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Solving a Deterministic Multi-product Single-machine EPQ Model with Partial Backordering, Scrapped Products and Rework

Mehrnaz Najafi [a], Ali Ghodratnama [a,*] and Seyed Hamid Reza Pasandideh [a]

[a] *Department of Industrial Engineering, Faculty of Engineering, Kharazmi University, Tehran, Iran*

Abstract
In this paper, an economic production quantity (EPQ) inventory model with scrap and rework is developed. The inventory model is for multiple products and all products are manufactured in a single machine. Clearly, the existence of one machine results in limited production capacity and thus in shortages. Therefore, shortages are permitted and partially backordered. We show that the model of the problem is a constrained non-linear program and use the GAMS modelling language to solve it. Our objective is to minimize the joint total cost of the system and the supply cost of the warehouse space, subject to capacity, service level, and budget and warehouse space constraints. Subsequently, a nonlinear programming solver BARON is used to solve the model. At the end, a numerical example is provided to demonstrate the applicability of the model to real-world manufacturing problems. To verify the solution obtained and to evaluate the performance of MCDM (Multi-Criteria Decision Making) methods, a TUKEY test is employed to compare the means of the primary objective values, the mean values of the second objective, and the mean of the CPU time needed for solving the problem using various methods of MCDM. Also, to compare the methods, we used the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution). The results show that the Torabi-Hasini method is the most efficient method to solve the model and the solving capacities of the methods differ significantly. Finally, some conclusions and future research are discussed.

Keywords: Production modeling; Economic production quantity; Rework, Multi-product; Backordering; Scrap.

1. Introduction

The first economic production quantity (EPQ) inventory model for a single product-single stage manufacturing system was proposed by Taft (1918). It has limitations and cannot be regarded as a universal inventory model. One limitation is that the production system involved in the classical EPQ inventory model can never manufacture defective products during the production cycle. However, doubtless defective products would be made in each production cycle in most real world situations. Therefore, it is worthwhile to study the presence of defective products in inventory models. Owing to this truth, there has recently been a growing interest in dealing with this issue in EPQ models. According to Haji et al. (2009), it is clear that there are several real world situations in which imperfect quality products should be remanufactured or repaired with an extra cost. In this regard, academicians and researchers have investigated the effects of imperfect quality production, rework, and breakdown on EPQ inventory models. Although, there has recently been a considerable emphasis on the implementation of quality control practices in manufacturing systems, we all know that until today it is difficult to guarantee one manufacturing system is defect-free. Thus, always some products would need a rework process. As mentioned earlier, obviously, the presence of defective products is common in many practical manufacturing environments. To tackle this issue, several studies have focused on the development of EPQ-type inventory models involving defective products. Jamal et al. (2004) proposed an EPQ in which defective products from each production cycle are accumulated until *N* equal cycles. Then, during a rework cycle, defective products are

Correspondig author email address: ghodratn@ut.ac.ir

reworked. It is important to point out that all reworked products are considered as good as new. Hence, they are able to satisfy this demand. It should be highlighted that Jamal et al (2004)'s paper has some errors in the numerical examples. Cárdenas-Barrón (2007) corrected the solutions to examples in Jamal et al. (2004). Later, Cárdenas-Barrón (2008) derived Jamal et al. (2004)'s two inventory policies in a simple way. In a subsequent paper, Cárdenas-Barrón (2009) developed an EPQ with rework process and planned backorders. It would be helpful to note that previous researches focused on single-stage manufacturing systems. Yet, the multi-stage manufacturing system with rework consideration is also dealt with in the inventory literature. Several other researches on EPQ inventory models that considered different variants of imperfect production processes are Sarker et al. (2008), Chung et al. (2009), Liu et al. (2009), Roy et al. (2009), Wang and Tang (2009), Hu et al. (2010), Khan et al. (2011), Sadjadi et al. (2012), Cárdenas-Barrón et al. (2012), Wee et al. (2013), and Cárdenas-Barrón et al. (2013), to name just a few recent research works. According to Chiu et al. (2007), in real manufacturing systems imperfect quality products are inevitable, and also defective products may be reworked or repaired. Hence, the overall production/inventory costs involved can be reduced well if reworking is done. There are several studies on the case of rework. For example, Chan et al. (2003) presented a new EPQ model with pricing, rework and reject situations. In another study, Chiu and Chiu (2006) developed an EPQ model with imperfect quality, backordering and failure. In the same year, Islam and Roy constructed an EPQ model with flexibility and reliability considerations. Later, Hou (2007) developed an EPQ model with setup cost and process quality as functions of capital expenditure. Liao et al. (2009) investigated an integrated maintenance and production system for the EPQ model with imperfect repair and rework. More recently, Chiu et al. (2012) developed a multi-delivery policy into an imperfect EPQ model with partial rework. As regards multi-product single-machine systems, Haji et al. (2008) studied an imperfect manufacturing process with rework in which several products are manufactured on a unique machine. In a subsequent article, Haji et al. (2009) investigated the optimum batch production with rework subject to a constraint on accumulated defective products. Taleizadeh et al. (2010) introduced a multi-product single-machine production system with stochastic scrapped production rate, incomplete backordering and service level constraint. Later, Taleizadeh et al. (2013) presented an imperfect multi-product production system with rework.

Based on the extensive literature on the economic production quantity model, it is perspicuously observed that there is still a growing strong interest in the issue of imperfect production systems, which is a real-world manufacturing section concern. Novel research has also been performed to which readers may refer (White and Censlive, 2013; Taleizadeh et al., 2012; Jiang et al., 2015). For instance, Chiu et al. (2015) accounted for a simplified approach to the multi-item economic production quantity model with scrap, rework, and multi-delivery. Recently, researchers have investigated this issue with the myopic analysis. For example, Pasandideh et al. (2015) examined a multi-product single-machine economic production quantity model for an imperfect production system under warehouse construction cost. In anotehr study, Al-Salamah (2016) took into account economic production quantity in batch manufacturing with imperfect quality, imperfect inspection, and destructive and non-destructive acceptance sampling in a two-tier market. Biel and Glock (2016) investigated the use of waste heat in a two-stage production system with controllable production rates.

Manna et al. (2016) considered an EPQ model with promotional demand in random planning horizon and solved it by the population varying genetic algorithm approach. Moussawi-Haidar et al. (2016) regarded production lot sizing with quality screening and rework. Öztürk (2017) presented a note on "production lot sizing with quality screening and rework". Pal et al. (2016) investigated a three-layer supply chain epq model for price- and stock-dependent stochastic demand with imperfect item under rework. Shah et al. (2016) considered an EPQ model for returned/reworked inventories during the imperfect production process under price-sensitive stock-dependent demand. Taleizadeh (2016) studied pricing and lot sizing for an EPQ inventory model with rework and multiple shipments. Viji and Karthikeyan (2016) developed an EPQ model for three levels of production with Weibull distribution deterioration and shortage.

Tsao et al. (2017) provided an imperfect production model under Radio Frequency Identification adoption and trade credit. Abdel-Aleem, et al. (2017) investigated a surface response optimization model for an EPQ system with imperfect production process under rework and shortage. Aldurgam et al. (2017) considered a single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate. Bhunia et al. (2017) examined a partially integrated production-inventory model with interval valued inventory costs, variable demand, and flexible reliability.

Chan et al. (2017) built an integrated production-inventory model for deteriorating items with considering the optimal production rate and deterioration during delivery. Manna et al. (2017a) developed a two-layer green supply chain imperfect production inventory model under a bi-level credit period. Later, Manna et al. (2017b) offered an imperfect production inventory model with production rate dependent defective rate and advertisement dependent demand. De et al. (2018) investigated green logistics under an imperfect production system and solved their model by a rough age based multi-objective genetic algorithm approach. Fakher et al. (2018) integrated production, maintenance, and quality in a multi-period multi-product profit-maximization model. Sadeghi et al. (2018) contributed to devising a lagrangian relaxation approach for a fuzzy random EPQ problem with shortages and redundancy allocation. Shaikh accounted for closed-form solutions for the EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. The rest of this paper is organized as follows. Section 2 presents the problem definition and notations. In section 3 the model depiction, the derivation procedure, and the mathematical model are presented.

Section 4 contains the complete and analytic solution procedure to locate and insure the optimal solutions to the EPQ inventory model proposed. In section 5, a numerical example is solved and the results are compared. Finally, the conclusion and some suggestions for further research come in Section 6.

2. Problem definition

2.1. Assumptions

- A system with imperfect production processes is considered.
- Defective items of n different kinds, at a rate, u_i; $i = 1, 2, \ldots, n$ are generated per cycle, and among these products, a sc_iportion is taken into account to be scrap and the other portion can be reworked.
- It is assumed that the production rate of the i-th item is p_i per cycle, and the quantity of the good items produced, that satisfies a corresponding demand, d_i, in every cycle.
- The production rate is constant and known.
- All of the parameters are considered constant in every cycle.
- All of the products are manufactured on one machine and the capacity is finite.
- A constant cycle length for all items is considered, i.e. $T_1 = T_2 = \cdots = T_n = T$.
- We assume that the entire quantity of imperfect quality items will be reworked, and a $m_i$portion as scrap will be left at the end of the rework period.
- The manufacturer uses the same resource for production and rework processes at the same time.
- For the common production system, budget and capacity are finite and a fraction of the shortage is backordered.
- In this research, the basic assumption of the EPQ inventory model with the rework process is that the rate of production minus defectives must always be larger than or equal to the demand.

2.2. The mathematical model

2.2.1. Parameters

d_i The demand rate of the i-th product,
p_i The production rate of the i-th product,
u_i The proportion of produced defective items of the i-th product,
v_i The proportion of shortage of the i-th product in the cycle backordered,
A_i The setup cost of a production run for the i-th product,
ts_iThe machine setup time to produce the i-th product,
o_iThe supply cost of per unit of storage space,
f_i The space occupied by each unit of product i,
c_iThe production cost of the i-th product per item,
k_i The rework cost of the i-th product per item,
h_i The holding cost of the i-th product per item per unit time,
b_i The backordered cost of the i-th product per item per unit time,
l_i The lost sale cost of the i-th product per item per unit time,
sc_i The proportion of scrap of the i-th product in production,
m_i The proportion of scrap of the i-th product in rework,
SL The safety factor of total allowable shortages,
W The total available budget per period,
H_i The maximum level of inventory of the i-th product when the regular production process stops,
H_i^{max} The maximum level of on-hand inventory of the i-th product when ends,
q_i^s The production lot size of the i-th product in each cycle (for $i = 1,2,\ldots,n$).

2.2.2. Decision variables

T The cycle length,
N Number of cycles per year, Cl_i
X_iThe continuous random variable represents the storage area of product i,
s_iThe total shortage quantity of the i-th product in a cycle.

3. Modeling

The production cycle length is the sum of the production up times for the good and defective items, t_i^1 and t_i^5 , respectively, the reworking time, t_i^2, and the production down times, t_i^3 and t_i^4 . Therefore, one has a total production cycle length of:

$$T = \sum_{j=1}^{5} t_i^j \tag{1}$$

Since all products are manufactured on a single machine with a limited capacity, the cycle length for all products is shown in Figure 1. Therefore, we have:

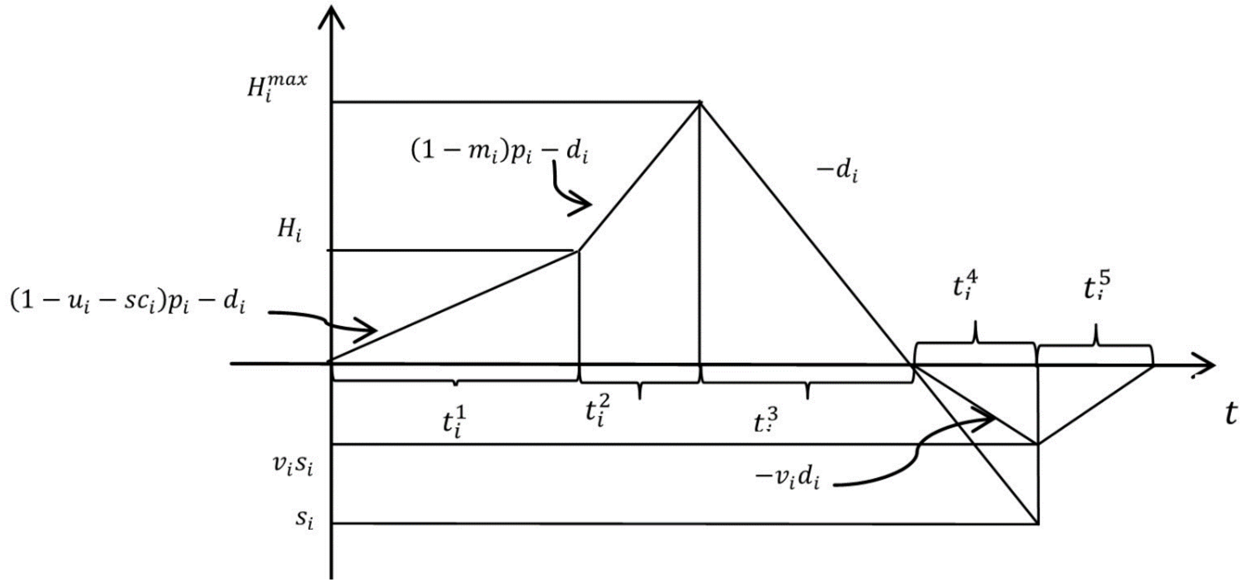

Figure 1. The on-hand inventory for perfect quality items

$$t_i^1 = \frac{q_i^s}{p_i} - \frac{v_i s_i}{(1-u_i-sc_i)p_i - d_i} \tag{2}$$

$$t_i^2 = u_i \frac{q_i^s}{p_i} \tag{3}$$

$$t_i^3 = \frac{H_i^{max}}{d_i} = \left(\frac{(1-sc_i-m_iu_i)}{d_i} - \frac{(1+u_i)}{p_i}\right) q_i^s - \frac{v_i s_i}{d_i} \tag{4}$$

$$t_i^4 = \frac{s_i}{d_i} \tag{5}$$

$$t_i^5 = \frac{v_i s_i}{(1-u_i-sc_i)p_i - d_i} \tag{6}$$

$$H_i = \big((1-u_i-sc_i)p_i - d_i\big)\frac{q_i^s}{p_i} - v_i s_i \tag{7}$$

$$H_i^{max} = H_i + u_i\big((1-m_i)p_i - d_i\big)\frac{q_i^s}{p_i} \tag{8}$$

$$H_i^{max} = ((1-sc_i-m_iu_i)p_i - (1+u_i)d_i)\frac{q_i^s}{p_i} - v_i s_i \tag{9}$$

Hence, from Eq. (1), the cycle length for a single product is:

$$T = \frac{(1-v_i)s_i + (1-sc_i-m_iu_i)q_i^s}{d_i} \tag{10}$$

$$q_i^s = \frac{Td_i - (1-v_i)s_i}{(1-sc_i-m_iu_i)} \tag{11}$$

3.1. The total cost function

The total cost function of the model becomes:

$$TC = CA + CP + CR + CH + CB + CL \tag{12}$$

3.1.1. Setup cost

The cost of a setup is A_i which occurs N times per year. So, the annual setup cost will be:

$$CA = \sum_{i=1}^{n} NA_i \tag{13}$$

$$N = \frac{1}{T} \tag{14}$$

3.1.2. Production cost

The total production cost is the summation of the product of production cost per unit and the quantity per period for all *i*-th products, which are c_i and q_i^s, respectively. The annual production cost is:

$$CP = \frac{1}{T}\sum_{i=1}^{n} c_i\, q_i^s \tag{15}$$

3.1.3. Rework cost

The annual rework cost is the summation of the product of the rework cost per unit of the *i*-th product and the quantity of the *i*-th product that is to be reworked, which are k_i and $u_i q_i^s$, respectively.The annual rework cost is obtained by multiplying the total rework cost by *N*. The cost for this joint policy will be:

$$CR = \frac{1}{T}\sum_{i=1}^{n} k_i u_i\, q_i^s \tag{16}$$

3.1.4. Holding cost

From Figure 1, the holding costs of the inventory system in independent and joint production policies are shown in Eq.(17):

$$CH = \frac{1}{T}\sum_{i=1}^{n} h_i \left[\frac{H_i}{2}(t_i^1) + \frac{H_i + H_i^{max}}{2}(t_i^2) + \frac{H_i^{max}}{2}(t_i^3)\right] \tag{17}$$

3.1.5. Backorder cost

Based on Fig.1, the backordered and lost sale costs per cycle are shown in (18) and (19), respectively.

$$CB = \frac{1}{2T}\sum_{i=1}^{n} b_i v_i s_i\,(t_i^4 + t_i^5) \tag{18}$$

3.1.6. Lost sale cost

$$CL = \frac{1}{2T}\sum_{i=1}^{n} l_i(1 - v_i)s_i \tag{19}$$

As a result, the objective function of the model becomes:

$$\begin{aligned} TC &= CA + CP + CR + CH + CB + CL \\ &= \frac{1}{T}\sum_{i=1}^{n} A_i + \frac{1}{T}\sum_{i=1}^{n} c_i\, q_i^s + \frac{1}{T}\sum_{i=1}^{n} k_i u_i\, q_i^s \\ &\quad + \frac{1}{T}\sum_{i=1}^{n} h_i\left[\frac{H_i}{2}(t_i^1) + \frac{H_i + H_i^{max}}{2}(t_i^2) + \frac{H_i^{max}}{2}(t_i^3)\right] \\ &\quad + \frac{1}{2T}\sum_{i=1}^{n} b_i v_i s_i\left(t_i^4 + t_i^5\right) \\ &\quad + \frac{1}{2T}\sum_{i=1}^{n} l_i(1 - v_i)s_i \end{aligned} \tag{20}$$

3.2. The supply cost of the warehouse space

The supply cost of the warehouse space is the summation of the product of the supply cost of per unit of storage space and the continuous random variable representing the storage area of product *i*, which are o_i and X_i , respectively.

$$G = \sum_{i=1}^{n} o_i X_i \tag{21}$$

3.3. The constraints

3.3.1. Capacity constraint

In the joint production systems having rework, the overall production, rework, and setup times ought to be smaller than the cycle length. In our problem, $\sum_{i=1}^{n}(t_i^1 + t_i^2 + t_i^5) + \sum_{i=1}^{n} ts_i$ must be less than or equal to T. Hence, the model with capacity constraint is:

$$\sum_{i=1}^{n}(t_i^1 + t_i^2 + t_i^5) + \sum_{i=1}^{n} ts_i \le T \tag{22}$$

From Eqs. (2), (3) and (6), the capacity constraint model becomes:

$$\sum_{i=1}^{n}(1+u_i)\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)p_i} + \sum_{i=1}^{n} ts_i \le T \tag{23}$$

3.3.2. Budget constraint

Since the production quantity is q_i^s , the total available budget is W and $u_i q_i^s$is the number of the *i*-th product which needs reworking .The budget constraint then becomes:

$$\sum_{i=1}^{n}(c_i q_i^s + k_i u_i q_i^s) \le W \tag{24}$$

3.3.3. Service level constraint

For the service level constraint, the *i*-th product shortage quantity per period, the annual demand of the *i*-th product, the number of periods in every year, and the factor of safety of allowable shortage are s_i, d_i , N, and SL, severally. With this, the service level constraint becomes:

$$\sum_{i=1}^{n}\frac{s_i}{Td_i} \le SL \tag{25}$$

3.3.4. Warehouse-space constraint

The space of the warehouse to store the products is limited

$$f_i(\left((1-sc_i - m_i u_i)p_i - (1+u_i)d_i\right)\frac{q_i^s}{p_i} - v_i s_i) \le X_i \tag{26}$$

3.4. The final model

$$Min\ Z = \frac{1}{T}\sum_{i=1}^{n} A_i + \frac{1}{T}\sum_{i=1}^{n} c_i \frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)} + \frac{1}{T}\sum_{i=1}^{n} k_i u_i \frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)}$$

$$+\frac{1}{T}\sum_{i=1}^{n} h_i \left[\begin{array}{c} \frac{1}{2}\left(\left((1-u_i - sc_i)p_i - d_i\right)\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)p_i} - v_i s_i\right)\left(\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)p_i} - \frac{v_i s_i}{(1-u_i - sc_i)p_i - d_i}\right) + \\ \left(\left((1-0.5u_i - sc_i - 0.5m_i u_i) - (1+0.5u_i)\frac{d_i}{p_i}\right)\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)} - v_i s_i\right)\left(u_i \frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)p_i}\right) + \\ \frac{1}{2d_i}(\left((1-sc_i - m_i u_i)p_i - (1+u_i)d_i\right)\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)p_i} - v_i s_i)^2 \end{array}\right]$$

$$+\frac{1}{2T}\sum_{i=1}^{n} b_i v_i s_i \left(\frac{s_i}{d_i} + \frac{v_i s_i}{(1-u_i - sc_i)p_i - d_i}\right) + \frac{1}{2T}\sum_{i=1}^{n} l_i (1-v_i) s_i$$

$$Min\ G = \sum_{i=1}^{n} o_i X_i$$

s.t.

$$\sum_{i=1}^{n}(1+u_i)\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)p_i} + \sum_{i=1}^{n} ts_i \le T$$

$$\sum_{i=1}^{n}(c_i + k_i u_i)\frac{Td_i - (1-v_i)s_i}{(1-sc_i - m_i u_i)} \le W$$

$$\sum_{i=1}^{n} \frac{s_i}{Td_i} \le SL$$

$$f_i\left(\left((1 - sc_i - m_i u_i)p_i - (1 + u_i)d_i\right)\frac{Td_i - (1 - v_i)s_i}{(1 - sc_i - m_i u_i)p_i} - v_i s_i\right) \le X_i \tag{27}$$

4. The Solution approach

The model is solved using the Gams software with seven methods of MCDM that are explained below.

4.1. Solution methods

If the problem is as follows:

$$\begin{aligned} & max\ f_1\ (x) \\ & max\ f_2\ (x) \\ & \vdots \\ & max\ f_k\ (x) \\ & s.t. \\ & \underline{X} \in \mathrm{X} \end{aligned} \tag{28}$$

to solve it we use the following methods.

4.1.1. Lp metric

First, according to the method of optimizing, optimal values of f_j objective function (f_j^*) are obtained.

$$\begin{aligned} & Min\ Dp = \left(\sum_{j=1}^{k}\left(\frac{f_j^* - f_j(x)}{f_j^*}\right)^p\right)^{1/p} \\ & s.t. \\ & \underline{X} \in \mathrm{X} \end{aligned} \tag{29}$$

4.1.1. Lp (p=1) method

$$\begin{aligned} Min\ Dp = & \left(\frac{f_1^* - f_1(x)}{f_1^*}\right) + \left(\frac{f_2^* - f_2(x)}{f_2^*}\right) \\ & + \cdots + \left(\frac{f_k^* - f_k(x)}{f_k^*}\right) \\ & s.t. \\ & \underline{X} \in \mathrm{X} \end{aligned} \tag{30}$$

4.1.2. Lp(p=∞) method

$$\begin{aligned} & Min\ Dp = \max\left\{\begin{matrix} \left(\frac{f_1^* - f_1(x)}{f_1^*}\right), \\ \left(\frac{f_2^* - f_2(x)}{f_2^*}\right), \dots, \left(\frac{f_k^* - f_k(x)}{f_k^*}\right) \end{matrix}\right\} \\ & s.t. \\ & \underline{X} \in \mathrm{X} \end{aligned} \tag{31}$$

4.1.3. BOM

In the BOM (Bounded Objective Method), the most important objective is in the objective function:

$$\begin{aligned} & \mathrm{Max}\ f_r(x) \\ & s.t. \\ & m_j \le f_j(x) \le M_j\ \ , j = 1, \dots, k\ , j \ne r \\ & \underline{X} \in \mathrm{X} \end{aligned} \tag{32}$$

4.1.4. GP method

In the GP (Goal Programming), the objective function is always minimization:

$$\begin{aligned} & min \sum_{i=1}^{k} h_i\ (d_i^+, d_i^-) \\ & \textstyle\sum_{i=1}^{k} h_i\ (d_i^+, d_i^-) = \{d_i^+\ , d_i^-, d_i^+ + d_i^-\} \\ & s.t. \\ & f_i + d_i^- - d_i^+ = b_i \\ & d_i^+ * d_i^- = 0 \end{aligned} \tag{33}$$

$d_i^+, d_i^- \geq 0$
$\underline{X} \in X$

Adverse deviation in the objective function and the objective in the restrictions are placed. When the goal type is as follows:
$f_i \leq b_i$, variable of deviation from the ideal that should be minimized is d_i^+.

4.1.5. GA method

In the GA (Goal Attainment) method, the maximum deviation from the ideal is minimized.

$$\begin{aligned} &\min Z \\ &s.t. \\ &f_j + w_j Z \geq b_j \quad j = 1, \ldots, k \\ &\underline{X} \in X \\ &Z \quad free \end{aligned} \tag{34}$$

4.1.6. TH method

Determine the α-positive ideal solution (α-PIS) and α-negative ideal solution (α-NIS) for each objective function and α-feasibility level. To obtain the α-positive ideal solutions, i.e., $(W_1^{\alpha-\mathrm{PIS}}, x_1^{\alpha-\mathrm{PIS}})$ and $(W_2^{\alpha-\mathrm{PIS}}, x_2^{\alpha-\mathrm{PIS}})$, the model should be solved for each objective function separately, and then the α-negative ideal solution for each objective function can be estimated as follows:

$$\begin{aligned} &W_1^{\alpha-NIS} = W_1(x_2^{\alpha-PIS}), \\ &W_2^{\alpha-NIS} = W_2(x_1^{\alpha-PIS}) \end{aligned} \tag{35}$$

Determine a linear membership function for every objective function as follows:

$$\mu_1(x) = \begin{cases} 1 & \text{if} W_1 \leq W_1^{\alpha-\mathrm{PIS}} \\ \frac{W_1^{\alpha-NIS} - W_1}{W_1^{\alpha-NIS} - W_1^{\alpha-PIS}} & \text{if} \quad W_1^{\alpha-\mathrm{PIS}} \leq W_1 \leq W_1^{\alpha-\mathrm{NIS}} \\ 0 & \text{if} \quad W_1 > W_1^{\alpha-NIS} \end{cases} \tag{36}$$

$$\mu_2(x) = \begin{cases} 1 & \text{if} W_2 \leq W_2^{\alpha-\mathrm{PIS}} \\ \frac{W_2^{\alpha-NIS} - W_2}{W_2^{\alpha-NIS} - W_2^{\alpha-PIS}} & \text{if} \quad W_2^{\alpha-\mathrm{PIS}} \leq W_2 \leq W_2^{\alpha-\mathrm{NIS}} \\ 0 & \text{if} \quad W_2 > W_2^{\alpha-NIS} \end{cases} \tag{37}$$

where $\mu_h(x)$ denotes the satisfaction degree of the h-th objective function.
Convert the multi-objective model into a single-objective MILP model using Torabi and Hassini (2008)'s method. It should be noted that both of these methods ensure obtaining the efficient answer. The TH aggregation function is as follows:

$$\begin{aligned} &\max \quad \lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_h \theta_h \mu_h(x) \\ &\text{s.t.} \\ &\lambda_0 \leq \mu_h(x) \ , \quad h = 1\,,2 \\ &x \in F(x), \ \lambda_0 \ \text{and} \ \lambda \in [0,1] \end{aligned} \tag{38}$$

where $F(x)$ denotes the feasible region involving the constraints of the equivalent crisp model. Also, $\theta_h$and γ denote the importance of the h-th objective function and the coefficient of compensation, respectively. Notably, the optimal value of variable $\lambda_0 = min_h\{\mu_h(x)\}$ indicates the minimum satisfaction degree of objective functions and the TH aggregation function actually looks for a compromise value between the min operator and the weighted sum operator based on the value of γ. In other words, the decision makers can obtain both balanced and unbalanced compromised solutions via manipulating the value of parameters θ_h and γ, based on their preferences.

4.1.7. FGP method

We propose to use Tiwari et al. (1987)'s weighted additive approach. With this approach, the weighted sum of the achievement levels of fuzzy goals is maximized. Formulation of the weighted additive model is as follows:

$$\begin{aligned} &\max \sum_h w_h \, \mu_h(x) \\ &\text{s.t.} \\ &\qquad\qquad \mu_h(x) \in [0,1] \ , \forall h \end{aligned} \tag{39}$$

$$x \geq 0$$

where w_h denotes the weight of the i-th goal. As can be seen, the weighted additive approach allows for assigning different weights to the individual goals in the simple additive fuzzy achievement function to reflect their relative importance levels.

4.2. Comparison of the methods

To verify the solution obtained and to evaluate the performance of MCDM (Multi Criteria Decision Making) methods, a TUKEY test is employed to compare the means of the first objective value, the means of the second objective values, and and the mean of the CPU time needed for solving the problem using various methods of MCDM. Moreover, to compare the methods we used the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution).

Multiple Comparison Tests

If we reject the null hypothesis, *Ho:* $\mu_1 = \mu_2 = \mu_3 = ... = \mu_k$, we usually want to know where inequalities exist among different *k* means. This idea is analogous to that of subdividing chi-square tables. Many methods are available to detect differences among individual means:

1. Tukey test
2. Student-Newman-Keul (SNK) test
3. Dunnett's test
4. Scheffe's Multiple Contrast.

Multiple comparison tests have the same assumptions of ANOVA: normality and homogeneity of variance. Though these tests are somewhat robust, nonparametric multiple comparison tests exist if the assumptions are seriously violated. All multiple comparison tests work best if sample sizes are equal.

4.2.1. Tukey test

We will look at one of the multiple comparison tests. The Tukey test is probably the most "conservative" multiple comparison test. It tests the two-tailed null hypothesis, *Ho:* $\mu_a = \mu_b$, where *a* & *b* represent all possible combinations of *k* sample means. Here is the procedure:

1. Compute the standard error:

a. For equal *n*: $s_{\bar{x}} = \sqrt{\frac{s_p^2}{n}} = \sqrt{\frac{MSE}{n}}$ (40)

b. For unequal n: $s_{\bar{x}} = \sqrt{\frac{s_p^2}{2}(\frac{1}{n_a} + \frac{1}{n_b})}$ (41)

2. Rank the sample means from the lowest to the highest.
3. Compare $|\bar{X}_a - \bar{X}_b|$ to $q_{\alpha,v,k} * s_{\bar{x}}$ and reject equality of means if $|\bar{X}_a - \bar{X}_b| \geq q_{\alpha,v,k} * s_{\bar{x}}$

An equivalent expression is to reject *Ho:*$\mu_a = \mu_b$ *if* $\frac{|\bar{X}_a - \bar{X}_b|}{s_{\bar{x}}} \geq q_{\alpha,v,k}$ (see Table 13 for critical q-values). Note that the proper procedure is to compare the largest mean against the smallest, and then proceed to compare the next to the smallest mean to the smallest. If any non-significant differences are detected between two means, then no comparisons are made for other means within the interval.

4.2.2. The TOPSIS method

The TOPSIS method consists of the following steps:

(1) Design a set of attributes
$C = \{C_j | j = 1,..,n\}$;

(2) Generate a set of possible alternatives $X = \{X_i | i = 1,..,m\}$;

(3) Construct the decision matrix $L = [a_{ij}]_{m \times n}$, where a_{ij} is the rating of alternative X_i with respect to attribute C_j ;

(4) Decision maker elicits weights for attribute C_j as w_j , where $0 < w_j < 1$, $j = 1,..,n$ and

$$\sum_{j=1}^{n} w_j = 1 ; \tag{42}$$

(5) The Normalized decision matrix is constructed by

$$r_{ij} = X_{ij} / (\sum_{i=1}^{m} X_{ij}{}^2) \tag{43}$$

(6) The weighted normalized decision matrix is constructed by

$$V_{ij} = w_j r_{ij} \tag{44}$$

(7) Positive ideal and negative ideal solutions are determined by

Positive Ideal solution:

$A^* = \{v_1^*, \dots, v_n^*\}$ Where

$$V_j^* = \begin{Bmatrix} max_i(V_{ij}) \ if \ j \in J \ ; \\ min_i(V_{ij}) \ if \ j \in J' \end{Bmatrix} \tag{45}$$

Negative ideal solution:

$A' = \{v_1^*, \dots, v_n^*\}$ Where

$$V_j^* = \begin{Bmatrix} min_i(V_{ij}) \ if \ j \in J \ ; \\ max_i(V_{ij}) \ if \ j \in J' \end{Bmatrix} \tag{46}$$

(8) Separation measures for each alternative are calculated by

$$S_i^* = \left[\sum_j (v_j^* - v_{ij})^2 \right]^{1/2} \quad i = 1, \dots, m \tag{47}$$

$$S_i' = \left[\sum_j (v_j' - v_{ij})^2 \right]^{1/2} \quad i = 1, \dots, m \tag{48}$$

(9) Relative closeness to the ideal solution C_i^*

$$C_i^* = \frac{S_i'}{(S_i^* + S_i')} \quad , \quad 0 < C_i^* < 1 \tag{49}$$

(10) Rank all alternatives according to the closeness coefficient and select the best one.

5. Computational results

The presented model is solved by the GAMS 23.6 software. The model is run with a computer of 2.40 GHz and 4.00 GB capability. All of the test problems are generated at random and then solved.

5.1. Input parameters

This section presents the input parameters given in Tables 1 and 2 introducing parameters related to each problem.

Table 1. General data of the Example

Problem	***i***	***d***	***p***	***u***	***v***	***A***	***ts***	***c***	***f***
1	10	(150,850)	(5000,12000)	(0.05,0.25)	(0.5,0.7)	(500,1900)	(0.0025,0.0045)	(6,34)	(2,4)
2	8	(150,800)	(5000,13000)	(0.05,0.35)	(0.5,0.75)	(500,1950)	(0.0025,0.0049)	(6,40)	(2,5)
3	6	(150,790)	(5000,12000)	(0.05,0.25)	(0.5,0.7)	(800,1900)	(0.0025,0.0045)	(9,34)	(2.5,4)
4	4	(250,750)	(5000,15000)	(0.05,0.25)	(0.5,0.7)	(500,1500)	(0.0025,0.0045)	(13,34)	(2,5.5)
5	5	(350,850)	(5000,11000)	(0.08,0.25)	(0.5,0.7)	(500,1300)	(0.0025,0.0055)	(8,33)	(2.7,4)
6	6	(150,1150)	(5000,17000)	(0.05,0.25)	(0.5,0.78)	(560,1600)	(0.0015,0.0049)	(17,35)	(2,4.8)
7	7	(150,1450)	(5000,17000)	(0.05,0.25)	(0.5,0.7)	(530,1600)	(0.0025,0.0045)	(8,34)	(2.8,4.9)
8	9	(150,850)	(5000,12000)	(0.05,0.25)	(0.5,0.7)	(500,1900)	(0.0025,0.0045)	(6,34)	(2,4)
9	10	(150,850)	(5600,15500)	(0.05,0.25)	(0.5,0.7)	(500,1760)	(0.0025,0.0045)	(6,34)	(2,6.7)
10	8	(150,850)	(4000,13000)	(0.05,0.25)	(0.5,0.7)	(500,1540)	(0.0025,0.0045)	(8,34)	(1.3,4)

Table 2. General data of the Example

Problem	***i***	***o***	***k***	***h***	***b***	***l***	***sc***	***m***	***SL***	***W***
1	10	(2,4)	(1,15)	(2,30)	(5,33)	(1,29)	(0.01,0.02)	(0.01,0.015)	0.9	400000

2	8	(2,6)	(1,25)	(2,35)	(5,40)	(1,39)	(0.01,0.028)	(0.01,0.018)	0.93	410000
3	6	(2.4,4)	(6,15)	(5,30)	(5,33)	(1,29)	(0.0104,0.02)	(0.0107,0.015)	0.95	420000
4	4	(1.5,4)	(1,15)	(12,30)	(12,33)	(1,35)	(0.01,0.02)	(0.01,0.015)	0.91	430000
5	5	(2.3,4)	(6,15)	(4,30)	(7,35)	(1,23)	(0.016,0.025)	(0.01,0.023)	0.92	440000
6	6	(2.1,4.3)	(6,22)	(3,27)	(6,31)	(1,26)	(0.01,0.027)	(0.01,0.017)	0.94	450000
7	7	(2,4.5)	(2,18)	(5,33)	(8,38)	(7,29)	(0.01,0.02)	(0.01,0.015)	0.96	460000
8	9	(2,4)	(1,15)	(2,30)	(5,33)	(1,29)	(0.01,0.02)	(0.01,0.015)	0.9	405000
9	10	(2.4,4)	(1,15)	(2,30)	(5,33)	(1,26)	(0.01,0.02)	(0.01,0.015)	0.98	430000
10	8	(2,6.4)	(1,15)	(4,30)	(7,33)	(1,27)	(0.01,0.02)	(0.01,0.015)	0.94	500000

5.2. The Tukey test

In this section, the results obtained by the application of the GAMS 23.6 software are compared with the Tukey test:

Table 3. The differences of the methods

Difference	Z	G	CPU time
lp(1), lp(∞)	29844.58	544.9557	14.6155
lp(1),BOM	105539.8	17013.94	14.4305
lp(1),GP	102704.4	9205.698	8.7632
lp(1),GA	96458.65	5673.766	14.4222
lp(1),FGP	87351.37	4000.44	14.6733
lp(1),TH	85965.68	3735.314	13.8073
lp(∞) ,BOM	75695.25	16468.99	0.185
lp(∞),GP	72859.78	8660.742	5.8523
lp(∞),GA	66614.06	5128.81	0.1933
lp(∞),FGP	57506.79	3455.484	0.0578
lp(∞),TH	56121.1	3190.358	0.8082
BOM,GP	2835.47	7808.246	5.6673
BOM,GA	9081.192	11340.18	0.0083
BOM,FGP	18188.47	13013.5	0.2428
BOM,TH	19574.16	13278.63	0.6232
GP,GA	6245.721	3531.932	5.659
FGP,TH	1385.691	265.1264	0.866
FGP,GA	9107.274	1673.326	0.2511
FGP,GP	15353	5205.258	5.9101
TH,GA	10492.97	1938.452	0.6149
TH,GP	16738.69	5470.384	5.0441

$\alpha = 0.05$, $k = 7$, $n = 10$, $MSE = 10.7$, $df = N - k = 70 - 7 = 63$. Table 13 shows (the q-th critical values for q corresponding to alpha.

Table 4. The critical values of q corresponding to alpha =0.05 (top) and alpha =0.01 (bottom)

Df for Error Term	*k*= Number of Treatments								
	2	3	4	5	6	7	3	9	10
5	3.64 5.70	4.60 6.98	5.22 7.80	5.67 8.42	6.03 8.91	6.33 9.32	6.58 9.67	6.80 9.97	6.99 10.24

6	3.46 5.24	4.34 6.33	4.90 7.03	5.30 7.56	5.63 7.97	5.90 8.32	6.12 8.61	6.32 8.87	6.49 9.10
7	3.34 4.95	4.16 5.92	4.68 6.54	5.06 7.01	5.36 7.37	5.61 7.68	5.82 7.94	6.00 8.17	6.16 8.37
8	3.26 4.75	4.04 5.64	4.53 6.20	4.89 6.62	5.17 6.96	5.40 7.24	5.60 7.47	5.77 7.68	5.92 7.86
9	3.20 4.60	3.95 5.43	4.41 5.96	4.76 6.35	5.02 6.66	5.24 6.91	5.43 7.13	5.59 7.33	5.74 7.49
10	3.15 4.48	3.88 5.27	4.33 5.77	4.65 6.14	4.91 6.43	5.12 6.67	5.30 6.87	5.46 7.05	5.60 7.21
11	3.11 4.39	3.82 5.15	4.26 5.62	4.57 5.97	4.82 6.25	5.03 6.48	5.20 6.67	5.35 6.84	5.49 6.99
12	3.08 4.32	3.77 5.05	4.20 5.50	4.51 5.84	4.75 6.10	4.95 6.32	5.12 6.51	5.27 6.67	5.39 6.81
13	3.06 4.26	3.73 4.96	4.15 5.40	4.45 5.73	4.69 5.98	4.88 6.19	5.05 6.37	5.19 6.53	5.32 6.67
14	3.03 4.21	3.70 4.89	4.11 5.32	4.41 5.63	4.64 5.88	4.83 6.08	4.99 6.26	5.13 6.41	5.25 6.54
15	3.01 4.17	3.67 4.84	4.08 5.25	4.37 5.56	4.59 5.80	4.78 5.99	4.94 6.16	5.08 6.31	5.20 6.44
16	3.00 4.13	3.65 4.79	4.05 5.19	4.33 5.49	4.56 5.72	4.74 5.92	4.90 6.08	5.03 6.22	5.15 6.35
17	2.98 4.10	3.63 4.74	4.02 5.14	4.30 5.43	4.52 5.66	4.70 5.85	4.86 6.01	4.99 6.15	5.11 6.27
18	2.97 4.07	3.61 4.70	4.00 5.09	4.28 5.38	4.49 5.60	4.67 5.79	4.82 5.94	4.96 6.08	5.07 6.20
19	2.96 4.05	3.59 4.67	3.98 5.05	4.25 5.33	4.47 5.55	4.65 5.73	4.79 5.89	4.92 6.02	5.04 6.14
20	2.95 4.02	3.58 4.64	3.96 5.02	4.23 5.29	4.45 5.51	4.62 5.69	4.77 5.84	4.90 5.97	5.01 6.09
24	2.92 3.96	3.53 4.55	3.90 4.91	4.17 5.17	4.37 5.37	4.54 5.54	4.68 5.69	4.81 5.81	4.92 5.92
30	2.89 3.89	3.49 4.45	3.85 4.80	4.10 5.05	4.30 5.24	4.46 5.40	4.60 5.54	4.72 5.65	4.82 5.76
40	2.86 3.82	3.44 4.37	3.79 4.70	4.04 4.93	4.23 5.11	4.39 5.26	4.52 5.39	4.63 5.50	4.73 5.60
60	2.83 3.76	3.40 4.28	3.74 4.59	3.98 4.82	4.16 4.99	4.31 5.13	4.44 5.25	4.55 5.36	4.65 5.45
120	2.80 3.70	3.36 4.20	3.68 4.50	3.92 4.71	4.10 4.87	4.24 5.01	4.36 5.12	4.47 5.21	4.56 5.30
infinity	2.77 3.64	3.31 4.12	3.63 4.40	3.86 4.60	4.03 4.76	4.17 4.88	4.29 4.99	4.39 5.08	4.47 5.16

$q_{\alpha,v,k} * s_{\bar{x}}$ =29342.167
Our final result is:

Table 5. Ranking (Z)

BOM	GP	GA	FGP	TH	lp(p=∞)	lp(p=1)
89316.359	92151.83	98397.551	107504.8	108890.52	165011.6134	194856.2

We can write it as:
BOM=GP=GA=FGP=TH ≠ lp(p=∞)=lp(p=1)
$q_{\alpha,\mathrm{v,k}} * s_{\bar{\mathrm{x}}}$ =5861.50254
Our final result is:

Table 6. Ranking (S)

lp(p=1)	lp(p=∞)	TH	FGP	GA	GP	BOM
1610.6648	2155.6205	5345.9783	5611.105	7284.4307	10816.3625	18624.609

We can write it as:
lp(p=1)=lp(p=∞)≠GP≠BOM
$q_{\alpha,\mathrm{v,k}} * s_{\bar{\mathrm{x}}}$ =7.20868601
Our final result is:

Table 7. Ranking (time)

FGP	lp(p=∞)	BOM	GA	TH	GP	lp(p=1)
1.3235	1.3813	1.5663	1.5746	2.1895	7.2336	15.9968

We can write it as:
FGP=lp(p=∞)=BOM=GA=TH=GP ≠lp(p=1)

5.3. TOPSIS

In this section, the TOPSIS is used to compare the methods.Tables 3 and 8-11 show the decision matrix, the normalized decision matrix, the weighted normalized decision matrix, ideal solutions, and separation measures for the numerical example, respectively. At the end, we calculate the closeness coefficient of each alternative in Table 12. In the third column of the table is the ranking order of alternatives.

Table 8. The decision matrix and weights of five attributes of the Example

	lp(p=1)	lp(p=∞)	BOM	GP	GA	FGP	TH	Weight
Average(Z)	194856.2	165011.61	89316.3594	92151.83	98397.551	107504.8254	108890.52	0.5
Average(S)	1610.6648	2155.6205	18624.6085	10816.36	7284.4307	5611.1047	5345.9783	0.4
Average(time)	15.9968	1.3813	1.5663	7.2336	1.5746	1.3235	2.1895	0.1

Table 9. The normalized decision matrix

0.575190947	0.487093496	0.263650642	0.272020592	0.29045718	0.317340702	0.321431087
0.06663623	0.089182072	0.770535059	0.447493246	0.30137059	0.232141948	0.22117317
8.92E-01	7.70E-02	0.087339558	0.403357868	0.08780238	0.073800616	0.122090253

Table 10. The weighted normalized decision matrix

0.287595474	0.243546748	0.131825321	0.136010296	0.14522859	0.158670351	0.160715544
0.026654492	0.035672829	0.308214024	0.178997299	0.12054824	0.092856779	0.088469268
8.92E-02	7.70E-03	0.008733956	0.040335787	0.00878024	0.007380062	0.012209025

Table 11. Ideal solutions

Attribute	Positive Ideal solution(PIS)	Negative ideal solution(NIS)
Z	0.131825321	0.287595474
G	0.026654492	0.308214024
Time	0.00738	8.92E-02

Table 12. Separation measures

Alternative Separation measure	lp(p=1)	lp(p=∞)	BOM	GP	GA	FGP	TH
d_i^+	0.175951662	0.112085287	0.281562787	0.155922816	0.094855907	0.071438074	0.068403438
d_i^-	0.281559532	0.28785587	0.175326173	0.205092237	0.248906124	0.263998252	0.26516791

Table 13. The Closeness coefficient and ranking

Alternatives	Cl_i	Ranking
TH	0.79493611	1
FGP	0.787029404	2
GA	0.7240652	3
lp(p=∞)	0.719745555	4
lp(p=1)	0.615415613	5
GP	0.568098852	6
BOM	0.383739133	7

6. Conclusions

Since in real world situations defective products are manufactured by imperfect production systems, there has been widespread research into imperfect quality products and imperfect production processes in recent years. In this regard, this study embarked on building an inventory that considers the presence of defective products where some of them are scrapped and the others are reworked.

In this study, an EPQ model with partial backordering, rework, and scrap was considered. The problem was formulated in a multi-objective nonlinear programming framework, where the goal was to find the optimal production period, order quantity, and backorder quantities so that both the joint total cost of the system and the supply cost of the warehouse space, subject to capacity, service level, budget and warehouse space constraints, are minimized.

For future research, we suggest developing inventory models with scrap, rework and backordering for the following cases:

- ✓ Multi products in multi-machine system,
- ✓ Multi products multi-stage manufacturing system,
- ✓ Multi products multi-stage with multiple constraints, i.e. number of runs, space, other limitations,
- ✓ Considering fuzzy or stochastic parameters.

The model was solved using the GAMS software. To compare the performances of the MCDM methods, a Tukey test and the TOPSIS method were used to compare the differences among the means of the first objective, the second objective, and the CPU time. The results revealed that the Torabi-Hasini method was efficient to solve the model.

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