

## An Optimization Framework for Combining the Petroleum Replenishment Problem with the Optimal Bidding in Combinatorial Auctions

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### Abstract

This paper addresses a periodic petroleum station replenishment problem (PPSRP) that aims to plan the delivery of petroleum products to a set of geographically dispatched stations. It is assumed that each station is characterized by its weekly demand and by its frequency of service. The main objective of the delivery process is to minimize the total travelled distance by the available trucks over an extended planning horizon. The problem configuration is described through a set of trucks with several compartments each and a set of stations with demands and prefixed delivery frequencies. Given such input data, the minimization of the total distance is subject to assignment and routing constraints that express the capacity limitations of each truck's compartment in terms of the frequency and the pathways' restrictions. In this paper, we develop and solve the full space mathematical formulation for the PPSRP with application to the Omani context. Our ultimate aim is to include such a model into an integrated framework having the objective of advising petroleum distribution companies on how to prepare bids in case of participation in combinatorial auctions of the transportation procurement.

**Keywords:** Vehicle routing; Petroleum station replenishment; Combinatorial auctions; Transportation procurement.

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## **1. Introduction**

The 1-day petroleum station replenishment problem (PSRP) has attracted the interest of several scholars during the last decades by developing several optimization models and algorithmic approaches for its solution. Different decision aspects of the PSRP have been considered such as sizing the transportation fleet, defining the routing of each truck, assigning the stations to be served to the appropriate trucks, assigning the compartments to the products, etc. Interested readers can refer, for example, to the paper (Cornillier F., Boctor F. and Renaud J. (2012)) that includes a broad discussion on the main characteristics of the PSRP that have been considered in the literature.

Our focus here will be devoted to the same problem but when defined over an extended time horizon of several days. The importance of covering an extended horizon is due to the fact that most of the stations do not need to be replenished every day but rather at a specified number of times during, say, a one week planning horizon. We believe that, by recognizing the periodicity nature of the replenishment service, distribution companies can achieve high level of efficiency in minimizing the delivery costs. Yet, such a cost saving will inevitably be paid by the necessity of facing more complex optimization problems.

To the best of our knowledge, only few works in the scientific literature have recognized the importance of incorporating the extended nature of the PSRP. Most of the works belong to the family of multi-period planning problems and only one paper takes into account the periodic nature of the petrol delivery and solves the PPSRP. However, no one has considered both the periodicity aspect and the multi-compartment within the same optimization framework.

In this paper we try to fill this gap by developing a mathematical model of the PPSRP with multi-compartment trucks, followed by a computational phase related to the distribution of petroleum products in the city of Muscat, the capital of Oman. The computational performance of the PPSRP turned out to be quite efficient when a limited size testbed is used. Indeed, the Omani case study illustrates the incentive behind using such an exact approach that allows the management of the problem from the data acquisition to the visualization of possible simulation scenarios in a more realistic way. However, since the problem is known to be NP-hard, exact method runs are costly in terms of computation time.

The remainder of this paper is organized as follows. In section 2, we review the literature related to the PPSRP. Section 3 is devoted to the formal description of the problem and the development of our optimization formulation. Our novel framework for the participation of the petroleum combinatorial framework is introduced in Section 4. Our computational results are presented in Section 5, followed by the some concluding remarks and future developments.

## **2. Related Literature**

The periodic petrol station replenishment problem (PPSRP) consists in designing a set of minimal-cost routes to serve the demands for different types petroleum products to a set of geographically spread stations over a given planning horizon. In most PSRP contributions a vehicle routing plan is determined for one single day. However, solving the PSRP over an extended

planning horizon can help in efficiently consolidating the deliveries and in yielding delivery savings for the distribution company (see Cornillier F., Boctor F. and Renaud J. (2012)). A high number of papers related to the 1-day PSRP has appeared during the last decades in the scientific literature. A non-comprehensive list may include the collection of papers by Cornillier et al. (2012, 2008b, 2008a, 2009) and the contributions of Brown and Graves (1981); Brown et al. (1987); Ben Abdelaziz et al. (2002); Rizzoli et al. (2003); Ng et al. (2008); Surjandari et al. (2011) and Boctor et al. (2011).

However, only a limited number of papers have considered the multi-period aspect and, thus, solved the PSRP over an extended planning horizon. Among those papers only one paper has explicitly included the periodicity aspect into the problem. For a complete overview of the different modelling features, optimization models and solution approaches developed for the both the periodic and multi-period PSRP the reader is referred to survey due to Triki and Al-Hinai (2016).

More specifically, Taqa Allah et al. (2000) have considered the multi-period case of the PSRP over an extended planning horizon. They examined the variant of the problem having a single depot and an unlimited homogeneous truck fleet and proposed several heuristic methods for its solution.

Malepart et al. (2003) have investigated the multi-period petrol station replenishment problem (MPSRP) in a real-life context and have included a variant of the problem in which the distribution company would decide for some stations the quantity to be delivered at each visit.

Cornillier et al. (2008b) proposed a heuristic for the MPSRP that maximizes the profit, which is calculated as the difference between the delivery revenue deriving from selling the petroleum products and the sum of regular and overtime drivers' costs. The heuristic was extensively tested on randomly generated data and the computational results reported confirm the efficiency of the proposed methodology.

To the best of our knowledge, only one contribution proposed by Triki (2013) has considered the periodic nature of the petrol delivery and has solved the PPSRP. Triki studied the PPSRP where each station  $i$  must be served  $f$  times within the time horizon by choosing the replenishment days among the feasible patterns for station  $i$  with the objective of minimizing the total distance travelled by the trucks. For the solution of the PPSRP, Triki has developed several heuristic methods based on the cluster-first and route-second paradigm. His construction heuristics as well as an improvement procedure have been tested on a real-life application related to an Italian distribution company.

However, the above-mentioned work considers trucks that are characterized by a single tank and a single product as well. Including the multi-compartment and multi-product case will increase the complexity of the problem since it adds an additional level of decision that assigns the products to the compartments and then the compartments to the stations. This aspect represents the main contribution of this paper with respect to the work of Triki (2013).

### 3. Problem Description and Mathematical Model

The PPSRP can be stated as follows. Given a set of geographically dispersed stations, and a set of trucks with limited number of compartments  $V_k$  having fixed capacities, the objective is to find a minimum distance set of vehicle routes that visit all stations. Three petroleum products are to be delivered in our case, namely, Super, Regular and Diesel. Note that the trucks are not equipped with flow meters, for this reason each compartment that contains one product should be fully delivered to one of the stations station.

Moreover, given the periodic nature of the problem, each station is characterized by its frequency of service  $f$  that can assume, considering a 6-day delivery plan, one of the following values:  $\{1,2,3,6\}$ . Such a frequency will, thus, define a set of possible pattern where (the symbol  $|\mathcal{A}|$  along all the paper means the cardinality of  $\mathcal{A}$ , i.e. the number of elements of  $\mathcal{A}$ ):

$$|\mathcal{P}_f| = \frac{\text{Number of working days}}{f} = \frac{6}{f}$$

The PPSRP can be stated in terms of the following components:

- A time horizon of  $T$  days indexed by  $t$ ;
- A set of patterns  $\mathcal{P}_f$  that define the feasible combinations of service days along the horizon;
- A set of  $P$  petroleum products indexed by  $p$ ;
- A set of  $N$  stations having each  $f_i$  as frequency and  $r_{ipt}$  as replenishment of product  $p$  during the delivery visit on day  $t$ ;
- A set of  $K$  trucks, characterized by its unit cost  $c_k$  and a prefixed number of compartments  $V_k$  each. The compartments are assumed to be homogeneous, i.e. having all the same known capacities;
- A distance matrix  $D=(d_{ij})$  of order  $N+1$  that reports all the shortest direct paths between any pair of locations (including the depot that is indexed as “0”).

Hence, the input related to the PPSRP can be summarized as depicted by the flowchart shown in Figure 1.

The objective function of the PPSRP consists in minimizing the total cost incurred by the transportation of  $P$  types of petroleum products to be delivered to  $N$  stations within a specific area over an extended planning horizon of  $T$  days. Hence, the PPSRP can be formally defined in terms of an undirected graph  $G=(V,E)$  where  $V=\{0, \dots, N\}$  denotes the vertex set so that 0 corresponds to the supplier's depot and  $1, \dots, N$  are the petrol stations and  $E=\{(i,j)$ , such that  $i,j \in V$ , and  $i \neq j\}$  corresponds to the edge set. Based on the above assumptions, the problem data are structured as follows:

**Parameter settings:**

- Horizon:  $T=6$  days
- Frequency:  $f_i \in \{1, 2, 3, 6\}$
- Patterns: each of the above frequencies give rise to the following possible patterns:

$$f = \begin{cases} 1 & \mathcal{P}_1 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}\} \\ 2 & \mathcal{P}_2 = \{\{1, 4\}, \{2, 5\}, \{3, 6\}\} \\ 3 & \mathcal{P}_3 = \{\{1, 3, 5\}, \{2, 4, 6\}\} \\ 6 & \mathcal{P}_6 = \{\{1, 2, 3, 4, 5, 6\}\} \end{cases}$$

**Indices:**

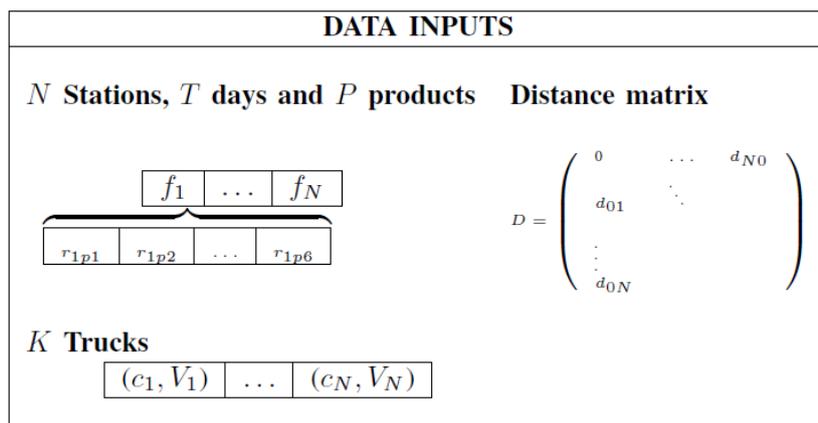
- $i, j$ : petrol station index  
 $p$ : product index  
 $k$ : truck index  
 $v$ : compartment index  
 $f$ : frequency index  
 $l$ : pattern index within each  $\mathcal{P}_f$   
 $t$ : time index

**Input data:**

- $N$ : Number of stations  
 $K$ : Number of trucks  
 $V_k$ : Number of compartments per truck  
 $F_i$ : Frequency of delivery per station  
 $r_{ipt}$ : Replenishment of product  $p$  to station  $i$  on day  $t$   
 $c_k$ : Unit cost for truck  $k$

**Output data (decision variables):**

- $x_{ijkt}$  Binary variable equal to 1 only if truck  $k$  travels from station  $i$  to station  $j$  using truck  $k$  at period  $t$   
 $y_{ipkvt}$  Binary variable that takes 1 only if product  $p$  to be delivered to station  $i$  is loaded in compartment  $v$  of truck  $k$  at period  $t$   
 $z_{il}$  Binary variable equal to 1 if frequency pattern  $l$  is assigned to station  $i$



**Figure 1.** Input Data related to our PPSRP

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*Min*    *Total travel cost*

*s.t.*

- Sub-tour elimination*
- Start and end at the depot*
- Serve each station by one truck*
- Path contiguity of each truck*
- Truck arriving to a destination should leave on the same day*
- $|Used\ trucks| \leq |available\ trucks|$
- The amount delivered meets the station's demand of each product*
- $|Used\ compartments| \leq |available\ compartments|$
- $|Visits\ that\ can\ occur\ in\ one\ trip| \leq |compartments| + 1$
- $|Trips\ to\ deliver\ product\ p| \leq |compartments|$  are filled with  $p$
- One pattern is selected for every station*
- Visit days are defined in terms of frequency patterns*

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**Figure 2.** Textual Formulation of the PPSRP

$$\text{Min } z(x) = \sum_{i=0}^N \sum_{j=0, j \neq i}^N \sum_{t=1}^6 \sum_{k=1}^K c_k d_{ij} x_{ijkt}$$

s.t.

$$\sum_{i \in S} \sum_{j \in S} x_{ijkt} \leq |S| - 1 \quad \forall k, \forall t, \quad (1)$$

$$S \subseteq \{2, \dots, N\}$$

$$\sum_{j=1}^N x_{0jkt} = 1 \quad \forall k, \forall t \quad (2)$$

$$\sum_{i=1}^N x_{i0kt} = 1 \quad \forall k, \forall t \quad (3)$$

$$\sum_{i=0}^N \sum_{k=1}^K x_{ijkt} = 1 \quad j \in \{1, \dots, N\}, \quad (4)$$

$$\forall k, \forall t$$

$$\sum_{i=0}^N x_{ijkt} - \sum_{i=0}^N x_{jikt} = 0 \quad j \in \{1, \dots, N\}, \quad (5)$$

$$\forall k, \forall t$$

$$\sum_{i=0}^N \sum_{k=1}^K x_{ijkt} = \sum_{h=0}^N \sum_{k=1}^K x_{jhkt} \quad j \in V, \forall t \quad (6)$$

$$\sum_{j=0}^N \sum_{k=1}^K x_{0jkt} \leq K \quad \forall t \quad (7)$$

$$\sum_{k=1}^K \sum_{v=1}^V y_{ipkvt} = r_{ipt} \quad i \in \{1, \dots, N\}, \quad (8)$$

$$\forall p, \forall t$$

$$\sum_{i=1}^N \sum_{p=1}^P \sum_{v=1}^V y_{ipkvt} \leq V_k \quad \forall k, \forall t \quad (9)$$

$$\sum_{i=0}^N \sum_{j=1}^N x_{ijkt} \leq V_k + 1 \quad \forall k, \forall t \quad (10)$$

$$y_{ipkvt} \leq \sum_{j=0}^N x_{ijkt} \quad i \in \{1, \dots, N\}, \quad (11)$$

$$\forall p, \forall k, \forall v, \forall t$$

$$\sum_{l \in P_f} z_{il} = 1 \quad i \in \{1, \dots, N\} \quad (12)$$

$$\sum_{j=0}^N \sum_{k=1}^K x_{ijkt} - \sum_{l \in P_f} a_{lt} z_{il} = 0 \quad i \in \{1, \dots, N\}, \quad (13)$$

$$\forall k, \forall t$$

$$x_{ijkt} \in \{0, 1\}, y_{ipkvt} \in \{0, 1\}, z_{il} \in \{0, 1\} \quad \forall i, \forall j, \forall k, \forall t, \forall v, \forall l \quad (14)$$

In the formulation (1)—(14), the objective function, expressed by means of equation (1) aims at minimizing the total travel cost that depends on the travelled distance and the unit cost (per kilometre) related to each available truck. The Structural constraints have the following meaning:

- Constraints (1) are sub-tour elimination requirements
- Constraints (2) and (3) make sure that each truck starts the delivery task from the depot,

- visits a subset of stations one by one and then returns the depot
- Constraints (4) ensure that each station is served by exactly one truck
  - Constraints (5) impose the continuity of each truck route
  - Constraints (6) force each truck arriving to a destination  $j$  to leave the same node on the same day
  - Constraints (7) guarantee that the number of trucks that can make departure from the depot as first trip cannot exceed the number of available trucks
  - Constraints (8) ensure that the amount of product  $p$  delivered to a station is equal to the station's demand.
  - Constraints (9) make sure that the number of used compartments must be less than or equal to the number of available compartments of truck  $k$
  - Constraints (10) limit the number of visits that can occur in one trip to be at most the number of compartments plus the depot.
  - Constraints (11) ensure that the number of trips carrying petrol cannot exceed the number of compartments that are filled with petrol
  - Constraints (12) make sure that exactly one pattern is selected for every station in such a way that within this pattern the station is visited according to its frequency
  - A station then will be visited on the days of the selected pattern. This is ensured by constraints (13)
  - Finally, Constraints (14) impose the binary requirements of all the decision variables.

The resulting model is a pure binary linear programming model having a number of variables and constraints that increases quickly with the number of stations, horizon days, trucks, patterns and compartments.

#### 4. A Combinatorial Auction Framework for the PPSRP

The distribution company ensuring the petrol distribution will be surely faced with covering useless movements, with waste of trucks capacity and with trips with some unloaded compartments. For this reason, the company may be interested in participating in some combinatorial auction that allows it to bid on a subset of the auctioned delivery to some stations. The combinatorial auction is characterized by the attractive feature of allowing the transportation companies to bid on a bundle of deliveries rather than on single delivery. Typically, the number of possible bundles is very high since it is of the order of  $2^{|\text{deliveries}|}$ . Such a huge number represents a serious challenge for defining a successful bid to be submitted to the auction. In order to define a successful bid, the company has to offer, for the bid to be submitted, a price that is lower than all the competitors. The decision variables consist thus in (i) selecting the bundle to be submitted among the possible ones and (ii) suggest the price corresponding to that bundle. Such a problem is known as the *Bid Generation Problem* (BGP). Once all the interested companies have submitted their bids, the auctioning organization will solve the so-called *Winner Determination Problem* (WDP) in order to identify the successful delivery bids. Sometimes, the auctioning organization can even be a competing petrol distribution company that is seeking to satisfy part of its deliveries through a more-effective low cost mechanism. From this point of view, the participation in the combinatorial auction could be even seen as an efficient tool for collaborative logistics. For more insights on the combinatorial

auctions for the transportation procurement the interested readers can see, for example, the surveys ((Sheffi Y. (2004)), (Elmaghraby W. and Keskinocak P. (2003))) and those specifically interested in the BGP and WDP are referred to ((Triki C., Oprea S., Beraldi P. and Crainic T. (2014)), (An N., Elmaghraby W. and Keskinocak P. (2005)), (Kuyzu G., Akyol Ç. G., Ergun Ö. and Savelsbergh M. (2015)), (Lee C.-G., Kwon R. H. and Ma Z. (2007)), (Triki, C. (2016))) and (Ma Z., Kwon R. and Lee C.-G. (2010)), (Remli N. and Rekik M. (2013)), respectively.

The distribution company's objective consists in maximizing the net profit behind participating in the auction. Specifically, the bundle to be selected should ensure that the revenue deriving from serving the successful deliveries must be higher than the extra-cost necessary to include those deliveries within the company's routing. The new optimization model should include all the constraints (2)–(14) related to the above PPSRP since the bid generation problem is a routing-based model. Moreover, the bid generation model should include the constraints that ensure the selection of the most profitable bundle and its price. The detailed textual description of the model can be summarized as follows:

*Max (Revenue from serving the submitted bundle of deliveries) – (New routing cost including the auctioned deliveries)*

*S.t:*

*Constraints (2)–(14) that define feasible routes including stations  $\{1, \dots, N\}$  and also auctioned deliveries*

*Only one bundle should be selected among the possible ones*

*Offered price for every bundle  $\leq$  Price submitted by all the other competitors for that bundle*

*The stations belonging to the successful bundle should be included in the routing planning*

*The variable related to the price of the submitted bundle should be non-negative*

*The variable related to the selection/non-selection of any bundle should belong to  $\{0,1\}$*

The introduction of such a framework within this paper represents rather a novel seminal idea in the context of the petrol replenishment that deserves more investigation, implementation and discussion. It will open new frontiers, Define new challenges and introduce new optimization problems for the scientific community involved in dealing with the petrol replenishment. More detailed models and solution approaches will be left for future investigation.

## **5. Computational Experiments**

As we are addressing an NP-hard optimization problem, the combinatorial structure of the proposed model makes the generation of the solution difficult and time consuming. Therefore, the tentative to solve at optimality the PPSRP covers few instances of limited size. We randomly generated several instances with a number of stations ranging from 5 to 22 and a number of vehicles that varies from 3 to 5 having either 3 or 5 compartments each.

The mathematical formulation (1)-(14) has been coded in IBM ILOG CPLEX 12.1. It has run on an Intel(R) core(TM) i3 processor, 2.53 GHz computer with 4 GB RAM.

We consider in what follows a national distributor of petroleum products in the Sultanate of Oman. The main petroleum sources are Al-Fahud and Murmul areas. The crude oil is then extracted and moved to the terminal in order to be refined and shipped to the petrol stations. The input data calibrated in terms of the addressed area are:

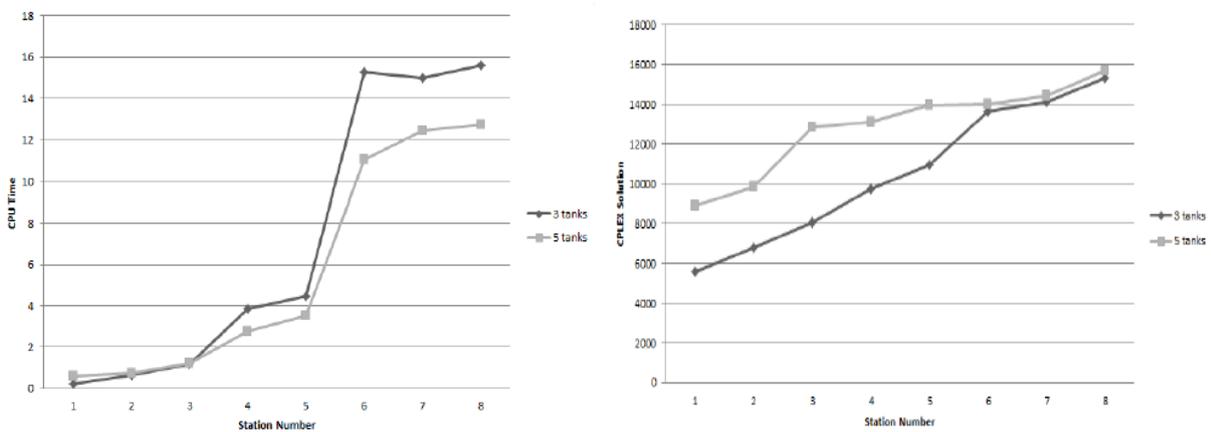
- $N \in \{5, \dots, 22\}$
- $K \in \{3, \dots, 7\}$
- $V \in \{3, 5\}$
- $r_{ipt}$  that denotes the demands per station  $i$  of products  $p$ , given that the delivery happens in day  $t$  belonging to pattern  $l$ . All demands are expressed here as a multiple of 8000 litres, i.e. the capacity of any single.
- $D$ , the distance matrix related to the set of stations in the city of Muscat covers up to 25 petrol stations plus the depot. The geographical coordinates vary in the interval [47,400] kilometres.
- $K$ , two truck types have been used when generating the instances having the following characteristics
  - Type 1:  $c_k = 0.5$  and  $V_k = 3$
  - Type 1:  $c_k = 0.8$  and  $V_k = 5$

To assess the effectiveness of the CPLEX optimizer in solving the PPSRP, we report in Table 1 the computational results of the PPSRP over 18 test problems while providing the total cost and the CPU time.

On the basis of the results reported in Table 1, we can note that the travelling cost as well as the CPU time are highly influenced by the number of stations  $N$ . This can be also clearly seen in Figure 3. When the number of stations  $N$  reaches the value of 22, the exact method is not able anymore to solve the problem. In this case, exact methods should make use of some decomposition techniques that tackles the structural characteristics of the combinatorial optimization problem. An alternative approach consists in developing heuristics and metaheuristic approaches.

**Table 1.** Experimental results for the city of Muscat

| $(N \times K)$  | $V$ | CPLEX solution | CPU time |
|-----------------|-----|----------------|----------|
| $(5 \times 3)$  | 3   | 5574           | 00:22    |
|                 | 5   | 8918           | 00:60    |
| $(7 \times 3)$  | 3   | 6775           | 00:66    |
|                 | 5   | 9865           | 00:79    |
| $(9 \times 4)$  | 3   | 8034           | 01:15    |
|                 | 5   | 12854          | 01:20    |
| $(11 \times 5)$ | 3   | 9762           | 03:85    |
|                 | 5   | 13132          | 02:77    |
| $(13 \times 5)$ | 3   | 10972.5        | 04:46    |
|                 | 5   | 13980          | 03:55    |
| $(17 \times 6)$ | 3   | 13637          | 15:28    |
|                 | 5   | 14030          | 11:02    |
| $(18 \times 6)$ | 3   | 14140          | 14:99    |
|                 | 5   | 14429          | 12:46    |
| $(20 \times 7)$ | 3   | 15338          | 15:63    |
|                 | 5   | 15690          | 12:73    |
| $(22 \times 4)$ | 3   | -              | -        |
|                 | 5   | -              | -        |



**Figure 3.** Variation of the cost (right) and CPU (left) with the problem size

## 6. Conclusions

The PPSRP is a challenging problem that points out the delivery of petrol products at customer locations while integrating vehicle scheduling and dispatching, over an extended time horizon. In this paper, we developed the mathematical model of the PPRSP as a period vehicle routing problem specifically designed for the petroleum framework. The novelty of our work is that we include two

major complexities within the same optimization framework: the periodicity and the multi-compartment case within the trucks.

As the literature review revealed, such a modelling is NP-hard, we tried our best to solve at optimality the PPSRP using CPLEX. The addressed instances of limited sizes are computationally evaluated in terms of the run time. The generated testbed applied to the city of Muscat (Oman) demonstrates how costly is obtaining the optimal solution. Moreover, this paper introduces a new framework that allows the petrol distribution companies to widen their business by participating in possible combinatorial auctions and give some guidelines on how to generate bids to be submitted to the auction and that can result to be successful.

Further research on this topic can be developed along several directions. First, the investigation of advanced decomposition techniques in order to split the overall PPSRP in several less complex sub-problems should be explored. Alternatively, petrol distribution companies may take advantage from developing heuristic and metaheuristic algorithms in order to be able to solve large scaled PPSRP instances. Second, this paper has presented just a general framework for the use of combinatorial auctions in the context of petrol distribution. Such a framework deserves more investigation in order to discover its real potentialities for improving the operation of petrol delivery companies.

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