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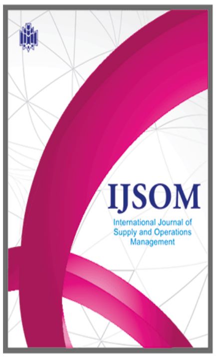

An EPQ Model with Increasing Demand and Demand Dependent Production Rate under Trade Credit Financing

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Abstract

This paper investigates an EPQ model with the increasing demand and demand dependent production rate involving the trade credit financing policy, which is seldom reported in the literatures. The model considers the manufacturer was offered by the supplier a delayed payment time. It is assumed that the demand is a linear increasing function of the time and the production rate is proportional to the demand. That is, the production rate is also a linear function of time. This study attempts to offer a best policy for the replenishment cycle and the order quantity for the manufacturer to maximum its profit per cycle. First, the inventory model is developed under the above situation. Second, some useful theoretical results have been derived to characterize the optimal solutions for the inventory system. The Algorithm is proposed to obtain the optimal solutions of the manufacturer. Finally, the numerical examples are carried out to illustrate the theorems, and the sensitivity analysis of the optimal solutions with respect to the parameters of the inventory system is performed. Some important management insights are obtained based on the analysis.

Keywords: EPQ; trade credit financing; increasing demand; demand dependent production rate.

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1. Introduction

Trade credit is a kind of credit which is offered by the buyer, allowing the buyer to buy the goods from the seller without the immediate payment. Since the globalization of the market and increased competition, the enterprises always adopt trade credit financing policy to boost sales, increase the market share and reduce the on-hand inventory levels. As a result, the trade credit financing plays an important role as the source of business financing after banks or other financial institute. For example, Wal-Mart has adopted the trade credit as a larger source of capital than bank borrowings (Zhang and Zhou, 2013). In the traditional inventory EPQ (Economic Production Quantity, EPQ) or EOQ (Economic Order Quantity, EOQ) model, it is assumed that the manufacturer must pay for the products when receiving them. However, in practice, the suppliers often provide the trade credit for the payment of the amount owed. Therefore, the assumptions of the EPQ or EOQ will be relaxed considering the trade credit. The optimal decisions for the inventory system will be influenced.

On the other hand, the optimal replenishment policies are also affected by the market demand and the production rate. In the traditional EPQ or EOQ model, it is assumed that the demand is a constant and the production rate is infinite or a constant. But in reality, for the market demand, it will be influenced by many factors such as the time for the short life cycle products. And when the market demand is better, the manufacturer will provide a higher production rate; when the market demand is shrinking, the supplier will reduce the production rate. Therefore, the production rate should be demand dependent production rate.

Therefore, in-depth research is required on the inventory replenishment decisions with the trade credit considering the increasing demand and the demand dependent production rate to extend the traditional EPQ model.

2. Literature Review

The inventory replenishment policies under trade credit financing have been researched intensively. The EOQ model with the trade credit financing is first proposed by Goyal (1985), which is extended by Chu et al. (1998) considering the deteriorated products. The order strategies are studied by Jamal et al. (1997) and Chang and Dye (2001) allowing the shortage under delayed payment and deteriorating conditions. The economic production quantity model is researched by Chung and Huang (2003) considering the manufacturer offering the retailer the delayed payment policy. Huang (2003) first extended Goyal's model to analyse the two levels of the trade credit policy. Qiu et al. (2003), Huang (2007), and Chung (2010) extend the models of Huang (2003). Tsao and Sheen (2008) use EPQ to model the decisions under system maintenance and trade credit. Teng et al. (2012) develop a supply chain inventory model with trade credit financing linked to order quantity, and then study the optimal policies for both the vendor and the buyer under a non-cooperative environment first, and then under a cooperative integrated situation. Liao et al. (2012) attempt to determine economic order quantity for deteriorating items with two-storage facilities (one is an owned warehouse and the other is a rented warehouse) where trade credit is linked to order quantity. It is clear that the issue of trade credit is very popular in this field of research.

The papers above discussed the EOQ or EPQ inventory models under trade credit financing all based on the assumptions that the demand rate is constant over time. But in practice, the market demand is always changing fast and effected by many factors such as the price, the inventory level, the stage of the product life time, etc. Some researchers realize this phenomenon and extend the studies above to establish the inventory models by assuming that the demand is variable. Chung and Liao (2011) discussed the inventory replenishment problems with trade credit financing considering a price sensitive demand. Kreng and Tan (2012) propose a production model for a lot-size inventory system considering the defective quality under the condition of two-level trade credit policy with a constant production rate and demand. Sarkar (2012) analyzed the EOQ model with delay in payments considering demand dependent on time and a constant replenishment rate. Teng *et al*. (2012) build the economic quantity model with trade credit financing for the non-decreasing demand with time and infinite replenishment rate. Min *et al*. (2012) develop an inventory model under conditions of permissible delay in payments, assuming that the items are replenished at a known constant rate and the demand rate of the items is dependent on the current inventory level. Soni (2013) discusses the optimal replenishment policies with the price and stock sensitive demand and infinite production rate under trade credit. Wu et al. (2014) explore the optimal credit period and lot size considering the demand dependent on the delayed payment time and the infinite production rate.

However, the infinite or a constant replenishment rate of the inventory models is inconsistent with the actual industrial practices. When the market demand is better, the manufacturer will provide a higher production rate; if the market demand is shrinking, the manufacturer will reduce the production rate. In the traditional EOQ or EPQ models without the trade credit financing, Manna and Chaudhuri (2006) presented a production-inventory system for deteriorating items with demand rate being a linearly ramp type function of time and production rate being proportional to the demand rate. The two models without shortages and with shortages were discussed. Both models were studied assuming that the time point at which the demand is stabilized occurs before the production stopping time. Skouri *et al.* (2011) extends this model by considering that: a) for the model with no shortages; the demand rate is stabilized after the production stopping time, and b) for the model with shortages; the demand rate is stabilized after the production stopping time or after the inventory level reaches zero or after the production restarting time. Recently, Soni and Patel (2012) developed a more general integrated supplier- retailer inventory model with a demand rate that is sensitive to the retailing price and a demand dependent production rate with two level of trade credit financing. But they do not consider the situation that the demand increases with time from the perspective of the product life cycle.

Therefore, based on the literatures above, this paper found that none of the above models explore the optimal replenishment policies of the manufacturer under trade credit financing combining with the increasing demand and demand dependent production rate. This study extends the EPQ models in the several ways following. First, trade credit financing is introduced to the traditional EPQ models. The manufacturer is offered by the supplier with a delayed payment time. Second, the increasing demand is introduced to the EPQ models with the trade credit financing. The demand is nearly a constant in the maturity stage from the perspective of the product life cycle. During the growth time, most of the product demand increases with time. Furthermore, the production rate dependent on demand is introduced to the EPQ models with trade credit financing considering the increasing demand for the first time.

Given the analysis above, this paper developed an inventory model under trade credit financing with increasing demand and demand dependent production rate. The study then demonstrates some easy-to-use theorems to identify the optimal replenishment policies for the manufacturer. Numerical examples are provided to illustrate the theorems proposed. Finally, sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out and some important management insights are obtained.

3. Mathematical modeling

The inventory system can be considered as follows. At the beginning, the stock level is zero. The production begins just after $t=0$, continues up to $t=t_1$ and stops as soon as the stock level becomes S. Then the inventory level decreases due to demand, until it becomes zero at $t = T$. Hence, the variation of the inventory level $I(t)$, with respect to time can be described by the following differential equations.

$$\frac{dI_1(t)}{dt} = K(t) - f(t),\ 0 \le t \le t_1. \tag{1}$$

And

$$\frac{dI_2(t)}{dt} = -f(t), t_1 \le t \le T. \tag{2}$$

Where $K(t) = rf(t)$, $f(t) = bt$. Using the values, the equation (1) and (2) become respectively as follows:

$$\frac{dI_1(t)}{dt} = (r-1)(a+bt),\ 0 \le t \le t_1. \tag{3}$$

With the conditions $I_1(0) = 0$ and $I_1(t_1) = S$.

$$\frac{dI_2(t)}{dt} = -(a+bt),\ t_1 \le t \le T. \tag{4}$$

With the conditions $I_2(t_1) = S$ and $I_2(T) = 0$.

Solving equation (3) and (4), the values of $I_1(t)$ and $I_2(t)$ are as follows:

$$I_1(t) = (r-1)(at + \frac{1}{2}bt^2), 0 \le t \le t_1. \tag{5}$$

$$I_2(t) = (aT + \frac{1}{2}bT^2) - (at + \frac{1}{2}bt^2), t_1 \le t \le T. \tag{6}$$

In addition, using the boundary condition $I_1(t_1) = I_2(t_1) = S$, we obtain

$$(r-1)(at + \frac{1}{2}bt^2) = (aT + \frac{1}{2}bT^2) - (at + \frac{1}{2}bt^2). \tag{7}$$

By solving the equation (7), we can obtain:

$$t_1 = \frac{-ar + \sqrt{a^2r^2 + 2br(aT + \frac{1}{2}bT^2)}}{br}. \tag{8}$$

$$\frac{\partial t_1}{\partial T} = \frac{a+bT}{\sqrt{br(2aT + bT^2)}}. \quad \frac{\partial^2 t_1}{\partial T^2} = \frac{b^2 r(2aT + bT^2) - (a+bT)^2}{[br(2aT + bT^2)]^{3/2}}.$$

The total profit per cycle consists of the following elements: revenue, ordering cost, holding cost, interest payable and interest earned. The components are evaluated as follows:

(1) Revenue per cycle: $R = \frac{s}{T}\int_0^T D(\mathrm{t})\,\mathrm{dt} = \frac{s}{T}(aT + \frac{1}{2}bT^2) = s(a + \frac{1}{2}bT)$.

(2) Ordering cost per cycle: $\frac{A}{T}$.

(3) Stock holding cost per cycle (excluding the interest charges):

$$\frac{c_h}{T}[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt] = \frac{c_h}{T}\{r(\frac{1}{2}at_1^2 + \frac{1}{6}bt_1^3) + (aT + \frac{1}{2}bT^2)(T - t_1) - \frac{1}{2}aT^2 - \frac{1}{6}bT^3\} \quad (9)$$

(4) There are three cases to occur in interest earned and interest charged for the items kept in stock per cycle.

Case 1: $M < t_1 < T$, i.e. $M < \frac{-a + \sqrt{a^2 + brM(bM + 2a)}}{b} < T$, as shown in Fig.1

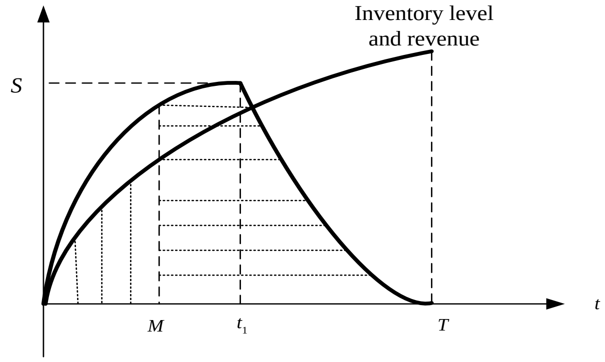

Figure1. The total interest earned and interest charged when $M < t_1 < T$

Interest earned per cycle is:

$$\frac{sI_e}{T}\int_0^M (M - t)D(t)dt = \frac{sI_e}{T}\int_0^M (M - t)(a + bt)dt = \frac{sI_e}{T}(\frac{1}{2}aM^2 + \frac{1}{6}bM^3). \quad (10)$$

Interest charged per cycle for the manufacturer from the supplier is:

$$\frac{c_pI_c}{T}[\int_M^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt] = \frac{c_pI_c}{T}[\frac{r}{6}bt_1^3 + \frac{r}{2}at_1^2 - (aT + \frac{1}{2}bT^2)t_1 + \frac{1}{2}aT^2 + \frac{1}{3}bT^3 - (r-1)(\frac{1}{2}aM^2 + \frac{1}{6}bM^3)] \quad (11)$$

Case 2: $t_1 < M < T$ **i.e.** $M < T < \frac{-a + \sqrt{a^2 + brM(bM + 2a)}}{b}$**,** as shown in Fig.2

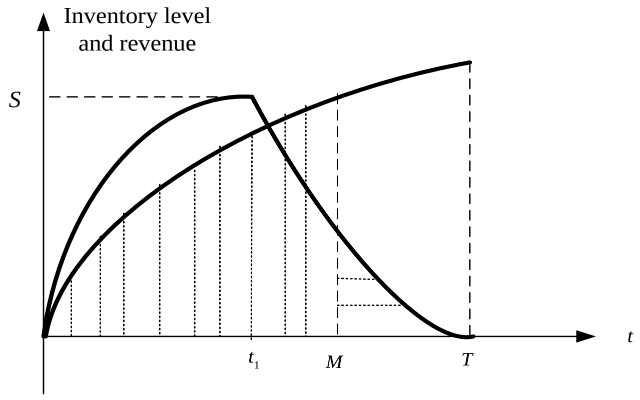

Figure2. The total interest earned and interest charged when $t_1 < M < T$

Interest earned per cycle is:

$$\frac{sI_e}{T}\int_0^M (M-t)D(t)dt = \frac{sI_e}{T}(\frac{1}{2}aM^2 + \frac{1}{6}bM^3). \tag{12}$$

Interest charged per cycle for the manufacturer from the supplier is:

$$\frac{c_p I_c}{T}\int_M^T Q_2(t)dt = \frac{c_p I_c}{T}[(aT + \frac{1}{2}bT^2)(T-M) - \frac{1}{2}a(T^2 - M^2) - \frac{1}{6}b(T^3 - M^3)]. \tag{13}$$

Case 3: $T \le M$ **,** as shown in Figure.3

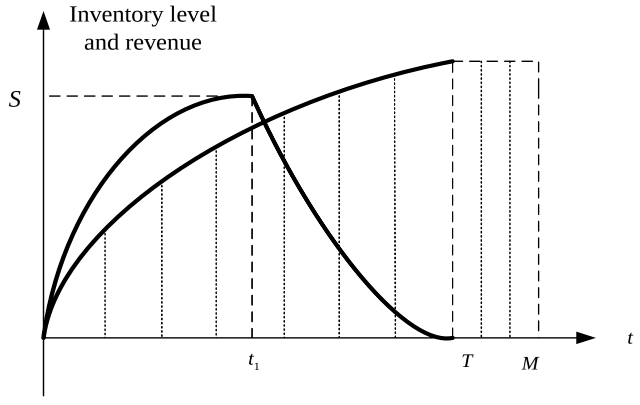

Figure3. The total interest earned and interest charged when $T \le M$

Interest earned per cycle due to the credit balance is:

$$\frac{sI_e}{T}[\int_0^T (T-t)D(t)dt + \int_0^{t_1} K(t)(M-T)dt] = \frac{sI_e}{T}[\frac{1}{2}aT^2 + \frac{1}{6}bT^3 + r(M-T)(at_1 + \frac{1}{2}bt_1^2)] . \tag{14}$$

Interest charged per cycle is zero.
From the above arguments, the total profit per cycle for the manufacturer can be expressed as: $\pi(T) =$ Revenue–Ordering cost – stock holding cost – interest payable + interest earned.

$$\pi(T) = \begin{cases} \pi_1(T); & if \ \ t_m \le T \\ \pi_2(T); & if \ \ M \le T \le \ t_m \\ \pi_3(T); & if \ \ 0<T \le M \end{cases} . \tag{15}$$

With

$$\pi_1(T) = s(a + \frac{1}{2}bT) - \frac{A}{T} - \frac{c_h + c_p I_c}{T}[-\frac{r}{3}bt_1^3 - \frac{r}{2}at_1^2 + \frac{1}{2}aT^2 + \frac{1}{3}bT^3] + \frac{c_p I_c (r-1) + sI_e}{T}(\frac{1}{2}aM^2 + \frac{1}{6}bM^3) . \tag{16}$$

$$\pi_2(T) = s(a + \frac{1}{2}bT) - \frac{A}{T} - \frac{c_h}{T}[-\frac{r}{3}bt_1^3 - \frac{r}{2}at_1^2 + \frac{1}{2}aT^2 + \frac{1}{3}bT^3] + \frac{sI_e}{T}(\frac{1}{2}aM^2 + \frac{1}{6}bM^3) - \frac{c_p I_c}{T}[(aT + \frac{1}{2}bT^2)(T-M) - \frac{1}{2}a(T^2 - M^2) - \frac{1}{6}b(T^3 - M^3)] . \tag{17}$$

$$\pi_3(T) = s(a + \frac{1}{2}bT) - \frac{A}{T} - \frac{c_h}{T}[-\frac{r}{3}bt_1^3 - \frac{r}{2}at_1^2 + \frac{1}{2}aT^2 + \frac{1}{3}bT^3] + \frac{sI_e}{T}[\frac{1}{2}aT^2 + \frac{1}{6}bT^3 + r(M-T)(at_1 + \frac{1}{2}bt_1^2)] . \tag{18}$$

$$t_m = \frac{-a + \sqrt{a^2 + brM(bM + 2a)}}{b} . \tag{19}$$

4. The optimal replenishment policy

Most strikingly, in order to optimize the solutions to Eq. (16-18), it is necessary to study $\pi_i(T)$ $(i = 1, 2, 3)$, respectively.

Case1. $M < t_1 < T$

To find the optimal replenishment policy, take the first-order partial derivative of $\pi_{11}(\mathrm{N,T})$ with respect to *T*. We can obtain

$$\frac{\partial \pi_1(\mathrm{N,T})}{\partial T} = \frac{1}{T^2} f_1(\mathrm{T}). \tag{20}$$

Where

$$f_1(\mathrm{T}) = \frac{1}{2}sbT^2 + A - (c_h + c_p I_c)[\frac{r}{3}bt_1^3 + \frac{r}{2}at_1^2 + \frac{1}{2}aT^2 + \frac{2}{3}bT^3 - \frac{\partial t_1}{\partial T}T(rbt_1^2 + rat_1)] - (c_p I_c(r-1) + sI_e)(\frac{1}{2}aM^2 + \frac{1}{6}bM^3). \tag{21}$$

It is easy to obtain

$$f_1^{'}(\mathrm{T}) = sbT - (c_h + c_p I_c)[aT + 2bT^2 - \frac{\partial^2 t_1}{\partial T^2}T(rbt_1^2 + rat_1) - (\frac{\partial t_1}{\partial T})^2 T(2rbt_1 + ra)]. \tag{22}$$

Taking the second-order partial derivative of $\pi_{11}(\mathrm{T})$ with respect to *T*, we can obtain

$$\frac{\partial^2 \pi_1}{\partial T^2} = -\frac{2}{T^3} f_1(T) + \frac{1}{T^2} f_1^{'}(T) = -\frac{2}{T^3}(f(T) - Tf_1^{'}(T)).$$

If $\tilde{T}_1$ is the root of $\frac{\partial \pi_1}{\partial T} = 0$ (this may or may not exist) and $f_1^{'}(\tilde{T}_1) < 0$, then

$$\left.\frac{\partial^2 \pi_1}{\partial T^2}\right|_{T=\tilde{T}_1} = \frac{1}{T^2} f_1^{'}(T) < 0.$$

And this $\tilde{T}_1$ corresponds to a maximum value of π_1.

We can obtain $\lim_{T\to\infty} f_1(\mathrm{T}) = -\infty < 0$

$\lim_{T\to 0} f_1(\mathrm{T}) = A - (c_p I_c(r-1) + sI_e)(\frac{1}{2}aM^2 + \frac{1}{6}bM^3)$. If $\lim_{T\to 0} f_1(\mathrm{T}) > 0$, then $\tilde{T}_1$ exists for $T \in [0,\infty)$. If $\tilde{T}_1$ is feasible, i.e. $t_m \le \tilde{T}_1$, the optimal replenishment cycle T_1 corresponds to $\max\{\pi_1(\tilde{T}_1), \pi_1(t_m)\}$.On the other hand, if $\tilde{T}_1$ does not exist or is infeasible, then the optimal replenishment cycle T_1 corresponds to $\max \pi_1(t_m)$. The optimal order quantity is $Q_1 = aT_1 + bT_1^2/2$. Therefore, based on the analysis above, it is easy to obtain the Theorem 1 to decide the optimal replenishment cycle for Case 1.

Theorem1.

(1) If $t_m \le \tilde{T}_1$, the optimal replenishment cycle T_1 corresponds to $\max\{\pi_1(\tilde{T}_1), \pi_1(t_m)\}$.

(2) If $\tilde{T}_1$ does not exist or is infeasible, then the optimal replenishment cycle T_1 corresponds to $\max \pi_1(t_m)$.

Case2. $t_1 < M < T$

The first derivative of $\pi_2(T)$ with respect to T is

$$\frac{\partial \pi_2(T)}{\partial T} = \frac{f_2(T)}{T^2}. \quad (23)$$

Where

$$\begin{aligned} f_2(T) &= \frac{1}{2}sbT^2 + A - c_h[\frac{r}{3}bt_1^3 + \frac{r}{2}at_1^2 + \frac{1}{2}aT^2 + \frac{2}{3}bT^3 - \frac{\partial t_1}{\partial T}T(rbt_1^2 + rat_1)] - \\ &(sI_e + c_pI_c)(\frac{1}{2}aM^2 + \frac{1}{6}bM^3) - c_pI_c[-\frac{1}{2}bMT^2 + \frac{1}{2}aT^2 + \frac{2}{3}bT^3] \end{aligned}. \quad (24)$$

It is easy to obtain

$$\begin{aligned} f_2^{'}(\mathrm{T}) &= sbT - c_h[aT + 2bT^2 - \frac{\partial^2 t_1}{\partial T^2}T(rbt_1^2 + rat_1) - (\frac{\partial t_1}{\partial T})^2 T(2rbt_1 + ra)] \\ &- c_pI_c[-bMT + aT + 2bT^2] \end{aligned}. \quad (25)$$

Taking the second-order partial derivative of $\pi_2(\mathrm{T})$ with respect to T, we can obtain

$$\frac{\partial^2 \pi_2}{\partial T^2} = -\frac{2}{T^3} f_2(T) + \frac{1}{T^2} f_2^{'}(T) = -\frac{2}{T^3}(f(T) - Tf_2^{'}(T)).$$

If $\tilde{T}_2$ is the root of $\frac{\partial \pi_2}{\partial T} = 0$ (this may or may not exist) and $f_2^{'}(\tilde{T}_2) < 0$, then

$$\left.\frac{\partial^2 \pi_2}{\partial T^2}\right|_{T=\tilde{T}_2} = \frac{1}{T^2} f_2^{'}(T) < 0.$$

And this $\tilde{T}_2$ corresponds to a maximum value of π_2.

Therefore, if $\tilde{T}_2$ is feasible, i.e. $t_m \le \tilde{T}_2 \le M$, the optimal replenishment cycle T_2 corresponds to $\max\{\pi_2(\tilde{T}_2), \pi_2(t_m), \pi_2(\mathrm{M})\}$. On the other hand, if $\tilde{T}_2$ does not exist or is infeasible, then the optimal replenishment cycle T_2 corresponds to $\max\{\pi_2(t_m), \pi_2(\mathrm{M})\}$. The optimal order quantity is $Q_2 = aT_2 + bT_2^2/2$. Therefore, based on the analysis above, it is easy to obtain the Theorem 2 to decide the optimal replenishment cycle for Case 2.

Theorem2.

(1) If $t_m \le \tilde{T}_2 \le M$ exits, the optimal replenishment cycle T_2 corresponds to $\max\{\pi_2(\tilde{T}_2), \pi_2(t_m), \pi_2(\mathrm{M})\}$.

(2) If $\tilde{T}_2$ does not exist or is infeasible, then the optimal replenishment cycle T_{11} corresponds to $\max\{\pi_2(t_m), \pi_2(\mathrm{M})\}$.

Case 3. $T \le M$

The first derivative of $\pi_3(T)$ with respect to T is

$$\frac{\partial \pi_3(T)}{\partial T} = \frac{f_3(T)}{T^2}. \tag{26}$$

Where

$$f_3(T) = \frac{1}{2}sbT^2 + A - c_h[\frac{r}{3}bt_1^3 + \frac{r}{2}at_1^2 + \frac{1}{2}aT^2 + \frac{2}{3}bT^3 - \frac{\partial t_1}{\partial T}T(rbt_1^2 + rat_1)] - sI_e(\frac{1}{2}aT^2 - \frac{1}{2}bMT^2 + \frac{2}{3}bT^3).$$

(27)

It is easy to obtain

$$f_3^{'}(\mathrm{T}) = sbT - c_h[aT + 2bT^2 - \frac{\partial^2 t_1}{\partial T^2}T(rbt_1^2 + rat_1) - (\frac{\partial t_1}{\partial T})^2 T(2rbt_1 + ra)] - sI_e(aT - bMT + 2bT^2). \tag{28}$$

Taking the second-order partial derivative of $\pi_3(\mathrm{T})$ with respect to T, we can obtain

$$\frac{\partial^2 \pi_3}{\partial T^2} = -\frac{2}{T^3} f_3(T) + \frac{1}{T^2} f_3^{'}(T) = -\frac{2}{T^3}(f(T) - Tf_3^{'}(T)).$$

If $\tilde{T}_3$ is the root of $\frac{\partial \pi_3}{\partial T} = 0$ (this may or may not exist) and $f_3^{'}(\tilde{T}_3) < 0$, then

$$\left.\frac{\partial^2 \pi_3}{\partial T^2}\right|_{T=\tilde{T}_3} = \frac{1}{T^2} f_3^{'}(T) < 0.$$

And this $\tilde{T}_3$ corresponds to a maximum value of π_3.

It is easy to obtain $\lim_{T\to 0} f_3(T) = A > 0$, $\lim_{T\to\infty} f_3(T) = -\infty < 0$, therefore, $\tilde{T}_3$ exists for $T \in [0, \infty)$. If $\tilde{T}_3$ is feasible, i.e. $0 \le \tilde{T}_3 \le M$ for Eq. (19), the optimal replenishment cycle T_2 corresponds to $\max\{\pi_2(\tilde{T}_2), \pi_2(0), \pi_2(\mathrm{M})\}$ .On the other hand, if $\tilde{T}_2$ is infeasible, then the optimal replenishment cycle T_2 corresponds to $\max\{\pi_2(0), \pi_2(\mathrm{M})\}$. The optimal order quantity is $Q_3 = aT_3 + bT_3^2/2$. Based on the analysis, it is easy to obtain the Theorem 3 to decide the optimal replenishment cycle for Case 3.

Theorem3.

(1) $\tilde{T}_3$ exists for $T \in [0, \infty)$.

(2) If $0 \le \tilde{T}_3 \le M$, the optimal replenishment cycle T_2 corresponds to $\max\{\pi_2(\tilde{T}_2), \pi_2(0), \pi_2(\mathrm{M})\}$.

(3) If $\tilde{T}_3 > M$, then the optimal replenishment cycle T_{11} corresponds to $\max\{\pi_2(0), \pi_2(\mathrm{M})\}$.

Finally, combining the three cases above, we can obtain the Algorithm for determining the optimal replenishment policy in the following algorithm.

Algorithm1.

Step1. Input the values for all parameters

Step2. Find the global maximum of $\pi_1(T)$, says T_1 , as follows:

Step2.1 compute $\tilde{T}_1$, if $t_m \le \tilde{T}_1$ and $f_1^{"}(\tilde{T}_1) < 0$, find the $\max\{\pi_1(\tilde{T}_1), \pi_1(t_m)\}$. And accordingly set T_1. Compute Q_1 and $\pi_1(T_1)$. Else, go to step 1.2.

Step 2.2 find the $\max \pi_1(t_m)$ and accordingly set T_1. Compute Q_1 and $\pi_1(T_1)$.

Step3. Find the global maximum of $\pi_2(T)$, says T_2 , as follows:

Step3.1 compute $\tilde{T}_2$, if $M \le \tilde{T}_2 \le t_m$ and $f_2^{"}(\tilde{T}_2) < 0$, find the $\max\{\pi_2(\tilde{T}_2), \pi_2(t_m), \pi_2(\mathrm{M})\}$ and accordingly set T_2. Compute Q_2 and $TRC_2(T_2)$. Otherwise, go to step 3.2.

Step 3.2 find the $\max\{\pi_2(t_m), \pi_2(\mathrm{M})\}$ and accordingly set T_2. Compute Q_2 and $\pi_2(T_2)$.

Step4. Find the global minimum of $\pi_3(T)$, says T_3 , as follows:

Step4.1 compute $\tilde{T}_3$, if $0 \le \tilde{T}_3 \le M$, and $f_3^{"}(\tilde{T}_3) < 0$, find the $\max\{\pi_2(\tilde{T}_2), \pi_2(0), \pi_2(\mathrm{M})\}$ and accordingly set T_3. Compute Q_3 and $\pi_3(T_3)$. Otherwise, go to step 4.2.

Step 4.2 find the $\max\{\pi_2(0), \pi_2(\mathrm{M})\}$ and accordingly set T_3. Compute Q_3 and $\pi_3(T_3)$.

Step5. Find the $\max\{\pi_1(T_1), \pi_2(T_2), \pi_3(T_3)\}$ and accordingly select the optimal value for T^* and π^*.

Stop.

5. Numerical analysis

The purposes of the numerical analysis are as follows: give the numerical examples to obtain the optimal solutions of the profit functions for the manufacturer. Carry out the sensitivity analysis to highlight the influence of the parameters associated with the model on the optimal decisions. The sensitivity analysis is shown in Table 1.

Example1.

The input parameters are: $A = \$10$ per order, $a = 100$ units, $r = 2$, $s = \$\ 20$ per unit, $c_h = \$5$ per unit per year, $c_p = \$10$ per unit, $I_e = 0.09$, $I_c = 0.14$, $M = 0.5$ year, *and b*=5.

Using the Algorithm, we can calculate $\pi_1(T_1) = 1917.6$ with $T_1 = 0.9881$, $\pi_2(T_2) = 1918.1$ with $T_2 = 0.9881$, $\pi_3(T_3) = 1541.3$ with $T_{13} = 0.2828$. Therefore, the optimal order cycle for the manufacturer is $T^* = T_1 = 0.9881$ year and the optimal order quantity is $Q^* = 101.3$ units. The maximum profit is $\pi^* = \$1918.1$.

Example2.

The input parameters are the same as example 1, except $M = 0.1$ year. Using the Algorithm, we can calculate $\pi_1(T_1) = 1934.9$ with $T_1 = 0.3030$, $\pi_2(T_2) = 1917.0$ with $T_2 = 0.1995$, $\pi_3(T_3) = 1901.5$ with $T_3 = 0.1$. Therefore, the optimal order cycle for the manufacturer is $T^* = T_1 = 0.3030$ year and the optimal order quantity is $Q^* = 30.5$ units. The maximum profit is $\pi^* = \$1934.9$.

For the sensitivity analysis, the basic parameters are the same as example 2. The following management insights are consistent with the expectations.

1) As the ordering cost increases, the replenishment cycle time and the order quantity per cycle significantly increase, while the profit decreases. These analytical results suggest that the manufacturer needs to order more quantity to reduce the number of orders if the ordering cost is more costly.
2) If the initial demand increases, the replenishment cycle time per cycle decreases, while the order quantity and the profit increase. It implies that when the initial market demand is greater, the retailer can make more profits.
3) The larger the value of s ,the more the values of the optimal cycle time, the order quantity and the profit increase. That is, when the unit selling price increases, the manufacturer can make more profit.
4) When the holding cost and the purchasing cost increases, it is obtained the optimal cycle length, the order quantity and the optimal total profit per cycle decrease. Therefore, it is reasonable that when the relevant cost increases, the manufacturer will reduce the cycle time and order quantity to lower the on-hand inventory level or the purchasing cost to cut down the cost. It means the retailer can adopt some measurements to reduce the holding cost or the purchasing cost to make more profit.
5) The larger the value of M , the more the value of the optimal cycle time and the value of the profit increase. That is, when the manufacturer's trade credit period offered by the supplier increases, the manufacturer can use the trade credit period to accumulate more interest.
6) As the rate of production dependent on demand increases, the optimal replenishment time and the order quantity first decrease and then increase, while the profit decreases. That is, when the dependence between the production and the demand is weak, the manufacturer will order fewer products. But if the dependence between the production and the demand is strong, the manufacturer will improve the order quantity to meet the demand.
7) When the demand rate increases, the optimal replenishment cycle, the optimal order quantity and the optimal profit increase. This implies that when the market demand becomes larger fast, the manufacturer will improve the replenishment quantity to meet the market's demand and its profit is improved.

Table1. The sensitivity analysis of parameters

	parameters	T^*	Q^*	π^*
A	10	0.3030	30.5	1934.9
	12	0.3232	32.6	1928.5
	14	0.3434	34.6	1922.6
a	100	0.3030	30.5	1934.9
	120	0.2626	31.7	2326.3
	140	0.2424	34.1	2718.6
s	20	0.3030	30.5	1937.9
	25	0.3232	32.6	2483.1
	30	0.3434	34.6	2992.9
c_p	10	0.3030	30.5	1934.9
	12	0.2828	28.5	1933.0
	14	0.2828	28.5	1931.1
c_h	5	0.3030	30.5	1934.9
	7	0.2424	24.4	1921.3
	9	0.2222	22.3	1909.6
I_e	0.09	0.3030	30.5	1934.9
	0.11	0.3030	30.5	1935.1
	0.13	0.3030	30.5	1935.3
I_c	0.14	0.3030	30.5	1934.9
	0.16	0.2828	28.5	1933.5
	0.18	0.2828	28.5	1932.2
M	0.1	0.3030	30.5	1934.9
	0.3	0.5956	60.4	1930.8
	0.5	0.9881	101.3	1917.6
r	2	0.3030	30.5	1934.9
	3	0.2985	30.1	1924.9
	4	0.3971	40.1	1905.8
b	5	0.3030	30.5	1934.9
	8	0.3434	34.8	1944.3
	12	0.4444	45.6	1959.4

6. Conclusion

Most of the existing inventory models under trade credit financing are assumed a constant demand and the infinite or a constant replenishment rate. However, in practice, the demand rate is an increasing function of time in the growth stage from the perspective of the production life cycle. On the other hand, the production rate is related to the market demand in some situations. When the market is better, the production rate will be improved.

Therefore, in this paper, an EPQ model is built under trade credit financing with increasing demand and the demand dependent production rate. Subsequently, the optimal replenishment cycle and the optimal order quantity are discussed, and the Algorithm is proposed to obtain the optimal solutions of the model. Finally, the numerical analysis is demonstrated to illustrate the theorems and the sensitivity analysis is carried out to give some management insights.

The research presented in this paper can be extended in several ways. For example, the ramp demand or the quadratic demand can be further discussed considering the demand dependent production rate in the EPQ inventory model with the trade credit financing. The deterioration rate can be introduced for the items. Additionally, the models can be generalized to consider the shortage or the partial backlogging. Furthermore, the influence of the poor quality products on the EPQ model can be discussed to obtain some management insights.

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Appendix A. Notations and assumptions

Notations

A: ordering cost per order.
c_h: the inventory holding cost per unit per unit time (excluding interest charges).
c_p: the purchasing cost per unit.
I_e: the interest rate earned per unit per unit of time.
I_c: the interest rate charged per unit per unit of time.
M: the credit period offered by the supplier.
I(t): the inventory level at time $t \in [0, T]$.
t_1 : the time at which the inventory level increases to maximum value (a decision variable).
S: the maximum inventory level at each scheduling period (cycle).
T: the replenishment cycle time.
Q: the order quantity.
s : the unit selling price for the manufacturer.
$\pi(\mathrm{T})$: the total relevant profit per cycle.

Assumptions

(1) The lead time is zero. Shortage is not allowed.
(2) During the growth stage of a product life cycle especially for the high- tech product, the market demand for the item is assumed to be time dependent and is defined as $R = D(t) = a + bt$ at any time with $t \geq 0$, $a > 0$, $b > 0$.
(3) The production rate for the manufacturer is $K(t) = rD(t)$ where $r(>1)$ is a constant.
(4) The manufacturer would settle the account at $t = M$ and pay for the interest charges on items in stock at the rate I_c over the interval [M, T] as $T \geq M$. Alternatively, the manufacturer settles the account at the $t = M$ and is not required to pay any interest charges for items for the stock during the whole cycle as $T \leq M$. Before the settlement of the account, the manufacturer can accumulate the sales revenue to earn the interest up to the end of the period M at the rate I_e.