. In a multi-sourcing situation, the retailer may mitigate its supply risk by spreading orders across the suppliers.

With multiple retailers, the supplier’s allocation rule becomes nontrivial (Chen et al. 2013). This situation has a great impact on the SFR to each retailer. (Ray and Jenamani 2013) proposed multi-sourcing models for optimal order allocation in a newsvendor setting under supply disruption with stochastic demand where suppliers are capacity constrained.

(Teller et al. 2016) pointed out the necessity of focusing on the management of key supplier relationships and their importance for overall supply chain performance. According to (Fernie et al. 2010), many retailers have begun to collaborate closely with suppliers to maximize the efficiency of the retail supply chain as a whole. Many retailers use service level agreements (SLAs) to outline performance expectations for their suppliers and specify consequences for failing to meet those expectations. Research on SLAs has explicitly investigated the role of SLAs in coordinating supply chains by motivating suppliers to improve service (Liang and Atkins 2013) (Sieke et al. 2012).

c) Supply lead-time variability

In general, uncertain supply lead-time related to procurement has been discussed at length in the inventory management literature. There is a rich body of literature on supplier-retailer inventory models with uncertain supply lead-time and the effect of supply uncertainty on supply chain performance (Schmitt et al., 2010) (Gupta and Cooper, 2005), (Yang et al. 2012), (Gumus et al. 2012), (Tang and Kouvelis 2011), (Wang et al. 2010). According to (X. Fang et al. 2013), supply lead-time uncertainty has long been identified as a fundamental factor influencing inventory decisions. This research has focused on inventory models with stochastic lead-times. (Song and Zipkin 2009) studied the performances of inventory management systems having deterministic lead times that have been assumed constant, stochastic and exogenous.

Traditional inventory models assumed that lead-time is a constant or random variable, which is not a controllable factor (Nasrabadi and Mirzazadeh 2016) (Sundararajan and Uthayakumar 2015). However, in practice, lead-time could be a decision variable.

d) Concluding remarks

The problem presented in our research and the way it is addressed are different from similar problems in the literature. We do not merely assume the supplier lead-time to be a random exogenous variable, but we include the impact of the order size decision on the supply lead-time and we use the result to predict the SFR. Consequently, in our study we consider orders and lead times as linked factors that affect the SFR. The inclusion of these two dimensions represents a better fit with real-life situations.
According to (Palaro and Hotta 2006), several approaches to the estimation of risks demand the joint distribution of risk factors to be known, which in the analytical approach is frequently the normal distribution.

The models of multivariate probability laws have received particular attention in recent years for the significance they add to the modeling and simulation of events. (Chelbi et al., 2009) used the multivariate Gamma distribution to analyze system reliability. They emphasized the importance of using the multivariate approach to analyze various correlations between different factors. They worked on determining the optimal periodic replacement strategy taking into account the reliability of the system based on two variables of time and usage. According to authors, the system wears out after a predefined operating time or according to its use. In this context, according to manufacturers, a car tire is replaced after 5 years or after 50000 km. Similarly, (Ben Hmida et al. 2010) used a bi-dimensional probability law to determine the budget estimation for a product warranty.

In the supply chain literature, the use of multivariate distribution is not very common. One relevant research is (Kaki et al. 2015). They analyzed the impact of supply uncertainty on newsvendor decisions for interdependent demand and supply. They derived a solution for a newsvendor facing stochastic supply yield in addition to stochastic demand, and provided a closed-form solution for a specific copula-based dependence structure.

3. The proposed approach

In order to be closer to the practical case study, we are interested in the SFR for a single item. We studied the probability distribution function (p.d.f) of the delivery lead-time (L) and the p.d.f of the order size (Q). In our case, in order to estimate Q we made a Kolmogorov-Smirnov test based on the ordered quantities during one year. We concluded from the hypothesis testing that Q approximates a normal distribution. Similarly, we studied the lead-time (L) variable. Based on collected data we found that the delivery lead-time (in days) can be approximated using a random variable L that follows a normal distribution.

As described in the previous section, for each period T_j, the order quantity Q_j and the lead-time L_j are decided in order to minimize the OOS risk at the WC that may occur during the cycle period. L_j is estimated based on the inventory (I_j) to avoid OOS during the supply lead-time. If the lead-time (L) exceeds L_j, the order Q_j will be cancelled and another order will be sent to the supplier with other parameters (size and lead-time).

Let V(Q_j) denote the quantity received in time from the supplier, thus the amount of received units V can be expressed as:

\[
V(Q_j) = \begin{cases} 
0 & \text{if } L > L_j \\
Q_j - \varepsilon & \text{if } L \leq L_j \\
& \text{and } Q \leq Q_j 
\end{cases}
\]  

(2)

Accordingly, the expected value of the SFR during a cycle period is:

\[
SFR = \frac{V(Q_j)}{Q_j} = \begin{cases} 
0 & \text{if } L > L_j \\
1 - \frac{\varepsilon}{Q_j} & \text{if } L \leq L_j \\
& \text{and } Q \leq Q_j 
\end{cases}
\]  

(3)

To study the impact of lead-time and order size on the SFR, we often consider lead-times and delivered quantity to be exogenous, meaning that the delivered quantity is independent of the lead-time and of the ordered quantity. In real situations, the order quantity Q (which in our case is close to the demand cycle) and the lead-time L are dependent. We noted that the SFR increases when the delivery lead-time increases or the order size decreases. This situation is similar to the just-in-time context, so we assume that the supplier could provide the total order if the lead-time is extended or the order size is reduced for the same lead-time.

Therefore, we consider the case where order quantity Q and lead-time L follow a bivariate normal distribution:

\[(Q, L) \sim \text{N} \left( \mu, \Sigma \right) \]  

(4)

where

\[
\mu = \begin{bmatrix} \mu_q \\ \mu_l \end{bmatrix}
\]

\[
\Sigma = \text{covariance matrix} \ \text{cov}(Q, L)
\]

To calculate the cumulative distribution function (C.D.F) of (Q, L), we use the inputs sets Q and L as:

\[
F(q,l) = P(Q \leq q, L \leq l)
\]  

(5)

The expected (absolute) SFR during a cycle period T_j can be expressed using (3) and (5) as:
How to Estimate the Supplier Fill Rate When the Supply Order and ...

\[
E[SFR] = 0^* \cdot P(L > L_j) + (1 - \frac{\epsilon}{Q_j})^* \cdot P(Q \leq Q_j, L \leq L_j) \tag{6}
\]

\[
\Rightarrow E[SFR] = F(Q_j, L_j) - (\frac{\epsilon}{Q_j} \cdot F(Q_j, L_j)) \tag{7}
\]

Then the expected maximum SFR during a cycle period \(T_j\) can be approximated using (8):

\[
\Rightarrow \text{Max}(SFR) \approx F(Q_j, L_j) \tag{8}
\]

For this purpose, we can predict the SFR using Figure 3 below.

\[
F(q, l) = P(Q \leq q, L \leq l)
\]

Figure 3. SFR estimation

Then, the p.d.f of \((Q, L)\) is:

\[
f_{QL}(q, l) = \frac{1}{2\pi\sigma_q\sigma_l\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\times
\frac{(q-\mu_q)^2}{\sigma_q^2} - 2\rho \left(\frac{q-\mu_q}{\sigma_q}\right)\left(\frac{l-\mu_l}{\sigma_l}\right) + \frac{(l-\mu_l)^2}{\sigma_l^2}\right\} \tag{9}
\]

4. Application

We collected weekly observations about a cleaning product. The data cover a total of 12 months “P” and involve information about retailer orders “Q”, the supplier lead-time “L”, and the quantity received “V”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>381</td>
<td>890</td>
<td>622</td>
<td>530</td>
<td>620</td>
<td>514</td>
<td>523</td>
<td>710</td>
<td>701</td>
<td>702</td>
<td>615</td>
<td>397</td>
</tr>
<tr>
<td>Q</td>
<td>3.7</td>
<td>4.6</td>
<td>3.4</td>
<td>4.3</td>
<td>3.7</td>
<td>2.6</td>
<td>3.2</td>
<td>2.2</td>
<td>3.8</td>
<td>4.8</td>
<td>1.8</td>
<td>1.9</td>
</tr>
<tr>
<td>V</td>
<td>354</td>
<td>854</td>
<td>460</td>
<td>487</td>
<td>508</td>
<td>395</td>
<td>397</td>
<td>198</td>
<td>560</td>
<td>126</td>
<td>196</td>
<td>337</td>
</tr>
<tr>
<td>SFR</td>
<td>0.93</td>
<td>0.96</td>
<td>0.74</td>
<td>0.92</td>
<td>0.82</td>
<td>0.77</td>
<td>0.76</td>
<td>0.28</td>
<td>0.80</td>
<td>0.18</td>
<td>0.32</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 2. Data analysis

In our case, in order to estimate Q we made a Kolmogorov-Smirnov test based on the ordered quantities during one year. We concluded from the hypothesis testing that Q approximates a normal distribution with mean \(\mu_Q\) equal to 538 units and a standard deviation \(\sigma_Q\) equal to 116 units.

Similarly, we studied the lead-time (L) variable. Based on the collected data, we found that the delivery lead-time (in days) can be approximated using a random variable L that follows a normal distribution with the following parameter equal to 3 days and standard deviation \(\sigma_L\) equal to 1 day. \(\mu_L\).

Therefore, in order to predict the SFR based on the order quantity q and the lead-time l, using Matlab, we implemented the C.D.F. Figure 4 illustrates the result of the SFR based on delivery lead-time and order quantity using an analytical approach.

Therefore, we can predict the SFR using Figure 4. For example, for an order in which Q = 600 units and lead-time L = 2.5 days, the SFR will be about 50%.

In Figure 5 below, we compare the empirical values of the SFR (TS) to the estimated values using our proposed approach (TSA). We note that the proposed formulation for the SFR is a good approximation of the real-life SFR in our case studied.
5. Concluding remarks and future work

Customer demand, received quantity, etc. cannot be predicted in advance. Therefore, the assumptions of uncertain demand and received quantity may be appropriate for all industries in this world. Additionally, when the demand and lead-time are uncertain, SFR becomes an important issue and predicting it brings several benefits. Our research contributes to the body of work on retailers ordering from unreliable suppliers. It can enrich the existing discussions about estimating the SFR in a specific context, and in turn tackle the mutual dependency that arises in this context (orders are dependent on the lead-time distribution and vice versa). Moreover, the proposed approach can help the decision maker to estimate the SFR based on a bivariate distribution taking into account the order size variability and lead time uncertainty. Motivated by a real life observation of the ordering process and supplier behavior, we have presented a numerical application of the new SFR measure based on the bivariate normal distribution.

This paper is limited in the use of the normal supply lead-time distribution. In real life situations, we often encounter difficulties in providing a precise estimation of the probability density function due to the insufficiency of historical data. Therefore, for further consideration of this problem, it would be interesting to propose a distribution-free model according to the mean and standard deviation of supply lead-time. It would be also interesting to perform a global sensitivity analysis. Sensitivity analysis will investigate how variation in the output of the numerical model can be attributed to variations of its input factors.

Moreover, information sharing about sales data, inventories and promotion plans may effectively reduce the supply risks threatening the retail supply chains. Efficient Consumer Response (ECR), Vendor Managed Inventory (VMI), and Collaborative Planning Forecasting and Replenishment (CPFR) are strategies of supply chain collaboration that have received considerable attention in the research (Hosseinia and Mehrjerdi 2016). These strategies have been implemented in the retail supply chain in order to reduce supply risk. It is important to study how these collaboration strategies will improve the SFR and the ordering process. For instance, (Tannous and Yoon 2018) investigated the relationship between...
risk, sustainability, and collaboration in Global Supply Chain Management. They concluded that “delivering GSCM optimization between partners through sustainability initiatives mitigates reputational risk exposure from the collaborative efforts among SC stakeholders to increase intrinsic value”.

References


Priyan S. and Uthayakumar R. (2015). Continuous review inventory model with controllable lead time, lost sales rate and order processing cost when the received quantity is uncertain, Journal of Manufacturing Systems, Vol. 34, pp. 23–33.


