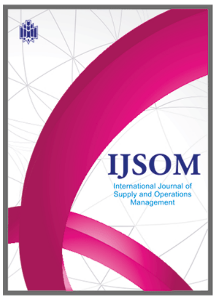
**International Journal of Supply and Operations Management**

# IJSOM

**May 2015, Volume 2, Issue 1, pp. 662-682**
**ISSN-Print: 2383-1359**
**ISSN-Online: 2383-2525**
**www.ijsom.com**




# A two-warehouse inventory model for deteriorating items with permissible delay under exponentially increasing demand

R.Sundara Rajan [a*] and R.Uthayakumar [b]

[a] Department of Mathematics, PSNA College of Engineering and Technology, Dindigul, Tamilnadu, India

[b] Department of Mathematics, Gandhigram Rural Institute - Deemed University, Gandhigram, amilnadu, India

## Abstract

In this study, a two-warehouse inventory model with exponentially increasing trend in demand involving different deterioration rates under permissible delay in payment has been studied. Here, the scheduling period is assumed to be a variable. The objective of this study is to obtain the condition when to rent a warehouse and the retailer’s optimal replenishment policy that minimizes the total relevant cost. An effective algorithm is designed to obtain the optimal solution of the proposed model. Numerical examples are provided to illustrate the application of the model. Based on the numerical examples, we have concluded that the single warehouse model is less expensive to operate than that of two warehouse model. Sensitivity analysis has been provided and managerial implications are discussed.

**Keywords**: Deterioration; Time-dependent demand; Two-warehouse, Permissible delay

[*] Corresponding author email address: dglsundar21579@gmail.com

## 1. Introduction

In classical inventory models, many researchers have considered that the demand rate is either constant or linear over time. An economic order quantity model with a constant demand rate under the condition of permissible delay in payments was established by Goyal (1985). Aggarwal and Jaggi (1995) extended Goyal's model to consider the deteriorating items. The above assumption is valid during the period of maturity stage of products like electronic goods or fashionable items. After gaining the customers goodwill, the demand of products will increase over time. Thus, demand of new products in the market during its growth as well its maturity period can be approximated with exponentially increasing function. For many reasons, retailers may purchase more goods than can be stored in their owned warehouse (OW), for example, permissible delay in payments, discount on bulk purchase, etc., to name a few. The excess units are stored in an additional storage place called a rented warehouse (RW). In this situation, deterioration of items are unavoidable. Also, to attract more number of customers, the retailers can be given permissible delay in payments to settle the account.

Ghare and Shrader (1963) proposed an economic order quantity model for items having a constant rate of deterioration horizon. Covert and Philip (1973) developed a model with Weibull distribution deterioration. Sarma (1987) discussed a deterministic order level inventory model for deteriorating items with two storage facility. Datta and Pal (1988) developed a model with variable rate of deterioration. A stochastic inventory model when delay in payments are permissible was considered by Shah (1993). Hwang and Shinn (1997) included the pricing strategy to the model, and developed the optimal price and lot-sizing for a retailer under the condition of permissible delay in payments. A deterministic order level inventory model for deteriorating items with two storage facilities was proposed by Benkherouf (1997). Bhunia and Maiti (1998) discussed a model for deteriorating items with linear trend in demand. Liao et al. (2000) proposed an inventory model with deteriorating items under inflation when a delay in payment is permissible. Huang et al. (2001) presented an inventory model for deteriorating items with linear trend under the condition of permissible delay in payments. Kar et al. (2001) proposed deterministic inventory model with linear trend in demand. Zhou and Yang (2005) presented a two-warehouse model with stock level dependent demand rate. Shah (2006) considered an inventory model for deteriorating items and time value of money under permissible delay in payments during a finite planning horizon. Soni et al. (2006) discussed an EOQ model for progressive payment scheme under discounted cash flow. Jaggi et al. (2006) proposed a model for deteriorating items with inflation induced demand. Ouyang et al. (2006) presented an inventory model for deteriorating items under permissible delay in payments. Chung and Huang (2007) investigated models for deteriorating items with limited storage facility. Chang et al. (2008) reviewed trade credit models. Wu et al. (2009) proposed an optimal payment time for deteriorating items under inflation and permissible delay in payments. Lee and Hsu (2009) discussed two-warehouse production model with time dependent demand rate. Liang and Zhou (2011) discussed a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. Ghoreishi et al. (2015) developed an economic ordering policy for non-instantaneous deteriorating items with selling price and inflation induced demand under permissible delay.

For seasonal products, fashionable commodities and electronic products with short product life cycle, customers' willingness to wait for backlogging during stock out period is diminishing with the length of the waiting period. Dave and Patel (1981) considered an inventory model for deteriorating items with time proportional demand and shortages. To allow shortages, Jamal

(1997) generalized Aggarwal and Jaggi's model. A partial backlogging inventory model for deteriorating items with Weibull distribution and permissible delay in payments was developed by Dye (2001). Yang (2006) considered a two-warehouse inventory model for deteriorating items with partial backlogging under inflation. Hui-Ling (2013) considered a two-warehouse partially backlogging inventory model for deteriorating items with permissible delay in payment under inflation. Ghoreishi et al. (2014) developed EPQ model for non-instantaneous deteriorating items under inflation. Maryam Ghoreishi (2014) developed an EOQ model for non-instantaneous deteriorating items with selling price dependent demand under inflation. Bhunia, Ali Akbar Shaikh (2015) developed two-warehouse inventory model for single deteriorating items with permissible delay in payments under partial backlogging. Chang et al. (2015) developed an inventory model with non-instantaneous deteriorating items with permissible delay linked to ordering quantity. Das et al. (2015) considered multi-item multi-warehouse inventory model for deteriorating items with price dependent demand under permissible delay.

**Table 1.** Summary of literature review for two-warehouse inventory model

| Authors and Year | Two warehouse deteriorating rate | Demand rate | Delay in payment |
|---|---|---|---|
| Sarma [1987] | $\alpha > \beta$ | Constant | No |
| Yang [2004] | $\alpha < \beta$ | Constant | No |
| Lee [2006] | $\alpha > \beta$ | Constant | No |
| Huang [2006] | $\alpha = \beta = 0$ | Constant | No |
| Dye et al. [2007] | $\alpha < \beta$ | Constant | No |
| Hsieh et al. [2007] | $\alpha < \beta$ | Constant | No |
| Chung [2007] | $\alpha = \beta \neq 0$ | Constant | Yes |

In all of the above mentioned-models, several researchers established their Economic Order Quantity (EOQ) inventory models by assuming that demand rate is constant. But in reality, during the maturity stage, only the demand rate is constant. Hence, the demand function increases with time during the growth stage of a product, when a new product is introduced in the market. In the present paper a two-warehouse inventory model with exponentially increasing trend in demand for deteriorating items under the condition of permissible delay is considered for both the two-warehouse and single-warehouse models. The scheduling period is taken to be a variable. An algorithm is provided to obtain an optimal solution which will make the retailer decide whether or not to rent a warehouse. Numerical examples are provided to compare the two proposed models and sensitivity analysis is carried out to substantiate the managerial insights.

## 2. Assumptions and notations

In developing the mathematical model of the inventory system for this study, the following assumptions are used.

### 2.1 Assumptions

1. The replenishment rate is in_nite
2. Lead time is zero
3. The inventory model deals with single item
4. Deterioration occurs as soon as items are received into inventory
5. There is no replacement or repair of deteriorating items during the period under consid-eration
6. Demand rate is deterministic

$$D(t) = a\ e^{bt} \quad 0 \le t < T \tag{1}$$

Where $a > 0, b > 0, a > b$

7. Shortages are not allowed to occur
8. The OW has a _xed capacity of W units and the RW has unlimited capacity
9. The RW is utilized only after OW is full, but stocks in RW are dispatched _rst
10. The holding cost is h per unit of time (excluding interest charges), when $h = h_o$ for items in OW and $h = h_r$ for items in RW and $h_r > h_o$
11. The items deteriorate at a constant rate _ in OW and at _ in RW.

## 2.2 Notations

In developing the mathematical model of the inventory system for this study, the following notations are used.

k ordering cost per order

c unit purchasing cost

s unit selling price (with s>c)

$h_r$ unit stock holding cost per unit of time in rented warehouse (excluding interest charges)

$h_o$ unit stock holding cost per unit of time in owned warehouse (excluding interest charges)

$I_e$ interest earned per \$ per unit of time by the retailer

$I_c$ interest charges per \$ in stocks per unit of time by the supplier

$I_m$ the maximum inventory level for each replenishment cycle

$w$ the capacity of the owned warehouse (OW)

$w_1$ The maximum inventory level

$t_w$ the time that inventory level reduce to W (decision variable)

T inventory cycle length (decision variable)

M the retailer's trade credit period offered by supplier in years

$I_r(t)$ the inventory level at time $t \in [0, t_w]$ in rented warehouse (RW)

$I_o(t)$ the inventory level at time $t \in [0,T]$ in (OW)

$Q$ the retailer's order quantity

$TC(t_w,T)$ the total relevant cost per unit time in a two-warehouse model

$\Pi(T)$ the total relevant cost per unit time in a single-warehouse model

$t_w^*, T^*$ are the optimal values

$TC^*, \Pi^*$ minimum total relevant costs

## 3. Model 1: Two-warehouse system

A lot size of particular units enter into the inventory system at time $t$=0. In OW $w$ units are kept and the remaining units are stored in RW. The items stored in OW are consumed only when the items in RW are consumed first. The stock in RW decreases owing to combined effects of demand and deterioration during the interval $[0,t_w]$, and it vanishes at $t=t_w$. However, the stock in OW depletes due to deterioration only during $[0,t_w]$. During $[t_w,T]$, the stock decreases due to combined effects of demand and deterioration. At time $T$, both the warehouses are empty. The entire process is repeated for every replenishment cycle which is depicted in Figure 1.

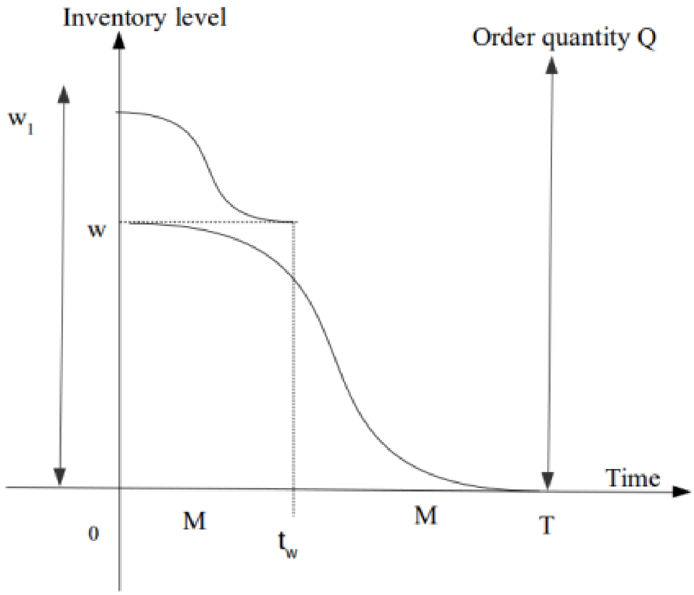



**Figure1.** Graphical representation of two-warehouse model

During the interval $[0, t_w]$, the inventory level in RW and OW is governed by the following differential equations:

$$\frac{dI_r(t)}{dt} = -\,ae^{bt} - \beta I_r(t) \qquad 0 < t \le t_w \tag{2}$$

With the boundary condition $I_r(t)$=0 and

$$\frac{dh_o(t)}{dt} = -\ \alpha h_o(t) \qquad\qquad 0 < t \le t_w \tag{3}$$

With the boundary condition $I_o(0)=w$, respectively.
While during $[t_w, T]$, the inventory level is governed by the following differential equation:

$$\frac{dI_o(t)}{dt} = -\ ae^{bt} - \alpha I_o(t) \qquad t_w < t \le T \tag{4}$$

With the boundary condition $I_o(T)$=0.
The solutions to the above differential equations (2) – (4) are:

$$I_r(t) = \frac{a}{\beta+b}\left[e^{(\alpha+\beta)t_w-\beta t} - e^{bt}\right] \tag{5}$$

$$I_0(t) = w\ e^{-\ \alpha t} \tag{6}$$

$$I_o(t) = \frac{a}{\alpha+b}\left[e^{(\alpha+b)T-\alpha t} - e^{bt}\right] \tag{7}$$

The maximum inventory level $w_1$ is given by

$$w_1 = I_r(0) + I_o(0) = a\ \left[e^{(b+\beta)t_w} - 1\right] + w \tag{8}$$

The total relevant costs, TC, comprise following elements:
the ordering cost = k
cumulative inventories of stock holding cost during [0,T] is

$$
\begin{aligned}
HC &= h_r \int_0^{t_w} I_r(t)dt + ho \int_0^{t_w} I_o(t)dt + ho \int_{t_w}^{T} I_o(t)dt \\
&= h_r \int_0^{t_w} \frac{a}{\beta + b} \left[e^{(a+\beta)t_w - \beta t} - e^{bt}\right]dt + h_o \int_0^{t_w} w\ e^{-\ \alpha t}dt \\
&\quad + \frac{h_o}{\alpha + b} \int_{t_w}^{T} a\ \left[e^{(\alpha+b)T - \alpha t} - e^{bt}\right]dt \\
&= -\ \frac{h_r a}{b + \beta}\left[\frac{e^{bt_w}}{\beta} + \frac{e^{bt_w}}{b} - \frac{e^{(b+\beta)t_w}}{\beta} - \frac{1}{b}\right] \\
&\quad +\ \frac{h_o a}{\alpha + b}\left[\frac{e^{bT}}{\alpha} + \frac{e^{bT}}{b} - \frac{e^{(\alpha+b)T - \alpha t_w}}{\alpha} - \frac{e^{bt_w}}{b}\right] \\
&\quad - \frac{h_o w}{\alpha}\left[e^{-\alpha t_w} - 1\right]
\end{aligned}
$$

the deteriorating cost during [0,T] is

$$
\begin{aligned}
DC &= \beta \int_0^{t_w} I_r(t)dt + \alpha \int_0^{T} I_o(t)dt \\
&= \beta \int_0^{t_w} \frac{a}{\beta + b}\left[e^{(a+\beta)t_w - \beta t} - e^{bt}\right]dt + \alpha \int_0^{t_w} w\ e^{-\ \alpha t}dt \\
&\quad +\ \alpha \int_{t_w}^{T} \frac{a}{\alpha + b}\left[e^{(\alpha+b)T - \alpha t} - e^{bt}\right]dt \\
&= -\ \frac{\beta a}{b + \beta}\left[\frac{e^{bt_w}}{\beta} + \frac{e^{bt_w}}{b} - \frac{e^{(b+\beta)t_w}}{\beta} - \frac{1}{b}\right] - w\left[e^{\alpha t_w} - 1\right] \\
&\quad -\ \frac{\alpha a}{\alpha + b}\left[\frac{e^{bT}}{\alpha} + \frac{e^{bT}}{b} - \frac{e^{(\alpha+b)T - \alpha t_w}}{\alpha} - \frac{e^{bt_w}}{b}\right]
\end{aligned}
$$

The interest payable.

Based on the parameters $t_w$, $T$ and $M$, there are three cases to be considered.

**Case1**. $M \leq t_w < T$

In this case, the interest payable is

$$\begin{aligned} IP_1 &= cI_c\int_0^{t_w} I_r(t)dt + cI_c\int_M^{t_w} I_o(t)dt + cI_c\int_{t_w}^{T} I_o(t)dt \\ &= -\frac{a\ cI_c}{b+\beta}[\frac{e^{bt_w}}{\beta}+\frac{e^{bt_w}}{b}-\frac{e^{(b+\beta)t_w-\beta M}}{\beta}-\frac{e^{bM}}{b}] \\ &\quad -\frac{a\ cI_c}{\alpha+b}[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}-\frac{e^{(\alpha+b)T-\alpha t_w}}{\alpha}-\frac{e^{bt_w}}{b}] \\ &\quad -\frac{wcI_c}{\alpha}[e^{-\alpha t_w}-e^{-\alpha M}] \end{aligned}$$

**Case2**. $t_w \le M < T$

In this case, the interest payable is

$$IP_2 = cI_c\int_M^T I_o(t)dt = -\frac{a\ cI_c}{\alpha+b}[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}-\frac{e^{(\alpha+b)T-\alpha M}}{\alpha}-\frac{e^{bM}}{b}]$$

**Case3**. $M > T$

In this case, the interest payable is zero.

the interest earned.

There are two cases to be considered.

**Case1**. $M < T$

$$IE_1 = sI_e\int_0^M ae^{bt}(M-t)dt = sI_e[\frac{ae^{bM}}{b^2}-\frac{aM}{b}-\frac{a}{b^2}]$$

**Case2**. $M > T$

$$\begin{aligned} IE_2 &= sI_e\int_0^T ae^{bt}(M-t)dt \\ &= sI_e a[\frac{e^{bT}}{b^2}+(M-T)\frac{e^{bT}}{b}-\frac{T}{b}-\frac{1}{b^2}-\frac{M-T}{b}] \end{aligned}$$

Thus, the annual total relevant costs for the retailer are given as:

$$TC(t_w,T) = \begin{cases} TC_1 & M \le t_w < T \\ TC_2 & t_w \le M < T \\ TC_3 & M > T \end{cases} \tag{9}$$

Where

$$TC_1 = (1/T)\left[k - \frac{a[h_r+\beta+cI_c]}{b+\beta}\left[\frac{e^{bt_w}}{\beta}+\frac{e^{bt_w}}{b}\right]+\frac{a[h_r+\beta]}{b+\beta}\left[\frac{e^{(b+\beta)t_w}}{\beta}+\frac{1}{b}\right]+\frac{acI_c}{b+\beta}\left[\frac{e^{(b+\beta)t_w-\beta M}}{\beta}+\frac{e^{bM}}{b}\right]\right]$$
$$+(1/T)\left[\frac{-w[h_o+\alpha]}{\alpha}[e^{-\alpha t_w}-1]-asI_e\left[\frac{e^{bM}}{b^2}-\frac{M}{b}\right]+\frac{asI_e}{b^2}-\frac{a[h_o+\alpha+cI_c]}{a+\alpha}\left[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}\right]\right]$$
$$+(1/T)\left[\frac{a[h_o+\alpha+cI_c]}{a+\alpha}\left[\frac{e^{(a+\alpha)T-\alpha t_w}}{\alpha}-\frac{e^{bt_w}}{b}\right]-\frac{wcI_c}{\alpha}[e^{-\alpha t_w}-e^{-\alpha M}]\right] \tag{10}$$

$$TC_2 = (1/T)\left[k - \frac{a[h_r+\beta]}{b+\beta}\left[\frac{e^{bt_w}}{\beta}+\frac{e^{bt_w}}{b}-\frac{e^{(b+\beta)t_w}}{\beta}-\frac{1}{b}\right]-\frac{a[h_o+\alpha]}{\alpha+b}\left[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}-\frac{e^{(\alpha+b)T-\alpha t_w}}{\alpha}-\frac{e^{bt_w}}{b}\right]\right]$$
$$+(1/T)\left[\frac{-w[h_o+\alpha]}{\alpha}[e^{-\alpha t_w}-1]-cI_c a\left[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}-\frac{e^{(\alpha+b)T-\alpha M}}{\alpha}\right]-\frac{acI_c}{\beta\, b}e^{bM}-\ asI_e\left[\frac{e^{bM}}{b^2}-\frac{M}{b}-\frac{1}{b^2}\right]\right] \tag{11}$$

$$TC_3 = (1/T)\left[k - \frac{a[h_r+\beta]}{b+\beta}\left[\frac{e^{bt_w}}{\beta}+e^{bt_w}b-\frac{e^{(b+\beta)t_w}}{\beta}-\frac{1}{b}\right]-\frac{a[h_o+\alpha]}{\alpha+b}\left[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}-\frac{e^{(\alpha+b)T-\alpha t_w}}{\alpha}-\frac{e^{bt_w}}{b}\right]\right]$$
$$+(1/T)\left[\frac{-acI_c}{\beta}\left[\frac{e^{bT}}{\alpha}+\frac{e^{bT}}{b}-\frac{e^{(\alpha+b)T-\alpha M}}{\alpha}-\frac{e^{bM}}{b}\right]-asI_e\left[\frac{e^{bT}}{b^2}+\frac{(M-T)e^{bT}}{b}-\frac{T}{b}-\frac{1}{b^2}-\frac{M-T}{b}\right]\right]$$
$$+(1/T)\left[\frac{-w(h_o+\alpha)}{\alpha}[e^{-\alpha t_w}-1]\right] \tag{12}$$

The optimal values of $t_w$ and T for minimum total relevant cost per unit time is any solution which satisfies simultaneously the equations $\frac{\delta TC_i(t_w,T)}{\delta t_w}$ = 0 and $\frac{\delta TC_i(t_w,T)}{\delta T}$ = 0 for i = 1, 2, 3 which also satisfies the conditions $\frac{\delta^2 TC_i(t_w,T)}{\delta t_w^2}$ > 0, $\frac{\delta^2 TC_i(t_w,T)}{\delta T^2}$ > 0 and ( $\frac{\delta^2 TC_i(t_w,T)}{\delta t_w^2}$ )( $\frac{\delta^2 TC_i(t_w,T)}{\delta T^2}$ ) - $\frac{\delta^2 TC_i(t_w,T)}{\delta t_w \delta T}$ )>0 .

Using these optimal values of $t_W$ and T, the optimal value of $w_1$ can be obtained from equation (8).

Graphically, we have shown below that the cost function is convex.

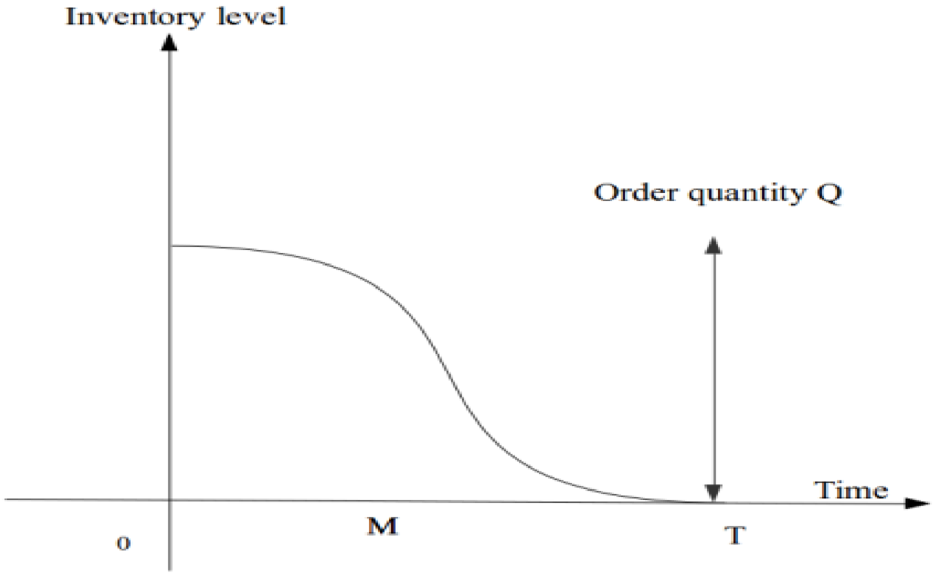



**Figure 2.** Graph of TC vs T for $\alpha > \beta$ when $t_w = 0.4691$

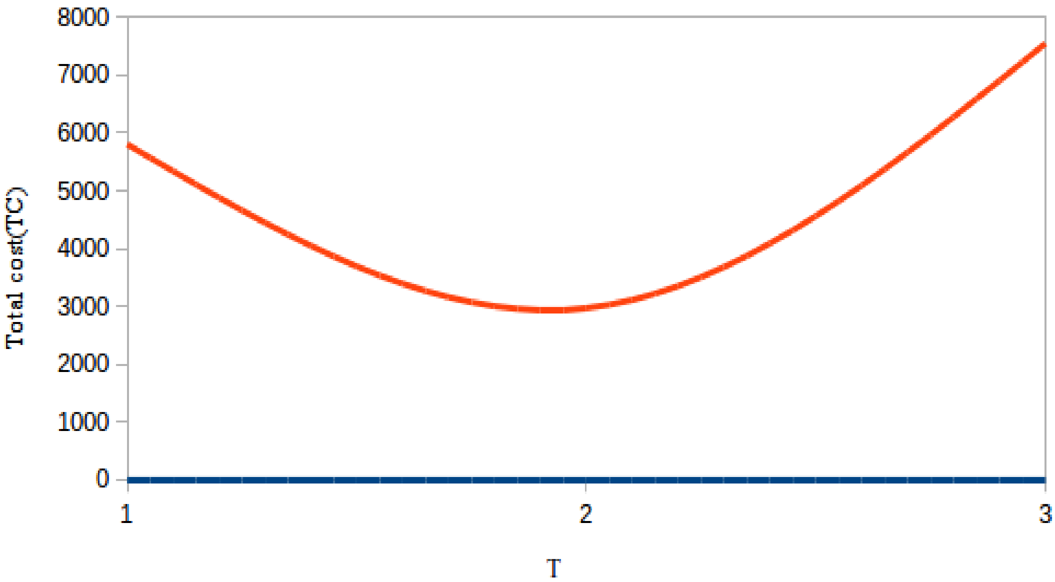



**Figure 3.** Graph of TC vs T for $\alpha > \beta$ when $T = 0.8011$

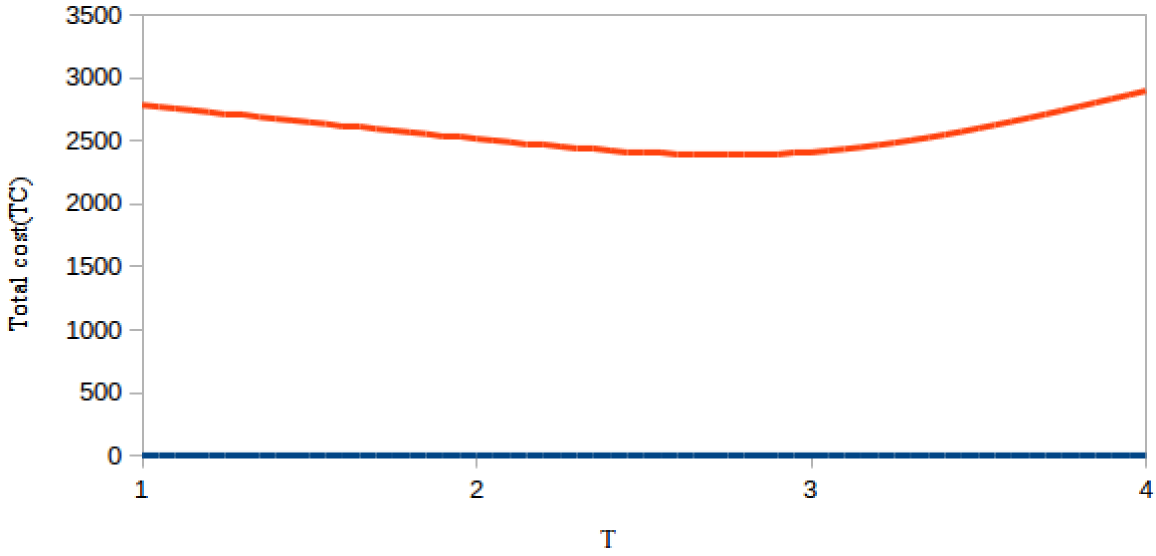



**Figure 4.** Graph of TC vs T for $\alpha < \beta$ when $t_w = 0.4641$

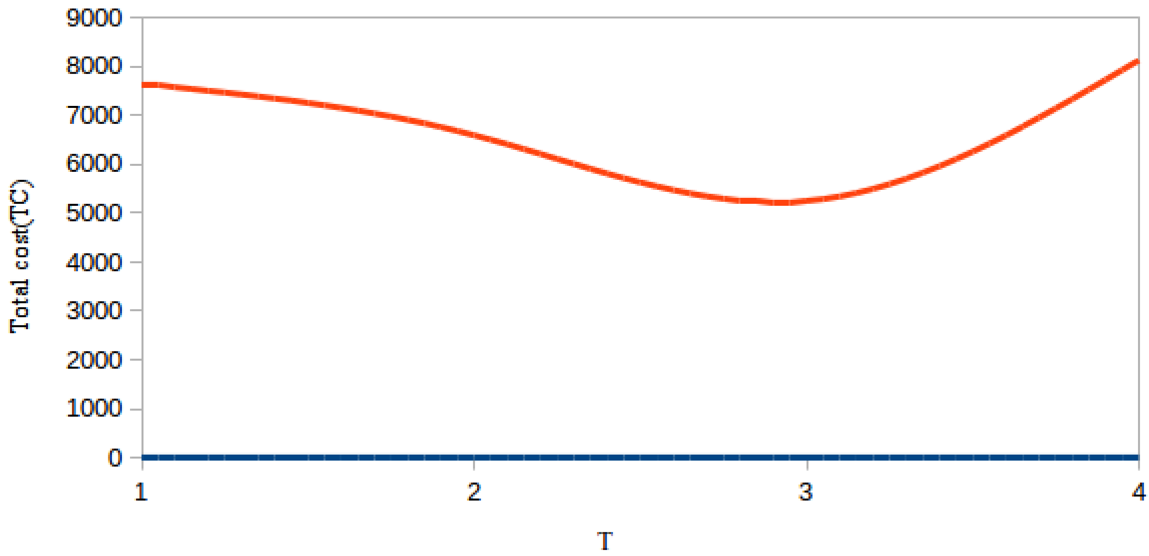



**Figure 5.** Graph of TC vs T for $\alpha < \beta$ when $T = 0.8017$

## 4. Model 2: Single-warehouse system

When the OW has enough space then the RW is not used,then the two-warehouse model reduces to single-warehouse inventory model. We remove the capacity constraint of the OW. At time t=0, a lot size of *w* units enters the system. By the time T, the inventory level reaches zero due to the combined effect of demand and deterioration. The entire process is repeated for every replenishment cycle and inventory scenario in single-warehouse model is depicted in Figure 6.

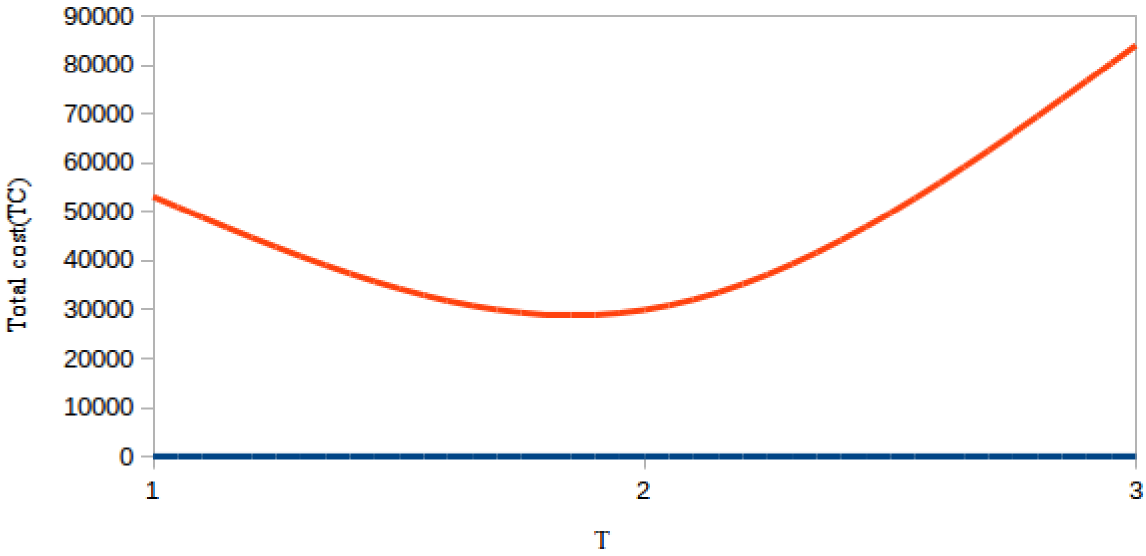



**Figure 6**. Graphical representation of single-warehouse system

As described in the two-warehouse model, the annual total relevant cost per unit time for the retailer can be obtained as follows:

$$\Pi(T) = \begin{cases} \Pi_1 & M \leq T \\ \Pi_2 & \text{M} > \text{T} \end{cases} \tag{13}$$

Where

$$\Pi_1(T) = (1/T)\left[k - \frac{ae^{bT}}{\alpha b}\left[h_o + \alpha + sI_p\right] + \frac{a[h_o+\alpha]}{\alpha+b}\left[\frac{e^{(\alpha+b)T}}{\alpha} + \frac{1}{b}\right]\right]$$
$$+(1/T)\left[\frac{acI_c}{\alpha+b}\left[\frac{e^{(\alpha+b)T-\alpha M}}{\alpha} + \frac{e^{bM}}{b}\right] - asI_e\left[\frac{e^{bM}}{b^2} - \frac{M}{b} - \frac{1}{b^2}\right]\right] \tag{14}$$

$$\Pi_2(T) = (1/T)\left[k - \frac{ae^{bT}}{\alpha b}\left[h_o + \alpha + sI_p\right] + \frac{a[h_o+\alpha]}{\alpha+b}\left[\frac{e^{(\alpha+b)T}}{\alpha} + \frac{1}{b}\right] + \frac{acI_c}{\alpha+b}\left[\frac{e^{(\alpha+b)T-\alpha M}}{\alpha} + \frac{e^{bM}}{b}\right]\right] \tag{15}$$

The maximum inventory level $Q$ is given by

$$Q = I_o(0) = \frac{a}{\alpha+b}[e^{(\alpha+b)T}] \tag{16}$$

## 5. Solution procedure

Depending on the total relevant costs obtained from single- and two-warehouse models, decision is made whether to rent a warehouse or not. In order to find the optimal solutions we propose the following algorithm.

Step 1.
Input the values of the parameters.
Step 2.
Solve the single-warehouse model and find the total relevant cost per unit time (Π1) using Equation (14), the ordering quantity Q using Equation (16) and the cycle time. Go to Step 3.
Step 3.
If the ordering quantity Q of single-warehouse model is less than the capacity of the OW, then solve the two-warehouse model using Step 4.
Step 4.
Obtain the total relevant cost per unit time (TC1) using Equation (10), ordering quantity w1 using Equation (8) and the cycle time, getting the optimal solution tw* and T* of the two-warehouse model. Go to Step 5.
Step 5.
If the total relevant cost per unit time of RW (TC1) is less than that of OW (Π1) and if M ≤ tw* < T* then it is economical to use the RW. Otherwise go to Step 6.
Step 6.
Solve the single-warehouse model and find the total relevant cost per unit time (Π2) using Equation (15), the ordering quantity Q using Equation (16) and the cycle time. Go to Step 7.
Step 7.
If the ordering quantity Q of single-warehouse model is less than the capacity of the OW, then solve the two-warehouse model using Step 8.
Step 8.
Obtain the total relevant cost per unit time (TC2) using Equation (11), ordering quantity w1 using Equation (8) and the cycle time, getting the optimal solution tw* and T*. Go to Step 9.
Step 9.
If the total relevant cost per unit time of RW (TC2) is less than that of OW (Π2) and if tw* < M ≤ T* then it is economical to use the RW. Otherwise go to Step 10.
Step 10.
Solve the single-warehouse model and find the total relevant cost per unit time using Equation (15), the ordering quantity Q using Equation (16) and the cycle time. Go to Step 11.
Step 11.
Obtain the total relevant cost per unit time (TC3) using Equation (12), ordering quantity Q using Equation (8) and the cycle time, getting the optimal solution tw* and T* of the two-warehouse model. Go to Step 12.
Step 12.
If the total relevant cost per unit time of RW (TC3) is less than that of OW (Π2) and if tw* < T*≤ M then it is economical to use the RW. Otherwise go to Step 13.
Step 13.
Let tw*,T* = min{(TC1,Π1),(TC2,Π2),(TC3,Π2)}, output the optimal tw*,T*.

## 6. Numerical examples

Numerical examples are carried out using SCILAB (5.5.0). The comparison between two models is illustrated by the following numerical examples with $\alpha > \beta$ and $\alpha < \beta$.

**Example 1:** $k$=1500, $a$=50, $b$=5, $h_0$=1, $h_r$=3, $c$=10, $p$=15, $I_p$=0.15, $I_e$=0.12, $w$=100, $\alpha$=0.1, $\beta$=0.06 with suitable units. The optimal values are tabulated below.

**Table 2.** Numerical results for models 1 and 2

| M | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|
| 0.25 | 0.4691 | 0.8011 | w1=196 | TC1=2409.1824 |
| | | 0.7080 | Q=353 | Π1=2602.3893 |
| 0.0833 | 0.4506 | 0.7972 | W1=187 | TC1=2469.1158 |
| | | 0.6904 | Q=322 | TC1=2721.8474 |

**Example 2:** k=1500, a=75, b=10, h0=1, hr=3, c=10, p=15, $I_p$=0.15, $I_e$ =0.12, w=100, α=0.1, β=0.06 with suitable units. The optimal values are tabulated below.

**Table 3.** Numerical results for models 1 and 2

| M | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|
| 0.25 | 0.3043 | 0.4747 | w1=252 | TC1=3863.0854 |
| | | 0.4291 | Q=559 | Π1=4125.4868 |
| 0.0833 | 0.2871 | 0.4705 | w1=226 | TC1=3991.6487 |
| | | 0.4116 | Q=467 | Π1=4428.3822 |

**Example 3:** k=1500, a=75, b=10, h0=1, hr=3, c=10, p=15, $I_p$=0.15, $I_e$ =0.12, w=100, α=0.05, β=0.03 with suitable units. The optimal values are tabulated below.

**Table 4.** Numerical results for models 1 and 2

| M | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|
| 0.25 | 0.2872 | 0.4719 | w1=226 | TC1=3976.8764 |
| | | 0.4136 | Q=469 | Π1=4404.5063 |
| 0.0833 | 0.3046 | 0.4761 | w1=251 | TC1=3848.2547 |
| | | 0.4314 | Q=562 | Π1=4099.0193 |

**Example 4:** k=1500, a=50, b=5, h0=0.2, hr=0.6, c=10, p=15, $I_{p=}$0.15, $I_e$ =0.12, w=100, α=0.05, β=0.03 with suitable units. The optimal values are tabulated below.

**Table 5.** Numerical results for models 1 and 2

| M | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|
| 0.25 | 0.6073 | 0.8886 | w1=301 | TC1=2107.6148 |
| | | 0.7715 | Q=477 | Π1=2340.9370 |
| 0.0833 | 0.6712 | 0.9086 | w1=381 | TC1=1963.5056 |
| | | 0.8197 | Q=612 | Π1=2085.8697 |

**Example 5:** k=1500, a=75, b=10, h0=1, hr=3, c=10, p=15, $I_p$=0.15, $I_e$ =0.12, w=100, α=0.06, β=0.1 with suitable units. The optimal values are tabulated below.

**Table 6.** Numerical results for models 1 and 2

| M | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|
| 0.25 | 0.3022 | 0.4750 | w1=250 | TC1=3857.2856 |
| | | 0.4309 | Q=562 | Π1=4104.3340 |
| 0.0833 | 0.2852 | 0.4710 | w1=225 | TC1=3984.7313 |
| | | 0.4132 | Q=467 | Π1=4409.2968 |

**Example 6:** k=1500, a=75, b=10, h0=1, hr=3, c=10, p=15, $I_p$ =0.15, $I_e$ =0.12, w=100, with suitable units. The optimal values are tabulated below.

**Table 7.** Numerical results for models 1 and 2

| M | α | β | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|---|---|
| 0.5 | 0.06 | 0.1 | 0.2735 | 0.5026 | w1=220 | TC2=3116.4359 |
| | | | | 0.5404 | Q=1705 | Π2=3506.0938 |
| 0.5 | 0.1 | 0.06 | 0.2769 | 0.5015 | w1=213 | TC2=3116.2781 |
| | | | | 0.5325 | Q=1601 | Π2=3572.5254 |

It is evident from Tables 1 – 6 that the total relevant cost per unit time for Model 2 is less expensive to operate than that of Model 1.

**Example 7:** k=1500, a=50, b=5, h0=1, hr=3, c=10, p=15, $I_p$ =0.15, $I_e$ =0.12, w=100, with suitable units. The optimal values are tabulated below.

**Table 8.** Numerical results for models 1 and 2

| M | α | β | $t_w$* | T* | Quantity | TC |
|---|---|---|---|---|---|---|
| 0.5 | 0.1 | 0.06 | 0.1208 | 0.5341 | w1=108 | TC2=2913.0075 |
| | | | | 0.9669 | Q=1348 | Π2=1717.4726 |
| 0.5 | 0.06 | 0.1 | 0.1395 | 0.5523 | w1=110 | TC2=3116.4202 |
| | | | | 0.9941 | Q=1502 | Π2=1648.3592 |

## 7. Sensitivity analysis

**Example 1**: k=1500, w=100, h0=0.2, hr=0.6, c=10, p=15, $I_p$ =0.15, $I_e$ =0.12, α=0.1, β=0.06 with suitable units. The optimal values are tabulated below.

**Table 9.** Effects of M on the optimal replenishment policy

| M | $t_w$* | T* | w1 | Q | TC1 | Π1 |
|---|---|---|---|---|---|---|
| 1/12 | 0.5689 | 0.8338 | 266 | - | 2200.7279 | - |
| | | 0.7391 | - | 415 | - | 2509.4619 |
| 2/12 | 0.5868 | 0.8790 | 283 | - | 2161.3400 | - |
| | | 0.7516 | - | 443 | - | 2438.1253 |

**Table 9.** Continued

| M | tw* | T* | w1 | Q | TC1 | Π1 |
|---|---|---|---|---|---|---|
| 3/12 | 0.6057 | 0.8845 | 302 | - | 2120.4469 | - |
| | | 0.7650 | - | 475 | - | 2363.4071 |
| 4/12 | 0.6255 | 0.8096 | 324 | - | 2076.3534 | - |
| | | 0.7795 | - | 512 | - | 2284.7065 |
| 5/12 | 0.6464 | 0.8972 | 350 | - | 2028.8704 | - |
| | | 0.7950 | - | 555 | - | 2201.2513 |
| 6/12 | 0.6684 | 0.9043 | 381 | - | 1977.2152 | - |
| | | 0.8119 | - | 606 | - | 2112.0352 |

**Example 2:** $h_0$=1, $h_r$=3, $c$=10, $p$=0.15, $I_e$=0.12, $I_p$=0.15, $\alpha$=0.1, $\beta$=0.06, M=0.5 with suitable units. The optimal values are tabulated below.

**Table 10.** Effects of changes in parameter of the model when α > β

| k | w | a | b | $t_w$* | T* | w1 | Q | TC2 | Π2 | Use RW? |
|---|---|---|---|---|---|---|---|---|---|---|
| 1500 | 100 | 50 | 5 | 0.1208 | 0.5341 | 108 | - | 2913 | - | |
| | | | | - | 0.9669 | - | 1348 | - | 1717.4276 | Yes |
| | 120 | 55 | 7 | 0.2053 | 0.5098 | 145 | - | 3022.3368 | - | |
| | | | | - | 0.7324 | - | 1396 | - | 2372.4415 | Yes |
| | 140 | 60 | 9 | 0.2547 | 0.5030 | 200 | - | 3098.5655 | - | |
| | | | | - | 0.5932 | - | 1451 | - | 3060.9454 | Yes |
| | 160 | 65 | 11 | 0.2848 | 0.501 | 291 | - | 3224.641 | - | No |
| | | | | - | 0.5019 | - | 1532 | - | 3869.9983 | |
| 2000 | 100 | 50 | 5 | 0.1331 | 0.5425 | 109 | - | 3841.757 | - | |
| | | | | - | 0.9977 | - | 1579 | - | 2226.2255 | Yes |
| | 120 | 55 | 7 | 0.2083 | 0.5124 | 146 | - | 4001 | - | |
| | | | | - | 0.7563 | - | 1655.6442 | - | 3043.7724 | Yes |
| | 140 | 60 | 9 | 0.2553 | 0.5038 | 200 | - | 4092 | - | |
| | | | | - | 0.6124 | - | 1729.0079 | - | 3889.924 | Yes |
| | 160 | 65 | 11 | 0.285 | 0.5013 | 292 | - | 4222.3401 | - | No |
| | | | | - | 0.5173 | - | 1822 | - | 4850.4936 | |
| 2500 | 100 | 50 | 5 | 0.1439 | 0.5503 | 111 | - | 4756.74 | - | |
| | | | | - | 1.0222 | - | 1792 | - | 2721.1321 | Yes |
| | 120 | 55 | 7 | 0.211 | 0.515 | 147 | - | 4973.8527 | - | |
| | | | | - | 0.7752 | - | 1896 | - | 3696.4896 | Yes |
| | 140 | 60 | 9 | 0.256 | 0.5046 | 201 | - | 5083.535 | - | |
| | | | | - | 0.6277 | - | 1987 | - | 4696.0285 | Yes |
| | 160 | 65 | 11 | 0.2852 | 0.5015 | 292 | - | 5219.5976 | - | No |

**Table 10.** Continued

| k | w | a | b | tw* | T* | w1 | Q | TC2 | Π2 | Use RW? |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | - | 0.53 | - | 2096 | - | 5804.8987 | |
| 3000 | 100 | 50 | 5 | 0.1534 | 0.5574 | 112 | - | 5659.445 | - | |
| 3000 | | | | - | 1.0428 | - | 1991 | - | 3205.2651 | Yes |
| | 120 | 55 | 7 | 0.2137 | 0.5174 | 147 | - | 5942.4595 | - | |
| | | | | - | 0.791 | - | 2121 | - | 4334.7882 | Yes |
| | 140 | 60 | 9 | 0.2567 | 0.5053 | 201 | - | 6073.7821 | - | |
| | | | | - | 0.6404 | - | 2233 | - | 5484.7882 | Yes |
| | 160 | 65 | 11 | 0.2854 | 0.5017 | 292 | - | 6216.4164 | - | No |
| | | | | - | 0.5404 | - | 2354 | - | 6738.9082 | |

**Table 11.** Effects of changes in parameter of the model when $\alpha < \beta$

| k | w | a | b | $t_w$* | T* | w1 | Q | TC2 | Π2 | Use RW? |
|---|---|---|---|---|---|---|---|---|---|---|
| 1500 | 100 | 50 | 5 | 0.1395 | 0.5523 | 110 | - | 2849.4202 | - | |
| | | | | - | 0.9941 | - | 1502 | - | 1648.3591 | Yes |
| | 120 | 55 | 7 | 0.2063 | 0.5159 | 146 | - | 2995.34 | - | |
| | | | | - | 0.7473 | - | 1516 | - | 2309.1158 | Yes |
| | 140 | 60 | 9 | 0.2518 | 0.5050 | 199 | - | 3081.2414 | - | |
| | | | | - | 0.6027 | - | 1552 | - | 2999.1616 | Yes |
| | 160 | 65 | 11 | 0.2814 | 0.5018 | 287 | - | 3203.6964 | - | No |
| | | | | - | 0.5086 | - | 1624 | - | 3806.7733 | |
| 2000 | 100 | 50 | 5 | 0.1546 | 0.5639 | 112 | - | 3745.1523 | - | |
| | | | | - | 1.0247 | - | 1755 | - | 2143.4273 | Yes |
| | 120 | 55 | 7 | 0.2107 | 0.5199 | 147 | - | 3960.5572 | - | |
| | | | | - | 0.7711 | - | 1794 | - | 2967.3164 | Yes |
| | 140 | 60 | 9 | 0.2529 | 0.5063 | 199 | - | 4070.0573 | - | |
| | | | | - | 0.622 | - | 1848 | - | 3815.2395 | Yes |
| | 160 | 65 | 11 | 0.2817 | 0.5021 | 282 | - | 4199.8131 | - | No |
| | | | | - | 0.5242 | - | 1931 | - | 4774.4333 | |
| 2500 | 100 | 50 | 5 | 0.1674 | 0.5743 | 113 | - | 4623.5802 | - | |
| | | | | - | 1.0491 | - | 1987 | - | 2625.4791 | Yes |
| | 120 | 55 | 7 | 0.2146 | 0.5237 | 148 | - | 4918.8915 | - | |
| | | | | - | 0.79 | - | 2052 | - | 3607.6599 | Yes |
| | 140 | 60 | 9 | 0.254 | 0.5075 | 200 | - | 5056.476 | - | |
| | | | | - | 0.6372 | - | 2122 | - | 4609.1545 | Yes |
| | 160 | 65 | 11 | 0.2820 | 0.5025 | 288 | - | 5195.2041 | - | No |

Table 11. Continued

| k | w | a | b | tw* | T* | w1 | Q | TC2 | Π2 | Use RW? |
|---|---|---|---|---|---|---|---|---|---|---|
| 2500 | | | | - | 0.5367 | - | 2218 | - | 5716.6731 | |
| 3000 | 100 | 50 | 5 | 0.1786 | 0.5836 | 115 | - | 5487.0775 | - | |
| | | | | - | 1.07 | - | 2204 | - | 5870.3082 | Yes |
| | 120 | 55 | 7 | 0.2184 | 0.5273 | 149 | - | 5870.3082 | - | |
| | | | | - | 0.8059 | - | 2295 | - | 4234.1442 | Yes |
| | 140 | 60 | 9 | 0.2552 | 0.5087 | 201 | - | 6040.5649 | - | |
| | | | | - | 0.65 | - | 2384 | - | 5385.8613 | Yes |
| | 160 | 65 | 11 | 0.2824 | 0.5029 | 289 | - | 6189.8767 | - | No |
| | | | | - | 0.5472 | - | 2492 | - | 6639.0223 | |

### 7.1 Summary of findings and managerial insights

Based on the computational results as shown in the above Tables 8, 9 and 10, we obtain the following managerial insights.

1. The optimal values are highly sensitive to the changes in the values of w, a, b for keeping k fixed.
2. It is evident from the Tables that the total relevant cost increases when w, a, b increases. It is advisable for the retailer to rent a warehouse.
3. When the values w, a, b increase keeping k fixed, it is observed that the ordering quantity and total relevant cost increases.
4. On increasing the values of w, a, b the values of tw* increase but decreasing the values of T*.
5. All the optimal values increase except the values of T* on increasing the values of w, a, b for fixed k.
6. Keeping all the parameters fixed except the permissible delay period M increases, the optimal values tw*, T*, w1, Q increases.
7. Keeping all the parameters fixed, it is observed that a higher permissible delay period M results in a lower value of the total relevant cost but higher value of optimal replenishment cycle T*.
8. The total relevant cost decreases when the permissible delay period M increases by keeping all the parameters fixed.

## 8. Conclusion

In this study, a two-warehouse inventory model for deteriorating items is developed under permissible delay in payment period. This is considered to attract more sales, generally, retailers have been given permissible delay in payment and the same model is compared with a single-warehouse inventory model. Holding cost in the RW is found to be higher than that of the OW with different rates of deterioration at both warehouses. The present model differs from the existing models, as exponentially increasing demand is considered here. Our aim is to obtain the optimal replenishment policy for minimizing the total relevant costs of the retailer. The optimal values are highly sensitive to the changes in the parameters. In particular, the total cost of the model increases whenever the values of the parameters increase. And hence, the retailor ought to rent a warehouse. The solution procedure is developed to obtain the optimal solution to decide on the feasibility of renting a warehouse. Numerical examples are provided to compare the two proposed models and sensitivity analysis is carried out to substantiate the managerial insights.

## 9. Acknowledgement

This research work is supported by UGC-SAP (DRSII), Department of Mathematics, Gandhigram Rural Institute-Deemed University, Gandhigram-624 302, Tamilnadu, India.

## References

Aggarwal S.P., Jaggi C.K., (1995). Ordering policies of deteriorating items under permissible delay in payments, *Journal of the Operational Research Society*, 46, 658-662.

Benkherouf I., (1997). A Deterministic Order Level Inventory Model for Deteriorating Items with Two Storage Facilities, *International Journal of Production Economics*, 48, 167-175.

Bhunia A.K., Maiti, (1998). A Two-warehouse Inventory Model for Deteriorating items with a Linear Trend in Demand and Shortages, *Journal of Operational Research Society*,1998; 49, 287-292.

Bhunia, A. K., Ali Akbar Shaikh., (2015). An application of PSO in a two-warehouse inventory model for deteriorating item under permissible delay in payment with different inventory policies. *Applied Mathematics and Computation*, 256 , 831-850.

Chang C.T., Teng J.T., GoyalS.K., (2008). Inventory lot-size models under trade credits: a review, *Asia-Pacific Journal of Operational Research*, 25, 89-112.

Chang C.T., Wu S.J., Chen L.C., (2009).Optimal payment time with deteriorating items under inflation and permissible delay in payments, *International Journal of Systems Science*, 40, 985-993.

Chang H.J., Dye C.Y., (2001). An inventory model for deteriorating items with partial backlogging and permissible delay in payment, *International Journal of Systems Science*, 32, 345-352.

Chang H.J., Hung C.H., Dye C.Y., (2001). An inventory model for deteriorating items with linear trend demand under condition of permissible delay in payments, *Production Planning & Control*, 12, 274-282.

Chang., Chun-Tao., Mei-Chuan Cheng., Liang-Yuh Ouyang.,(2015). Optimal pricing and ordering policies for non-instantaneously deteriorating items under order-size-dependent delay in payments. *Applied Mathematical Modelling.* 39.2 , 747-763.

Chung K.J., Huang T.S., (2007). The Optimal retailer's ordering policies for deteriorating items with limited storage capacity under trade credit financing, *International Journal of Production Economics*, 106, 127-145.

Covert R.P., Philip G.C., (1973). An EOQ Model for Items with Weibull Distribution, *American Institute of Industrial Engineering Transactions*, 5, 323-326.

Das., Debasis., Arindam Roy., and Samarjit Kar.,(2015). A multi-warehouse partial backlogging inventory model for deteriorating items under inflation when a delay in payment is permissible. *Annals of Operations Research.* 226.1, 133-162.

Datta T.K. and Pal A.K., (1988). Order Level Inventory System with Power Demand Pattern for Items with Variable Rate of Deterioration, *Indian Journal of Pure and Applied Mathematics*, 19, 1043-1053.

Dave U.,Patel L.K.,(1981).(T,Si) Policy Inventory Model for Deteriorating items with Time Proportional Demand, *Journal of Operational Research Society,* 32, 137-142.

Ghare P.M.,Shrader G.F.,(1963). A Model for Exponentially Decaying Inventories, *Journal of Industrial Engineering*, 14, 238-243.

Ghoreishi., Maryam., Abolfazl Mirzazadeh., Isa N. Kamalabadi.,(2014). Optimal pricing and lot-sizing policies for an economic production quantity model with non-instantaneous deteriorating items, permissible delay in payments, customer returns, and inflation. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture* .

Ghoreishi., Maryam., Gerhard-Wilhelm Weber., Abolfazl Mirzazadeh.,(2015). An inventory model for non-instantaneous deteriorating items with partial backlogging, permissible delay in payments, inflation-and selling price-dependent demand and customer returns. *Annals of Operations Research* 226.1 , 221-238.

Goyal S.K., (1985). Economic order quantity under conditions of permissible delay in payment, *Journal of the Operational Research Society*, 36, 335-378.

Hui-Ling Yang, Chun-Tao Chang, (2013). A two-warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation, *Applied Mathematical Modeling*, 37, 2717-2726.

Hwang H., Shinn S.W., Wang S., (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the conditions of permissible delay in payments, *Computers & Operations Research*, 24, 539-547.

Jaggi C.K.,Aggarwal K.K. and Goel S.K.,(2006). Optimal Order Policy for Deteriorating Items with Inflation Induced Demand, *International Journal of Production Economics*, 34, 151-155.

Jamal A.M.M, (1997). An ordering policy for deteriorating items with allowable shortages and permissible delay in payments, *Journal of the Operational Research Society*, 48, 826-833.

Kar S., Bhunia A.K. and Maiti, (2001). Deterministic Inventory Model with Two Levels of Storage, a Linear Trend in Demand and a Fixed Time Horizon, *Computers and Operations Research*, 28, 1315-1331.

Liang Y., Zhou F., (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payments, *Applied Mathematical Modeling*, 35, 2221-2231.

Lee C.C., Hsu S.L., (2009). A two-warehouse production model for deteriorating inventory items with time-dependent demands, *European Journal of Operational Research*, 194, 700-710.

Liao H.C., Tsai C.H., Su C.T., (2000). An inventory model with deteriorating items under inflation when a delay in payment is permissible, *International Journal of Production Economics*, 63, 207-214.

Maryam Ghoreishi, Abolfazl Mirzazadeh., Gerhard-Wilhelm Weber., Isa Nakhai-Kamalabadi., (2014). Joint pricing and replenishment decisions for non-instantaneous deteriorating items with partial backlogging, inflation-and selling price-dependent demand and customer returns, *Journal of Industrial and Management Optimization.* 11, 933-949.

Ouyang L.Y., Wu K.S., Yang C.T., (2006). A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments, *Computers and Industrial Engineering*, 51, 637-651.

Sarma K.V.S., (1987). A deterministic order level inventory model for deteriorating items with two storage facility, *European Journal of Operational Research*, 29, 70-73.

Shah N.H., (1993). Probabilistic order quantity under conditions of permissible delay in payments, I*nternational Journal of Production Economics*, 32, 77-82.

Shah N.H., (2006). Inventory model for deteriorating and time value money for a finite time horizon under permissible delay in payments, *International Journal of Systems Science*, 37, 9-15.

Soni H., Gor A.S., Shah N.H., (2006). An EOQ model for progressive payment scheme under DCF approach, *Asia-Pacific Journal of Operational Research*, 23, 500-524.

Yang H.L., (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation, *International Journal of Production Economics,* 103, 362-370.

Zhou Y.W., Yang S.L., (2005). A two-warehouse inventory model for items with stock-level-dependent demand rate, *International Journal of Production Economics*, 95, 215-228.