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The Optimization of a Multi-Product Three-Echelon Supply Chain

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Abstract

This paper aims at the single-objective optimization of multi-product for three-echelon supply chain architecture consisting of production plants, distribution centers (DCs), and customer zones (CZs). The key design decisions considered in the study include the quantity of products to be shipped from plants to DCs, from DCs to CZs, cycle length, and the production quantity so as to minimize the total cost .To optimize the objective, three-echelon network model is mathematically represented considering the associated constraints, production, capacity and shipment costs and solved using genetic algorithm (GA) and Simulated Annealing (SA).Some numerical illustrations are provided at the end to not only show the applicability of the proposed methodology, but also to select the best method using a t-test along with the simple additive weighting (SAW) method.

Keywords: Three echolon supply chain; Genetic algorithm; Simulated annealing Algorithm.

1. Introduction

Supply chain is a system of facilities and activities that functions to procure, produce, and distribute goods to the customers. 'Supply chain management is basically a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores so that the merchandise is produced and distributed at the right quantities, to the right locations, and at the right time in order to minimize system-wide costs (or maximize profits) while satisfying service-level requirements' (Simchi et al., 2000, p. ?). The above definition reveals that there are many independent entities in a supply chain, each of which tries to maximize its own inherent objective functions in business transactions. This is a complicated problem as too many factors are involved and need more than one objective to be satisfied simultaneously. Since today, although the success measures for the companies are determined by lesser costs, shorter production time, shorter lead time, less stock, larger product range, more reliable delivery time, higher quality, better customer services, and efficient coordination between the customer demand, supply, and production, the trade-off between cost investment and service levels may change over time. Hence the supply chain performance needs to be evaluated continuously and the supply chain managers should make timely and right decisions (Shen, 2007).

The key issues in supply chain management can broadly be divided into three main categories: (i) supply chain design, (ii) supply chain planning, and (iii) supply chain control. In the supply chain design phase, strategic decisions, like facility location decisions and technology selection decisions play major roles. It is very important to design an efficient supply chain to facilitate the transfer of goods. Environmental changes during the facility's lifetime can drastically alter the appeal of a particular site, turning today's optimal location into tomorrow's investment blunder. Assigning the best locations for the new facilities is thus an important strategic challenge (Owen and Daskin, 1998).

In traditional supply chain management, the focus of the supply chain network designs is usually on single objective,

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minimum cost, or maximum profit. But the design, planning, and scheduling projects usually involve the trade-offs among different incompatible goals, such as fair profit distribution among all members, customer service levels, fill rates, safety stocks, etc. (Chen & Lee, 2004). Therefore, real supply chains are to be optimized simultaneously by considering more than a single objective.

Many of the problems that occur in supply chain optimization are combinatorial in nature and picking a set of optimal solutions in case of multi-objective formulations requires a algorithm that can efficiently search the entire objective space using small amounts of computation time. The literature shows that evolutionary algorithms perform well in this respect and give optimal results when applied to many combinatorial problems. This work proposes the utility of genetic algorithm for the simultaneous optimization of one objective, minimizing total supply chain cost for a three-echelon supply chain architecture so as to arrive at an efficient supply chain design and optimal shipment plan which can be used as decision support system. Weber et al. (2000) proposed a method for suppliers' selection which involves developing the supplier order quantity solutions using a SA method and evaluating these solutions using data envelopment analysis (DEA).

Proposing the idea that supply chain design should be considered in terms of cooperating systems involved in networks, Chandra and Kumar (2001) developed two single-objective linear models for SCN design: (1) a decomposition model identifying common constraints and (2) a dynamic process flow model using the flow of components in a network.

In most of the classical supply chain network designs, the aim has been to send products from one layer to another in order to supply demands such that the sum of strategic and tactical/operational cost is minimized. For instance, Amiri (2006) developed a SC model to obtain the best strategic decisions on locating production plants and distribution warehouses in order to dispatch the products from plants to customers with the goal of minimizing the total costs of the distribution network. Gebennini et al. (2009) suggested a three-stage production-distribution system to minimize costs.

Intricacy involved in mutual relations between various supply chain components together with risks and uncertainties throughout the chain have turned the SC decision-making process into a challenging problem, where newer? Aims are propounded. The uncertainties in supply chain networks are divided into three classes based on the supplier layer, receiver layer, and in the middle layers. Because reversing the decisions in relation to the SC network configuration is very costly and difficult, the importance of the interactions between these decisions is largely enhanced under uncertainty.

Bidhandi and Yusuff (2011) modeled a stochastic supply chain network as a two-stage program under strategic and tactical decisions. They also investigated how the demands of customer, operational costs, and the capacity of the facilities might be highly uncertain as all of them can severely affect the strategic decisions. In the strategic level, Snyder (2006) investigated a problem called "reliable facility location problem (RFLP)" to locate the facilities at the distributer level of a SC under uncertainty when the facilities were subject to random failures. Murthy et al. (2004) indicated that the uncertainty at the strategic level is the most important and difficult issue to be considered. In the tactical level, Van Landeghem and Vanmaele (2002) worked on a supply chain planning problem that involved the distribution of raw materials and products. Moreover, Jamshidi et al. (2012) suggested a bi-objective multi-echelon SCN design model considering several transportation options at each level of the chain with different costs and a capacity constraint.

The SC network design under demand uncertainty has received significant attention in the last decade. For instance, Cardona-Valdés et al.(2011) purposed the design of a bi-objective two-echelon production distribution network under demand uncertainty to minimize both the total cost and the total service time. El-Sayed et al.(2010) developed a multiperiod three-echelon forward-reverse logistics network design under demand uncertainty in the forward direction and deterministic customer demand in the reverse direction in order to maximize the total expected profits. In the strategic level, Schüt et al. (2009) presented another SC problem as a two-stage stochastic program under the short-term operations and demand uncertainty to minimize the total expected costs. Georgiadis et al. (2011) investigated a supply chain network design problem under time-varying uncertain demand in terms of a number of probable scenarios.

Wang et al. (2011) studied a two-echelon SC with stochastic demand to make decisions at both strategic and operational levels to maximize profit, where a genetic algorithm (GA) with efficient greedy heuristics was employed to solve the problem. Moreover, the optimization of a bi-criteria multi-echelon SC in the presence of demand uncertainty with the goals of maximizing the net present value and minimizing the expected lead time was investigated by You and Grossmann (2008). They proposed a ε -constraint method to solve the multi-period mixed-integer nonlinear programming (MINLP) problem.

Furthermore, Olivares-Benitez et al. (2012) modeled a two-echelon single-product SC design problem as a bi-objective mixed-integer programming and studied the three variations of the classical ε -constraint methods to generate Pareto-optimal solutions. Ruiz-Femenia et al. (2013) analyzed the influence of demand uncertainty on the multi-objective

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optimization of chemical supply chains, simultaneously considering their economic and environmental performance. Moreover, Rodriguez et al. (2014) proposed an optimization model to redesign the supply chain of spare part delivery under demand uncertainty from strategic and tactical perspectives in planning horizon consisting of multiple periods. They addressed the uncertain demand by defining the optimal amount of safety stock that would guarantee a certain service level at a customer site. Moreover, the risk-pooling effect was taken into account when defining inventory levels in distribution centers and customer zones.

Mirzapour et al. (2011) addressed a multi-site, multi-period, multi-product three-echelon SC under uncertainties of cost parameters and demand fluctuations. They utilized the LP-metric method to solve the proposed bi-objective problem as a single- objective mixed integer programming. Azaron et al. (2008) proposed a multi-objective stochastic programming approach for the supply chain network design problem in which demands, supplies, processing time, shortage, transportation, and capacity expansion costs were purposed uncertain. They applied the goal attainment method to obtain the minimum values of the financial risk, the total expected costs, and the variance of the total cost as Pareto-optimal solutions.

Mele et al. (2007) proposed an agent-based approach to solve problems involved in SCs that are either driven by pull strategies or operate under uncertain environments. Song et al. (2014) considered a manufacturing supply chain problem with multiple suppliers in the presence of multiple uncertainties, such as uncertain material supplies, random customer demands, and stochastic production times. Pishvaee et al. (2014) developed a multi-objective possibilistic programming model to design a sustainable medical supply chain network under uncertainty by considering the conflicting economic, environmental, and social objectives. Moreover, Wu et al. (2013) developed a stochastic fuzzy multi-objective programming model for supply chains that would outsource risk in the presence of both random and fuzzy uncertainties.

Many researchers have widely applied GA to solve SCM (stands for?) problems. Tsai and Chao (2009) developed a dynamic adaptive GA using a chromosome refinement procedure designed to adapt the ordinal structure of the genes within a chromosome. Prakash et al. (2012) provided a knowledge-based GA (KBGA) to optimize a SC network. Bandyopadhyay and Bhattacharya fbu(2014) proposed a tri-objective optimization problem for a two echelon serial supply chain. They considered the modification of nondominated sorting genetic algorithm-II (NSGA-II) with a mutation algorithm that has been embedded into the modified NSGA-II to solve the problem. A Lagrangian heuristic approach was also utilized in their research to compare the obtained results.

Zegordi et al. (2010) used a GA to solve a mixed integer programming problem developed for a two-stage SC problem that involved the scheduling of products and vehicles. Costa et al. (2010) presented a new efficient encoding/decoding procedure used within a GA to minimize the total current cost of a single-product three-stage SCN design under strategic decisions. Moreover, Wang and Hsu (2010) considered a spanning-tree-based GA to minimize the relevant cost associated with strategic decisions in a closed-loop logistic network problem formulated into an integer linear programming model.

Naimi Sadigh et al. (2012) investigated a multi-product manufacturer-retailer supply chain, where demand of each product is jointly influenced by price and advertising expenditure. They proposed several solution procedures including imperialist competitive algorithm, modified imperialist competitive algorithm, and evolution strategy. Pasandideh et al. (2010) considered a multi-product economic production quantity problem with limited warehouse-space in which the orders are delivered discretely in form of multiple pallets and the shortages are completely backlogged. Baykasoglu and Gocken (2010) presented a direct solution method that is based on ranking methods of fuzzy numbers and Tabu search to solve fuzzy multi-objective aggregate production planning problem.

There are other approaches in the literature to solve different SC problems. For instance, Cardona-Valdés et al. (2011) investigated the design of a two-echelon SCN with uncertain demand. An important contribution of their study was the development of a Tabu search within the framework of multi-objective adaptive memory programming to find optimal Pareto fronts of a two-stage stochastic bi-objective programming problem.

However, in recent years, many researcher have investigated some relevant issue as well. For instance, Zhou et al. (2013) introduced a multi-product multi-echelon inventory control model with joint replenishment strategy. Alimardani et al. (2013) considered bi-product inventory planning in a three-echelon supply chain with backordering, poisson demand, and limited warehouse space. Miranbeigi et al. (2015) investigated demand satisfaction in supply chain management system using a full online optimal control method. Maghsoudlou et al. (2016) optimized a bi-objective three-echelon multi-server supply-chain problem in congested systems. Modak et al. (2016) considered three-echelon supply chain coordination considering duopolistic retailers with perfect quality products. Dai et al. (2017) optimized multi-echelon inventory with three types of demand in supply chain. Mohammadi et al. (2017) found optimal design of a multi-echelon supply chain in a systems thinking framework. Stenius et al. (2017) developed sustainable multi-echelon inventory control with shipment consolidation and volume dependent freight costs. Panda et al. (2017) scrutinized coordination and benefit sharing in a three-echelon distribution channel with deteriorating product. Ross et al. (2017) investigated

integrated location-inventory modelling under forward and reverse product flows in the used merchandise retail sector. Topan et al. (2017) considered sustainable multi-echelon inventory control with shipment consolidation and volume dependent freight costs. Belgin et al. (2018) developed a mathematical model related to the two-echelon vehicle routing problem with simultaneous pickup and delivery and solved it by a heuristic approach. Ghasemy Yaghin (2018) investigated integrated multi-site aggregate production-pricing planning in a two-echelon supply chain with multiple demand classes. Habibi-Kouchaksaraei (2018) designed a robust bi-objective multi-echelon mathematical model for blood supply chain in a disaster. Johansson and Olsson (2018) considered age-based inventory control in a multi-echelon system with emergency replenishments. Kayvanfar et al. (2018) studied a multi-echelon multi-product stochastic model to supply chain of small-and-medium enterprises in industrial clusters. Liu et al. (2018) devised a branch-and-cut algorithm for the two-echelon capacitated vehicle routing problem with grouping constraints. Zhou et al. (2018) took into account multi-depot two-echelon vehicle routing problem with delivery options arising in the last mile distribution.

The current study addresses single-objective optimization of a multi-product three-echelonsupply chain network. The network consists of certain manufacturing plants, distribution centers (DCs), and customer nodes.

The remainder of this paper is organized as follows. In Section 2, the problem is stated and the assumptions are explained in more details. Section 3 provides the mathematical model of the problem at hand. In Section 4, the solution approaches are described. Section 5 contains numerical examples for a set of designed problems of different sizes. In this section, we apply the t-test and the simple additive weighting (SAW) method, as a statistical and a multi-attribute decision-making (MADM) approach respectively, to compare the algorithms. Finally, the conclusion and suggestions for future studies are presented Section 6.

2. The problem

Consider a three-echelon supply chain network shown in Fig. 1. This network consists of manufacturing plants to the left, distribution centers (DCs) in the middle of the figure, and customer nodes to the right.

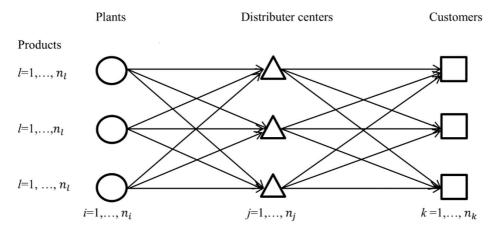


Figure 1. A three-echelon supply chain net

The main assumptions of this problem are:

- The supply chain has an integrated structure consisting of manufacturing plants and potential DCs designed to supply customer demands for several products in single period.
- Shortage can not occur at customer nodes.
- More than one plant can replenish the demand of a given warehouse.
- More than one distributor center can replenish the demand of each customer.
- All the internal parameters, such as production and transportation costs, and holding costs of inventory for products all are considered as the uniform random variables.
- > The capacities of the manufacturers, the distributors, and the retailers are known.
- > The warehouses operate perfectly.
- > The manufacturing plants have limited production per period.
- A production system with imperfect production processes.
- > Incomplete items of *n* different kinds, are generated at a rate, u_i ; i = 1, 2, ..., n, per cycle and among these products a sc_i portion is taken into account to be scrap and the other portion can be reworked.
- > All of the parameters are considered as stochastic in every cycle.
- > All of the products are manufactured by one machine.
- A definite cycle length for all items is perceived: $T_1 = T_2 = \cdots = T_n = T$.
- > It is assumed that the production rate of the *i*th item is p_i per cycle, and the quantity of the good items produced, denoted by d_i , in every cycle.

- We assume that the entire quantity of imperfect quality items will be reworked, and at the end of the rework period, a m_i portion as scrap will be left.
- > The manufacturer at the same time utilizes the same resource for production and rework processes.
- > For the common production system, budget and capacity are finite.
- In this research, the following basic assumption of EPQ inventory model with the rework process is that the rate of production minus defectives must always be greater than or equal to the demand.
- \succ The inspection process for the products is perfect, and the inspection time is zero.
- > The locations of plants, distributors, retailers and suppliers are fixed.
- > Products are temporarily stored at the distributors before delivery to the retailers.
- > There is no inventory at the distributor centers and customer zones at the beginning or end of the cycle.

Under the above-mentioned conditions, the present study attempts to make decisions so as to determine a proper SCN configuration in order to minimize the total cost.

The proposed mathematical model can determine the economic production quantity and the quantity of products produced by each plant in a period, and assign products to the transportation channels between the SC entities in a period.

Furthermore, several limitations were considered in the proposed model as follows:

- > The capacities of the manufacturer, distributor, and retailers are limited.
- > The storage capacities for each perfect product are limited.
- All demands must be satisfied during the planning horizon.
- > The production and reworking times are limited.

3. Mathematical model

A single-objective mixed-integer non-linear mathematical formulation of the problem at hand is derived in this section using the following notations including indices, parameters, and decision variables. *Sets of indices:*

- *i* The set of indices $\{i = 1, 2, ..., ni\}$ used for manufacturing plants
- *j* The set of potential distribution centers (DCs) or warehouses $\{j = 1, 2, ..., n_j\}$
- k The set of customer nodes $\{k = 1, 2, ..., n_k\}$
- *l* The set of finished products $\{l = 1, 2, ..., n_l\}$

Parameters:

- c_{il} Unit production cost of product *l* produced by manufacturing plant *i*
- p_{il} The production rate of *i*th product at manufacturing plant *i*, units/unit time
- db_{il} The demand rate of *i* th productby all distribution center, units/unit time
- dr_{kl} Demand of product *l* by customer *k*, units
- re_{il} Rework cost per defective good *l* by manufacturer *i*
- hm_{il} Holding cost of products *l* for defective goods stored at manufacturer *i*
- u_{il} Percentage of defective goods *l* produced by manufacturer *i*
- *sc_{il}* Percentage of scrap products*l* produced by factory *i*
- f_l Volume of one unit of product $l(m^3)$
- m_{il} The proportion of scrap of *i*th product in rework at manufacturing plant *i*
- h_{il} Unit inventory holding cost of product *l* by plant *i*
- ic_{il} Unit inspection cost of product l by plant i
- A_{il} Set up cost of producing product *l* by manufacturing plant *i*
- CM_{ijl} Shipping cost of each product *l* from manufacturer *i* to distributor *j*
- CD_{ikl} Shipping cost of each product *l* from distributor *j* to retailer *k*

 Tdf_{il} Shipping cost of each defective goods l at manufacturer i (from production to defective goods storage and vice versa).

- dc_{il} Discount cost per scrap products *l* for sale by manufacturing plant *i*
- S_i Total storage capacity of plant *i*
- W_i The total available budget of manufacturing plant *i* per period
- E_i Total storage capacity of distributor j
- Es_{il} Storage capacity of distributor *j* for product *l*
- R_k Total storage capacity of retailer k
- Rs_{kl} Storage capacity of retailer k for product l
- G_{il} Production capacity of factory *i* for product *l*

Decision variables:

Q_{il}	EPQ of products <i>l</i> by manufacturing plant <i>i</i>
Q_i	Quantity of all products produced by manufacturing plant <i>i</i>
INV _{il}	Inventory of product <i>l</i> at manufacturing plant <i>i</i> per period
perfect _{il}	Quantity of perfect products <i>l</i> produced by factory <i>i</i> per period
Т	Cycle lenght
X _{ijl}	Quantity of products <i>l</i> transported from factory <i>i</i> to distributor <i>j</i>
Y_{jkl}	Quantity of products l transported from distributor j to retailer k

The production cycle length is the sum of the production uptimes for the good and incomplete items, t_i^1 and t_i^5 , respectively, the reworking time, t_i^2 , and the production downtimes, t_i^3 and t_i^4 . Hence, one has a total production cycle length of:

$$T = \sum_{j=1}^{3} t_{il}^{j} \tag{1}$$

Since all products are produced on a single machine with a limited capacity, the cycle length for all products is shown in Figure 2. Therefore, we have:

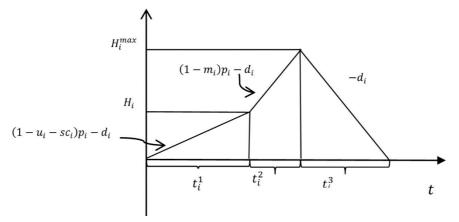


Figure 2. On-hand inventory for perfect quality items

$$t_{il}^{1} = \frac{Q_{il}}{p_{il}} \tag{2}$$

$$t_{il}^2 = u_{il} \frac{Q_{il}}{p_{il}}$$
(3)

$$t_{il}^{3} = \frac{H_{i}^{max}}{d_{i}} = \left(\frac{(1 - sc_{il} - m_{il}u_{il})}{db_{il}} - \frac{(1 + u_{il})}{p_{il}}\right)Q_{il}$$
(4)

$$H_{il} = \left((1 - u_{il} - sc_{il})p_{il} - db_{il} \right) \frac{Q_{il}}{p_{il}}$$
(5)

$$H_{il}^{max} = H_{il} + u_{il} ((1 - m_{il})p_{il} - db_{il}) \frac{Q_{il}}{p_{il}}$$

$$H_{il}^{max} = ((1 - m_{il})p_{il} - db_{il}) Q_{il} \qquad (6)$$

$$H_{il}^{max} = \left((1 - sc_{il} - m_{il}u_{il})p_{il} - (1 + u_{il})db_{il}\right)\frac{q_{il}}{p_{il}}$$
Hence, from Eq. (1), the cycle length for a single product is:
$$(6)$$

Hence, from Eq. (1), the cycle length for a single product is: $(1 - sc_{ii} - m_{ii}u_{ii})\Omega_{ii}$

$$T = \frac{(1 - sc_{il} - m_{il}u_{il})Q_{il}}{db_{il}}$$
(7)

$$Q_{il} = \frac{T \times db_{il}}{\left(1 - sc_{il} - (u_{il} \times m_{il})\right)}$$
(8)

$$TC = \sum_{i=1}^{ni} \sum_{l=1}^{nl} c_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{l=1}^{nl} hm_{il}u_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{j=1}^{nj} \sum_{l=1}^{nl} X_{ijl}(CM_{ijl}) + \sum_{j=1}^{nj} \sum_{k=1}^{nk} \sum_{l=1}^{nl} Y_{jkl}(CD_{jkl}) + \sum_{i=1}^{ni} \sum_{l=1}^{nl} re_{il}u_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{l=1}^{nl} Tdf_{il}u_{il}Q_{il} + \sum_{i=1}^{ni} \sum_{l=1}^{nl} dc_{il}((sc_{il} + m_{il}u_{il})Q_{il})$$

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$$+\sum_{i=1}^{ni}\sum_{l=1}^{nl}A_{il} + \sum_{i=1}^{ni}\sum_{l=1}^{nl}ic_{il}Q_{il} + \sum_{i=1}^{ni}\sum_{l=1}^{nl}h_{il}INV_{il}$$
(9)

Constraints:

$$Q_{il} \leq G_{il} \tag{10}$$

$$\sum_{i=1}^{N} \sum_{l=1}^{N} X_{ijl} \le E_j \tag{11}$$

$$\sum_{i}^{ni} X_{ijl} \le E s_{jl} \tag{12}$$

$$\sum_{j=1}^{\overline{l=1}} \sum_{l=1}^{nl} Y_{jkl} \le R_k$$
(13)

$$\sum_{j=1}^{n_j} Y_{jkl} \le Rs_{kl} \tag{14}$$

$$perfect_{il} = Q_i (1 - sc_{il} - (u_{il} \times m_{il}))$$

$$\sum_{i=1}^{ni} X_{ijl} = \sum_{k=1}^{nk} Y_{jkl}$$
(15)
(16)

$$\sum_{\substack{j=1\\n_{j}}}^{n_{j}} Y_{jkl} = dr_{kl}$$
(17)

$$\sum_{j=1}^{N} X_{ijl} = perfect_{il} \tag{18}$$

$$\sum_{\substack{l=1\\n'}}^{nl} (t_{il}^1 + t_{il}^2) + \sum_{l=1}^{nl} ts_{il} \le T$$
(19)

$$\sum_{\substack{l=1\\n_i}}^{m} (c_{il} + re_{il} \times u_{il})Q_i \le W_i$$
(20)

$$\sum_{\substack{j=1\\nl}}^{j} X_{ijl} = T \times db_{il}$$
(21)

$$\sum_{l=1}^{m} f_l(((1 - sc_{il} - m_{il}u_{il})p_{il} - (1 + u_{il})db_i)\frac{Q_{il}}{p_{il}}) \le S_i$$
(22)

$$Q_{il}, Q_i, T, X_{ijl}, Y_{jkl} \ge 0, integer$$
(23)

Eq.(9) represents an objective function that minimizes the total costs of the supply chain, including the costs of production, holding at the distributor, defective good storage, rework of defective goods, transportation from the manufacturers to the distributors, transportation from the distributors to the retailers, transportation from the manufacturers to defective good storage and vice versa, the discount cost of scrap goods for sale, and the retailer shortages due to defective goods production. Eq. (10) states the restriction of production capacity. Eqs. (11) and (12) denote the delivery capacity limitations of the distributors for all types of products and each type of product, respectively. Eqs. (13) and (14) consider the delivery capacity limitations of the retailers for all product types and each product type, respectively. Eq. (15) expresses the total amount of perfect goods after reworking. Eq. (16) shows the balance between the total inputs and outputs of goods moving to and from the distributors during the cycle. Eq. (17) explains how the total demands during the cycle are supplied. Eq. (18) assures that the total goods shipped from the manufacturers to the distributors are of perfect quality. In other words, the scrap goods are removed from the system. In the joint production, systems having rework, the overall production, rework, and setup times ought to be smaller than the cycle length.

Eq. (19) assures $\sum_{l=1}^{nl} (t_{il}^1 + t_{il}^2) + \sum_{l=1}^{nl} ts_{il}$ must be less than or equal to T. Since the production quantity is q_i^s , the total available budget is W, and $u_i q_i^s$ is the number of the *i*-th product which requires rework. Eq. (20) shows the budget constraint.

Eq. (21) expresses the total goods shipped from the manufacturers to the distributors that are equal to the product of the length of each period and the demand rate of *i*th product by all distribution center. Eq. (22) shows the space of the

warehouse to store the products at manufacturing plants is limited. Eq.(23) indicates that the production amount, deliveries to warehouses and retailers, and perfect goods after reworking should all have positive values.

4. Solution algorithms

4.1. GA

Genetic algorithm (GA) is introduced first by Holland (1975). The formulation of the model is a non-linear integerprogramming (NIP) model. This characteristic causes the model to be hard enough to be solved by an exact method (Gen, 1997). Genetic algorithms are stochastic search techniques based on the mechanism of natural selection and natural genetics. Each individual in the population is called a chromosome, deputizing a solution to the problem at hand. The chromosomes evolve through consecutive iterations, called generations.

During every generation, the chromosomes are evaluated, utilizing some measures of fitness. To create the subsequent generation, new chromosomes, called offspring, are built by either crossover operator or mutation operator. A new generation is built according to the fitness values of the chromosomes. After some generations, the algorithm converges to the best chromosome (Pasandideh and Niaki, 2008).

The usual form of GA was described by Goldberg (1989). GA is a stochastic search technique whose solution process mimics natural evolutionary phenomena: genetic inheritance and Darwinian strife for survival (Gen, 2000). Recently, the GA has been receiving great attention and it has successfully been applied to other problems in the supply chain environment (Chen et al., 1998; Park, 2001).

GA is known as a problem-independent approach; however, the chromosome representation is one of the critical issues when applying it to optimization problems.

4.1.1. GA algorithm in initial and general conditions

The required initial information to start a GA is:

- (i) Population size (*nPop*): It is the number of the chromosomes or scenarios that are kept in each generation.
- (ii) Crossover rate (Pc): This is the probability of performing a crossover in the GA method.
- (iii) Mutation rate (*Pm*): This is the probability of performing mutation in the GA method.
- (iv) maximum number of iterations (N)
- (v) The general steps involved in a GA algorithm are as follows:

1. Initialization.

1.1. Set the parameters (*N*, *Pc*, *Pm*, stopping criteria, selection strategy, crossover operation, mutation operation, and number of generation).

- 1.2. Initialize the population (randomly).
- 2. Evaluate the fitness. Repeat
- 3. Select individuals for the mating pool (size of mating pool = N)
- 4. For each consecutive pair, apply crossover with probability Pc.
- 5. For each new generation, apply mutation with probability Pm.
- 6. Replace the current population by the resulting mating pool.
- 7. Evaluate the fitness.

Until stopping criteria is met.

4.1.2. The chromosomes

One of the important components of the GA is the selection of chromosomes. We attempted to select the best chromosomes in the proposed GAs that would give us good results and require a low run-time. In the studied mathematical model, the variable Y_{jkl} has both direct and indirect relationships with the variables Sc_{il} , X_{ijl} , and Q_{il} . Therefore, any change in variable Y_{jkl} leads to certain changes in other variables, and thus, the variable Y_{jkl} was chosen as the chromosome. The chromosomes are displayed in a matrix with the columns denoting the number of products (*l*) and rows (*j* * *k*). Each gene in the matrix was randomly created by Matlab software, as shown in Eq. (24).

 $Matrix Y_{(j*k)*l} = \begin{pmatrix} Y_{111} & \cdots & Y_{11l} \\ \vdots & \ddots & \vdots \\ Y_{1k1} & \cdots & Y_{jkl} \end{pmatrix}_{(j*k)*l}$ (24)

4.1.3. Evaluation

A fitness function is required to evaluate the chromosomes of each generation. In most GA applications, the objective function of the optimization model at hand is considered a fitness function. However, as explained before, the inventory model of this research has some constraints. These characteristics make the probability of a generated chromosome being feasible very low. In order to promote this chance, a penalty function is defined to be a positive and known sum of

squared violation of each constraint. As a result, the penalty and the fitness function in which violation per constraint is denoted by E_i is defined as follows:

Penalty function = $\begin{cases} \sum_{i=1}^{4} E_i/4 & \text{If the chromosome} \in \text{infeasible region} \\ 0 & \text{If the chromosome} \in \text{feasible region} \end{cases}$

Fitness function = Penalty function +Objective function

4.1.4. Initial population

In this step, a collection of chromosomes is randomly generated.

4.1.5. Crossover

To perform the crossover, two chromosomes must be merged. First, the chromosomes to be combined should be identified and allowed to mix. The columns will be combined for each chromosome selected for the crossover, and the intersection point will be used to combine the chromosomes.

For example, if we work with two chromosomes with four periods and three rows, the combination of the chromosomes will proceed as follows. First, the intersection point is chosen. Thereafter, the values of both sides of the matrix are exchanged. The intersection point shown in this example denotes the first period (Figure 3).

Y ₁₁₁	Y ₁₁₂	Y ₁₁₃	Y ₁₁₄		Y'_{111}	Y'112	Y'113	Y'114
Y ₂₁₁	Y ₂₁₂	Y ₂₁₃	Y ₂₁₄		Y'_{211}	Y'212	Y'213	Y'214
Y ₃₁₁	Y ₃₁₂	Y ₃₁₃	Y ₃₁₄		Y'_{311}	Y'312	Y'313	Y'314
	Selected	Parent 1				Selected	Parent 2	
					\bigcup			
Y ₁₁₁	Y'112	Y'113	Y'114		Y'111	Y ₁₁₂	Y ₁₁₃	Y ₁₁₄
Y211	Y'212	Y'213	Y'214	1	Y'211	Y212	Y213	Y214
Y311	Y'312	Y'313	Y'314	1	Y'311	Y ₃₁₂	Y ₃₁₃	Y ₃₁₄
Produced Child 1					Produce	d Child 2		

Figure 3. Display of the cross over operation.

4.1.6. Mutation

The mutation probability refers to the probability of change in any gene. In this research, each chromosome receives a certain number of genes that are assumed to change, and this number is derived by multiplying the total number of genes by the mutation probability. Accordingly, a number of genes must be selected to undergo mutation.

The resulting value is rounded off to the nearest whole number. If four genes should undergo mutation, these four genes are randomly selected, and their values are changed (Figure 4).

Y ₁₁₁	Y ₁₁₂	Y ₁₁₃	Y ₁₁₄	Y'111	Y112	Y ₁₁₃	Y'114
Y ₂₁₁	Y ₂₁₂	Y ₂₁₃	Y214	Y ₂₁₁	Y'212	Y213	Y214
Y ₃₁₁	Y ₃₁₂	Y ₃₁₃	Y ₃₁₄	Y ₃₁₁	Y ₃₁₂	Y'313	Y ₃₁₄
Parent and selected genes			Produced child				

Figure 4. Display of the mutation operation

(25)

4.1.7. Chromosomes selection

In genetic algorithms, the selection operator is used to guide the search process towards more promising regions in a search space. Some selection methods, like roulette wheel, tournament, ranking, and elitist are discussed in Michalewicz (1996). Furthermore, a detailed explanation of the operation of roulette wheel selection can be found in Goldberg (1989). Since in the roulette wheel selection, better solutions get higher chance to become parents of the next generation, it is used to select the chromosomes of this research. The selection in this method is based on the fitness value of the chromosomes. We select N chromosomes among the parents and the offspring with the best fitness values.

4.1.8. Stopping criterion

The last step in a GA methodology is to check if the method has found a solution that is good enough to meet a stopping criterion. Stopping criteria is a set of conditions such that when the method satisfies them, a good solution is obtained. In this study, a pre-specified maximum number of iterations (N) is considered to stop the algorithms.

4.2. SA

Simulated annealing (SA) is a generic probabilistic meta-heuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space (Kirkpatrick et al 1983). The method was independently described by Kirkpatrick et al (1983) and Cerny (1985). The title and the inspiration come from annealing in metallurgy, a technique that involves heating and controlled cooling of a material to increase the size of its crystals and reduce their deficiencies.

Both of them are attributes of the material that depend on its thermodynamic free energy. Heating and cooling the material impress both the temperature and the thermodynamic free energy. While the very amount of cooling brings the same amount of decrease in temperature, it will afford a bigger or smaller decrease in the thermodynamic free energy depending on the rate that it occurs, with a slower rate generating a bigger decrease.

This notion of slow cooling is implemented in the Simulated Annealing algorithm as a slow decrease in the probability of accepting worse solutions as it explores the solution space. Accepting worse solutions is a basic property of meta-heuristics because it allows for a more extensive search for the optimal solution.

The procedure of simulated annealing algorithm is given below for the minimization problem:

- > Get an initial temperature T > 0.
- \blacktriangleright Get an initial solution *S*.
- > Initial solution (S) is considered as the best solution.
- > The following loop repeats until stopping criteria is met.
- Perform the following loop L times.
- ➢ Generate solution S'in neighborhood of S.
- $\blacktriangleright \quad \text{Let } \Delta = \cot(S') \cot(S).$
- ► If $(\Delta < 0)$ (downhill move), set S = S'.
- ► If (Δ >0) (uphill move), Create a random number (*x*), *x* ∈ [0 1]
- $Find x < \exp(-\Delta/T), \text{ set } S = S'.$
- End loop
- Set $T = \alpha \times T$ (reduce temperature).
- > If stopping criteria is met, the algorithm is terminated.

The required initial information to start a SA is:

- Population size (*nPop*): It is the number of the initial solutions
- Final Temperature (*TF*)
- Temperature Decrement Rate (α)
- Iterations at each temperature (*IT*)

4.2.1. Determining starting temperature

There are two key parameters in the cooling process that determine how firm or amorphous the result of the metal in its frozen state will be. The first one is the temperature starting point from which the cooling begins, and the second is the rate by which the temperature is falling. Concerning the rate of decay, it should be low enough to allow the atoms in the molten metal to line themselves up and give enough time to form a crystal lattice with the minimum internal energy. Evidently, a slow decay will lead to a long-time - solidifying process.

To reduce the run time, one may think of a low starting temperature. However, on the other hand, if the initial temperature is not copious enough, atoms of the molten metal would not have enough freedom to rearrange their positions in a very regular minimum energy structure.

In this algorithm, starting temperature is calculated based on Equation (26).

$$temprature = \frac{TF}{\alpha^{\Lambda} IT}$$
(26)

4.2.2. Initial solution structure

The quality of the initial solution affects the performance of the meta-heuristics. To reduce this dependency, several methods start their search from multiple primary solutions. The structure presentation of the chromosome in this algorithm is similar to GA algorithm that was explained previously.

In this step, a chromosomes is generated. Equation (25) is considered as the fitness function.

4.2.3. Neighbourhood search

The way in which a meta-heuristic moves from one solution to its neighbor is another critical component in SA algorithm. For every iteration, the neighbourhood solution is generated using the mutation operator used in GA algorithm. This process is called neighbourhood generation.

4.2.4. Temperature Decrement

There are some rules for temperature decrement, where the most common method is classic temperature decrement $(T=T \times \alpha)$. The temperature decrement rate is α .

4.2.5. Stopping criteria

The assumed criteria in this algorithm is reaching to Final Temperature (TF).

4.3. Tuning the parameters of the two solution algorithms

In this paper, a regression approach is utilized for calibrating the main parameters of the algorithms. First, each algorithm is employed 30 times. Each time their parameters are changed in their relevant intervals. The values of the related responses are first normalized by the linear norm.

Then, a quadratic regression function for each measure is estimated using the Design Expert software to find the significant relationships between the parameters and their responses. Next, the estimation of the results utilizing the estimated regression function is taken to be optimized by the GAMS software in order to find the optimal combinations of the parameters. The quadratic regression function consists of interaction, linear, and quadratic coefficients shown in Equation (27).

$$\begin{split} E(Y) &= \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_{11} X_1^2 + \beta_{22} X_2^2 + \beta_{33} X_3^2 + \beta_{44} X_4^2 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{14} X_1 X_4 + \beta_{23} X_2 X_3 + \beta_{24} X_2 X_4 + \beta_{34} X_3 X_4 \end{split}$$

Where E(Y) is the expected value of a given response, β_0 is a constant coefficient representing the intercept, β_i , i = 1,2,3,4 are the linear coefficients, β_{ii} , i = 1,2,3,4 are the quadratic coefficients, β_{ij} , i & j = 1,2,3,4, $i \neq j$ are the interaction coefficients, and X_i , i = 1,2,3,4 are the four parameters of the algorithms. This process is explained in the next section, where input data of the problem instances are given. Furthermore, the results obtained using the parameter-tuned algorithms are analyzed.

Numerical examples evaluate the performances of the two meta-heuristic algorithms in the next section. Then, they are solved employing GA and SA. Numerical examples are provided in the next section to evaluate the performances of the two single-objective meta-heuristic algorithms in the next section.

5. Numerical results

Consider a multi-product, multi-period, three-echelon SC problem, in which the internal parameters are randomly generated based on Uniform distributions in their corresponding ranges for different problem instances shown in Table 1. And the ranges of the parameters are given in Table 2.

		Problem No.				
	1	2	3	4	5	
Manufacturing plants	3	6	8	9	13	
Distributor centers	6	8	10	12	16	
Customers	3	9	10	11	17	
Products	3	5	6	7	8	

Table 1. Generated problem instances

Table 2. Parameter range

<i>d</i> ~uniform [500 700]	<i>Tcm</i> ~uniform [9,15]	<i>S</i> ~uniform [2000 3000]	
<i>pc</i> ~uniform [4,8]	<i>Tcd</i> ~uniform [6,10]	<i>p</i> ~uniform [5000 12000]	
<i>u</i> ~uniform [0,0.09]	<i>Tcdef</i> ~uniform [3,7]	<i>db</i> ~uniform [4000 5000]	
A~uniform [1,4]	disc~uniform [8,14]	tscd~uniform [24000 30000]	
<i>ts</i> ~uniform [0.0025,0.0045]	<i>cb</i> ~uniform [2,3]	scd~uniform [10000 18000]	
<i>re~</i> uniform [4,8]	<i>sc</i> ~uniform [0.04,0.06]	tscr~uniform [15500 28900]	
<i>h</i> ~uniform [2,30]	<i>m</i> ~uniform [0,0.9]	<i>scr</i> ~uniform [4000 8500]	
<i>f</i> ~uniform [0.1,3] <i>ci</i> ~uniform [1,3]		sto~uniform [10000 20000]	
<i>hd</i> ~uniform [7,13]	Inv~uniform [2800 3900]	W~uniform [100000000,400000000]	
	<i>hm</i> ~uniform [4,8]		

The parameter ranges of both algorithms along with their levels are shown in Table 3.

Table 3	Algorithm	narameter	ranges	along	with 1	hoir	lovele
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Single-objective	Algorithm parameter	Parameter range	Low(-1)	Medium (0)	High (1)
algorithms					
GA	nPop(A)	10-50	10	30	50
	Pc (B)	0.6-0.8	0.6	0.7	0.8
	Pm (C)	0.01-0.1	0.01	0.055	0.1
	Ngen (D)	5-15	5	10	15
SA	nPop (A)	1-11	1	6	11
	IT (B)	31-59	31	45	59
	Alpha (C)	0.8-0.9	0.8	0.85	0.9
	TF (D)	0.1-0.3	0.1	0.2	0.3

As mentioned in the previous section, the parameters of the algorithms is first tuned to obtain better solutions. Therefore, each algorithm is employed 30 times, each time their parameters change in their corresponding ranges. Then, the fitness function is recorded in each run. As an example, Table 4 contains the experimental results of employing GA.

No.	nPop	Ν	Pc	Pm	Cost
1	10	15	0.8	0.01	741571
2	10	5	0.6	0.1	729150
3	50	10	0.7	0.055	724177
4	30	10	0.7	0.055	730092
5	30	10	0.7	0.055	725932
6	30	15	0.7	0.055	736817
7	30	10	0.6	0.055	719168
8	50	5	0.8	0.01	719685
9	50	15	0.6	0.1	714431

Table 4. Experimental results of employing GA

Table 4. Continued								
No.	nPop	Ν	Pc	Pm	Cost			
10	30	5	0.7	0.055	737146			
11	10	15	0.6	0.01	737620			
12	10	15	0.8	0.1	736120			
13	10	10	0.7	0.055	728026			
14	30	10	0.7	0.055	725094			
15	50	5	0.6	0.01	719957			
16	30	10	0.8	0.055	727127			
17	50	5	0.8	0.1	744086			
18	30	10	0.7	0.055	721584			
19	30	10	0.7	0.055	733371			
20	30	10	0.7	0.1	725031			
21	10	5	0.6	0.01	745728			
22	50	15	0.6	0.01	712495			
23	50	15	0.8	0.1	720066			
24	10	5	0.8	0.01	733738			
25	50	15	0.8	0.01	729551			
26	50	5	0.6	0.1	725417			
27	10	15	0.6	0.1	733302			
28	30	10	0.7	0.055	728543			
29	30	10	0.7	0.01	740144			
30	10	5	0.8	0.1	742045			

Tabl	e 4.	Continue

To investigate the relationship between the fitness value and the parameters, a regression analysis using the Design Expert software is then employed. The quadratic regression coefficients of the fitness function of the GA are given in Table 5.

Table	Table 5. The Quadratic regression coefficients of the fitness function of the GA								
Term	Coefficient(β)	Standard error	F-value	p-value					
Model	-	-	3.21	0.0161					
Constant	7.283E+005	1895.34	-	-					
nPop	-6524.19	1438.16	20.58	0.0004					
N	-1943.33	1438.16	1.83	0.1966					
Pc	3151.17	1438.16	4.80	0.0446					
Pm	-602.20	1438.16	0.18	0.6813					
nPop × N	-1909.55	1525.40	1.57	0.2298					
nPop × Pc	2088.34	1525.40	1.87	0.1911					
nPop × Pm	2521.94	1525.40	2.73	0.1190					
N × Pc	634.84	1525.40	0.17	0.6832					
$N \times Pm$	-2431.72	1525.40	2.54	0.1318					
$Pc \times Pm$	1954.59	1525.40	1.64	0.2195					
nPop × nPop	-3074.92	3790.68	0.66	0.4299					
$N \times N$	7805.23	3790.68	4.24	0.0573					
$Pc \times Pc$	-6028.67	3790.68	2.53	0.1326					
Pm × Pm	3411.03	3790.68	0.81	0.3824					

This relationship is obtained as:

 $Pm - 1909.54674 \times nPop \times IT + 2088.34096 \times nPop \times Pc + 2521.94017 \times nPop \times Pm + 634.84159 \times IT \times nPop \times Pm + 634.84159 \times nPop \times Pm + 634.84159 \times nPop \times Pm + 634.84159 \times nPop \times$ $Pc - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times Pc \times Pm - 3074.91626 \times nPop^2 + 7805.22589 \times It^2 - 2431.72137 \times IT \times Pm + 1954.58958 \times It^2 + 2431.72137 \times It^2 + 2431.7217 \times It^2 + 243177 \times It^2 + 243$ $6028.67381 \times Pc^2 + 3411.02944 \times Pm^2$ (28)

Afterwards, the regression function is optimized using the GAMS software to obtain the best values of the parameters in their corresponding ranges. The results of parameter setting process of GA and SA are presented in Table 6 and 7, respectively.

Table 6.	The results of	parameter	setting process	of GA algorithm
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Algorithms	nPop	Ν	Рс	Pm
GA	50	6	0.6	0.01

Table 7. The results of parameter setting pro-	rocess of SA algorithm
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Algorithms	nPop	Alpha	TF	IT
SA	4	0.8	0.3	59

In order to validate the results obtained using GA, not only a *t*-test is used, but also a multiple attribute decision-making (MADM) approach is employed. The *t* test is employed using the SPSS software. The null hypothesis in this test is the equality of the means, i.e., $H_0: \mu l = \mu 2$, where μ_l and μ_2 are the response means. To do this, both algorithms are first implemented *n*=30 times, each time with their optimal parameter setting.

Then, the values of objective functions and the CPU time of problem instances are obtained. The means of the 30 runs for each of the 5 problem instances solved by GA and SA are provided in Tables 8 and 9, respectively.

Table 8. The mean of the objective function and the mean of CPU TIME in GA algorithm				
Problem No.	Cost	CPU Time		

Problem No.	Cost	CPU Time
1	114032.0426	1.5871297
2	548930.3058	15.8196912
3	722018.301	35.9244424
4	930298.327	48.297758
5	1642840.303	207.424508

Table 9. 1	The mean of the obj	ective function and the mea	an of CPU TIME in S	SA algorithm

Problem No.	Objective function	CPU Time
1	106836.7178	1.059103
2	532068.1821	9.838433
3	717157.165	22.81341
4	922899.1441	48.1613
5	1611441.6	222.3112

Based on 95% confidence level, i.e., a=0.05, the results of the *t*-test on the means of fitness function and CPU time obtained using the two algorithms on the first problem are shown in Table 10.

Table 10. CPU TIME and Cost mean comparison of employing the two algorithms in solving the first problem
Paired Differences

	Turice Directors							
Criteria Mean Std. Std. Error 95% Confidence Interval of the Deviation Mean Difference				t	df	Sig		
		Deviation	Mean	Lower	Upper			.(2-tailed)
Objective function	-7399.18286	11484.72311	2096.81397	-11687.64895	-3110.71677	-3.529	29	0.001
CPUTI	-0.13645	13.85549	2.52965	-5.31018	5.03727	-0.054	29	0.957

As seen in this table, there are significant differences between the fitness function means of the two algorithms, since the related value of significance (two-tailed) are less than 0.05.

The values of significance (two-tailed) of the measure comparisons in all problem instances are shown in Table 11.

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Problem No.	Objective function	CPU Time
1	0.000	0.000
2	0.000	0.000
3	0.000	0.000
4	0.001	0.957
5	0.000	0.108

 Table 11. The p-values of the t-tests on the equality of the means of the measures in all problem instances

As seen in this table, there are significant differences between the objective function means of the two algorithms obtained in all problem instances and there are significant differences between the CPU time means of the two algorithms obtained in fourth and fifth problem instances.

To select the better algorithm, the SAW method (Hwang and Yoon, 1981), as one of the multiple attribute decisionmaking approach, is utilized here. As the SAW method starts with normalizing a decision matrix based on the linear method, a decision matrix $D = [d_{ij}]_{m \times n}$ is first organized for *m* (here 2) alternatives and *n* indices (here 2) based on the desirability of alternative *i* towards index *j* denoted by d_{ij} . In the next step, an un-scaled weight matrix is made the weights obtained of Entropy method. The weights are shown in Table 12.

Problem No.	Objective function	CPU TIME
1	0.02579788	0.9742021
2	0.004417	0.995582961
3	0.00022701	0.999773
4	0.8884662	0.111533849
5	0.071977388	0.928023

Table 12. The weight vector of the means of the two measures in all problem instances

Then, the total sum of each row is computed. At the end, an algorithm with the largest total sum of weights (TSW) is selected for each of the problem instances. Table 13 shows TSWs of each algorithm in all problem instances.

Problem No.	GA	SA	Applicability
1	0.674262303	1	SA
2	0.623444931	1	SA
3	0.635120081	1	SA
4	0.992618417	1	SA
5	0.998624336	0.937856412	GA

Table 13. Comparison results using the SAW method

As seen in this table, SA is the better algorithm for problem instances with small and medium sizes, while GA works satisfactorily. In short, based on the results obtained using the *t*-test and the SAW method, it can be concluded that both algorithms work satisfactorily to optimize the SCN problem at hand.

Both algorithms are coded and implemented in MATLAB 8.20 and run on a computer with core (TM) i3-CPU 2.40 GHz, RAM 4.00 GB.

7. Conclusion

In this paper, we have proposed a new mathematical model for a three-echelon defective goods supply chain network (DGSCN) that not only minimizes the costs of production, holding, transport, defective goods, and scrap products, but also determines the economic production quantity (EPQ) and the appropriate cycle length. The GA and SA methods were used to solve the proposed mathematical model. To model the real states accurately, the values of the parameters were treated as probabilities with uniform distributions.

Five problem instances of small, medium, and large were used to demonstrate the application of the proposed methodology as well as to compare the performances in terms of the means of two performance measures. After the parameter calibration process of both algorithms based on a statistical method using 30 samples, the algorithms were employed to solve the problem instances, each 30 times. The results of *t*-test and the SAW method at the end showed SA acted better than GA for most of problem instances.

For future researches in this area, we recommend the followings:

- In addition to the storage capacity limitation, we may consider other constraints too.
- Other meta-heuristic search algorithms may also be employed and a comparison may be made among the algorithms.
- We can consider backorder or lost sale for shortages.
- Some parameter of the model may be either fuzzy or random variable. This being the case, the model has either fuzzy or stochastic nature.
- Researchers could apply other mutation and crossover operators.
- For the further remarks, discounts or inflation for system costs can be took into account.
- A distance parameter between the layers as well as travel times to transport the products to customer nodes can appropriately be considered.
- The SCN using the De-Novo programming approach can also be designed.

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