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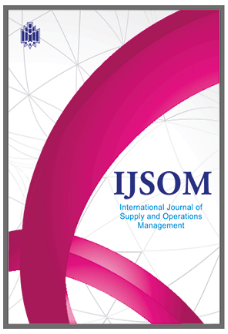

Studies on some inventory model for deteriorating items with Weibull replenishment and generalized pareto decay having demand as function of on hand inventory

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Abstract
Inventory models play an important role in determining the optimal ordering and pricing policies. Much work has been reported in literature regarding inventory models with finite or infinite replenishment. But in many practical situations the replenishment is governed by random factors like procurement, transportation, environmental condition, availability of raw material etc., hence, it is needed to develop inventory models with random replenishment. In this paper, an EPQ model for deteriorating items is developed and analyzed with the assumption that the replenishment is random and follows a Weibull distribution. It is further assumed that the life time of a commodity is random and follows a generalized Pareto distribution and demand is a function of on hand inventory. Using the differential equations, the instantaneous state of inventory is derived. With suitable cost considerations, the total cost function is obtained. By minimizing the total cost function, the optimal ordering policies are derived. Through numerical illustrations, the sensitivity analysis is carried. The sensitivity analysis of the model reveals that the random replenishment has significant influence on the ordering and pricing policies of the model. This model also includes some of the earlier models as particular cases for specific values of the parameters.

Keywords: Random replenishment; Generalized Pareto decay; Stock on hand; EPQ model; Weibull distribution.

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1. Introduction

Inventory models create a lot of interest due to their ready applicability at various places like market yards, warehouses, production process, transportation systems cargo handling, etc., several inventory models have been developed and analyzed to study various inventory systems. Much work has been reported in literature regarding Economic Production Quantity (EPQ) models during the last two decades. The EPQ models are also a particular case of inventory models. The major constituent components of the EPQ models are 1) Demand 2) production (Production) (Replenishment) and 3) Life time of the commodity. Several EPQ models have been developed and analyzed with various assumptions on demand pattern and life time of the commodity. In general, it is customary to consider that the replenishment is random in production inventory models.

Several researchers have developed various inventory models with stock dependent demand. Silver and Peterson (1985) mentioned that the demand for many consumer items is directly proportional to the stock on hand. Gupta and Vrat (1986) have pointed the inventory models with stock dependent demand. Later, Baker and Urban (1988), Mandal and Phaujdhar (1989), Datta and Pal (1990), Venkat Subbaiah, et al. (2004), Teng and Chang (2005), Arya, et al. (2009), Mahata and Goswami (2009a), Panda, et al. (2009c), Roy, et al. (2009), Uma Maheswara Rao, et al. (2010), Yang, et al. (2010), Yang, et al. (2011), Srinivasa Rao and Essay (2012), Jasvinder Kaur, et al. (2013), Santanu Kumar Ghosh, et al. (2015) and others have developed inventory models for deteriorating items with stock dependent demand. In all these models they assumed that the replenishment is instantaneous or having fixed finite rate, except Sridevi, et al. (2010) that developed and analyzed an inventory model with the assumption that the rate of production is random and follows a Weibull distribution. However, in many practical situations arising at production processes, the production (replenishment) rate is dependent on the stock on hand. But in some other situations such as textile markets, seafood’s industries, etc., the demand is a function of stock on hand. Levin et al. (1972) has have observed that at times the presence of inventory has a motivational effect on demand. It is also generally known that large pails of goods displayed in the markets encourage customers to buy more. Thus, in certain items, the demand increases if large amount of stock is on hand.

Another important consideration for developing the EPQ models for deteriorating items is the life time of the commodity. For items like food, processing the life time of the commodity is random and follows a generalized Pareto distribution. (Srinivasa Rao, et al. (2005), Srinivasa Rao and Begum (2007), Srinivasa Rao and Eswara Rao (2015)). Very little work has been reported in the literature regarding EPQ models for deteriorating items with random replenishment and generalized Pareto decay having stock dependent demand, even though these models are more useful for deriving the optimal production schedules of many production processes. Hence, in this paper, we develop and analyze an economic production quantity model for deteriorating items with Weibull rate of replenishment and generalized Pareto decay having demand is a function of on hand inventory. The generalized Pareto distribution is capable of characterizing the life time of the commodities which have a minimum period to start deterioration, and the rate of deterioration is inversely proportionate to time.

Using the differential equations, the instantaneous state of inventory is derived. With suitable cost considerations, the total cost function is derived. By minimizing the total cost function, the optimal ordering quantity, optimal replenishment down time and optimal replenishment uptime are derived. A numerical illustration is also discussed. The sensitivity of model with respect to parameters and costs is also discussed. This model is extended to the case of without shortages.

2. Notations and assumptions

2.1. Notations

The following assumptions are made for developing the model.

i) The demand rate is a function of production, which is

$$\lambda(t) = \emptyset_1 + \emptyset_2 I(t); \; 0 \le \emptyset_2 \le 1 \tag{1}$$

Where $\emptyset_1$, $\emptyset_2$ are positive constants, I(t) is the on hand inventory

ii) The replenishment is finite and follows a two parameter Weibull distribution with probability density function

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}; \; \alpha > 0, \beta > 0, t > 0$$

Therefore, the instantaneous rate of replenishment is

$$k(t) = \frac{f(t)}{1-F(t)} = \alpha\beta t^{\beta-1}; \; \alpha > 0, \beta > 0, t > 0 \tag{2}$$

iii) Lead time is zero

iv) Cycle length, T is known and fixed

v) Shortages are allowed and fully backlogged

vi) A deteriorated unit is lost

vii) The deterioration of the item is random and follows a generalized Pareto distribution. Then the instantaneous rate of deterioration is

$$h(t) = \frac{1}{\alpha - \gamma t} \; ; \; 0 < t < \frac{\alpha}{\gamma} \tag{3}$$

2.2. Assumptions

The following notations are used for developing the model.

Q: Ordering quantity in one cycle

A: Ordering cost

C: Cost per unit

h: Inventory holding cost per unit per unit time

π: Shortages cost per unit per unit time

s: Selling price per unit

3. Inventory model with shortages

Consider an inventory system in which the stock level is zero at time t=0. The Stock level increases during the period (0, t_1), due to excess of replenishment after fulfilling the demand and deterioration. The replenishment stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2, the inventory reaches zero and back orders accumulate during the period (t_2, t_3). At time t_3, the replenishment starts again and fulfils the backlog after satisfying the demand. During (t_3, T), the back orders are fulfilled and inventory level reaches zero at the end of the cycle T. The Schematic diagram representing the instantaneous state of inventory is given in Figure 1.

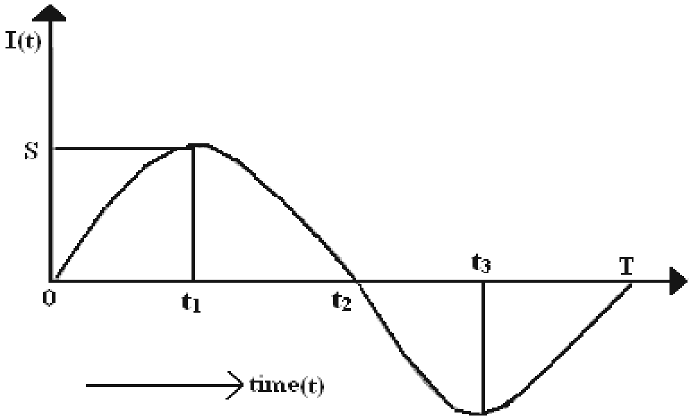


Figure 1. Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{d}{dt}I(t) + h(t)I(t) = \alpha\beta t^{\beta-1} - (\emptyset_1 + \emptyset_2 I(t)) \; ; \quad 0 \le t \le t_1 \tag{4}$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -(\emptyset_1 + \emptyset_2 I(t)) \; ; \qquad t_1 \le t \le t_2 \tag{5}$$

$$\frac{d}{dt}I(t) = -(\emptyset_1 + \emptyset_2 I(t)) \; ; \qquad t_2 \le t \le t_3 \tag{6}$$

$$\frac{d}{dt}I(t) = \alpha\beta t^{\beta-1} - (\emptyset_1 + \emptyset_2 I(t)); \qquad t_3 \le t \le T \tag{7}$$

The solution of differential equations (4) – (7) using the initial conditions, I(0) = 0, I(t_1) = S, I(t_2) = 0 and I(T) = 0, the on hand inventory at time 't' is obtained as

$$I(t) = S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2(t_1 - t)} - (a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \int_t^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)\,(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du;$$

$$0 \le t \le t_1 \tag{8}$$

$$I(t) = S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2(t_1-t)} - \emptyset_1 (a-\gamma t)^{1/\gamma} e^{-\emptyset_2 t} \int_{t_1}^{t} (a-\gamma u)^{-1/\gamma} e^{\emptyset_2 u}\, du;$$

$$t_1 \le t \le t_2 \tag{9}$$

$$I(t) = -\frac{\emptyset_1}{\emptyset_2}\left(1 - e^{\emptyset_2(t_2-t)}\right);$$

$$t_2 \le t \le t_3 \tag{10}$$

$$I(t) = -e^{-\emptyset_2 t}\alpha\beta \int_{t}^{T} u^{\beta-1} e^{\emptyset_2 u} du + \frac{\emptyset_1}{\emptyset_2}\left(e^{\emptyset_2(T-t)} - 1\right); \qquad t_3 \le t \le T \tag{11}$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t k(t)dt - \int_0^t \lambda(t)dt - I(t)\,, 0 \le t \le t_2$$

This implies

$$L(t) = \begin{cases} \alpha t^{\beta} - \left(\emptyset_1 t + \emptyset_2 \int_0^t I(t)dt\right) - S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2(t_1-t)} \\ \quad + (a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \int_{t}^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du; \; 0 \le t \le t_1 \\ \alpha t_1^{\beta} - \left(\emptyset_1 t + \emptyset_2 \int_0^t I(t)dt\right) - S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2(t_1-t)} \\ \quad + \emptyset_1 (a-\gamma t)^{1/\gamma} \int_{t_1}^{t} (a-\gamma u)^{-1/\gamma} e^{\emptyset_2 u}\, du; \qquad t_1 \le t \le t_2 \end{cases}$$

Stock loss due to deterioration in the cycle of length T is

$$L(T) = \alpha t_1^{\beta} - \left(\emptyset_1 t + \emptyset_2 \int_0^t I(t)dt\right)$$

Ordering quantity Q in the cycle of length T is

$$Q = \int_0^{t_1} k(t)dt + \int_{t_3}^{T} k(t)dt = \alpha\left(t_1^{\beta} + T^{\beta} - t_3^{\beta}\right) \tag{12}$$

From equation (8) and using the condition I (0) = 0, we obtain the value of ‘S’ as

$$S = (a - \gamma t_1)^{1/\gamma} e^{-\emptyset_2 t_1} \int_0^{t_1} (\alpha\beta u^{\beta-1} - \emptyset_1)(a - \gamma u)^{-1/\gamma} e^{\emptyset_2 u} du \tag{13}$$

When t = t_3, then equations (10) and (11) become

$$I(t_3) = -\frac{\emptyset_1}{\emptyset_2}\left(1 - e^{\emptyset_2(t_2 - t_3)}\right)$$

$$and\ I(t_3) = -e^{-\emptyset_2 t_3} \alpha\beta \int_{t_3}^{T} u^{\beta-1} e^{\emptyset_2 u} du + \frac{\emptyset_1}{\emptyset_2}\left(e^{-\emptyset_2 t_3} - 1\right)$$

Equating the equations and on simplification, one can get

$$t_2 = \frac{1}{\emptyset_2}\left[log\left[e^{\emptyset_2 T} - \frac{\emptyset_2}{\emptyset_1}\int_{t_3}^{T} \alpha\beta u^{\beta-1} e^{\emptyset_2 u}\right]\right] \tag{14}$$

Let $K(t_1, t_2, t_3)$ be the total cost per unit time. Since the total cost is the sum of the set up cost, cost of the units, the inventory holding cost and shortage cost, the total cost per unit time becomes

$$K(t_1, t_2, t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right) + \frac{\pi}{T}\left(\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt\right) \tag{15}$$

Substituting the values of I(t) and Q in equation (12), one can obtain $K(t_1, t_2, t_3)$ as

$$\begin{aligned} K(t_1, t_2, t_3) = {} & \frac{A}{T} + \frac{C}{T}\alpha\left(t_1{}^{\beta} + T^{\beta} - t_3{}^{\beta}\right) \\ & + \frac{h}{T}\left\{\int_0^{t_1}\left[S\left(\frac{a - \gamma t}{a - \gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2(t_1 - t)} - (a - \gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t}\right.\right. \\ & \left.\times \int_t^{t_1} (\alpha\beta u^{\beta-1} - \emptyset_1)(a - \gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du\right] dt \\ & + \int_{t_1}^{t_2}\left[S\left(\frac{a - \gamma t}{a - \gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2(t_1 - t)} - \emptyset_1 (a - \gamma t)^{1/\gamma} e^{-\emptyset_2 t}\right. \end{aligned}$$

$$\times \int_{t_1}^{t} (a-\gamma u)^{-1/\gamma} e^{\emptyset_2 u}\, du \Bigg] dt \Bigg\} + \frac{\pi}{T}\Bigg\{ \int_{t_2}^{t_3} \left[\frac{\emptyset_1}{\emptyset_2}\left(1-e^{\emptyset_2(t_2-t)}\right)\right] dt + \int_{t_3}^{T} \left[e^{-\emptyset_2 t}\alpha\beta \int_{t}^{T} u^{\beta-1} e^{\emptyset_2 u}\, du - \frac{\emptyset_1}{\emptyset_2}\left(e^{\emptyset_2(T-t)}-1\right)\right] dt \Bigg\} \tag{16}$$

Substituting the values of 'S' and 't_2' from equations (13) and (14) in the total cost equation (16), we obtain

$$K(t_1,t_3) = \frac{A}{T} + \frac{C}{T}\alpha\left(t_1{}^{\beta} + T^{\beta} - t_3{}^{\beta}\right)$$
$$+\frac{h}{T}\Bigg\{ \int_0^{t_1} \left[(a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \left[\int_0^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du \right]\right] dt$$
$$- \int_0^{t_1} \left[(a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \left[\int_t^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du \right]\right] dt$$
$$+ \int_{t_1}^{t_2} \left[(a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \left[\int_0^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du \right]\right] dt$$
$$-\emptyset_1 \int_{t_1}^{t_2} \left[(a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \left[\int_{t_1}^{t} (a-\gamma u)^{-1/\gamma} e^{\emptyset_2 u}\, du \right]\right] dt \Bigg\}$$
$$+\frac{\pi}{T}\Bigg\{ \frac{\emptyset_1}{\emptyset_2}\left[T - \frac{1}{\emptyset_2}\left[\log\left[e^{\emptyset_2 T} - \frac{\emptyset_2}{\emptyset_1}\alpha\beta \int_{t_3}^{T} u^{\beta-1} e^{\emptyset_2 u}\, du \right]\right]\right]$$
$$- \frac{e^{-\emptyset_2 t_3}}{\emptyset_2} \int_{t_3}^{T} \alpha\beta u^{\beta-1} e^{\emptyset_2 u}\, du + \int_{t_3}^{T} \left[e^{-\emptyset_2 t} \left[\alpha\beta \int_t^{T} u^{\beta-1} e^{\emptyset_2 u}\, du \right]\right] dt \Bigg\} \tag{17}$$

4. Optimal pricing and ordering policies of the model

In this section we obtain the optimal policies of the inventory system under study. To find the optimal values of t1 and t3, we obtain the first order partial derivatives of $K(t_1,t_3)$ given in equation (17) with respect to t1 and t3 and equate them to zero. The condition for minimization of $K(t_1,t_3)$ is

$$D = \begin{vmatrix} \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1^2} & \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} \\ \dfrac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} & \dfrac{\partial^2 K(t_1,t_3)}{\partial {t_3}^2} \end{vmatrix} > 0$$

Differentiating $K(t_1,t_3)$ given in equation (17) with respect to t_1 and equating to zero, one can obtain

$$C\alpha\beta t_1^{\beta-1} + h\alpha\beta t_1^{\beta-1}(a-\gamma t_1)^{-\frac{1}{\gamma}} e^{\emptyset_2 t_1} \int_{t_1}^{t_2} (a-\gamma t)^{\frac{1}{\gamma}}\, e^{-\emptyset_2 t} dt = 0 \tag{18}$$

Differentiating $K(t_1,t_3)$ given in equation (17) with respect to t_3 and equating to zero, one can obtain

$$-C\alpha\beta t_3^{\beta-1} + \pi\left[\alpha\beta t_3^{\beta-1} - \frac{\alpha\beta t_3^{\beta-1} e^{\emptyset_2 t_3}}{\left(e^{\emptyset_2 T} - \frac{\emptyset_2}{\emptyset_1}\int_{t_3}^{T} \alpha\beta u^{\beta-1}\, e^{\emptyset_2 u}\, du\right)}\right] = 0 \tag{19}$$

Solving equations (18) and (19) simultaneously, we obtain the optimal time at which replenishment is to be stopped t_1^* of t_1 and the optimal time t_3^* of t_3 at which the replenishment should be restarted after accumulation of backorders is obtained.
The optimum ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1^*, t_3^* in equation (12).

5. Numerical illustration

To expound the model developed, consider the case of deriving an optimal ordering quantity, replenishment down time, replenishment uptime and total cost for an edible oil manufacturing unit. Here, the product is deteriorating type and has random life time and assumed to follow a generalized Pareto distribution. Based on the discussions held with the personnel connected with the production and marketing of the plant and the records, the values of different parameters are considered as A = Rs.300/- C = Rs.10/- h =Re. 0.2/- π =Re. 0.3/-, T = 12 months. For the assigned values of replenishment parameters (α, β) = (12, 0.5), deterioration parameters (a, γ) = (10, 0.04) and production parameters (φ_1, φ_2) = (4, 0.4). The values of parameters above are varied further to observe the trend in optimal policies, and the results obtained are shown in Table 1. Substituting these values, the optimal ordering quantity Q^*, replenishment uptime, replenishment down time and total cost are computed and presented in Table 1.
From Table 1 it is observed that the deterioration parameters and replenishment parameters have a

tremendous influence on the optimal replenishment times, ordering quantity and total cost.

When the ordering cost 'A' increases from 300 to 360, the optimal ordering quantity Q^* decreases from 15.18 to 14.67, the optimal replenishment down time t_1^* increases from 1.22to 1.30, the optimal replenishment uptime t_3^* increases from 10.92 to 11.44, the total cost per unit time K^*, increases from 36.74 to 41.39. As the cost parameter 'C' increases from 10 to 11.5, the optimal ordering quantity Q^* increases from 15.18 to 16.61, the optimal replenishment down time t_1^* increases from 1.22 to 1.65, the optimal replenishment uptime t_3^* increases from 10.92 to 11.39, the total cost per unit time K^*, increases from 36.74 to 40.24.

As the holding cost 'h' increases from 0.2 to 0.23, the optimal ordering quantity Q^* decreases from 15.18 to 13.27, the optimal replenishment down time t_1^* decreases from 1.22 to 0.69, the optimal replenishment uptime t_3^* decreases from 10.92 to 10.20, the total cost per unit time K^*, decreases from 36.74 to 34.41. As the shortage cost 'π' increases from 0.3 to 0.36, the optimal ordering quantity Q^* decreases from 15.18 to 15.17, the optimal replenishment down time t_1^* increases from 1.22 to 1.23, the optimal replenishment uptime t_3^* increases from 10.92 to 10.93. The total cost per unit time K^*, decreases from 36.74 to 36.73.

As the replenishment parameter 'α' varies from 12 to 13.8, the optimal ordering quantity Q^* increases from 15.18 to 17.33, the optimal replenishment down time t_1^* increases from 1.22 to 1.29, the optimal replenishment uptime t_3^* increases from 10.92 to 11.19, the total cost per unit time K^*, increases from 36.74 to 38.62. Another replenishment parameter 'β' varies from 0.5 to 0.53, the optimal ordering quantity Q^* increases from 15.18 to 16.31, the optimal replenishment down time t_1^* increases from 1.23 to 1.75, the optimal replenishment uptime t_3^* decreases from 10.92 to 11.32, the total cost per unit time K^*, increases from 36.74 to 39.08.

As the deteriorating parameter 'γ' varies from 0.04 to 0.046, the optimal ordering quantity Q^* increases from 15.18 to 15.18, the optimal replenishment down time t_1^* increases from 1.22 to 1.23, the optimal replenishment uptime t_3^* increases from 10.92 to 10.93, the total cost per unit time K^*, increases from 36.74 to 36.75. Another deteriorating parameter 'a' varies from 10 to 11.5, the optimal ordering quantity Q^* increases from 15.18 to 16.55, the optimal replenishment down time t_1^* increases from 1.22 to 1.69, the optimal replenishment uptime t_3^* increases from 10.92 to 11.46, the total cost per unit time K^*, increases from 36.74 to 38.25.

As the production parameter 'φ_1' increases from 4 to 4.6, the optimal ordering quantity Q^* decreases from 15.18 to 13.33, the optimal replenishment down time t_1^* decreases from 1.22 to 0.69, the optimal replenishment uptime t_3^* decreases from 10.92 to 10.17, the total cost per unit time K^*, decreases from 36.74 to 34.45. Another production parameter'φ_2' increases from 0.4 to 0.48, the optimal ordering quantity Q^* increases from 15.18 to 16.83, the optimal replenishment down time t_1^* increases from 1.22 to 1.73, the optimal replenishment uptime t_3^* increases from 10.92 to 11.42, the total cost per unit time K^*, increases from 36.74 to 38.42.

Table 1. Optimal values of t_1^*, t_3^*, Q^* and K^* for different values of parameters

A	C	h	π	α	β	Γ	a	φ_1	φ_2	t_1	t_3	Q	K
300	10	0.2	0.3	12	0.5	0.04	10	4	0.4	1.22	10.92	15.18	36.74
320										1.25	11.08	15.05	38.33
340										1.28	11.26	14.88	39.89
										1.30	11.44	14.67	41.39
	10.5									1.47	11.07	15.83	38.47
	11.0									1.56	11.23	16.33	39.33
	11.5									1.65	11.39	16.61	40.24
		0.21								1.02	10.66	14.54	35.97
		0.22								0.86	10.42	13.93	35.22
		0.23								0.69	10.20	13.27	34.41
			0.32							1.22	10.92	15.18	36.74
			0.34							1.22	10.92	15.17	36.73
			0.36							1.23	10.93	15.17	36.73
				12.6						1.23	11.00	15.86	37.33
				13.2						1.26	11.09	16.57	37.96
				13.8						1.24	11.19	17.33	38.62
					0.51					1.61	11.31	15.57	38.20
					0.52					1.74	11.38	15.80	38.81
					0.53					1.75	11.32	16.31	39.08
						0.042				1.22	10.92	15.18	36.74
						0.044				1.22	10.92	15.18	36.75
						0.046				1.23	10.93	15.18	36.75
							10.5			1.38	11.11	15.66	37.29
							11.0			1.54	11.29	16.12	37.79
							11.5			1.69	11.46	16.55	38.25
								4.2		1.07	10.64	14.50	35.93
								4.4		0.84	10.39	13.87	35.14
								4.6		0.69	10.17	13.33	34.45
									0.44	1.26	10.89	15.43	36.92
									0.46	1.74	11.42	16.83	38.44
									0.48	1.74	11.42	16.83	38.42

6. Sensitivity analysis of the model

To study the effects of changes in the parameters on the optimal values of replenishment down time, replenishment uptime, optimal ordering quantity and total cost, sensitivity analysis is performed taking the values of the parameters as A = Rs.300/- C = Rs.10/- h =Re. 0.2/-, π =Re. 0.3/-, T = 12 months. For the assigned values of replenishment parameters (α, β) = (12, 0.5), deterioration parameters (a, γ) = (10, 0.04) and production parameters (φ_1, φ_2) = (4, 0.4). Sensitivity analysis is performed by changing the parameter values by -15%, - 10%, -5%, 0%, 5%, 10% and 15%. First changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values of replenishment down time, replenishment uptime, optimal ordering quantity and total cost are computed. The results are presented in Table2. The relationships between parameters, costs and the optimal values are shown in Fig.2.

From Table 2, it is observed that the deteriorating parameters (a, γ) have less effect on replenishment down time $\mathbf{t_1}^*$, replenishment up time $\mathbf{t_3}^*$ and significant effect on optimal ordering quantity and total cost. Decrease in unit cost C results decrease in replenishment down time $\mathbf{t_1}^*$, replenishment up time $\mathbf{t_3}^*$, increase in optimal ordering quantity Q* and total cost K*. The increase in production rate parameters (φ_1, φ_2) has less effect on replenishment down time $\mathbf{t_1}^*$, replenishment up time $\mathbf{t_3}^*$, moderate effect on optimal ordering quantity Q* and total cost K* , respectively. Increase in holding cost h results significant variation in optimal ordering quantity Q* and decrease in total cost K*. The increase in shortage cost results less effect on optimal ordering quantity Q* and total cost K*.

Table 2. Sensitivity analysis of the model – with shortages

Variation Parameters	**Optimal policies**	**Change in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	$\mathbf{t_1^*}$	1.15	1.175	1.19	1.22	1.24	1.27	1.28
	$\mathbf{t_3^*}$	10.58	10.68	10.80	10.92	11.04	11.17	11.30
	$\mathbf{Q^*}$	15.41	15.34	15.26	15.18	15.08	14.97	14.84
	$\mathbf{K^*}$	33.12	34.33	35.54	36.74	37.94	39.11	40.27
C	$\mathbf{t_1^*}$	0.93	1.02	1.12	1.22	1.47	1.55	1.65
	$\mathbf{t_3^*}$	10.49	10.63	10.77	10.92	11.07	11.23	11.39
	$\mathbf{Q^*}$	14.31	14.60	14.89	15.18	15.83	16.33	16.61
	$\mathbf{K^*}$	33.96	34.86	35.79	36.74	37.97	39.33	40.24
h	$\mathbf{t_1^*}$	1.71	1.67	1.45	1.22	1.02	0.85	0.69
	$\mathbf{t_3^*}$	11.57	11.47	11.20	10.92	10.66	10.42	10.20
	$\mathbf{Q^*}$	16.91	16.59	15.89	15.18	14.54	13.93	13.27
	$\mathbf{K^*}$	38.38	38.34	37.56	36.74	35.97	35.22	34.41
$\boldsymbol{\pi}$	$\mathbf{t_1^*}$	1.22	1.22	1.22	1.22	1.22	1.22	1.22
	$\mathbf{t_3^*}$	10.91	10.91	10.91	10.92	10.92	10.92	10.93
	$\mathbf{Q^*}$	15.19	15.19	15.18	15.18	15.1	15.17	15.17
	$\mathbf{K^*}$	36.75	36.75	36.75	36.74	36.74	36.73	36.73
$\boldsymbol{\alpha}$	$\mathbf{t_1^*}$	1.26	1.24	1.22	1.22	1.23	1.26	1.29
	$\mathbf{t_3^*}$	10.71	10.77	10.84	10.92	11.00	11.09	11.19
	$\mathbf{Q^*}$	13.43	14.00	14.54	15.18	15.86	16.57	17.33
	$\mathbf{K^*}$	35.30	35.75	36.20	36.74	37.33	37.96	38.62
$\boldsymbol{\beta}$	$\mathbf{t_1^*}$	1.01	1.12	1.17	1.22	1.41	1.64	1.75
	$\mathbf{t_3^*}$	10.14	10.39	10.62	10.92	11.20	11.38	11.49
	$\mathbf{Q^*}$	15.07	15.10	15.13	15.18	15.87	16.82	17.51
	$\mathbf{K^*}$	34.68	35.35	35.99	36.74	37.50	38.41	39.08

Table 2. continued

γ	**t_1^***	1.22	1.22	1.22	1.22	1.22	1.22	1.22
	t_3^*	10.92	10.92	10.92	10.92	10.92	10.92	10.92
	Q^*	15.18	15.18	15.18	15.18	15.18	15.18	15.18
	K^*	36.74	36.74	36.74	36.74	36.74	36.75	36.75
a	**t_1^***	0.74	0.90	1.06	1.22	1.38	1.53	1.69
	t_3^*	10.24	10.48	10.71	10.92	11.11	11.29	11.46
	Q^*	13.51	14.11	14.66	15.18	15.66	16.12	16.55
	K^*	34.82	35.51	36.15	36.74	37.29	37.79	38.25
φ_1	**t_1^***	1.79	1.61	1.47	1.22	1.01	0.84	0.69
	t_3^*	11.57	11.48	11.22	10.92	10.64	10.39	10.17
	Q^*	16.98	16.60	15.93	15.18	14.50	13.87	13.33
	K^*	38.92	38.37	37.61	36.74	35.93	35.14	34.45
φ_2	**t_1^***	1.71	1.61	1.41	1.22	1.16	1.10	0.95
	t_3^*	11.61	11.46	11.17	10.92	10.68	10.42	10.32
	Q^*	14.37	14.64	14.21	15.18	15.63	16.13	16.83
	K^*	38.10	36.67	37.33	36.74	36.62	36.21	35.82

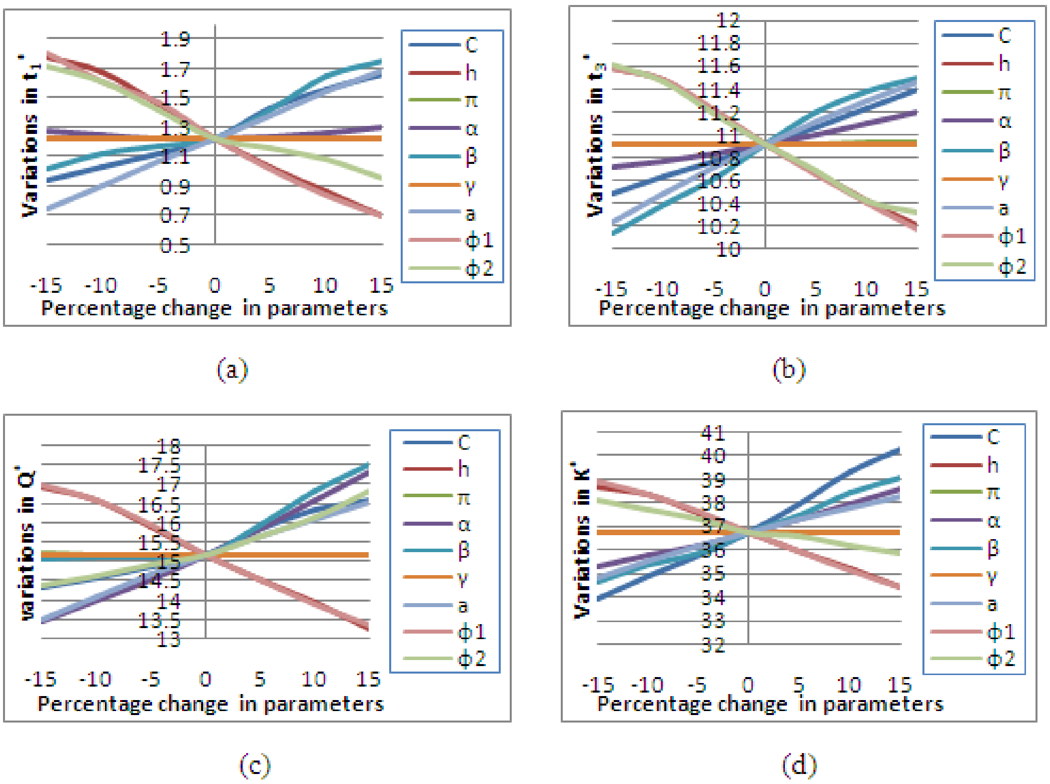


Figure 2. Relationship between optimal values and parameters with shortages

7. Inventory model without shortages

In this section, the inventory model for deteriorating items without shortages is developed and analyzed. Here, it is assumed that shortages are not allowed and the stock level is zero at time t = 0. The stock level increases during the period (0, t_1), due to excess replenishment after fulfilling the demand and deterioration. The replenishment stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, T). At time T, the inventory reaches zero. The Schematic diagram representing the instantaneous state of inventory is given in Figure 3.

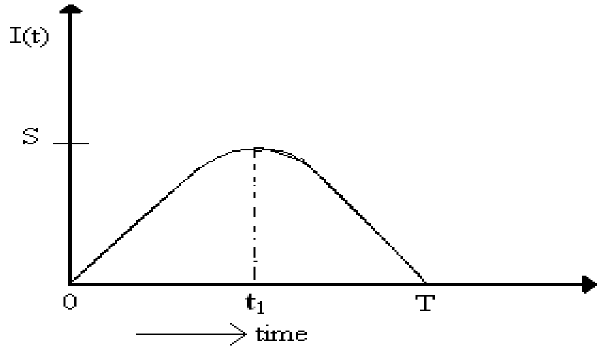


Figure 3. Schematic diagram representing the inventory level.

The differential equations governing the system in the cycle time [0, T] are:

$$\frac{d}{dt}I(t)+h(t)I(t)= \alpha\beta t^{\beta-1}-(\emptyset_1+\emptyset_2 I(t)); \qquad 0\le t\le t_1 \tag{20}$$

$$\frac{d}{dt}I(t)+h(t)I(t)= -(\emptyset_1+\emptyset_2 I(t)); \qquad t_1\le t\le t_2 \tag{21}$$

where, h(t) is as given in equation (3),with the initial conditions, I(0) = 0, I(t_1) = S, and I(T) = 0. Substituting h (t) given in equation (3) in equation (20) and (21) and solving the differential equations, the on hand inventory at time‘t’ is obtained as

$$I(t)=S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}}e^{\emptyset_2(t_1-t)}-(a-\gamma t)^{\frac{1}{\gamma}}e^{-\emptyset_2 t}\int_t^{t_1}\left(\alpha\beta u^{\beta-1}-\emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}}e^{\emptyset_2 u}du; \qquad 0\le t\le t_1 \tag{22}$$

$$I(t)= S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}}e^{\emptyset_2(t_1-t)}-\emptyset_1(a-\gamma t)^{1/\gamma}e^{-\emptyset_2 t}\int_{t_1}^{t}(a-\gamma u)^{-1/\gamma}e^{\emptyset_2 u}\,du; \qquad t_1\le t\le T \tag{23}$$

Stock loss due to deterioration in the interval (0, t) is

$$L(t) = \int_0^t k(t)dt - \int_0^t \lambda(t)dt - I(t)\ , 0 \le t \le T$$

This implies

$$L(t) = \begin{cases} \alpha t^{\beta} - \left(\emptyset_1 t + \emptyset_2 \int_0^t I(t)dt \right) - S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2 (t_1 - t)} \\ + (a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \int_t^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du;\ 0 \le t \le t_1 \\ \alpha t_1{}^{\beta} - \left(\emptyset_1 t + \emptyset_2 \int_0^t I(t)dt \right) - S\left(\frac{a-\gamma t}{a-\gamma t_1}\right)^{\frac{1}{\gamma}} e^{\emptyset_2 (t_1 - t)} \\ + \emptyset_1 (a-\gamma t)^{1/\gamma} \int_{t_1}^{t} (a-\gamma u)^{-1/\gamma} e^{\emptyset_2 u}\, du; \qquad t_1 \le t \le T \end{cases}$$

Ordering quantity Q in the cycle of length T is

$$Q = \int_0^{t_1} k(t)dt = \alpha t_1{}^{\beta} \tag{24}$$

From equation (22) and using the condition I (0) = 0, we obtain the value of ‘S’ as

$$S = (a-\gamma t_1)^{1/\gamma} e^{-\emptyset_2 t_1} \int_0^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-1/\gamma} e^{\emptyset_2 u} du \tag{25}$$

Let $K(t_1)$ be the total cost per unit time. Since the total cost is the sum of the set up cost, cost of the units, the inventory holding cost. Therefore, the total cost is

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt \right) \tag{26}$$

Substituting the value of I (t), Q and S given in equation’s (22), (23), (24) and (25) in equation (26) and on simplification, we obtain $K(t_1)$ as

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\alpha t_1{}^{\beta}$$

$$+ \frac{h}{T}\left\{ \int_0^{t_1} \left[(a-\gamma t)^{\frac{1}{\gamma}} e^{-\emptyset_2 t} \left[\int_0^{t_1} \left(\alpha\beta u^{\beta-1} - \emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}} e^{\emptyset_2 u} du \right] \right] dt \right.$$

$$-\int_0^{t_1}\left[(a-\gamma t)^{\frac{1}{\gamma}}e^{-\emptyset_2 t}\left[\int_t^{t_1}\left(\alpha\beta u^{\beta-1}-\emptyset_1\right)\,(a-\gamma u)^{-\frac{1}{\gamma}}e^{\emptyset_2 u}du\right]\right]dt$$

$$+\int_{t_1}^{T}\left[(a-\gamma t)^{\frac{1}{\gamma}}e^{-\emptyset_2 t}\left[\int_0^{t_1}\left(\alpha\beta u^{\beta-1}-\emptyset_1\right)(a-\gamma u)^{-\frac{1}{\gamma}}e^{\emptyset_2 u}du\right]\right]dt$$

$$\left.-\emptyset_1\int_{t_1}^{T}\left[(a-\gamma t)^{\frac{1}{\gamma}}e^{-\emptyset_2 t}\left[\int_{t_1}^{t}(a-\gamma u)^{-1/\gamma}e^{\emptyset_2 u}\,du\right]\right]dt\right\} \tag{27}$$

8. Optimal pricing and ordering policies of the model

In this section, we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, we equate the first order partial derivatives of $K(t_1)$ with respect to t_1 equate them to zero. The condition for minimum of $K(t_1)$ is

$$\frac{d^2K(t_1)}{dt_1^2}>0$$

Differentiating $K(t_1)$ with respect to t_1 and equating to zero we get

$$C\alpha\beta t_1^{\beta-1}+h\alpha\beta t_1^{\beta-1}(a-\gamma t_1)^{-\frac{1}{\gamma}}e^{\emptyset_2 t_1}\int_{t_1}^{T}(a-\gamma t)^{\frac{1}{\gamma}}\,e^{-\emptyset_2 t}dt=0 \tag{28}$$

Solving the equation (28), we obtain the optimal time at which the replenishment is to be stopped at t_1^* of t_1.

The optimal ordering quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal value of t_1 in equation (24).

9. Numerical illustration

To expound the model developed, consider the case of deriving an optimal ordering quantity, replenishment time and total cost for an edible oil manufacturing unit. Here, the product is deteriorating type and has random life time and assumed to follow a generalized Pareto distribution. Based on the discussions held with the personnel connected with the production and marketing of the plant and the records, the values of different parameters are considered as A = Rs.1500/- C = Rs.10/- h =Re. 0.2/-, T = 12 months. For the assigned values of replenishment parameters (α, β) = (10, 0.5), deterioration parameters (a, γ) = (10, 0.04) and production parameters (φ_1, φ_2) = (100, 0.4). The values of above parameters are varied further to observe the trend in optimal policies and the results obtained are shown in Table 3. Substituting these values the optimal ordering quantity Q^*, replenishment uptime, replenishment down time and total cost are computed and presented in Table 3.

From Table 3 it is observed that the deterioration parameters and replenishment parameters have a tremendous influence on the optimal replenishment time, optimal ordering quantity and total cost.

Table 3. Optimal values of t_1^*, Q^* and K^* for different values of parameters

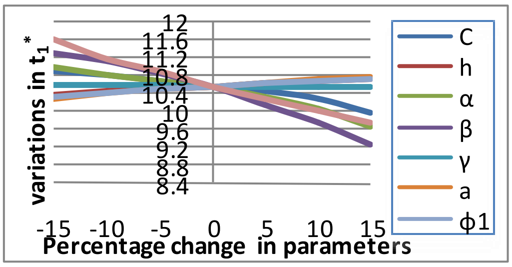
A	**C**	**h**	**Α**	**β**	**γ**	**a**	**φ1**	**φ2**	**t1**	**Q**	**K**
1500	**10**	**0.2**	**10**	**0.5**	**0.04**	**10**	**100**	**0.4**	10.57	32.52	131.26
1600									9.85	31.38	134.35
1700									8.98	29.98	138.35
1800									8.86	29.76	146.16
	10.5								10.44	32.31	131.51
	11.0								10.30	32.09	131.71
	11.5								9.98	31.78	131.93
		0.3							10.63	32.60	130.72
		0.4							10.68	32.68	130.19
		0.5							10.72	32.75	129.65
			10.5						10.34	33.93	131.57
			11.0						10.06	34.89	131.57
			11.5						9.67	35.01	132.07
				0.51					10.50	33.18	131.31
				0.52					10.43	33.84	131.38
				0.53					9.98	33.86	128.80
					0.042				10.57	32.51	131.26
					0.044				10.57	32.51	131.27
					0.046				10.57	32.51	131.27
						10.5			10.65	32.64	131.13
						11.0			10.72	32.75	131.00
						11.5			10.79	32.85	130.89
							105		10.63	32.61	130.71
							110		10.68	32.69	130.16
							115		10.73	32.76	129.62
								0.42	10.28	32.06	131.80
								0.44	10.00	31.63	132.34
								0.46	9.75	31.23	132.88

10. Sensitivity analysis of the model

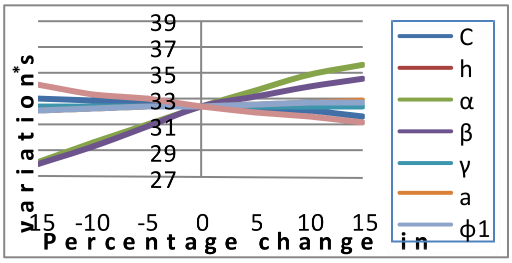
To study the effects of changes in the parameters on the optimal values of replenishment time, optimal ordering quantity and total cost, sensitivity analysis is performed taking the values of the parameters as A = Rs.1500/- C = Rs.10/- h =Re. 0.2/-, T = 12 months. For the assigned values of replenishment parameters (α, β) = (10, 0.5), deterioration parameters (a, γ) = (10, 0.04) and production parameters (φ_1, φ_2) = (100, 0.4). Sensitivity analysis is performed by changing the parameter values by -15%, - 10%, -5%, 0%, 5%, 10% and 15%. First, changing the value of one parameter at a time while keeping all the rest at fixed values and then changing the values of all the parameters simultaneously, the optimal values of replenishment time, optimal ordering quantity and total cost are computed. The results are presented in Table 4. The relationships between parameters, costs and the optimal values are shown in Fig.3. From Table 4, it is observed that variation in the deterioration parameters (a, γ) has considerable effect on replenishment time t_1^* , optimal ordering quantity Q* and total cost K*.Similarly , variation in deterioration parameters (a, γ) has slight effect on replenishment time t_1^*, optimal ordering quantity Q* and significant effect on total cost K*.The decrease in unit cost 'C' results in an increase in replenishment time t_1^* , optimal ordering quantity Q*and total cost K*. The increase in production rate parameters (φ_1, φ_2) result in variation in replenishment time t_1^*, slight increase in optimal ordering quantity Q* and total cost K*.The increase in holding cost h has significant effect on optimal values of replenishment time t_1^*, optimal ordering quantity Q* and total cost K*.

Table 4. Sensitivity analysis of the model – without shortages

Variation Parameters	**Optimal Policies**	**Change in parameters**						
		-15%	**-10%**	**-5%**	**0%**	**5%**	**10%**	**15%**
A	t_1^*	11.55	11.28	10.98	10.57	10.05	9.41	8.82
	Q*	33.98	33.58	33.15	32.52	31.70	30.68	29.70
	K*	123.99	126.54	128.96	131.26	133.55	136.13	139.78
C	t_1^*	10.89	10.80	10.69	10.57	10.44	10.30	9.98
	Q*	33.01	32.86	32.70	32.52	32.31	32.09	31.74
	K*	130.14	130.58	130.95	131.26	131.51	131.71	131.93
h	t_1^*	10.36	10.44	10.51	10.57	10.63	10.68	10.72
	Q*	32.19	32.31	32.42	32.52	32.60	32.68	32.75
	K*	132.90	132.34	131.80	131.26	130.72	130.19	129.65
α	t_1^*	10.98	10.84	10.70	10.57	10.34	10.06	9.67
	Q*	28.17	29.63	31.08	32.52	33.73	34.89	35.61
	K*	130.90	130.88	131.02	131.26	131.57	131.88	132.07
β	t_1^*	11.32	11.12	10.85	10.57	10.14	9.74	9.27
	Q*	28.01	29.32	30.89	32.52	33.22	33.98	34.60
	K*	131.53	131.35	131.27	131.26	130.85	129.90	128.74
γ	t_1^*	10.58	10.58	10.57	10.57	10.57	10.57	10.57
	Q*	32.53	32.52	32.52	32.52	32.51	32.51	32.51
	K*	131.25	131.25	131.26	131.26	131.26	131.27	131.27
a	t_1^*	10.30	10.40	10.49	10.57	10.65	10.72	10.79
	Q*	32.09	32.25	32.39	32.52	32.64	32.75	32.85
	K	131.76	131.57	131.41	131.26	131.13	131.00	130.89
φ_1	t_1^*	10.35	10.43	10.51	10.57	10.63	10.68	10.73
	Q*	32.17	32.30	32.42	32.52	32.61	32.69	32.76
	K*	132.92	132.36	131.81	131.26	130.71	130.16	129.62
φ_2	t_1^*	11.62	11.18	10.89	10.57	10.28	10.00	9.75
	Q*	34.08	33.43	33.01	32.52	32.06	31.63	31.23
	K*	129.58	130.16	130.72	131.26	131.80	132.34	132.88



(a)



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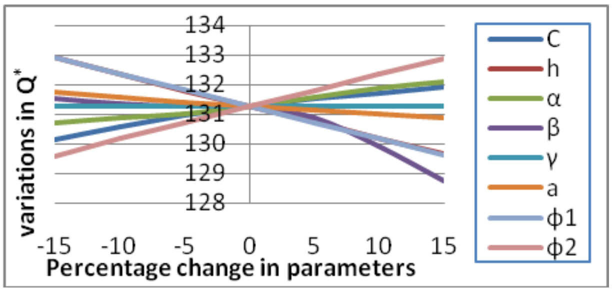

(c)

Figure 4. Relationship between optimal values and parameters without shortages

11. Conclusions

In this paper, production level inventory models for deteriorating items with Weibull replenishment and generalized Pareto decay with and without shortages are developed and analyzed. By minimizing the total cost function, the optimal values of the ordering quantity, replenishment down time, and replenishment uptimes are derived. The sensitivity model with respect to the parameters and costs revealed that the change in replenishment parameters and deteriorating parameters have significant influence on optimal production schedule. By suitably estimating the parameters and costs the production manager can optimally derive the production schedule and reduce waste and variation of resources. The model developed in this paper is much useful for scheduling the production time inventories, warehouses, market yards where the demand is a function of production level inventory and production is governed by several uncertainties. The operational manager of these systems can estimate the production and life time distribution parameters from the historical data. The demand parameters can be estimated from the market records. This model is having potential applications in manufacturing and production industries like edible oil mills, sugar factories, seafood industries, warehouses, etc, where the deterioration of the commodity is random and follows generalized Pareto distribution. This model also includes some of the earlier models as particular cases for specific values of the parameters.

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