Defining Robust Recovery Solutions for Preserving Service Quality during Rail/Metro Systems Failure

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Abstract

This paper proposes a sensitivity analysis for evaluating the effectiveness of recovery solutions in the case of disturbed rail operations. Indeed, when failures or breakdowns occur during daily service, new strategies have to be implemented so as to react appropriately and re-establish ordinary conditions as rapidly as possible. In this context, the use of rail simulation is vital: for each intervention strategy, it provides the evaluation of interactions and performance analysis prior to actually implementing the corrective action. However, in most cases, simulation tasks are deterministic and fail to allow for the stochastic distribution of train performance and delays. Hence, the strategies adopted might not be robust enough to ensure effectiveness of the intervention. We therefore propose an off-line procedure for disruption management based on a microscopic and stochastic rail simulation which considers both service operation and travel demand. An application in the case of a real metro line in Naples (Italy) shows the benefits of the proposed approach in terms of service quality.

Keywords: Sensitivity Analysis; Public Transport Management; Rail System; Travel Demand Estimation; Quality of Service.

1. Introduction

The management of a rail system, especially in the event of disruption, is a complex task which significantly affects service attractiveness. Railway undertakings often struggle to find strategies which minimise delays and increase the reliability and stability of the planned timetable. Basically, the approaches adopted for reacting to service disruptions vary greatly and are based on the experience of dispatchers involved in this management phase (Pender et al., 2013). Moreover, the infrastructure characteristics and the signalling system technology strongly influence these decisions (Schmocker et al., 2005). However, the adoption of rail simulation models could be useful to find solutions for promptly reacting to unforeseen events (Hansen and Pachl, 2008).

These models can be classified according to the assumption on the level of detail considered for network representation, into macroscopic, mesoscopic and microscopic. Macroscopic simulation models (Prinz et al., 2001; Kettner and Sewcyk, 2002) represent the railway network with a low level of detail. Hence a node can depict a station or a junction of the real network regardless of the complexity of the station or of the junction itself.

Macroscopic models are usually adopted to support long-term planning tasks or special routing problems since they provide reliable results in a short computational time. Moreover, this method facilitates the adoption of integer programming models and hence promotes the implementation of algorithms for optimising both the planning and rescheduling process (De Shutter et al., 2002; Meng et al., 2010). Ease of computation also gives the possibility to consider different parameters to optimise by means of a more complex objective function. Thus rolling stock capacity, crew requirements and empty movements can also be taken into account (Schöbel, 2007; Cadarso and Marín, 2011, 2014).Mesoscopic models (Marinov and Viegas, 2011; De Fabris et al., 2014) are based on the simulation of network performance by means of aggregate variables such us capacity, flow and density. Moreover, traffic flow is represented by vehicle or convoy packets with identical characteristics (destination, routing behaviour, etc.) which propagate on the network. Thanks to the possibility to focus on specific factors, leaving aside the others, according to the aim of the analysis, this approach allows problems related to complex networks to be addressed in a simplified manner. Such models are generally used for both strategic and tactical planning.

Microscopic simulation models (Nash and Huerlimann, 2004; Siefer and Radtke, 2005), by contrast, require a highly detailed description of the infrastructure, the signalling system and the rolling stock. As a consequence, simulation needs a longer computational time than when using macroscopic models, but provides extremely precise results. However, there are ever more examples of the combination between macro and micro approaches which draw on the benefits of the two methods and actually represent a good compromise for searching, in order to find optimal dispatching solutions (Eickmann et al., 2003; Placido et al., 2014).

Consequently, optimization models may be classified as macroscopic, mesoscopic or microscopic according to the assumption related to the network model on which they are based. In order to take into account the large amount of interactions existing among the components of a railway system, a microsimulation approach is certainly the most appropriate. Based on this approach, operational

tools have been developed to support dispatchers in the management of networks during disrupted conditions (D'Ariano, 2009; Corman and D'Ariano, 2012).

In the literature, this issue is called a 'rescheduling problem' and has been dealt with by adopting different approaches (Cacchiani et al., 2014), most of which are based on the Alternative Graph model proposed by Mascis and Pacciarelli (2002). Therefore, these models can be based on different failure severity (see, for instance, Corman et al., 2010a; Dollovoet et al., 2012; Ghaemi et al., 2016); different kinds of network (see, for instance, Corman et al. 2010b; Veelenturf et al., 2016; Botte et al., 2017); and different perspectives such as passenger needs, operational costs or energy consumptions (see for instance Corman et al., 2011; D'Acierno et al., 2016; Umiliacchi et al., 2016). Obviously, operational aspects during perturbed operations are not the only factors to be considered. Another important issue to be taken into account is the interaction between travel demand and rail service which occurs during the passenger boarding/alighting process and produces the so-called snowball effect (Kanai et al., 2011): the greater the headway, the higher the crowding level on the platform will be. The more passengers are waiting on the platform, the longer are the dwell times of trains within the station. The longer the dwell time, the greater is the train delay which results in an increase in headway.

The study of this phenomenon is vital to simulate the rail system faithfully and carry out feasible analyses regarding management of the network (Kunimatsu et al., 2012). Indeed, in the literature there are numerous contributions which address the close relationship between dynamic passenger demand, the timetable and the reliability of the rail service (see, for instance, Stato et al., 2013; Pouryousef and Lautala, 2015; Barrena et al., 2016).

Hence, design strategies influence passenger behaviour and discomfort, and this effect strongly modifies the attractiveness of the service. Indeed, simulation of the network without considering customer needs is not sufficient to ensure a high level of service quality. To this end, both macroscopic and microscopic models have recently been proposed (Corman et al., 2012; Kecman et al., 2013). However, rescheduling tasks are generally based on deterministic simulations and fail to consider the randomness of some events such as driver behaviour or the boarding/alighting process at each station. This means that the solutions provided by the above-mentioned models may not be robust enough to implement in a real context. Therefore, automatic systems for real-time dispatching of rail traffic have not yet been put into practice and are still a matter for research (Quaglietta et al., 2013). For this reason, the aim of this paper is to evaluate the robustness of intervention strategies during perturbed service conditions, considering the user generalised cost perceived by customers as a service quality index. Importantly, the concept of robustness addressed in the paper does not concern the capacity of the rail system to absorb delays and avoid their propagation (as proposed by Goverde, 2005; Cacchiani et al., 2009), but the more general notion of robustness as the ability of a model to provide the same output when input data change. To this aim, we tested the proposed optimisation model by considering input data first as deterministic values and then as random variables in order to compare the optimal intervention strategies provided by the model in these two cases. Therefore, the suggested methodology is based on a two-step off-line procedure. First of all, the optimal solutions for re-establishing ordinary conditions and minimising passenger discomfort are evaluated by adopting a deterministic microscopic

simulation. Then, by means of numerous stochastic microscopic simulations, a sensitivity analysis of these solutions was carried out so as to evaluate their feasibility and reliability. The paper is structured as follows. In the second section, the notation and terminology adopted in the paper are summarised. The architecture of the proposed model is described in detail in section 3. An application on a real metro network in the city of Naples (Italy) is then presented. Finally, conclusions and possible research prospects are summarised in section 5.

2. Notation and terminology

In the next sections the following notation will be adopted:

$\boldsymbol{X}$	The considered multivariate random variable (i.e. a vector of random variables);
$\overline{\boldsymbol{X}}$	The fixed vector whose elements are the mathematical expectations (i.e. first moments or means) of elements of $\boldsymbol{X}$;
$\boldsymbol{\varepsilon}_X$	The random residual of $\boldsymbol{X}$;
$\boldsymbol{\Omega}_X(\cdot)$	The statistical distribution of $\boldsymbol{\varepsilon}_X$;
$\boldsymbol{h}_X$	The vector of parameters of statistical distribution $\boldsymbol{\Omega}_X(\cdot)$;
$\boldsymbol{y}$	The vector of parameters identifying the generic intervention action;
$\hat{\boldsymbol{y}}_{STOC}$	The optimal value of vector $\boldsymbol{y}$, by adopting a stochastic approach;
$\hat{\boldsymbol{y}}_{DET}$	The optimal value of vector $\boldsymbol{y}$, by adopting a deterministic approach;
$\boldsymbol{S}_y$	The definition set of vector $\boldsymbol{y}$, which identifies all feasible intervention strategies;
$\boldsymbol{fc}$	The vector of parameters describing failure contexts;
$\boldsymbol{tnp}$	The vector of parameters identifying the transportation network performance (i.e. travel times and monetary costs);
$\boldsymbol{rnp}$	The vector of parameters identifying network performance (i.e. headways and travel times) of the rail system;
$\boldsymbol{td}$	The vector identifying travel demand (i.e. user flows) on the network;
$\boldsymbol{\Lambda}(\cdot)$	The function simulating the whole rail system;
$\boldsymbol{in}^0$	The vector of parameters describing the rail infrastructure in a non-perturbed condition;
$\boldsymbol{rs}^0$	The vector of parameters describing rolling stock performance in a non-perturbed condition;
$\boldsymbol{ss}^0$	The vector of parameters describing the signalling system in a non-perturbed condition;
$\boldsymbol{pt}$	The vector describing the planned timetable;
i	The generic user category;
β^i_{VOT}	A parameter which expresses the monetary value associated by user category i to a time unit and indicated in the literature as *Value of Time* (*VOT*);
β^i_w	A parameter which expresses the relevance (i.e. relative weight) associated by user category i to waiting times;
$tw^{i,r}_{s,p}$	The average waiting time for user category i at station s on platform p between run $(r-1)$ and run r;
$fw^{i,r}_{s,p}$	The number of passengers of category i waiting at station s on platform p between run $(r-1)$ and run r;
β^i_b	A parameter which expresses the relevance (i.e. relative weight) associated by user category i to on-board times;

$tb_l^{i,r}$	The time spent by user category i for travelling on the rail convoy associated to run r while crossing link l;
$fb_l^{i,r}$	The number of passengers of category i travelling on the rail convoy associated to run r while crossing link l;
$\boldsymbol{\Gamma_{FSM}}(\cdot)$	The function describing the *Failure Simulation Model* (*FSM*) which provides reduced performance of the rail system for each possible failure context;
$\boldsymbol{\Gamma_{SeSM}}(\cdot)$	The function describing the *Service Simulation Model* (*SeSM*) which provides rail system performances depending on rail infrastructure, rolling stock, signalling system, planned timetable and travel demand;
$\boldsymbol{\Gamma_{SuSM}}(\cdot)$	The function describing the *Supply Simulation Model* (*SeSM*) which provides user generalised costs on all transportation systems depending on the outputs from service and travel demand simulation models;
$\boldsymbol{\Gamma_{PPSM}}(\cdot)$	The function describing the *Pre-Platform Simulation Model* (*PPSM*) which provides passenger flow arrivals at each station depending on the performance of all transport modes;
$\boldsymbol{\Gamma_{OPSM}}(\cdot)$	The function describing the *On-Platform Simulation Model* (*OPSM*) which assigns passengers waiting at platforms to trains according to capacity constraints;
$N(\hat{\boldsymbol{y}}_{DET})$	A neighbourhood of vector $\hat{\boldsymbol{y}}_{DET}$, which consists of all configurations $\boldsymbol{y}$ providing objective function values close to the objective function calculated in the case of strategy $\hat{\boldsymbol{y}}_{DET}$;
mc^r	The capacity (in terms of the maximum number of passengers per rail convoy) of run r;
$obp_{(s-1:s)}$	The number of on-board passengers between stations *s-1* and *s*;
ap_s	The number of passengers alighting at station *s*.
$\overline{rnp}$	The rail network performance calculated by the deterministic approach;
$\overline{td}$	The travel demand in the case of the average level.

3. Analysis of the disruption management model

In this section, we describe our proposal for determining optimal intervention strategies in the case of a rail or metro system breakdown which are robust to variations in input data such as rail service performance (speeds, accelerations and delays). In particular, Section 3.1 provides an innovative analytical formulation which can be implemented by adopting four simulation models proposed in the literature by D'Acierno et al. (2013). Commercial software and an ad-hoc tool developed for managing real-dimension networks are described in Section 3.2. Finally, in Section 3.3 the proposed procedure for analyzing the rail service by jointly adopting a deterministic and stochastic approach is described in detail.

3.1. Framework of the model

The aim of this study is to provide a methodology in order to perform a stability analysis of intervention strategies in the case of rail system disruptions. As shown in the literature (D'Acierno et al., 2013), it is possible to identify a virtual analytical application which, for any possible Defining Robust Recovery Solutions for Preserving Service Quality … Int J Supply Oper Manage (IJSOM), Vol.3, No.3 breakdown of the rail system, is able to provide the optimal intervention strategy, minimizing user discomfort.

Obviously, it is necessary to allow for the effects of corrective action on all users of the rail system. This means that, in the case of a single train failure, discomfort has to be computed for users on the faulty train, for users on other trains and for users on the platforms. The improvement provided by the current study consists in considering some variables of the problem as the realization of random (i.e. stochastic) processes in order to verify the stability of solutions obtained with a deterministic approach, that is:

$$X = \overline{X} + \varepsilon_X \tag{1}$$

With:

$$\overline{X} = E[X]$$
$$\varepsilon_X \sim \Omega_X(h_X)$$

Hence, the problem of determining the optimal intervention strategy in the case of rail system failure, adopting a random process approach, can be formulated as a multidimensional bi-level constrained optimisation problem where it is necessary to minimise an objective function which expresses user discomfort, that is:

$$\hat{y}_{STOC} = \underset{y \in S_y}{\arg \min} \; Z(y, fc, tnp, rnp, td) \tag{2}$$

Subject to:

$$[tnp, rnp, td]^{\mathrm{T}} = \Lambda(y, fc, tnp, rnp, td, in^0, rs^0, ss^0, pt) \tag{3}$$

With:

$$[y, fc, tnp, rnp, td, in^0, rs^0, ss^0, pt]^{\mathrm{T}} = \left[\overline{y}, \overline{fc}, \overline{tnp}, \overline{rnp}, \overline{td}, \overline{in}^0, \overline{rs}^0, \overline{ss}^0, \overline{pt}\right]^{\mathrm{T}} +$$
$$+ \left[\varepsilon_y, \varepsilon_{fc}, \varepsilon_{tnp}, \varepsilon_{rnp}, \varepsilon_{td}, \varepsilon_{in^0}, \varepsilon_{rs^0}, \varepsilon_{ss^0}, \varepsilon_{pt}\right]^{\mathrm{T}} \tag{4}$$

$$Z(y, fc, tnp, rnp, td) = \sum_i \beta_{VOT}^i \cdot \left(\beta_w^i \cdot \sum_s \sum_p \sum_r tw_{s,p}^{i,r}(y, fc, tnp, rnp) \cdot fw_{s,p}^{j,r}(td) + \right.$$
$$\left. + \beta_b^i \cdot \sum_l \sum_r tb_l^{i,r}(y, fc, tnp, rnp) \cdot fb_l^{i,r}(td) \right) \tag{5}$$

However, in the case of a deterministic approach, that is:

$$\varepsilon_X = 0 \qquad \forall \; x$$

Eq. (4) trivially degenerates into:

$$[y, fc, tnp, rnp, td, in^0, rs^0, ss^0, pt]^{\mathrm{T}} = \left[\overline{y}, \overline{fc}, \overline{tnp}, \overline{rnp}, \overline{td}, \overline{in}^0, \overline{rs}^0, \overline{ss}^0, \overline{pt}\right]^{\mathrm{T}} \tag{6}$$

And Eq. (2) has to be reformulated as:

$$\hat{y}_{DET} = \underset{y \in S_y}{\arg \min} \; Z(y, fc, tnp, rnp, td) \tag{7}$$

It is worth noting that constraint (3) is a consistency constraint between travel demand flows and transportation system performance, whose formulation requires the definition of four kinds of simulation models (D'Acierno et al., 2013): *failure*, *service*, *supply* and *travel demand simulation models*.

The *Failure Simulation Model* (*FSM*), indicated as function $\Gamma_{FSM}(\cdot)$, can be considered a model which provides reduced performances of the rail system (i.e. infrastructure, rolling stock and/or signalling system) for each possible failure context. Basically, this model is based on the adoption of *RAMS* techniques (see CENELEC, 1999). The term RAMS is the acronym for *Reliability*, *Availability*, *Maintainability* and *Safety* where:

- *Reliability* can be considered the ability of a system to perform a specific function, and may be given as design reliability or operational reliability;

- *Availability* represents the ability of a system to be kept in a functioning state;

- *Maintainability* stands for the simplicity with which the product or system can be repaired or maintained;

- *Safety* is the requirement not to harm people, the environment, or any other assets during the life cycle of the system.

In our context, a RAMS technique aims to study the behaviour of the whole system, or just a component of it, in order to assess failure modes and their causes. In particular, RAMS analysis consists of the following three phases: data compilation, simulation and impact on system life cycle. The first phase is the basis of any RAMS simulation or process since it gives the possibility to obtain data on the failure rate and other reliability parameters which are the input for the simulation. In the second phase, the objective is to model the system in terms of reliability aspects. In other words, the simulation, given the values of failure rate and reliability of each single component, estimates causes and effects of failures of the same components, but by considering their mutual interaction in a system and a stated environment. Finally, in the last step, the goal is the assessment of the cost of breakdowns and corrective maintenance operations. Therefore, the failure model provides the possibility to determine weaknesses and defects of the rail system under study, selecting the failure scenarios which are worth analysing.

The *Services Simulation Model* (*SeSM*), indicated as function $\Gamma_{SeSM}(\cdot)$, can be considered a model which provides rail system performance as depending on rail infrastructure, rolling stock, signalling system, planned timetable and travel demand. It consists of a microscopic rail simulation model which simulates the service with the highest level of detail regarding infrastructure, rolling stock and the planned timetable.

The choice of adopting a microsimulation model, instead of a macrosimulation one, results from several reasons which is worth discussing. Microsimulation models, albeit not computationally efficient, can reproduce the dynamic of network loading and consider the evolution of rail traffic conditions during the whole daily service. Disruptions or breakdowns to the service are indeed dynamic events which cannot be simulated statically. Hence, a microsimulation model, due to the highly detailed description of the network, enables simulation of any kind of failure. Thus, the outputs provided by the *FSM*, whatever the component of the system affected (e.g. interruption of a line section, rail convoy breakdown, problem to the signalling system), can be set up in the *SeSM* for estimating related effects on the rail service.

The *Supply Simulation Model* (*SuSM*), indicated as function $\boldsymbol{\Gamma}_{SuSM}(\cdot)$, can be considered a model which provides user generalised costs on all transportation systems depending on the outputs from service and travel demand simulation models. The idea is to simulate the rail network in detail (i.e. microscopically through the *SeSM*) taking into account also the different transport modes within the study area so as to enable a more realistic estimation of the arrival rate at station. In order to make clearer this statement, an example is provided. Passengers waiting for a train on the platform can decide to modify their trip (adaptive choice) if no trains arrive after a long period. Likewise, if something negative happens within the road or bus systems, it is likely that the number of customers arriving at stations increases. Obviously, this approach is useful for the study of urban contexts, where public transport systems are strongly integrated. Conventional railway networks (e.g. long distance train), by contrast, can be analysed as a closed system without interactions with other transport modes.

The *Travel Demand Simulation Model* (*TDSM*) can be considered a model which provides user flows on all transportation networks (included the rail system) conditioned by all transportation system performance (i.e. outputs of *SeSM* and *SuSM*). This process is the result of two different choice levels, whose analysis requires two correspondent sub-models. First of all, the *Pre-Platform Simulation Model* (*PPSM*), indicated as function $\boldsymbol{\Gamma}_{PPSM}(\cdot)$, determines the flows of passengers who, according to the performances of all transport modes (evaluated by the *SuSM*), arrive at each station and hence decide to take the train (Cascetta, 2009). By contrast, the *On-Platform Simulation Model* (*OPSM*), indicated as function $\boldsymbol{\Gamma}_{OPSM}(\cdot)$, assigns these passengers to trains according to their capacity limit. Basically, the *OPSM* analyses, for each train approaching the station, whether there is enough capacity to host all the customers willing to board the train. If this condition is not verified, passengers on the platform behave differently depending on the characteristics of the simulated rail service.

In case of high frequency rail services, passengers do not know the exact train schedule but go to the station considering, as only information, the headway (e.g. a train every two minutes). Hence, due to the high frequency, it is likely that the surplus of passengers (i.e. users who do not manage to board the train) remain on the platform waiting for the next train. On the contrary, when the model is applied on conventional rail lines, where frequencies of a specific train category can be very low (e.g. long distance trains), except from particular situations where passengers do not have any alternative for reaching their destination, it is likely that no one remain on the platform waiting for a following convoy in case the capacity of the train is reached (very uncommon event on conventional lines) or in case the line is interrupted.

According to these assumptions, the travel demand assignment is performed. Thus, all information about passenger trips (e.g. the time spent waiting the train, the number of train waited before boarding, the total travel time) is known and the objective function (i.e. Eq. 5) can be calculated.

Taking the four simulation models explicitly into account, Eq. (3) can be rewritten as:

$$
\begin{cases}
\boldsymbol{tnp} = \boldsymbol{\Gamma}_{SuSM}\left(\boldsymbol{rnp}, \boldsymbol{td}\right) \\
\boldsymbol{rnp} = \boldsymbol{\Gamma}_{SeSM}\left(\boldsymbol{y}, \boldsymbol{\Gamma}_{FSM}\left(\boldsymbol{fc}, \boldsymbol{in}^0, \boldsymbol{rs}^0, \boldsymbol{ss}^0\right), \boldsymbol{td}, \boldsymbol{pt}\right) \\
\boldsymbol{td} = \boldsymbol{\Gamma}_{OPSM}\left(\boldsymbol{rnp}, \boldsymbol{\Gamma}_{PPSM}\left(\boldsymbol{tnp}, \boldsymbol{rnp}\right), \boldsymbol{\Gamma}_{FSM}\left(\boldsymbol{fc}, \boldsymbol{in}^0, \boldsymbol{rs}^0, \boldsymbol{ss}^0\right)\right)
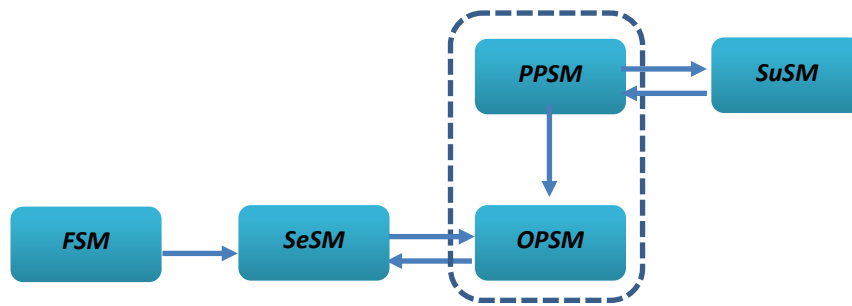\end{cases} \tag{8}
$$

Figure 1. Interaction among simulation models

Figure 1 shows the graphical representation of expression (6). Basically, given the failure data (*FSM*) and the passenger flows at each station (interaction between *SuSM* and *PPSM*), the kernel of the procedure is the simulation of the rail network (*SeSM*) and the assignment of travel demand to the service (*OPSM*).

This generates a dynamic interaction: rail system performances (mainly headways and running times) influence passenger flow arrivals at stations which, in turn, affect the service through the boarding/alighting process. Therefore, the proposed approach is able to capture the above-mentioned snowball effect (i.e. the increase in delays due to the interaction between rail service and travel demand).

3.2. Description of software and tools for the application of the model

Previous section shows the framework of the proposed approach and the analytical formulation of each model. However, there is no practical explanation about how the *SeSM* and the *OPSM* are implemented.

The microscopic rail simulation model adopted in this procedure is OPENTRACK® which is a commercial software programme (Nash et al., 2006). It reproduces in high detail the motion of the trains using a mixed discrete/continuous simulation process that calculates both the continuous numerical solution of the differential motion equations for the vehicles (trains) and the discrete processes of signal box states and delay distributions. All information regarding infrastructure, rolling stock and timetable can be set up so as to take any detail into account. In addition, according to the analysis targets, these data can be considered as deterministic or stochastic variables, giving the possibilities to simulate the effects of stochastic disturbances affecting real operations. Furthermore, in OPENTRACK® it is possible to manage input and output data also from outside the program by means of proper text files, enabling the combination with other tools as the one presented in the following.

The *OPSM* is performed by means of OPM 1.0, namely a specific application developed in C++ language (Placido, 2015) for the dynamic assignment of travel demand to the service. It consists of three modules which specify the passenger inflow to stations (as a result of the interaction between *PPSM* and *SuSM*), the rolling stock characteristics (e.g. the maximum number of passengers per train, number of doors per coach, the maximum number of sitting and standing passengers per coach) and the service simulation (e.g. itinerary of each simulated train, information regarding headways and running times).

As outputs, this tool provides the actual load diagram for each simulated train, all information about passengers' trips (e.g. waiting and boarding times), platform congestion and the user generalised cost of the strategy (i.e. objective function 5). OPM 1.0 can also estimate the dwell times at stations as flow dependent if integrated with another module, which is DwTE 1.0 (Placido et al., 2015). In particular, this module enables the analysis of the above mentioned '*snowball effect*' providing the departure delays at each station resulting from the dynamic interaction between passengers and rail service.

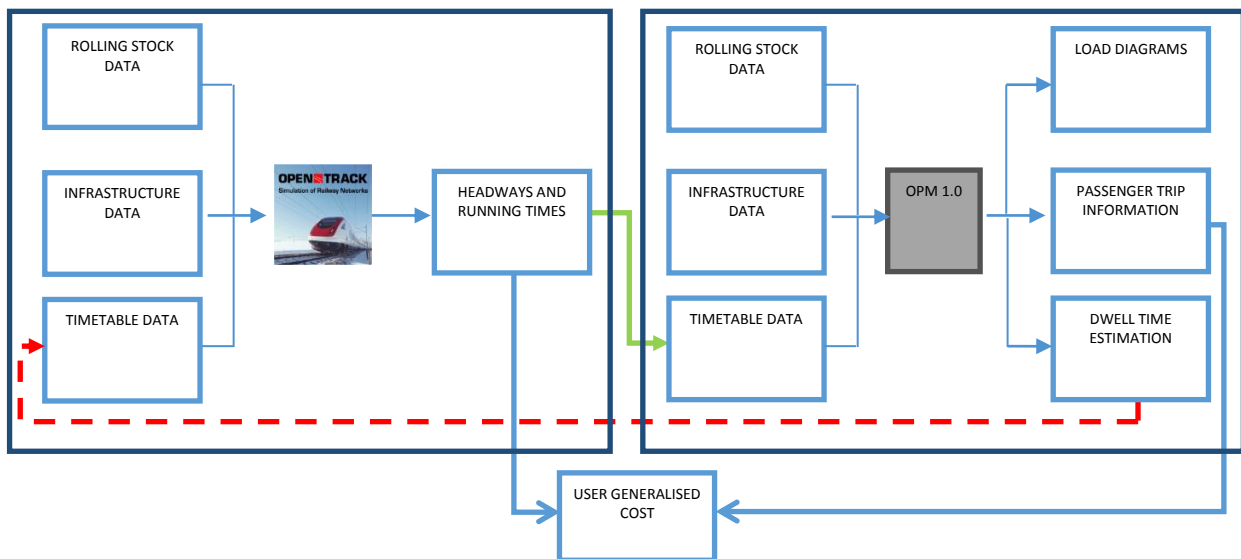The combination of OPENTRACK® and OPM 1.0 is shown in Figure 2.

Figure 2. Combination of OPENTRACK® and OPM 1.0

3.3. *Procedure description*

The resolution of problem (2), subject to constraint (3) and to objective function (5), requires the implementation of two different phases. The first is a deterministic phase and is divided into the following steps:

1. First of all, the *FSM* selects the breakdown context which has to be simulated and it determines the effects on the infrastructure, the rolling stock and on the signalling system;

2. According to these outputs, the *SeSM* simulates the rail service under fault conditions;

3. Different intervention strategies are then selected to re-establish ordinary service conditions by adopting an exhaustive approach (i.e. simulation of all possible implementable strategies) or a selective approach (i.e. simulation of a limited set of alternatives);

4. For each simulated strategy, different passenger inflows (i.e. variability of variable *td*) to stations are assigned dynamically to the service considering the capacity constraint of trains, so as to evaluate the answer of the system with different travel demand levels;

5. After completing the travel demand assignment, the user generalised cost of each recovery solution is calculated;

6. Then the costs are compared and the optimal intervention strategy $\hat{y}_{DET}$ (i.e. lower generalised cost) is identified, ending the deterministic phase.

The second phase assesses the feasibility and the robustness of the strategies and consists of the following additional steps:

1. At the end of the deterministic phase, a neighbourhood of $\hat{y}_{DET}$, indicated as $N(\hat{y}_{DET})$, which consists of all the corrective actions providing objective function values close to the minimum cost (i.e. objective functions calculated in the case of strategy $\hat{y}_{DET}$) is analysed;

2. n vectors describing random residual ε_X are extracted;

3. For each single extracted vector ε_X, the new objective function values for all the intervention strategies of set $N(\hat{y}_{DET})$ are calculated. Obviously, calculation of the objective function requires the solution of problem (8);

4. Finally, the distribution of the objective function values for each element of set $N(\hat{y}_{DET})$ is analysed in order to perform the analysis of stability.

Figure 3 summarises the whole procedure described above.

4. Application to a real metro line

In order to show the utility of the proposed approach, this methodology was applied in the case of Line 1 of the Naples metro system (Italy). The line, consisting of 17 stations along a total length of about 18 kilometres, plays a key role in the Naples public transportation system since it connects the high density suburbs with the city centre. For this reason, especially during peak hours, the line is extremely crowded since customers cannot rely on the performance of alternative means of transport (e.g. buses or trams) which is generally lower due to the high congestion level of the main roads.
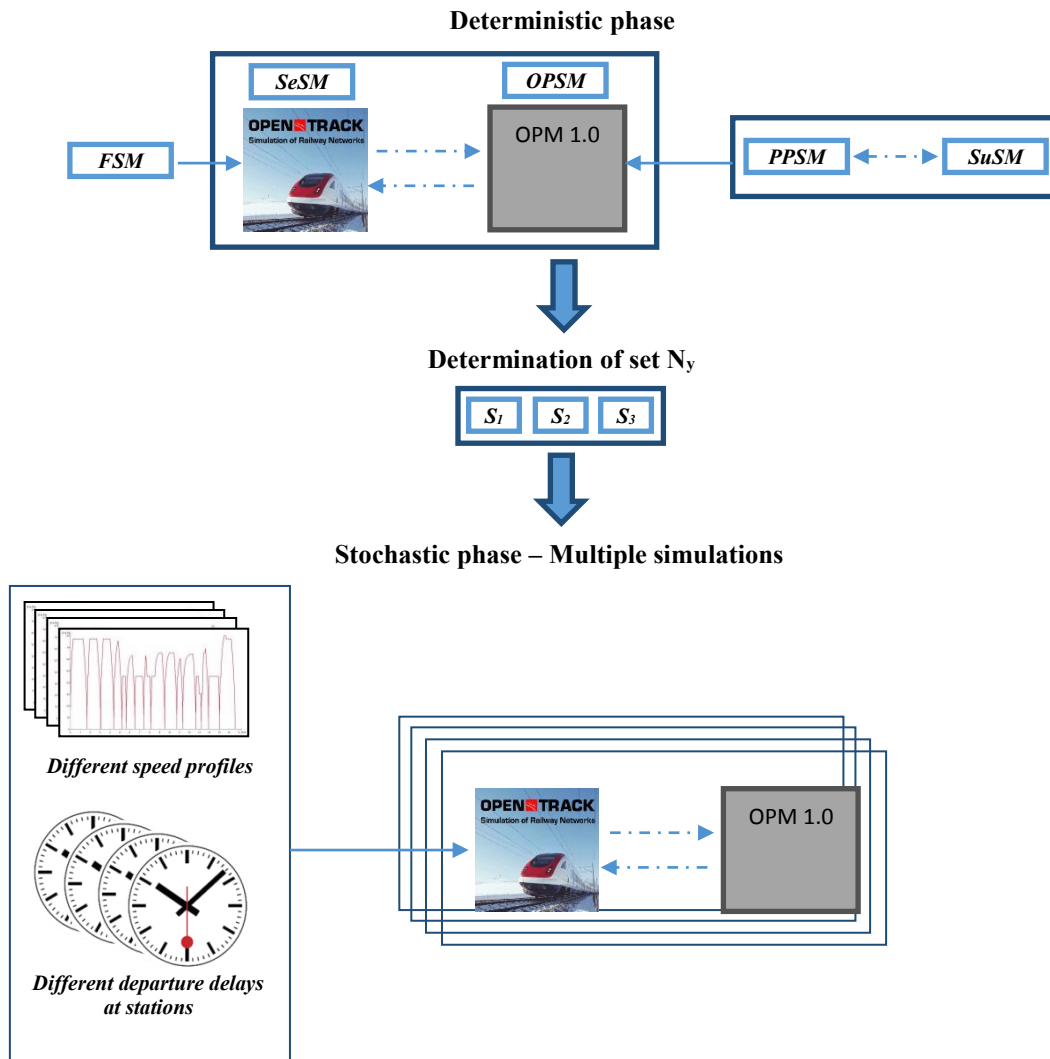
Deterministic phase

Determination of set N_y

Stochastic phase – Multiple simulations

Figure 3. Graphical description of both the deterministic and stochastic phases

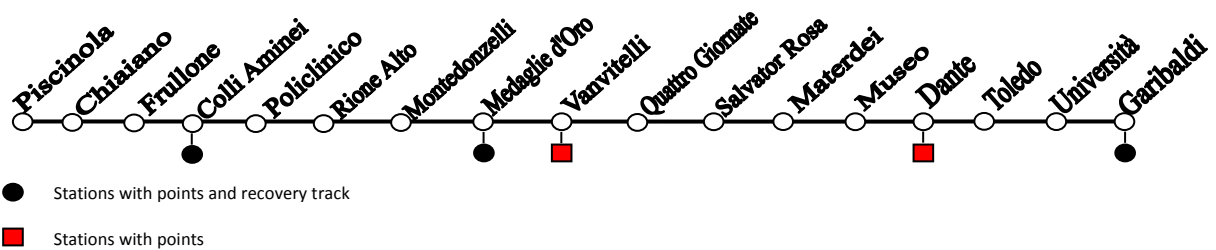● Stations with points and recovery track

■ Stations with points

Figure 4. Framework of Line 1 in Naples (Italy)

The Line 1 infrastructure is extremely complex because of the hilly terrain in the city. Indeed, steep slopes and low radius curves have led to the construction of two completely separate tunnels, one per direction. Only certain stations (indicated in Figure 4 with additional coloured dots and

squares) are equipped with points and/or recovery tracks, which reduces the elasticity of the system in the event of failure.

Furthermore, there is just one depot located near Piscinola station and spare trains are not always available due to the lack of rolling stock. Indeed, when there is a faulty train in the network, dispatchers prefer to close the whole line and remove the broken-down convoy or, should it be possible, just leave the service without any kind of intervention. Obviously, this results in great discomfort for passengers who are not considered at all. Indeed, although this strategy is the easiest to implement and is also optimal from an operational point of view, it does not fulfil customers' needs. The application of our approach will therefore show how it is possible to plan recovery intervention strategies whilst maintaining a high level of service quality.

4.1. The deterministic approach for selecting the optimal intervention strategies

As it is explained in the previous section, the deterministic approach is the first phase of the proposed procedure and it is necessary for the purpose of evaluating the optimal strategies form passengers' standpoint.

Hence, the whole line has been reproduced in OPENTRACK® including all details regarding infrastructure, signalling system and rolling stock.

The simulated scenario consists of assuming a breakdown to a train during the morning peak hours. To be precise, run 900 breaks down after leaving Chiaiano at 7:05 am and is forced to run at a maximum speed of 45 km/h. Obviously, this performance reduction constitutes a bottleneck for the line, since the faulty train cannot be easily overtaken.

In order to evaluate the propagation of the effects on the following time periods, our simulation concerned all the daily operations. In particular, the following timetable is planned:

- a train every 8 minutes from 6:00 am to 09:00 pm;

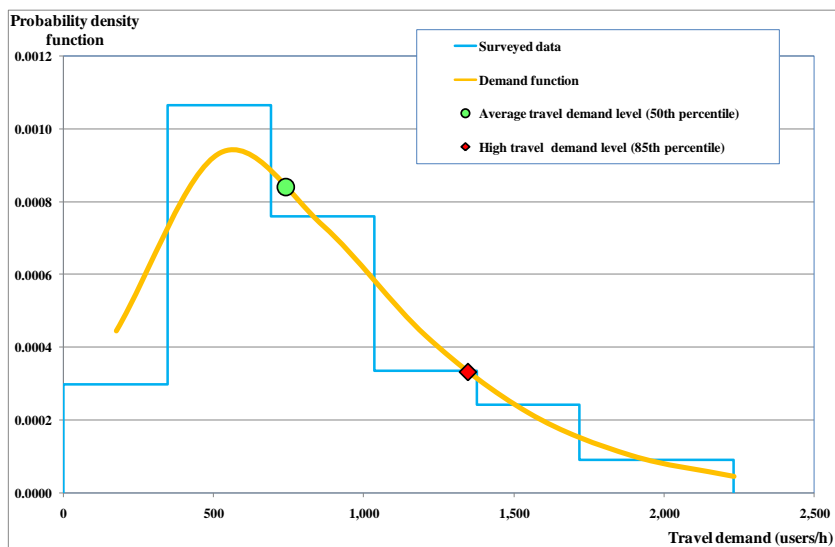- a train every 14 minutes from 09:00 pm to midnight.

Figure 5. Estimation of travel demand values for different congestion levels

The pre-platform simulation model (i.e. function Γ_{PPSM}), as already shown, estimates the travel demand matrices expressing passenger mode choices through the interaction with the *SuSM* (Cascetta, 2009). It is thus possible to ascertain the flow of passengers entering the system and going to the platform. As shown in the literature (Marzano et al., 2008; Cantelmo et al., 2014), these matrices are estimated by using a combination of models and surveys, and then used in this application. In particular, two flow levels are considered so as to evaluate whether the solutions are affected by different travel demands. Indeed, the procedure was applied by first considering the average travel demand flow corresponding to usual conditions and then a higher travel demand compatible with particular crowded days. Figure 5 shows the estimated distribution of travel demand which was determined by means of standard calibration techniques whose aim is to identify the statistical distribution which best fits the surveyed data (Cascetta, 2009). In particular, in our case, surveyed data are represented by passenger arrival flows at turnstiles which, in this way, are random variables, and thus the O-D matrices can be defined according to different travel demand percentiles. This estimation procedure was carried out for each station, travel direction and time period analysed.

Travel demand is then assigned by OPM 1.0 (i.e. function Γ_{OPSM}) to each train according to the network performance evaluated by OPENTRACK® (i.e. function Γ_{SeSM}). As far as the capacity constraint is concerned, in the case of Line 1 each train comprises two traction units coupled together for a total capacity of 864 passengers.

The objective function parameters of formula (5) express the 'disutility' perceived by the users during their journey. In particular, as shown in the literature, the waiting time (β_w^i = 2.5) is considered almost three times more burdensome than the travel time (β_b^i = 1) (Wardman, 2004; Cascetta, 2009). Moreover β_{VOT}^i, which reflects the user's willingness to pay for saving one hour of travel, is considered equal to 5.00 €/h.

The determination of the intervention strategies is based on the adoption of an exhaustive approach. Basically, according to the infrastructure characteristics, the set of strategies comprises the following alternatives:

1. The faulty train continues to perform its service all day;

2. The train stops at Colli Aminei during its outward trip (i.e. from Piscinola to Garibaldi) and is then driven onto the recovery track. No spare trains are considered;

3. The train stops at Medaglie d'Oro during its outward trip and is then driven onto the recovery track. No spare trains are considered;

4. The train stops at Garibaldi at the end of its outward trip and is then driven onto the recovery track. No spare trains are considered;

5. The train completes the outward trip and starts the return trip (i.e. from Garibaldi to Piscinola) up to Medaglie d'Oro where it is driven onto the maintenance track. No spare trains are considered;

6. The train completes the outward trip and starts the return trip up to Colli Aminei where it is driven onto the maintenance track. No spare trains are considered;

7. The train completes the outward trip and starts the return trip up to Piscinola where is driven to the depot. No spare trains are considered;

8.   The train stops at Colli Aminei during its outward trip and is then driven onto the recovery track. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

9.   The train stops at Medaglie d'Oro during its outward trip and is then driven onto the recovery track. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

10. The train stops at Garibaldi at the end of its outward trip and is then driven onto the recovery track. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

11. The train completes the outward trip and starts the return trip up to Medaglie d'Oro where it is driven onto the maintenance track. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

12. The train completes the outward trip and starts the return trip up to Colli Aminei where it is driven onto the maintenance track. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

13. The train completes the outward trip and starts the return trip up to Piscinola where it is driven to the depot. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

14. The train stops its run at Dante and, after changing direction, is driven empty to the depot. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

15. The train stops its run at Dante and, after changing direction, is driven empty to the depot. No spare trains are considered;

16. The train stops its run at Vanvitelli and, after changing direction, is driven empty to the depot. No spare trains are considered;

17. The train stops its run at Vanvitelli and, after changing direction, is driven empty to the depot. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

18. The train stops its run at Medaglie d'Oro and, after changing direction, is driven empty to the depot. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation;

19. The train stops its run at Medaglie d'Oro and, after changing direction, is driven empty to the depot. No spare trains are considered;

20. The train stops its run at Coli Aminei and, after changing direction, is driven empty to the depot. No spare trains are considered;

21. The train stops its run at Colli Aminei and, after changing direction, is driven empty to the depot. A spare train starts from Piscinola to replace the faulty rolling stock for the rest of the daily operation.
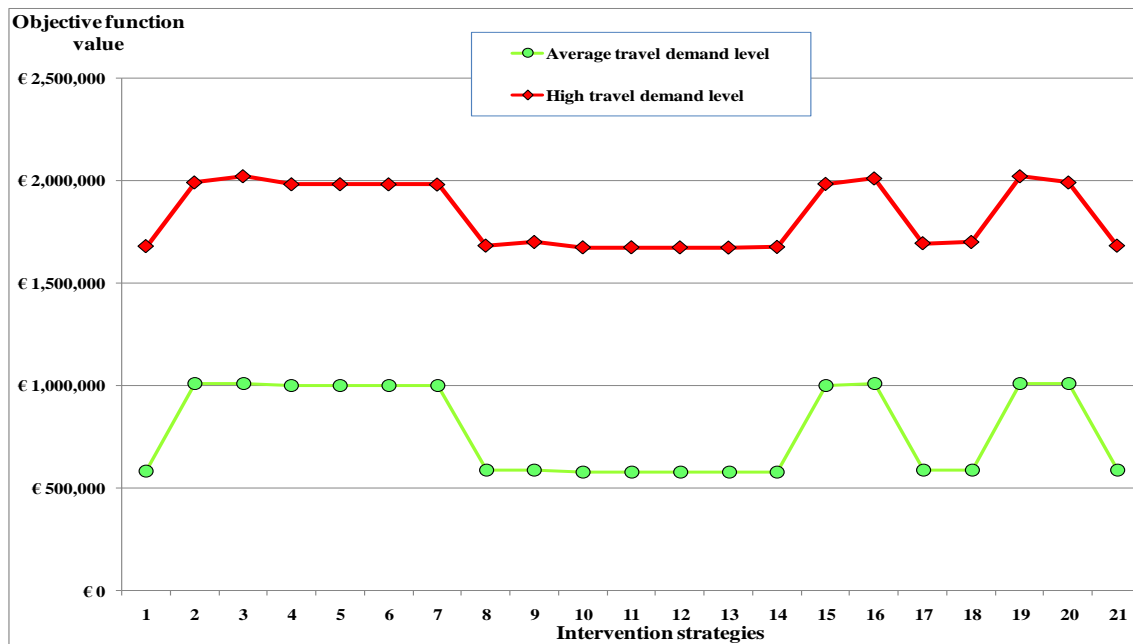
Figure 6. Objective function values for different intervention strategies

As can be seen, for any feasible strategy, the possibility of including a new spare train in the service has been considered. This assumption shows the importance of investing in rolling stock in order to increase the reliability and feasibility of the service, especially in the case of degraded conditions. It is important to underline the fact that, every time the train stops its run, passengers are forced to get off the coaches and wait for a following train on the platform.

Table 1. Objective function values for any feasible intervention strategy

Scenario	Average travel demand level User generalised cost	High travel demand level User generalised cost
1	€ 582,728	€ 1,679,960
2	€ 1,010,220	€ 1,991,610
3	€ 1,008,680	€ 2,021,960
4	€ 1,000,280	€ 1,982,610
5	€ 1,000,390	€ 1,982,780
6	€ 1,000,030	€ 1,982,290
7	€ 999,338	€ 1,981,330
8	€ 587,464	€ 1,682,420
9	€ 586,201	€ 1,700,280
10	**€ 578,208**	**€ 1,674,060**
11	€ 578,361	€ 1,674,280
12	**€ 578,001**	**€ 1,673,790**
13	**€ 577,305**	**€ 1,672,830**
14	€ 579,384	€ 1,676,050
15	€ 1,001,420	€ 1,984,550
16	€ 1,008,460	€ 2,011,190
17	€ 586,131	€ 1,694,210
18	€ 586,054	€ 1,700,090
19	€ 1,008,530	€ 2,021,760
20	€ 1,010,070	€ 1,991,410
21	€ 587,318	€ 1,682,220

It is worth noting that even in the case of a simple network as the one considered, the number of feasible intervention strategies is considerable. Hence, in the case of more complex networks (such as rail context with several interconnections) the adoption of an exhaustive approach has to be excluded and the optimisation problem (2) has to be solved by means of meta-heuristic algorithms in order to obtain sub-optimal solutions (i.e. selective approach) within suitable calculation times (D'Acierno et al., 2014). Figure 6 and Table 1 summarise the results of the deterministic approach considering both average and high travel demand. In order to make clearer how user generalised costs are evaluated, more details about the procedure are necessary. For each scenario, the *SeSM* provides the headways and the running times (i.e. $tb_l^{i,r}$) of each train performing the service. The number of passengers waiting for the approaching run r on the platform of the generic station s (i.e. $fw_{s,p}^{j,r}$) can be easily estimated multiplying the obtained headway by the passenger arrival rate at station. At the same time, the *OPSM* evaluates the number of passenger who can board the train (i.e. $fb^{i,r}$) according to its residual capacity, calculated as follows:

$$rc_s^{\ r} = mc^r - obp_{(s-1;s)} + ap_s$$

Obviously, adopting a *FIFO* (*First In – First Out*) assumption, the number of non-served passengers will have priority in boarding the following runs. Thus parameter $tw_{s,p}^{j,r}$ considers a precise estimate of the time wasted on the platform by user flow $fw_{s,p}^{j,r}$.

For both travel demand levels, the optimal solution which produces the lowest user generalised cost is that corresponding to '*strategy 13*'. The strategies which consider the introduction of the spare train are evidently preferable inasmuch as they give the possibility to calm down the negative effects of the breakdown in less time than the other alternatives. In addition, these solutions enable to keep the planned frequency levels during the rest of the day. As previously explained, it is worth considering a neighbourhood (in terms of objective function value) of the optimal strategy and analysing also other alternatives which provide similar results. For this reason, strategies 10, 12 and 13 are simulated again in the following stochastic approach. For all three strategies considered, the intervention solution suggests stopping the train after completing the outward trip. This is due to the fact that during the morning rush hours, many passengers travel from the suburbs to the city centre so as to reach the main places of the city. For the same reason, in the opposite direction, the travel demand matrix is almost unloaded. Therefore, the these strategies highlights the fact that users prefer travelling by a slower train and reaching their destinations rather than waiting more time for other crowded trains which run faster. During the return trip by contrast, the cancellation of the run does not provide the same inconvenience, since passenger levels are lower. These results can be obtained only by means of travel demand assignment to the service, meaning that a simple simulation of the rail service is not sufficient to yield recovery strategies which are really useful to satisfy customers' requirements.

As regards the computational time, the simulation of each scenario takes about 500 seconds. The OPM 1.0 tool, by contrast, needs just a few seconds to assign passengers to the network and obtain the results. However, since our proposal is an off-line procedure, time is not the main variable to consider.

4.2. The stochastic approach for analysing the robustness of the selected intervention strategies

Application of the stochastic analysis provides useful information about the feasibility and the robustness of the three optimal intervention strategies. Indeed, the variability in acceleration, the maximum speed and the dwell times are key factors which strongly influence the service and cannot be neglected.

For this purpose, 100 stochastic scenarios were constructed based on the following assumptions:

- Train performance (i.e. acceleration and speed) is defined according to a piecewise linear distribution function: 33% of the trains are supposed to perform at 85%–90%, 33% at 90%–95%, and 34% at 95%–100%;

- The departure delays at each station follow a negative exponential random variable whose average is 10 seconds. In this way, although implicitly, the variability of dwell time is taken into account.

Both assumptions correspond to considering that vector $\textbf{\textit{rnp}}$ is affected by variability. Hence, jointly with the assumption of two levels of demand, which corresponds to the adoption of variability also in the case of vector $\textbf{\textit{td}}$, in this real metro line application Eq. (4) can be expressed as:

$$\left[\textbf{\textit{rnp}},\textbf{\textit{td}}\right]^{\mathrm{T}} = \left[\overline{\textbf{\textit{rnp}},\textbf{\textit{td}}}\right]^{\mathrm{T}} + \left[\varepsilon_{rnp},\varepsilon_{td}\right]^{\mathrm{T}}$$

According to these data, the three selected strategies were evaluated again for all the stochastic scenarios, obtaining as output a user generalised cost vector comprising 100 elements. The results are then compared so as to highlight the optimal strategy for each case.

Table 2. Number of times (%) of optimality for each strategy (stochastic analysis)

	Strategy 10	Strategy 12	Strategy 13
Average travel demand	27%	32%	41%
High travel demand	34%	33%	33%

Table 2 shows the outputs of the above procedure. Obviously, different travel demand levels do not provide the same result since the number of passengers who are unable to board the first arriving train may differ considerably.

As can be seen, given average conditions, strategy 13 is the one which guarantees the highest level of robustness. It has to be preferred since it proves the optimal solution in most cases. Moreover, from an operational point of view, this alternative is better than the other two: after running at low speed up to the terminus (i.e. Piscinola), the train is driven directly to the depot and does not need to be hauled from a recovery track at the end of the daily operations.

In the case of crowded days, by contrast, all the three alternatives are equivalent and there is no evident advantage to choosing one over another. For this reason, strategy 13 is still the best since it guarantees fewer movements without passengers. However, even if this result may seem the same as that obtained without considering parameter variability, simulations show that only in 41% of cases the deterministic approach correspond to a 'real condition' scenario, while in the other 59% of cases the deterministic approach may provide a non-optimal intervention strategy.

5. Conclusions and research prospects

In this study we proposed an off-line procedure for implementing a stability analysis of intervention strategies in the case of breakdowns in rail/metro systems. Indeed, the adoption of traditional studies, based on considering the average values of variables (i.e. the deterministic approach), could generate alterations in the definition of corrective actions especially in extreme cases (such as days with high demand levels). For this reason, we showed a two-step procedure for identifying the optimal strategy and its neighbourhood (deterministic phase) and carrying out a stability analysis affecting involved variables by means of a Montecarlo approach. The proposed methodology was applied in the case of a metro line in Italy so as to highlight the advantages in a real context. Simulation results confirm the importance of allowing for stochasticity of both travel demand and train performance in order to reflect real conditions much better. In addition, the proposed approach provides an interesting methodology to interpret the outcome of the analysis more accurately since it allows estimation of the error degree which is made in the case of a purely deterministic procedure.

For future research we propose to apply this decision support system to a large number of disruption events and to test the model also on more complex networks and/or conventional rail lines. Indeed, although the results obtained (i.e. the deterministic solution performs well also in the stochastic process) are interesting, more complex networks or breakdown contexts could bring to completely different conclusions and have to be evaluated. In addition, in order to provide a more complete analysis, the objective function will be modified to allow for intervention costs for rail operators, such as energy consumption and efficiency indexes.

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