International Journal of Supply and Operations Management

IJSOM

May 2014, Volume 1, Issue 1, pp. 20-37
ISSN-Print: 2383-1359
ISSN-Online: 2383-2525
www.ijsom.com

EOQ Model for Deteriorating Items with exponential time dependent Demand Rate under inflation when Supplier Credit Linked to Order Quantity

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Abstract

In paper (2004) Chang studied an inventory model under a situation in which the supplier provides the purchaser with a permissible delay of payments if the purchaser orders a large quantity. Tripathi (2011) also studied an inventory model with time dependent demand rate under which the supplier provides the purchaser with a permissible delay in payments. This paper is motivated by Chang (2004) and Tripathi (2011) paper extending their model for exponential time dependent demand rate. This study develops an inventory model under which the vendor provides the purchaser with a credit period; if the purchaser orders large quantity. In this chapter, demand rate is taken as exponential time dependent. Shortages are not allowed and effect of the inflation rate has been discussed. We establish an inventory model for deteriorating items if the order quantity is greater than or equal to a predetermined quantity. We then obtain optimal solution for finding optimal order quantity, optimal cycle time and optimal total relevant cost. Numerical examples are given for all different cases. Sensitivity of the variation of different parameters on the optimal solution is also discussed. Mathematica 7 software is used for finding numerical examples.

Keywords: Inventory, inflation, exponential time dependent, credit, finance

1. Introduction

The main aim of any inventory control system is that when and how much to order. A large number of research papers and books have been published presenting models for doing this under various assumptions and conditions. Inventories are often replenished periodically at certain production rate, which is seldom infinite. Deterioration plays an important role in inventory management. In real life, almost all items deteriorate over time. Deterioration may be slow or fast. Thus, during the development of EOQ model, deterioration cannot be ignored.

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Some of deteriorating items are such as volatile liquids, blood bank, medicines, fashion goods, radioactive materials, photographic films, and green vegetables etc.

At present, most of the items have significant rate of deterioration. The analysis of deteriorating inventories began with Ghare and Schrader (1963), who established the classical inventory model with constant rate of deterioration. Covart and Philip (1973) extended Ghare and Schrader (1963) model and obtained an economic order quantity model for a variable rate of deterioration by assuming a two parameter Weibull distribution. Researchers like Philip (1974), Wee (1997), Misra (1975), Chakraborty et al. (1998), Mukhopadhyay et al.(2004), and Tadikamalla (2007) established inventory models that focused on deteriorating products. Goyal and Giri (2001) and Raafat (1991) established a complete survey of the published inventory literature for deteriorating inventory models. Balkhi and Benkherouf (1996) developed a method for obtaining an optimal production cycle time for deteriorating items in a model where demand and production rates are functions of time.

At present, great interest has been shown in developing mathematical models in the presence of trade credits. Kingsman (1983), Chapman et al. (1985) and Daellenbach (1986) have developed the effect of the trade credits on the optimal inventory policy. Trade credit has been a topic of interest for many authors in inventory policy like Hayley and Higgings (1973), Davis and Gaither (1985), Ouyang et al. (2004), Ward and Chapman et al. (1988). Recently, Khanna et al. (2011) developed an EOQ model for deteriorating items with time dependent demand under permissible delay in payments. Inventory models with permissible delay in payments were first studied by Goyal (1985). Shinn et al. (1996) extended Goyal's (1985) model by considering quantity discounts for freight cost. Chu et al. (1998) and Chung et al. (2001) also extended Goyal's model for the case of deteriorating items. Sana and Chaudhury (2008) developed a more general EOQ model with delay in payments, price discounting effect and different types of demand rate.

All the above articles are based on the assumption that the cost is constant over the planning horizon. This assumption may not be true in real life, as many countries have high inflation rate. Inflation also influences demand of certain products. As inflation increases, the value of money goes down. As a result, while determining the optimal inventory policy, the effect of inflation and time value of money cannot be ignored. Buzacott (1975) discussed EOQ model with inflation subject to different types of pricing policies. Wee and Law (1999) established the problem with finite replenishment rate of deteriorating items taking account of time value of money. Chang (2004) developed an inventory model for deteriorating items under inflation under a situation in which the supplier provides the purchaser with a permissible delay of payments if the purchaser orders a large quantity. Recently, Tripathi (2011) developed an inventory model under which the supplier provides the purchaser a permissible delay in payments if the purchaser orders a large quantity. This paper is the extension of Tripathi (2011) paper in which demand rate is time dependent and deterioration rate is zero. Liao (2007) established the inventory replenishment policy for deteriorating items in which the supplier provides a permissible delay in payments if the purchaser orders a large quantity. Hon and Lin (2009) developed an inventory model to determine an optimal ordering policy for deteriorating items with delayed payments permitted by the supplier under inflation and time discounting. Teng et al. (2012) proposed an EOQ model in which the constant demand to a linear, non- decreasing demand functions of time, which is suitable not only for the growth stage but also for the maturity stage of the product life cycle. Khanra et al. (2011) developed an EOQ model for deteriorating item having time dependent developed an EOQ model for deteriorating item having time- dependent demand when delay in payment is permissible.

Yang et al. (2010) developed an EOQ model for deteriorating items with stock- dependent demand and partial backlogging. Ouyang and Chang (2013) extended the effects of the reworking imperfect quality item and trade credit on the EPQ model with imperfect production process and complete backlogging. Soni (2013) developed an EOQ model considering (i) the demand rate as multivariate function of price and level of inventory (ii) delay in payment is permissible. Taleizadeh and Nematollahi (2014) investigated the effect of time- value of money and inflation on the optimal ordering policy in an inventory control system. Sarkar et al. (2014) developed an economic manufacturing quantity (EMQ) model for the selling price and the time dependent demand pattern in an imperfect production process. Teng et al. (2013) developed an EOQ model extending the constant demand to a linear, non-decreasing demand function of time and incorporate a permissible delay in payment in payment under two levels of trade credit into the model. Tripathi and Pandey (2013) presented an inventory model for deteriorating items with Weibull distribution time dependent demand rate under permissible delay in payments. Sarkar (2012) presented an EOQ model for finite replenishment rate where demand and deterioration rate are both time dependent. Tripathi (2011) established an inventory model for non- deteriorating item and time dependent demand rate under inflation when the supplier offers a permissible delay to the purchaser, if the order quantity is greater than or equal to a predetermined quantity. Some articles related to the inventory policy under delay in payments can be found in Mirzazadeh and Moghaddam (2013), Mirzazadah et al. (2009), Teheri et al. (2013) and their references.

In this paper, an attempt has been made to develop an inventory model for deteriorating items with exponential time dependent demand rate in which inflation and time value of money are taken into account. Optimal solution for the proposed model is derived by taking truncated Taylor's series approximation for finding closed form optimal solutions. Numerical examples and sensitivity analysis have been performed to observe the effect of different parameters on the optimal inventory replenishment policy.

The rest of this paper is organized as follows. In section 2 notations and assumptions are mentioned, which have been used throughout the manuscript. In section 3, the mathematical models are derived under four different circumstances in order to minimize the total cost in planning horizon. In section 4, determination of optimal solution is presented. Numerical examples are provided in section 5 to demonstrate the applicability of the proposed model. We characterize the effect of the values of parameters on the optimal replenishment cycle, order quantity and total relevant cost in section 6. In section 7 conclusion and future research is given.

2. Notations and Assumptions

The following notations are used throughout this paper:

h : holding cost rate per unit time

r : constant rate of inflation per unit time , where $0 \le \alpha < 1$

pe^{rt} : selling price per unit at time t, where p is the unit selling price at time zero

ce^{rt} : purchase cost per unit at time t, where c is unit purchase cost at time zero and p > c

Ae^{rt} : ordering cost per order at time t, where A is the ordering cost at time zero

H : length of planning horizon

m : permissible delay in settling account

I_c : Interest charged per \$ in stock per year

I_d : Interest earned per unit

Q : order quantity

Q_d : minimum order quantity at which the delay in payments is permitted

T : replenishment time interval

T_d : the time interval that Q_d units are depleted to zero due to time dependent demand

$I(t)$: level of inventory at time t

$R(t)$: annual demand as a $R(t) = \lambda e^{\alpha t}$, $\lambda > 0$, $0 < \alpha \leq 1$

$Z(T)$: total relevant cost over (0,H)

The total relevant cost consists of (i) cost of placing order (ii) cost of purchasing (iii) cost of carrying inventory excluding interest charges (iv) cost of interest charges for unsold items at the initial time or after credit period m and interest earned from sales revenue during credit period *m*.

2.1. Assumptions

1. The inflation rate is constant.
2. Shortages are not allowed.
3. The demand for item is exponentially increasing function of time.
4. Replenishment is instantaneous.
5. If $Q < Q_d$, then the payments for items received must be made immediately.
6. If $Q \geq Q_d$, then the delay in payments up to m is permitted.

During permissible delay period the account is not settled, generated sales revenue is deposited in an interest bearing account. At the end of credit period, the customer pays off all units ordered and begins payment of the interest charged on the items in stock.

3. Mathematical Formulation

Let us consider the length of horizon H = nT, where n is an integer for the number of replenishment to be made during period H, and T is an interval of time between replenishment. The level of inventory I(t) gradually decreases mainly to meet demand only. Hence the variation of inventory with respect to time is given by

$$\frac{dI(t)}{dt} + \theta . I(t) = -\lambda e^{\alpha t} \quad , 0 \leq t \leq T = H/n \ ; \ 0 \leq \alpha \leq 1 , \tag{1}$$

With boundary conditions I(0) = Q and I(T) = 0.

Solution of (1) with boundary condition I(T) = 0, is given by

$$I(t) = \frac{\lambda}{(\alpha+\theta)}\left\{e^{(\alpha+\theta)T-\theta t} - e^{\alpha t}\right\} \quad , 0 \leq t \leq T \tag{2}$$

And order quantity is

$$Q = \frac{\lambda}{(\alpha+\theta)}\left\{e^{(\alpha+\theta)T} - 1\right\} \tag{3}$$

Since the lengths of time interval are all the same, we have

$$I(kT+t) = \frac{\lambda}{(\alpha+\theta)}\left\{e^{(\alpha+\theta)T-\theta t} - e^{\alpha t}\right\} \quad , 0 \leq t \leq T \tag{4}$$

From the order quantity, we can obtain the time interval that Q_d units are depleted to zero due to demand only as

$$T_d = \frac{1}{(\alpha+\theta)}\log\left\{1+\frac{(\alpha+\theta)}{\lambda}Q_d\right\}$$

Or $$T_d = \frac{Q_d}{\lambda}\left(1-\frac{(\alpha+\theta)}{2\lambda}Q_d+\frac{(\alpha+\theta)^2}{3\lambda^2}Q_d^{\ 2}\right) \quad \text{(approximately)} \tag{5}$$

To obtain total relevant cost in [0, H], we obtain ordering cost, purchasing cost and holding cost as follows:

(a) Ordering cost

$$OC = \sum_{k=0}^{n-1} A(kT) = A\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \quad \text{(ref. appendix)} \tag{6}$$

(b) Purchasing cost

$$PC = \sum_{k=0}^{n-1} I(0)C(kT) = \frac{c\lambda}{(\alpha+\theta)}\left(e^{(\alpha+\theta)T}-1\right)\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \quad \text{(ref. appendix)} \tag{7}$$

(c) Holding cost

$$HC = h\sum_{k=0}^{n-1} C(kT)\int_0^T I(kT+t)dt = \frac{hc\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\theta+\alpha)T}-e^{\alpha T}}{\theta}-\frac{e^{\alpha T}-1}{\alpha}\right\}\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{8}$$

Now the following four cases arise based on the values of T, m and T_d for finding interest charged and interest earned.

Case I. $0 < T < T_d$

Since cycle time interval T is less than T_d i.e. $T < T_d$ (i.e. $Q < Q_d$), the delay in payments is not permitted in this case. The supplier must be paid for the items as soon as the customer receives them.

Since the interest is charged for all unsold items start at the initial time, the interest payable in (0, H) is given by

$$IC_1 = I_c\sum_{k=0}^{n-1} C(kT)\int_0^T I(kT+t)dt = \frac{cI_c\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\theta+\alpha)T}-e^{\alpha T}}{\theta}-\frac{e^{\alpha T}-1}{\alpha}\right\}\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{9}$$

Thus the total relevant cost in (0, H) is

$$Z_1(T) = OC + DC + HC + IC_1$$

$$= \left[A+\frac{c\lambda}{(\alpha+\theta)}\left(e^{(\alpha+\theta)T}-1\right)+\frac{c\lambda(h+I_c)}{(\alpha+\theta)}\left\{\frac{e^{(\theta+\alpha)T}-e^{\alpha T}}{\theta}-\frac{e^{\alpha T}-1}{\alpha}\right\}\right]\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{10}$$

Case II: $T_d \le m < T$

Since $T_d \le m < T$, we know that there is a permissible delay m which is greater than the cycle time interval T. As a result, there is no interest charged i.e. $IC_2 = 0$, but the interest earned in (0, H) is given by

$$IE_2 = I_d\sum_{k=0}^{n-1} P(kT)\int_0^T \lambda e^{\alpha t}t\,dt + (m-T)\int_0^T \lambda e^{\alpha t}dt$$

$$= \frac{pI_d\lambda}{\alpha}\left\{\left(e^{\alpha T}-1\right)\left(m-\frac{1}{\alpha}\right)+T\right\}\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \tag{11}$$

So the total relevant cost in this case in (0, H) is

$$Z_2(T) = OC + DC + HC - IE_2$$

$$= \left[A + \frac{c\lambda}{(\alpha+\theta)}\left(e^{(\alpha+\theta)T} - 1\right) + \frac{hc\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\alpha+\theta)T} - e^{\alpha T}}{\theta} - \frac{e^{\alpha T} - 1}{\alpha}\right\} - \frac{pI_d\lambda}{\alpha}\left\{\left(e^{\alpha T} - 1\right)\left(m - \frac{1}{\alpha}\right) + T\right\}\right]\left(\frac{e^{rH} - 1}{e^{rT} - 1}\right) \tag{12}$$

Case III: $T_d \le m \le T$

Since $T \ge m \ge T_d$, the delay in payments is permitted and the total relevant cost includes both the interest charged and the interest earned. The interest charged in (0, H) is

$$IC_3 = I_c \sum_{k=0}^{n-1} C(kT) \int_m^T I(kT+t)dt$$
$$= \frac{cI_c\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\alpha+\theta)T-\theta m} - e^{\alpha T}}{\theta} - \frac{e^{\alpha T} - e^{\alpha m}}{\alpha}\right\}\left(\frac{e^{rH} - 1}{e^{rT} - 1}\right) \tag{13}$$

The interest earned in [0, H] is

$$IE_3 = I_d \sum_{k=0}^{n-1} P(kT) \int_0^m \lambda e^{\alpha t} t dt = \frac{pI_d\lambda}{\alpha}\left(me^{\alpha m} - \frac{e^{\alpha m} - 1}{\alpha}\right)\left(\frac{e^{rH} - 1}{e^{rT} - 1}\right) \tag{14}$$

Hence the total relevant cost

$$Z_3(T) = OC + DC + HC + IC_3 - IE_3$$

$$= \left[A + \frac{c\lambda}{(\alpha+\theta)}\left(e^{(\alpha+\theta)T} - 1\right) + \frac{hc\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\theta+\alpha)T} - e^{\alpha T}}{\theta} - \frac{e^{\alpha T} - 1}{\alpha}\right\} + \frac{cI_c\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\alpha+\theta)T-\theta m} - e^{\alpha T}}{\theta} - \frac{e^{\alpha T} - e^{\alpha m}}{\alpha}\right\} - \frac{pI_d\lambda}{\alpha}\left(me^{\alpha m} - \frac{e^{\alpha m} - 1}{\alpha}\right)\right]\left(\frac{e^{rH} - 1}{e^{rT} - 1}\right) \tag{15}$$

Case IV: $m \le T_d \le T$

Since $T \ge T_d \ge m$, case IV is similar to case III.
Therefore, the total relevant cost in (0, H) is

$$Z_4(T) = \left[A + \frac{c\lambda}{(\alpha+\theta)}\left(e^{(\alpha+\theta)T} - 1\right) + \frac{hc\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\theta+\alpha)T} - e^{\alpha T}}{\theta} - \frac{e^{\alpha T} - 1}{\alpha}\right\} + \frac{cI_c\lambda}{(\alpha+\theta)}\left\{\frac{e^{(\alpha+\theta)T-\theta m} - e^{\alpha T}}{\theta} - \frac{e^{\alpha T} - e^{\alpha m}}{\alpha}\right\} - \frac{pI_d\lambda}{\alpha}\left(me^{\alpha m} - \frac{e^{\alpha m} - 1}{\alpha}\right)\right]\left(\frac{e^{rH} - 1}{e^{rT} - 1}\right) \tag{16}$$

The graphs of all four cases are given in the appendix for better explanation.
Since it is difficult to find the solutions for finding the exact value of T, therefore, we make use of the second order approximation for the exponential terms in equation (10) , (12) , (15) and (16), which follows as:

$$\begin{cases} e^{rT} = 1 + rT + \dfrac{r^2T^2}{2} \\ e^{rH} = 1 + rH + \dfrac{r^2H^2}{2} \\ e^{rm} = 1 + rm + \dfrac{r^2m^2}{2} \end{cases} \tag{17}$$

The above approximation is valid for smaller values of deterioration rate θ and inflation rate r such that rT <1, rm < 1 and rH < 1. In reality, the values of deterioration rate θ or inflation rate r is usually very small.
Hence, equations (10), (12), (15) and (16) reduces to

$$Z_1(T) = \left\{ \frac{A}{T} + c\lambda + \frac{c\lambda}{2}\left(h + I_c + \alpha + \theta\right)T \right\} \left(1 - \frac{rT}{2}\right) H \left(1 + \frac{rH}{2}\right) . \tag{18}$$

$$Z_2(T) = \left[\frac{A}{T} + c\lambda + \frac{c\lambda}{2}\left(h + \alpha + \theta\right)T - pI_d\lambda \left\{ m + \frac{(m\alpha - 1)T}{2} \right\} \right] \left(1 - \frac{rT}{2}\right) H \left(1 + \frac{rH}{2}\right) . \tag{19}$$

$$Z_3(T) = \left[\frac{A}{T} + c\lambda + \frac{c\lambda T}{2}\left(h + \alpha + \theta\right) + \frac{cI_c\lambda}{2T}\left(T - m\right)^2 - \frac{pI_d\lambda m^2(1+\alpha m)}{2T} \right] \left(1 - \frac{rT}{2}\right) H \left(1 + \frac{rH}{2}\right) . \tag{20}$$

$$Z_4(T) = \left[\frac{A}{T} + c\lambda + \frac{c\lambda T}{2}\left(h + \alpha + \theta\right) + \frac{cI_c\lambda}{2T}\left(T - m\right)^2 - \frac{pI_d\lambda m^2(1+\alpha m)}{2T} \right] \left(1 - \frac{rT}{2}\right) H \left(1 + \frac{rH}{2}\right) . \tag{21}$$

Similarities and Differences among case I, II, III and IV
If interest earned IE_3 becomes zero then case III becomes case I for limit t = 0 to t = T.
If interest charges IC_3 becomes zero then case III becomes II.

4. Determination of optimal solution

To find the optimal solution for the problem, we minimize $Z_i(T)$ for Case I, II, III, IV respectively and then compare them to obtain minimum value. Our aim is to find minimum relevant cost for all cases i.e. Case I, II, III, IV respectively with respect to T. The necessary and sufficient condition to minimise $Z_i(T)$; i =1,2,3,4 for given values of T are respectively $\frac{dZ_1(T)}{dT} = 0, \frac{dZ_2(T)}{dT} = 0, \frac{dZ_3(T)}{dT} = 0, \frac{dZ_4(T)}{dT} = 0$, and $\frac{d^2Z_1(T)}{dT^2} > 0, \frac{d^2Z_2(T)}{dT^2} > 0, \frac{d^2Z_3(T)}{dT^2} > 0, \frac{d^2Z_4(T)}{dT^2} > 0$
Differentiating equations (18), (19), (20) and (21) with respect to T and equating to zero, we get

$$c\lambda\left\{(h + I_c + \alpha + \theta)(1 - rT) - r)\right\}T^2 - A = 0 . \tag{22}$$

$$\lambda\left[(1 - rT)\left\{c(h + \alpha + \theta) - pI_d(m\alpha - 1)\right\} - r(c - pI_d m)\right]T^2 - 2A = 0 . \tag{23}$$

$$\mathrm{c}\lambda\left\{(\mathrm{h} + \mathrm{I_c} + \alpha + \theta)(1 - rT) - r(1 - I_c m)\right\}T^2 - \lambda m^2\left\{cI_c - pI_d(1 + \alpha m)\right\} - 2A = 0 . \tag{24}$$

And

$$\mathrm{c}\lambda\left\{(\mathrm{h} + \mathrm{I_c} + \alpha + \theta)(1 - rT) - r(1 - I_c m)\right\}T^2 - \lambda m^2\left\{cI_c - pI_d(1 + \alpha m)\right\} - 2A = 0 . \tag{25}$$

And second derivative of equations (18), (19), (20), (21) are given by

$$\frac{d^2Z_1}{dT^2} = \left\{ \frac{2\mathrm{A}}{T^3} - \frac{rc\lambda}{2}\left(h + I_c + \alpha + \theta\right) \right\} H \left(1 + \frac{rH}{2}\right) > 0. \tag{26}$$

$$\frac{d^2Z_2}{dT^2}=\left\{\frac{2\mathrm{A}}{\mathrm{T}^3}+\frac{pI_d\lambda(m\alpha-1)r}{2}-\frac{c\lambda r(h+\alpha+\theta)}{2}\right\}H\left(1+\frac{rH}{2}\right)>0. \tag{27}$$

$$\frac{d^2Z_3}{dT^2}=\left[\frac{2\mathrm{A}}{T^3}+\frac{\lambda m^2}{T^3}\left\{cI_c-pI_d(1+\alpha m)\right\}-\frac{c\lambda r}{2}\left(h+I_c+\alpha+\theta\right)\right]H\left(1+\frac{rH}{2}\right)>0\,. \tag{28}$$

And $$\frac{d^2Z_4}{dT^2}=\left[\frac{2\mathrm{A}}{T^3}+\frac{\lambda m^2}{T^3}\left\{cI_c-pI_d(1+\alpha m)\right\}-\frac{c\lambda r}{2}\left(h+I_c+\alpha+\theta\right)\right]H\left(1+\frac{rH}{2}\right)>0\,. \tag{29}$$

5. Numerical examples

Example 1: Let H = 1 year, λ= 500 units, h = \$2/unit/year, I_c= 0.10/\$/year, I_d = 0.05/\$/year, A= \$200 per order, r = 0.05 per unit, θ = 0.2/ unit/year, α = 0.5, c = \$ 25 per unit. Substituting these in (24), we get Optimal cycle time T = T_1^* = 0.0735746 year, optimal total relevant profit Z_1 (T) = $Z_1^*(T_1^*)$ = \$31124, and optimal order quantity Q = Q_1^* = 37.7322 units, which shows that if $Q^* < Q_d$ then $T_1 < T_d$, this proves case I.

Example 2: Let H = 1 year, λ= 50 units, h = \$2/unit/year, I_c= 0.16/\$/year, I_d = 0.05/\$/year, A= \$150 per order, r = 0.05 per unit, θ = 0.2/ unit/year, α = 0.8, c = \$ 10 per unit, m = 90 days, p = 40. Substituting these in Equation (24), we get Optimal cycle time T = T_2^* = 0.0974674 year, optimal total relevant profit Z_2 (T) = $Z_2^*(T_2^*)$ = \$5098.51, and optimal order quantity Q = Q_2^* = 10.2692 units, which shows that $T_2 \le m$, if $Q^*(T_2) \ge Q_d$, then $T_2 \ge T_d$, we get $T_d \le T_2 \le m$, this proves case II.

Example 3: Let H = 1 year, λ= 100 units, h = \$5/unit/year, I_c= 0.08/\$/year, I_d = 0.01/\$/year, A= \$50 per order, r = 0.05 per unit, θ = 0.2/ unit/year, α = 0.9, c = \$ 30 per unit, m = 90 days, p = 40. Substituting these in (24), we get Optimal cycle time T = T_3^* = 0.358109 year, optimal total relevant profit Z_3 (T) = $Z_3^*(T_3^*)$ = \$1350.84, and optimal order quantity Q = Q_3^* = 21.1115 units, which shows that $T_3 \le m$ if $Q^*(m) \ge Q_d$, then $m \ge T_d$, we get, $T_3 \le m \le T_d$, this proves case III.

Example 4: Let H = 1 year, λ= 100 units, h = \$5/unit/year, I_c= 0.08/\$/year, I_d = 0.01/\$/year, A= \$50 per order, r = 0.05 per unit, θ = 0.2/ unit/year, α = 0.9, c = \$ 30 per unit, m = 90 days, p = 40. Substituting these in (24), we get Optimal cycle time T = T_4^* = 0.358109 year, optimal total relevant profit Z_4 (T) = $Z_4^*(T_4^*)$ = \$1350.84, and optimal order quantity Q = Q_4^* = 21.1115 units, which shows that $T_4 \ge m$ if $Q^*(m) \ge Q_d$, then $m \ge T_d$, we get, $T_4 \le T_d \le m$, this proves case IV.

6. Sensitivity Analysis

We have performed sensitivity analysis by changing A, c, h, m and θ and keeping the remaining parameters at their original values. The corresponding variations in the cycle time, the order quantity and total relevant cost are given in Table 1 (Table 1.a, Table 1.b, Table 1.c, Table 1.d), Table 2 (Table 2.a, Table 2.b, Table 2.c, Table 2.d, Table 2.e), Table 3 (Table 3.a, Table 3.b, Table 3.c, Table 3.d, Table 3.e) for case I, II and III respectively.

Case I

Table 1.a. Variation of ordering cost 'A'.

A	$T = T_1 *$	$Q = Q_1 *$	$Z_1(T_1 *)$
100	0.0524258	26.6939	29498.0
150	0.0639399	32.6854	30378.7
250	0.0820095	42.1817	31782.7
300	0.0895929	46.2009	32380.0
350	0.0965318	49.8966	32930.7
400	0.1029610	53.3357	33444.5

Table 1.b. Variation of purchasing cost 'c'.

C	$T = T_1 *$	$Q = Q_1 *$	$Z_1(T_1 *)$
30	0.0942811	48.6961	19651.0
35	0.0874993	45.0895	22550.3
40	0.0820095	42.1817	25426.2
45	0.0774463	39.7728	28283.1
55	0.0702353	35.9809	33951.3
60	0.0673162	34.4511	36766.8

Table 1.c. Variation of holding cost'h'.

H	$T = T_1 *$	$Q = Q_1 *$	$Z_1(T_1 *)$
2.1	0.0723065	37.0682	31220.9
2.2	0.0711030	36.4362	31316.0
2.3	0.0699587	35.8358	31409.6
2.4	0.0688689	35.2645	31501.6
2.5	0.0678294	34.7198	31592.2

Table 1.d. Variation of deterioration rate 'θ'.

Θ	$T = T_1 *$	$Q = Q_1 *$	$Z_1(T_1 *)$
0.1	0.0748237	38.3916	31028.8
0.3	0.0723864	37.1102	31217.6
0.5	0.0701722	35.9478	31400.4
0.7	0.0681475	34.8865	31577.6
1.0	0.0654104	33.4539	31834.3

Case II

Table 2.a. Variation of ordering cost 'A'.

A	$T = T_2*$	$Q = Q_2*$	$Z_2(T_2*)$
20	0.0822673	8.59896	4756.48
30	0.0876780	9.19061	4877.10
40	0.0927246	0.74534	4990.73
60	0.1019520	10.7669	5201.30
70	0.1062130	11.2418	5299.78
100	0.1179100	12.5557	5574.13

Table 2.b. Variation of purchasing cost 'c'.

C	$T = T_2*$	$Q = Q_2*$	$Z_2(T_2*)$
10	0.1622620	17.6743	2218.25
20	0.1178480	12.5486	3713.49
40	0.0850864	8.90682	6426.73
50	0.0765314	7.97528	7718.72
60	0.0715870	7.28659	8985.01
100	0.0549052	5.65632	13890.5

Table 2.c. Variation of holding cost 'h'.

H	$T = T_2*$	$Q = Q_2*$	$Z_2(T_2*)$
1	0.1664190	18.1651	4259.86
2	0.1359600	14.6127	4521.19
4	0.1063130	11.2529	4927.72
6	0.0905773	9.50896	5255.16
8	0.0803771	8.39304	5537.28
10	0.0730573	7.59929	5788.95

Table 2.d. Variation of permissible delay period 'm'.

m (in days)	$T = T_2*$	$Q = Q_2*$	$Z_2(T_2*)$
100	0.0974674	10.2692	5098.51
110	0.0974675	10.2692	5098.51
120	0.0974675	10.2692	5098.51
130	0.0974675	10.2692	5098.51
140	0.0974676	10.2693	5098.50
150	0.0974676	10.2693	5098.50

Table 2.e. Variation of deterioration rate ‘θ’.

Θ	$T = T_2*$	$Q = Q_2*$	$Z_2(T_2*)$
0.1	0.0981622	10.3462	5083.23
0.3	0.0967921	10.1945	5113.64
0.4	0.0961342	10.1217	5128.63
0.5	0.0954921	10.0507	5143.50
0.6	0.0948646	9.98142	5158.24
0.8	0.0936497	9.84733	5187.41

Case III

Table 3.a. Variation of ordering cost ‘A’.

A	$T = T_3*$	$Q = Q_3*$	$Z_3(T_3*)$
100	0.310987	17.9672	1226.21
120	0.330855	19.2794	1278.07
160	0.366627	21.6971	1373.97
180	0.382944	22.8134	1418.80
200	0.398409	23.8887	1461.92
250	0.434006	26.4093	1563.50

Table 3.b. Variation of purchasing ‘c’.

C	$T = T_3*$	$Q = Q_3*$	$Z_3(T_3*)$
5	0.463878	28.5735	825.636
15	0.312238	18.0492	1844.16
20	0.285775	16.3304	2323.68
25	0.268332	15.2167	2795.61
30	0.255889	14.4314	3262.84

Table 3.c. Variation of holding cost ‘h’.

H	$T = T_3*$	$Q = Q_3*$	$Z_3(T_3*)$
1.0	0.467701	28.8537	1161.89
1.5	0.401256	24.088	1264.65
2.5	0.327002	19.0234	1427.65
3.0	0.303125	17.4534	1497.33
3.5	0.284012	16.2172	1561.56
4.0	0.268240	15.2108	1621.47

Table 3.d. Variation of permissible delay period 'm'.

M	$T = T_3*$	$Q = Q_3*$	$Z_3(T_3*)$
70	0.350959	20.6273	1324.55
80	0.354582	20.8273	1337.80
100	0.361537	21.3446	1363.66
110	0.364866	21.5715	1376.27
120	0.368095	21.7921	1388.66
130	0.371223	22.0063	1400.81

Table 3.e. Variation of deterioration rate 'θ'.

Θ	$T = T_3*$	$Q = Q_3*$	$Z_3(T_3*)$
0.1	0.352765	20.7493	1358.41
0.3	0.362931	21.4395	1346.48
0.4	0.367326	21.7395	1344.49
0.5	0.371368	22.0103	1344.29
0.6	0.375113	22.2734	1345.47
0.7	0.378609	22.5141	1347.74

All the above observations can be sum up as follows:

- From Table 1.a, increase of ordering cost 'A' results increase in optimal cycle time $T = T_1*$, order quantity $Q = Q_1*$, and total relevant cost $Z_1(T_1*)$.
- From Table 1.b, increase of unit purchasing cost 'c' results decrease in optimal cycle time $T = T_1*$, order quantity $Q = Q_1*$, and increase in total relevant cost $Z_1(T_1*)$.
- From Table 1.c, increase of holding cost 'h' results decrease in optimal cycle time $T = T_1*$, order quantity $Q = Q_1*$, and increase in total relevant cost $Z_1(T_1*)$.
- From Table 1.d, increase of deterioration rate 'θ' results decrease in optimal cycle time $T = T_1*$, order quantity $Q = Q_1*$, and increase in total relevant cost $Z_1(T_1*)$.
- From Table 2.a, increase of ordering cost 'A' results increase in optimal cycle time $T = T_2*$, order quantity $Q = Q_2*$, and total relevant cost $Z_2(T_2*)$.
- From Table 2.b, increase unit purchase cost 'c' results decrease in optimal cycle time $T = T_2*$, order quantity $Q = Q_2*$, and increase in total relevant cost $Z_2(T_2*)$.
- From Table 2.c, increase of holding cost 'h' results decrease in optimal cycle time $T = T_2*$, order quantity $Q = Q_2*$, and increase in total relevant cost $Z_2(T_2*)$.
- From Table 2.d, increase of credit period 'm' results slight increase in optimal cycle time $T = T_2*$, order quantity $Q = Q_2*$ and total relevant cost $Z_2(T_2*)$.
- From Table 2.e, increase of deterioration rate 'θ' results decrease in optimal cycle time $T = T_2*$, slight decrease in order quantity $Q = Q_2*$, and increase in total relevant cost $Z_2(T_2*)$.
- From Table 3.a, increase of ordering cost 'A' results increase in optimal cycle time $T = T_3*$, order quantity $Q = Q_3*$, total relevant cost $Z_3(T_3*)$.
- From Table 3.b, increase of unit purchasing cost 'c' results decrease in optimal cycle time $T = T_3*$, order quantity $Q = Q_3*$ and increase in total relevant cost $Z_3(T_3*)$.

- From Table 3.c, increase of holding cost 'h' results decrease in optimal cycle time $T = T_3^*$, order quantity $Q = Q_3^*$ and increase in total relevant cost $Z_3(T_3^*)$.
- From Table 3.d, increase of credit period 'm' results increase in optimal cycle time $T = T_3^*$, order quantity $Q = Q_3^*$, total relevant cost $Z_3(T_3^*)$.
- From Table 3.e, increase of deterioration rate 'θ' results increase in optimal cycle time $T = T_3^*$, order quantity $Q = Q_3^*$ and slight decrease in total relevant cost $Z_3(T_3^*)$.

Note: If $\alpha = 0$, this paper reduces to Chang [24] and if $\theta = 0$, demand rate is time dependent this paper reduces to Tripathi [41].

7. Conclusion and Future Research

The present model is based on exponential time dependent demand rate. Some researchers adopted an exponential functional form like $a.e^{bt}$, $a > 0$, b in not equal to zero implying exponential increase ($b > 0$) or decrease ($b < 0$) in the demand rate. An exponential rate being very high, it is doubtful whether the real market demand of any commodity can rise or fall exponentially, it means accelerated rise or fall in demand. Accelerated increase in the demand rate takes place in the case of the aircraft, computer, machines and their spare parts. Accelerated decrease in the demand rate is found to be in case of obsolete aircraft, computer, and spare parts in machines.

In this paper, we developed an inventory model for deteriorating items under inflation to determine the optimal ordering policy when the supplier provides a permissible delay in payment linked to order quantity. In order to obtain closed form solution of the optimal cycle time, truncated Taylor's series approximation is used in exponential terms. Numerical examples are studied to illustrate the theoretical results for all four cases. A sensitivity analysis is given to the variation of different parameters. There are some managerial phenomena (i) a higher value of purchasing cost causes, lower value of cycle time, order quantity and higher values of total relevant cost (ii) a higher value of holding cost causes lower values of cycle time, order quantity and higher values of total relevant cost. (iii) a higher value of deterioration rate causes, lower values of optimal cycle time, order quantity and higher values of total relevant cost.

The proposed model can be extended in several ways. For instance, we may extend the constant deterioration rate to a time dependent deterioration rate. In addition, we could consider the demand as a function of stock level, function of time, selling price and others. Finally, it can be generalized for shortages, quantity discount, freight changes and others.

Appendix

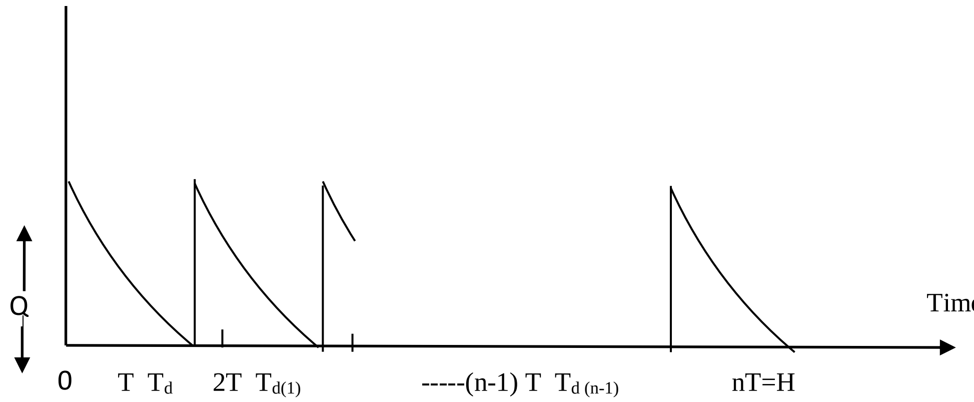
(i)
$$\sum_{k=0}^{n-1} C(kT) = \sum_{k=0}^{n-1} ce^{rkT} = c\sum_{k=0}^{n-1} e^{rkT} = c\left\{1 + e^{rT} + e^{2rT} + e^{3rT} + ----------- e^{(n-1)rT}\right\}$$
$$= c\left(\frac{e^{rnT}-1}{e^{rT}-1}\right) = c\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \qquad (\text{ since } nT = H)$$

(ii)
$$\sum_{k=0}^{n-1} A(kT) = \sum_{k=0}^{n-1} Ae^{rkT} = \mathrm{A}\sum_{k=0}^{n-1} e^{rkT} = \mathrm{A}\left\{1+e^{rT}+e^{2rT}+e^{3rT}+-----------e^{(n-1)rT}\right\}$$
$$= A\left(\frac{e^{rnT}-1}{e^{rT}-1}\right) = A\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \qquad \text{(since nT = H)}$$

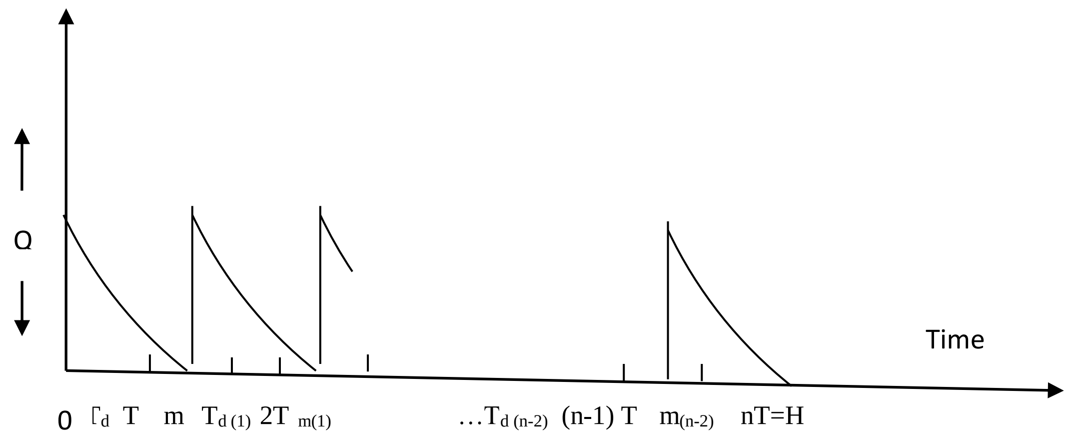
(iii)
$$\sum_{k=0}^{n-1} P(kT) = \sum_{k=0}^{n-1} pe^{rkT} = \mathrm{p}\sum_{k=0}^{n-1} e^{rkT} = \mathrm{p}\left\{1+e^{rT}+e^{2rT}+e^{3rT}+-----------e^{(n-1)rT}\right\}$$
$$= p\left(\frac{e^{rnT}-1}{e^{rT}-1}\right) = p\left(\frac{e^{rH}-1}{e^{rT}-1}\right) \qquad \text{(since nT = H)}$$

(iv) Taylor's series expansion of exponential terms is $e^{rT} = 1 + rT + \frac{r^2T^2}{2}$, etc.

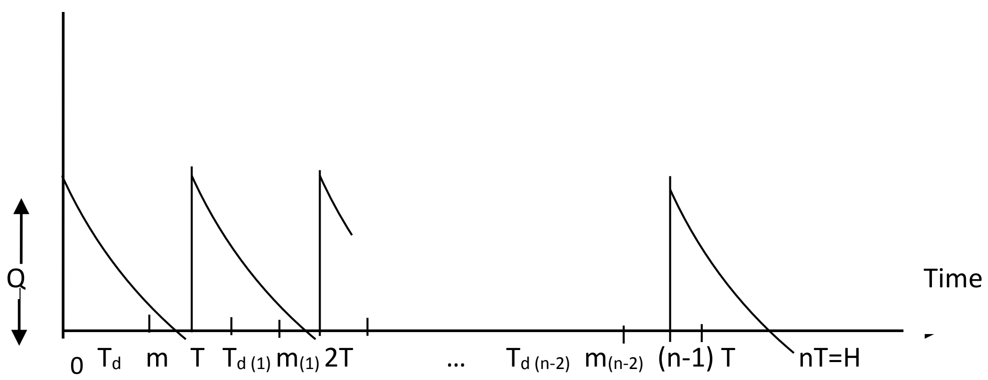
Inventory Level



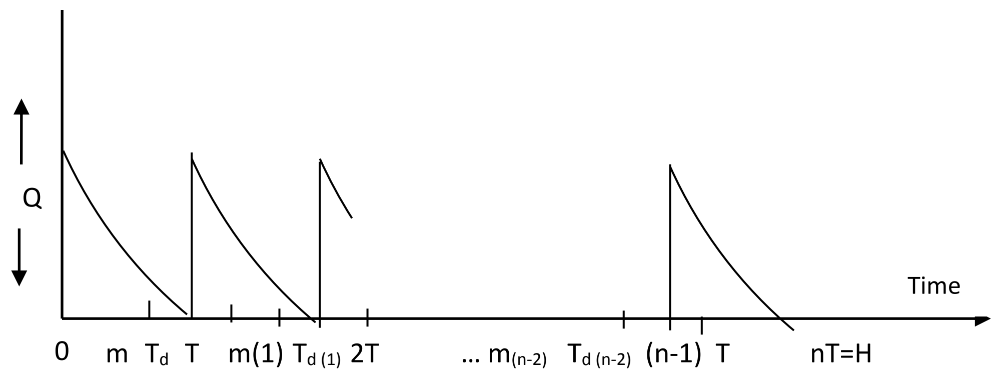
Inventory Level



Case II. $T_d \le m \le T$



Case III. $T_d \leq m \leq T$



Case IV. $m \leq T_d \leq T$

Figure 1 : Graph between inventory level and length of planning horizon

Acknowledgements

We would like to thank the editor and the referees for the valuable and constructive comments that have led to a significant improvement in the original paper.

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