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Vendor Managed Inventory of a Single-vendor Multiple-retailer Single-warehouse Supply Chain under Stochastic Demands

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Abstract

In this study, a vendor-managed inventory model is developed for a single-vendor multiple-retailer single-warehouse (SV-MR-SV) supply chain problem based on the economic order quantity in which demands are stochastic and follow a uniform probability distribution. In order to reduce holding costs and to help balanced on-hand inventory cost between the vendor and the retailers, it is assumed that all inventory is held at a central warehouse with the lowest cost among the parties. The capacity of the central warehouse is limited. The objective is to find the warehouse replenishment frequency, the vendor's replenishment frequency, the order points, and the order quantities of the retailers such that the total inventory cost of the integrated supply chain is minimized. The proposed model is a mixed integer nonlinear programming problem (MINLP); hence, a genetic algorithm (GA) is utilized to solve this NP-hard problem. The parameters of the GA are calibrated using the Taguchi method to find better solutions. Some numerical illustrations are solved at the end to demonstrate the applicability of the proposed methodology and to evaluate the performance of the solution method.

Keywords: Supply chain management; Vendor managed inventory; Probabilistic demand; Central warehouse; Genetic algorithm; Taguchi method.

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1. Introduction

Vendor managed inventory (VMI) is an industrial policy to integrate all parties of a supply chain to collaborate with each other. VMI is a well-known practice, in which the vendor manages inventory at the retailers and decides when and how much to replenish. Under a VMI policy, the vendor determines the time interval and the quantity of replenishments by accessing retailer's inventory and demand data (Darwish & Odah 2010). A VMI system that is designed well can reduce inventory levels and raise supply chain integration through reducing system costs (Achabal et al. 2000; Angulo et al. 2004; Cetinkaya & Lee 2000).

In the recent years, there has been an increasing interest in research on VMI to sustain its eminence performance practically as firms, suppliers, and vendors increasingly found out the interests of more adjacent collaboration and integration. In 1980s, Walmart and Procter and Gamble started their partnership under the VMI contractual agreements. Many retailers such as K-mart, Home Depot, and JC Penny then imitated it (Yao et al. 2007).

In a traditional supply chain, each member attempts to minimize its inventory cost. However, when they employ the VMI policy, they aim to show that partnership is a way to reach coordination that helps members to align their decisions and reach to the minimum total cost of the supply chain (Cachon & Fisher 2000). Another flow of research focuses on operational profits as unifying shipments (Cheung & Lee 2002) and adjusting recent delivery rate (Chaouch 2001). The concentration of the research performed in this paper is the second aspect, however under probabilistic demands.

Fry et al. (2001) showed how the VMI policy could be beneficial in coordination between production and delivery of a supply chain. Yao et al. (2007) investigated the advantages of using VMI in reducing a supply chain cost. Zhang et al. (2007) presented a single-vendor multi-retailer supply chain model under the VMI contract, in which the demand rate was assumed constant and the buyer`s ordering cycles were different. Liao et al. (2011) developed a multi-objective model for a location–inventory problem (MOLIP) under the VMI policy in a single-vendor multi-retailer supply chain and investigated the possibility of using a multi-objective version of the non-dominated sorting genetic algorithm (NSGA-II) to solve it. Coelho et al. (2012) examined the benefits of VMI in consistency requirements of a vehicle routing problem. They analyzed the effect of different inventory policies, routing decisions, and delivery sizes. Disney & Towill (2002) studied a supply chain under VMI, where vendor satisfies the retailer`s orders and controls retailer`s inventory by defining the order quantity and order time of the retailer. Yao & Dresner (2008) examined the benefits realized for manufacturers and retailers under VMI and compared the distribution of benefits between manufacturers and retailers. They showed that the distribution of benefits would depend on the replenishment frequency and the inventory holding cost parameters.

Yao et al. (2007) proposed an analytical model for a single vendor-single retailer supply chain based on EOQ and showed VMI would reduce the total cost. Dong & Xu (2002) modeled a retailer`s inventory system with deterministic demands using EOQ. Darwish & Odah (2010) presented a one vendor-multiple retailer supply chain model under VMI. In this model, a penalty cost on exceeded inventory the vendor sends to the retailer was assumed, where the retailers determined the upper bound. They solved the problem using a heuristic algorithm to reduce computational efforts. Pasandideh at el. (2011) extended Yao et al.'s (2007) model for several products, while the number of orders and the warehouse space were constrained. They solved the problem using a GA. Sadeghi et al. (2013) investigated a multi-vendor multi-retailer single-warehouse supply chain operating based on the VMI policy with considering certain demand at the retailers. In addition, to suite real-world inventory problems, Sadeghi et al. (2014) hybridized an inventory problem with a redundancy-allocation optimization problem.

Bichescu & Fry (2009) examined decentralized supply chains that follow the (Q, R) inventory policies under VMI agreements. Within the VMI scenario, they examined the effect of divisions of channel power on supply chain and individual operator performance by examining different game theoretic models. Song & Dinwoodi (2008) modeled a supply chain problem with uncertain replenishment lead times and demands. They used dynamic stochastic programming and heuristic methods in their study. Zhao & Cheng (2009) studied a two-level VMI system containing a distributer center and a retailer, both of which follow the order-up-to-level replenishment policy to maximize their overall system profit. They showed the benefits of VMI`s implementation at both strategic and operational levels.

In this paper, a single-vendor multiple-retailer single-warehouse model is developed wherein the vendor manages the inventory level of all the retailers. In this VMI, the demands at the retailers' level are uncertain, where the uncertainty is modeled by a uniform probability distribution. Moreover, in order to reduce holding costs and to help balanced on-hand inventory cost between the vendor and the retailers, it is assumed that all inventory is held at a central warehouse with the lowest cost among the parties. The capacity of the central warehouse is limited. We show that the proposed model is a mixed integer nonlinear programming problem (MINLP); an NP-hard problem for which exact methods is unable to solve. Hence, a genetic algorithm (GA) is employed to solve it. In addition, to find a better solution, the parameters of genetic algorithm are tuned using the Taguchi method.

The rest of the paper is organized as follows: In Section 2, the notations and the assumptions are stated. In Section 3, the mathematical formulation of the problem is described. The GA solution approach is given in Section 4. A numerical example is presented in Section 5 to demonstrate the applicability of the proposed methodology. In Section 6, the Taguchi method is applied to tune the parameters of the meta-heuristic. Finally, in Section 7, conclusions and some further research are presented.

2. The assumptions and notations

The assumptions involved in the modeling are:

1. A common replenishment cycle is assumed for all retailers. This is a reasonable assumption in which under the VMI policy the vendor makes decisions based on the inventory level, demand, and other supply chain data.
2. A single limited-capacity central warehouse is assumed.
3. All goods received by the retailers are sold to the customers. Hence, the annual demand of the retailers is equal to the one of the vendor ($D = \sum_{i=1}^{n} d_i$).

4. The vendor is responsible for the ordering cost of the retailers and determines their economic order quantities.
5. The demands are probabilistic and follow a uniform distribution.
6. The shortage is possible as backlogged.

The indices, parameters, and the decision variables are:

i : Index for retailers ($i = 1, 2, ..., n$)

n : Number of retailers

x_i : ith retailer`s demand during lead time; ($x_i \sim U(a_i, b_i)$)

$f(x_i)$: The probability density function of the demand during lead time

z_i : A binary parameter to model shortages at retailers

p_i : ith retailer`s shortage cost per unit inventory

D : Vendor`s expected demand rate

d_i : ith retailer`s expected demand rate

k_i : Ordering cost of retailer i

K : Ordering cost for the vendor

Y : Vendor's total order quantity

T : Retailers` planning period

y_i : ith retailer`s order quantity; $y_i = Td_i$ (a decision variable)

Y_W : Warehouse`s order quantity

m : Number of replenishments of a retailer by the vendor per unit time (a decision variable)

N : Replenishment frequency of vendor supplied by the warehouse per unit time (a decision variable)

I_V : Vendor's average inventory per unit time

R_i : Order point of retailer i (a decision variable)

H : The unit holding cost of the vendor per unit time

h_i : The unit holding cost of ith retailer per unit time

h_i : The unit holding cost of ith retailer per unit time

f : Space required storing one unit of the demand

F : The warehouse space

H_W : The unit holding cost of inventory in the warehouse per unit time

k_W : Ordering cost of the warehouse

THC_R : The expected total holding cost of retailers

TSC_R : The expected total shortage cost of retailers

TIC_R : The expected total inventory cost of the retailers

TIC_V : The expected total inventory cost of the vendor

TOC_V : The expected total ordering cost of the vendor

THC_W : The expected total holding cost of the warehouse

TIC_W : The expected total inventory cost of the warehouse

TIC_{VMI} : The expected total inventory cost under the VMI policy

As stated above, a vendor is assumed to supply several retailers in order to meet their customer`s demand. Also it is assumed that $D = \sum_{i=1}^{n} d_i$. In other words, as the vendor is responsible to determine the number of replenishments for the retailers under the VMI contract, he defines a unique T for all retailers, i.e. $\frac{y_i}{d_i} = \frac{y_1}{d_1}$. Moreover, in the three-echelon supply chain under investigation, a single central warehouse with limited capacity is assumed with a cost modeled in the next section.

3. The mathematical model

According to the VMI policy, the mathematical model should consist of three parts, one for the retailers, the other two for the vendor and the warehouse. Based on the common replenishment cycle, we have:

$$y_i/d_i = y_1/d_1 \Rightarrow T_i = T_1 \quad (1)$$

Assuming an equal annual demand for the vendor and all the retailers, the vendor`s order quantity is equal to the number of retailers' replenishments multiplied by their order quantities as

$$Y = m\sum_{i=1}^{n} y_1 d_i / d_1 \tag{2}$$

In other words,

$$DT = m\sum_{i=1}^{n} d_i T_i \tag{3}$$

The overall cost of the VMI system consists of the vendor, the warehouse, and retailers' inventory costs as

$$TIC_{VMI} = TIC_R + TIC_V + TIC_W \tag{4}$$

In what follows, TIC_R, TIC_V, and TIC_W are derived.

3.1. Retailers' inventory cost

In an integrated inventory system, the retailers' total inventory cost includes holding and shortage costs. The total holding costs of the retailers is:

$$THC_R = \sum_{i=1}^{n} h_i \left(\frac{d_i T}{2} + R_i - E\left(x_i\right) \right) \tag{5}$$

Where, based on the uniform distribution, the expected number of items a retailer holds in its storage is

$$E\left(x_i\right) = \frac{a_i + b_i}{2} \tag{6}$$

This cost is merged into the vendor's holding cost and it will be a part of vendor`s cost. Moreover, the total shortage cost of the retailers is:

$$TSC_R = \sum_{i=1}^{n} \left(\frac{z_i p_i}{T} \int_{R_i}^{b_i} (x_i - R_i) f(x_i) dx_i \right) \tag{7}$$

Where z_i is a binary variable used to ensure Eq. (7) is ignored if the maximum demand during lead time for *i*th retailer is smaller than his reorder point. Hence, using Eq.s 5-7, the total annual inventory cost of the retailers is obtained using the following equation.

$$TIC_{\text{Retailers}} = \left(\sum_{i=1}^{n} h_i \left(\frac{Td_i}{2} + R_i - E\left(x_i\right) \right) \right) + \left(\sum_{i=1}^{n} \left(\frac{z_i p_i}{T} \int_{R_i}^{b_i} (x_i - R_i) f(x_i) dx_i \right) \right) \tag{8}$$

3.2. Vendor’s inventory cost

As the vendor`s ordering cost includes the retailers ordering cost and that the annual number of replenishments is D/Y, the vendor's ordering cost is obtained by

$$TOC_V = \left(K + m\sum_{i=1}^{n}(k_i)\right)(D/Y) \tag{9}$$

Moreover, according to Darwish et al. (2010) the average annual inventory of the vendor in this case is:

$$I_V = \left((m+1)/2\right)DT \tag{10}$$

In which, $DT = \sum_{i=1}^{n} d_i T$. Therefore, the annual holding cost of the vendor is:

$$HI_V = H\left((m+1)/2\right)DT \tag{11}$$

Hence, the total annual inventory cost of the vendor is formulated as follows.

$$TIC_V = \left(K + \sum_{i=1}^{n} mk_i\right)\frac{1}{mT} + H\left(\frac{m+1}{2}\right)T\sum_{i=1}^{n} d_i \tag{12}$$

3.3. Warehouse's inventory cost

The order quantity of the warehouse (Y_W) is equal to summation of the shipped quantities to the vendor;

$$Y_W = NY \Rightarrow Y_W = NDT \tag{13}$$

Moreover, the warehouse`s ordering cost is obtained using Eq. (14).

$$TOC_W = \frac{k_W}{NmT} \tag{14}$$

To model the holding cost, the average inventory of the warehouse is equal to $I_W = \left((N+1)/2\right)Y$ (similar to the vendor`s average inventory.) As a result, the holding cost of the warehouse is formulated as shown in Eq. (15).

$$THC_W = \frac{H_W(N+1)}{2} m\sum_{i=1}^{n} Td_i \tag{15}$$

Consequently, the mathematical formulation of the problem at hand becomes:

$$\begin{aligned} Min\ TIC_{VMI} &= \left(K + \sum_{i=1}^{n} mk_i\right)\frac{1}{mT} + H\left(\frac{m+1}{2}\right)T\sum_{i=1}^{n} d_i \\ &+\left(\sum_{i=1}^{n} h_i\left(\frac{Td_i}{2} + R_i - E(x_i)\right)\right) + \left(\sum_{i=1}^{n}\left(\frac{z_i p_i}{T}\int_{R_i}^{b_i}(x_i - R_i)f(x_i)dx_i\right)\right) \\ &+\frac{k_w}{NmT} + \frac{H_w(N+1)}{2} m\sum_{i=1}^{n} Td_i \end{aligned} \tag{16}$$

Subject to:

$$T^{*}=\sqrt{\frac{\left(\frac{K}{m}+\sum_{i=1}^{n}k_i\right)+\sum_{i=1}^{n}\frac{p_i z_i\left(b_i-R_i\right)^2}{2\left(b_i-a_i\right)}+\frac{k_W}{NM}}{\frac{H}{2}\left(m+1\right)\sum_{i=1}^{n}d_i+\sum_{i=1}^{n}\frac{h_i d_i}{2}+\frac{H_W m\left(N+1\right)\sum_{i=1}^{n}d_i}{2}}} \quad (17)$$

$$fNm\sum_{i=1}^{n}Td_i \le F \quad ;\forall i\in\{1,2,...,n\} \quad (18)$$

$$y_i = Td_i \quad (19)$$

$$R_i \ge b_i - z_i M \quad (20)$$

$$R_i \le b_i + \left(1-z_i\right)M \quad (21)$$

$$m>0\ \&\ \text{Integer} \ ; \ \forall i \in\{1,2,3,...,n\} \quad (22)$$

$$R_i \le y_i;\ \forall i\in\{1,2,3,...,n\} \quad (23)$$

The mixed integer non-linear programming model presented above is derived for a single vendor-multiple retailer single-warehouse inventory system under the VMI policy in which there is a limited-capacity central warehouse. As a MINLP model is hard (if not impossible) to solve using an exact method, a GA is utilized in the next section for a near optimum solution.

4. A solution algorithm

Exact methods due to their time consuming computational processes are unable to solve MINLP problems of large sizes. This makes one to have no choice, except using an evolutionary algorithm (EA). For instance, Nachiappan & Jawahar (2007) employed a GA to find a near-optimum solution of a single-vendor multiple buyers supply chain problem under the VMI policy. Sue-Ann et al. (2012) compared the performance of a particle swarm optimization (PSO) algorithm to the one of a hybrid GA and artificial immune system (GA–AIS). Sadeghi et al. (2013) proposed a hybrid PSO as well as a GA to solve a multi-retailer multi-vendor single warehouse VMI inventory problem.

As the good performance of GA has been proved in the literature to solve MINLPs, it is used in this study to find the solution of the problem at hand. To reach better solution, the parameters of the algorithm are calibrated using the Taguchi method.

4.1. Genetic algorithm

GA is a type of evolutionary computation that mimics the principles of natural genetics. Holland (1962) was the first who introduced GA. This evolutionary search algorithm is based on the principles of evolution and heredity. In what follows, the steps involved in the GA developed in this research are described.

4.1.1. Initial conditions

In this step, the GA parameters, i.e. the population size N_{pop}, the crossover probability P_c, the mutation probability P_m, the stopping criterion, the selection policy, the crossover operation, the mutation operation, and the number of iteration are set. Some of these parameters are tuned using the Taguchi method described in Section 5.

4.1.2. Chromosomes

In GA, a chromosome is a series of genes that are possible appropriate or inappropriate solution of the problem. In this paper, a chromosome is a vector consisting of $(n+4)$ positive integer elements (genes). The first gene represents the order quantity of the first retailer, y_1, the second gene expresses the rate of replenishment, m, the third is the warehouse shipment rate to the vendor, N, and the other genes indicate the order points of all retailers. For a problem with four retailers, the chromosome structure is shown in Fig. 1.

$$\begin{bmatrix} z_1 & y_1 & m & N & R_1 & R_2 & R_3 & R_4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 22 & 43 & 12 & 11 & 7 & 8 & 12 \end{bmatrix}$$

Figure 1. A typical chromosome

4.1.3. Initial population and evaluation

In optimization problems, the fitness value used to evaluate a chromosome is the value of the objective function. Chromosomes are generated randomly to create the initial population consisting of N_{pop} chromosomes. However, some of them may be not feasible, i.e. may not satisfy the constraints. In order to generate feasible chromosomes, the death penalty approach is taken in this paper. In this method, a big value is added to the objective function value of any infeasible chromosome. In this case, the constrained optimization problem becomes a non-constraint problem.

4.1.4. Crossover

In a crossover operation, a pair of chromosomes mates to form offspring. The pair is selected randomly from the generation with probability Pc. While there are many different types of crossover operators, a two-point crossover operator is used in this paper. An example of this operation is shown in Fig. 2 for a 4-retailers problem. Note that two steps are shown in this figure; (1) selecting two random points for the cut, and (2) displacing the string between the cut-off points of the two parents; leading to the creation of two offspring.

$$\begin{array}{ccc} \textit{Parents} & & \textit{Offspring} \\ [0\quad 3\quad 22\!\downarrow\quad 44\quad 55\!\downarrow\quad 12\quad 13\quad 16] & \Rightarrow & [0\quad 3\quad 22\quad 23\quad 67\quad 12\quad 13\quad 16] \\ [1\quad 23\quad 1\!\downarrow\quad 23\quad 67\!\downarrow\quad 14\quad 13\quad 16] & & [1\quad 23\quad 1\quad 44\quad 55\quad 14\quad 13\quad 16] \end{array}$$

Figure 2. An example of the crossover operation

4.1.5. Mutation

Mutation is the second operation in a GA to prospect new solutions. It operates on each of the chromosomes resulted from the crossover operation. In mutation, a gene is replaced with another gene randomly with probability P_m. Fig. 3 shows a representation of the mutation operator for a 4-retailer problem.

$$\begin{array}{ccc} \textit{Parents} & & \textit{Offspring} \\ [0\quad 3\quad 22\quad \underline{44}\quad 55\quad 12\quad 13\quad 16] & \Rightarrow & [0\quad 3\quad 22\quad 78\quad 55\quad 12\quad 13\quad 16] \\ [1\quad 23\quad 1\quad 23\quad 67\quad \underline{14}\quad 13\quad 16] & & [1\quad 23\quad 1\quad 23\quad 67\quad 37\quad 13\quad 16] \end{array}$$

Figure 3. An example of the mutation operation

4.1.6. Chromosome selection

In this step of the GA methodology, chromosomes are selected for the next generation. The selection performs with respect to the fitness value of the chromosomes. The roulette wheel selection method is used in this paper to select N_{pop} chromosomes with the best fitness values among the parents and offspring.

4.1.7. Stopping criterion

In the last step of GA, we test whether the method has found a solution that meets the user's expectations. In this paper, we stop when a convergence is observed in 150 iterations. This is an arbitrary condition defined in this research. Note that this parameters of GA is calibrated using the Taguchi method (Taguchi et al. 2005.)

5. Parameter tuning

The quality of the solution obtained by employing a meta-heuristic algorithm is significantly impressed by its parameters. Hence, their calibration will improve the quality of the solution obtained. The parameters to be calibrated act as controllable factors in design of experiments (DOE) (Montgomery 2005). As the required number of experiments in the Taguchi method is less than the one in response surface methodology (RSM), the first is utilized in this paper. This is an usual approach taken by many authors such as Naderi et al. (2009) and Rahmati et al. (2013) to tune the parameters of their meta-heuristic algorithms.

5.1. The Taguchi method

Taguchi (1993) introduced a family of fractional factorial matrices to reduce the number of experiments required to determine the optimal levels of the factors that significantly affect a response. He categorized the factors into two main classes: 1) controllable factors, and 2) noise factors. While omitting the noise factors is impossible, Taguchi attempted to minimize the effects of the noise factors and to determine the optimal levels of the significant controllable factors. Taguchi changes the repetitive data to the values which measure the variation of the results, defined as the ratio of the signal (or controllable factors) to noise (S/N). According to the type of the problem, there are three standard values for this ratio, (S/N), including:

1. Nominal is the best with the aim of reducing the amount of variability around a specific objective value. In this case the S/N is defined as

$$SN_T = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}(\bar{y}-y_i)^2\right) \quad (24)$$

2. Smaller is the better; it is used for experiments whose objective function is the minimization type. In this case the S/N is defined as

$$SN_S = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}y_i^2\right) \quad (25)$$

3. Larger is the better; it is used for experiments whose objective function is the maximization type. The S/N is defined here as

$$SN_L = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right) \quad (26)$$

In Eq.s (24)-(26), n denotes the number of iterations, y_i represents the obtained response in *i*th iteration, and $\bar{y}$ is the average response in all iterations. Note that as the objective function of the problem at hand is of a minimization type, the smaller is the better-type in Eq. (25) is used in this research.

5.2. Taguchi method implementation

The Taguchi implementation is taken place in 5 steps. First, the parameters that affect the response significantly are defined. Second, the levels of the parameters are determined via a trial and error process. Third, in this step, the smallest orthogonal array is chosen to minimize the experimentation time. Fourth, the obtained design is used to find a solution. Finally, the results are analyzed based on the (*S/N*) measure.

The GA parameters that affect the solution significantly are the population size (N_{pop}), the maximum number of iterations (*It*), the mutation probability (P_m), and the crossover probability (P_c). In Table 1, the three levels of these parameters that are obtained using a trial and error procedure are shown.

Table 1. GA parameters and their levels

Variable	*Level*		
It	1000	1250	1500
N_{pop}	150	175	200
P_c	0.8	0.9	0.9
P_m	0.25	0.28	0.3

With reference to the Taguchi standard arrays table, the L_9 orthogonal arrays, as the most suitable design, is used to tune the GA parameters. Table 2 contains the input data for a 5-retailers problem as an example. For this example, the experimental results of five replications along with their *(S/N)* ratio are shown in Tables 3 for GA. In this table, the values 1, 2, and 3 correspond to the three levels of each parameter. The graph of the convergence path using the fitness values is presented in Fig. 4.

Table 2. Input data

K	a_i	b_i	k_i	d_i	h_i	H	P_i	f	F	K_W	H_W	n
639	35	77	198	827	15	4	57	3	64000	4	5	5
	36	74	172	864	14		53					
	50	79	179	983	9		30					
	28	69	213	825	14		40					
	22	76	118	995	9		57					

Table 3. Experimental results to tune the GA parameters

It	N_{pop}	P_c	P_m	y_1	y_2	y_3	y_4	y_5	*Mean*	*S/N*
1	1	1	1	266.5811	263.575	268.7814	271.9583	307.0514	275.5894	-888198
1	2	2	2	261.7221	262.0681	263.5078	262.421	272.4388	264.4316	-884473
1	3	3	3	261.1465	261.983	262.3587	301.9879	261.9876	269.8927	-886392
2	1	2	3	261.8148	272.0891	286.13	301.0232	273.3002	278.8715	-889182
2	2	3	1	267.8594	265.2474	301.5537	272.0477	264.793	274.3002	-887756
2	3	1	2	271.8853	262.1761	261.5101	261.3584	286.9312	268.7722	-885936
3	1	3	2	286.5286	273.949	286.2794	265.1224	288.676	280.1111	-889512
3	2	1	3	262.0594	315.1842	261.8402	261.7201	263.3491	272.8306	-88744
3	3	2	1	272.2148	261.4263	288.5894	262.4702	273.7274	271.6856	-88687

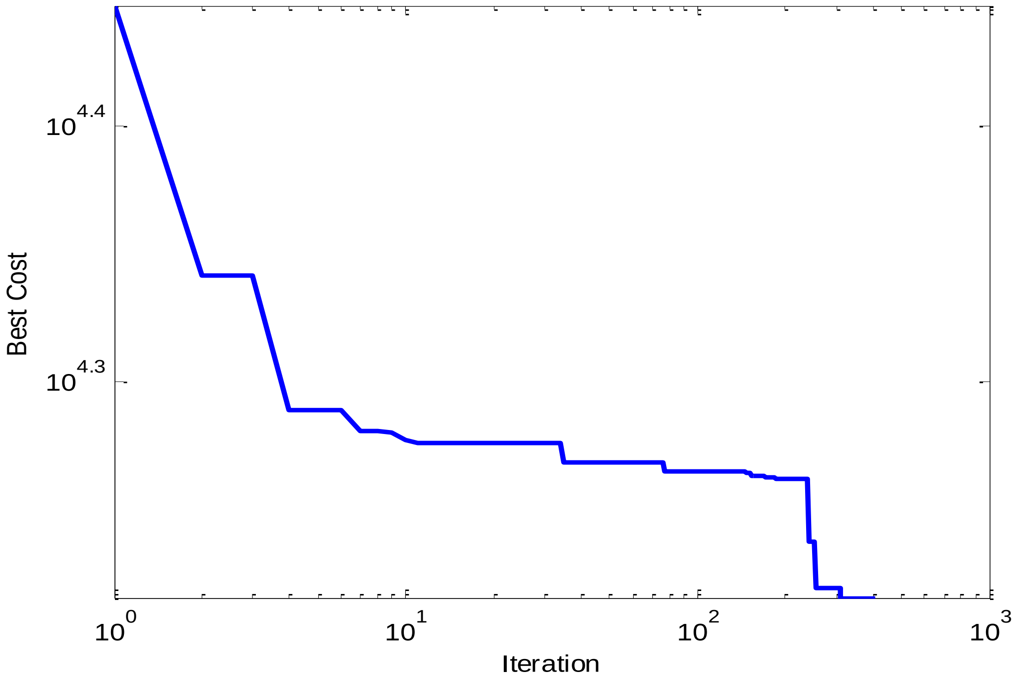

Figure 4. The graph of the convergence path

Figure 5 depicts the average *(S/N)* ratios obtained in Tables 3. As lesser values of *(S/N)* ratio is desired, then based on Fig. 5 the optimal values of GA parameters are obtained as: *It=1500; Npop=200;* $P_c=0.9$; $P_m=0.3$.

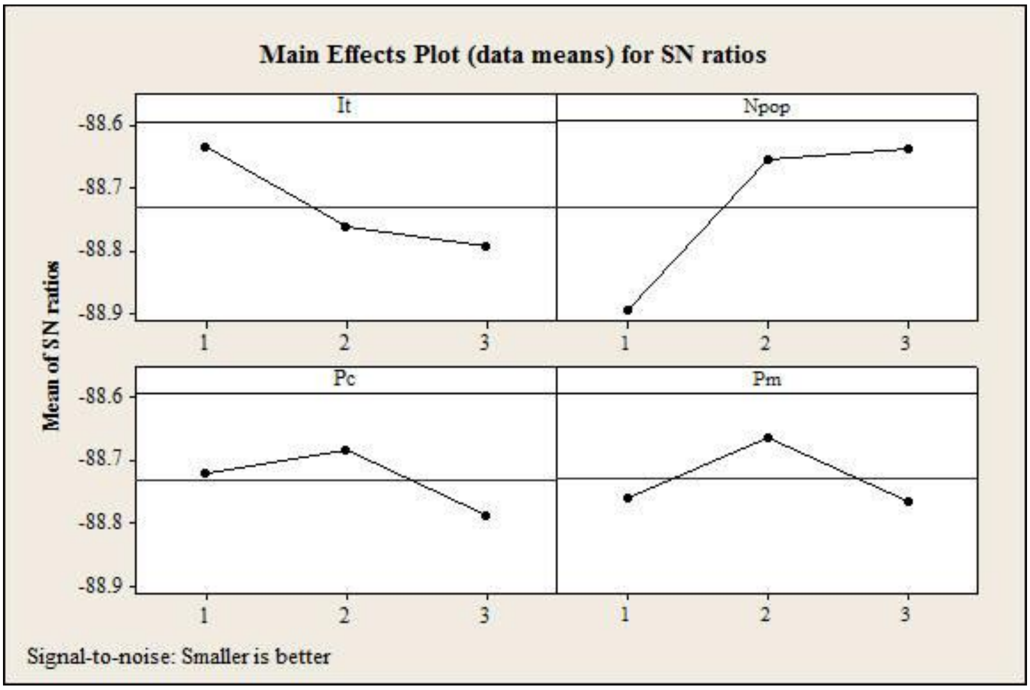

Figure 5. The average S/N ratio for GA vs. different values of its parameters

Consequently, the GA solution based on its tuned parameters is shown in Table 4.

Table 4. The solution obtained using parameter-tuned GA

Y_1	z_i	R_1	R_2	R_3	R_4	R_5	N	m
186	1	37	41	36	42	38	1	6
116	0	47	52	58	43	60	4	6
112	0	43	45	34	44	45	6	6
155	0	42	48	34	47	41	2	6
178	0	38	43	33	44	35	1	6

The total annual inventory costs obtained by GA in 10 independent replications are presented in Table 5.

Table 5. GA Result

	GA run 01	GA run 02	GA run 03	GA run 04	GA run 05
5	16431.11009	18374.28501	18308.956	16398.50824	16443.26325
10	27144.82293	27187.3042	26236.90028	26253.69165	28749.37022
15	40578.38375	40706.584	44255.09938	40351.27285	44107.20045
20	61850.41791	40548.7875	40481.19251	44105.86786	43998.93347
25	47401.53095	50635.52461	64090.84572	45487.56471	45081.25673
30	100223.8396	89801.04007	84177.46612	84689.62736	95499.41904
35	127155.2619	120407.7791	131338.3421	178871.3973	147169.3701
40	128933.9347	118091.7667	121167.1833	115788.9464	133507.7598
45	239711.9065	239711.9065	181383.8435	239711.9065	177140.7497

Table 5. Continued

	GA run 06	GA run 07	GA run 08	GA run 09	GA run 10
5	16495.57218	16571.66479	16430.28353	18351.2017	16387.69911
10	26247.15002	28601.48251	27101.00592	27315.2073	33192.4294
15	55474.25137	40436.79644	40665.05604	40458.16457	58435.20333
20	40596.29825	47967.48064	40390.40872	48312.2831	44028.44938
25	47348.22864	47342.22014	45347.37763	58449.25489	47406.06191
30	134310.9131	83787.73115	91462.65672	86494.10936	117457.7904
35	178908.0176	108313.0777	117480.1513	175268.7141	118349.4413
40	126765.707	120822.2536	129796.6	147703.1454	147557.1675
45	177584.4589	306463.0882	182516.2222	235360.3618	234326.4351

Note that in this research all programs are coded in MATLAB 2012 and that a PC with 4 HZ, Core i3 CPU is used to run the programs in Windows 7.

6. Conclusion and future research

In this paper, an integrated stochastic inventory model in a one-vendor multi-retailer single-warehouse three-echelon supply chain was developed with respect to the vendor managed inventory policy. In this model, retailers faced stochastic demands and there was a limited-capacity central warehouse. The aim was to determine the time, the number of the retailers' inventory replenishments, the replenishment quantities, the reorder points, and the replenishment frequency of the warehouse such that the total cost of the chain would be minimized. As this model shown to be of a non-linear mixed-integer programming; hard to be solved using exact methods, a genetic algorithm was utilized for a near-optimum solution. While the parameters of the algorithm were tuned using the Taguchi method, we showed that GA was an efficient algorithm to solve the problem.
For future research in this area, we recommend the following:

a. Calibrating the parameters of the algorithm using another statistical method such as RSM
b. Extending the model for multi-vendo, multi-retailer, multi-warehouse, three echelon supply chains
c. Using other algorithms such as imperialist competitive algorithm, to solve the problem
d. Investigating the effects of using discount or inflation

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